Electron emission from 2D novel materials and applications

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ENGINEERING PRODUCT DEVELOPMENT

11 April 2018, Michigan Institute for Plasma Science and Engineering (MIPSE), University of Michigan

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Y. B. Zhu (former PhD) – Assoc Prof (China)
L. Wu (former PhD) – Research Scientist (Singapore, IHPC)
P. Zhang (former MEng) – Asst Prof (Michigan State U)
M. Pant (former RA) – PhD in MIT
W. S. Koh (former PhD) – group leader (Singapore, IHPC)

Ministry of Education SINGAPORE

Agency for Science, Technology and Research SINGAPORE

Temasek Laboratories

SUTD-MIT INTERNATIONAL DESIGN CENTRE (IDC) Innovation Ready Design

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH UNITED STATES AIR FORCE

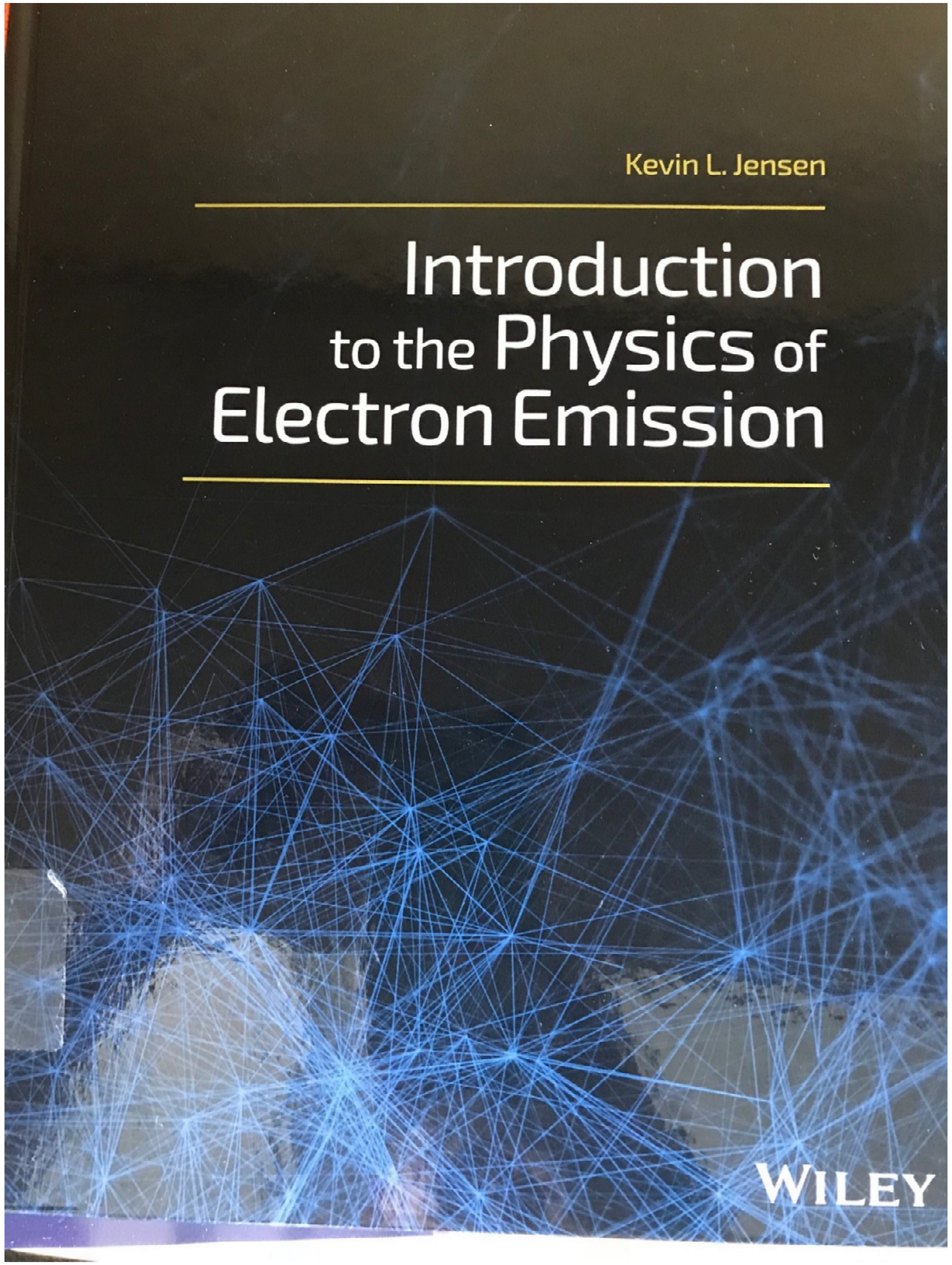
ASIAN OFFICE OF AEROSPACE R&D BRIDGING SCIENCE & TECHNOLOGY

INTRODUCTION: SCALING LAWS OF ELECTRON EMISSION

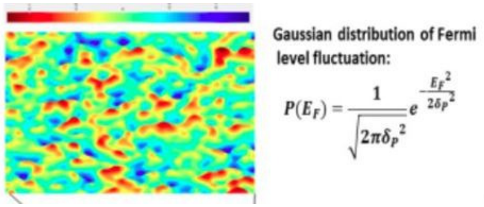
Processes	Old Scaling law of emission current density J (A/m^2)	New Scaling law for 2D materials, or ultrafast pulse or nano size gap
Thermionic Emission by Richardson (1901), Nobel Prize (Physics) 1928	$J_{RLD}(T) = A_{RLD} T^2 \exp\left(-\frac{\Phi}{k_B T}\right)$	1. T^2 scaling becomes T^3 for 2D materials and smooth transition (PR Applied 2015, 2016) 2. Universal scaling of 2D materials contact (PRL 2018 in press)
Photoelectric or photoemission by Einstein (1905), Nobel Prize (Physics) 1921	$J_{FD}(F) \propto (\hbar\omega - \Phi)^2$	3. The number of photons required is reduced at very fast time scale < 10 fs: n = 3 to n = 2 at 8 fs (PRB 2013) 4. laser-excited photo-emission for graphene
Field emission by Fowler and Nordheim (FN law) in 1928	$J_{FN}(F) = A_{FN} F^2 \exp\left(-\frac{B\Phi^{3/2}}{F}\right)$	5. For 2D materials, FN law is different (in review) 6. Fractional modelling of FN law (IEEE TED)
Child Langmuir(CL) law (1911) for vacuum diode, Nobel Prize (Chemistry) 1932 Mott-Gurney law (1940) for solid-state and organic electronics	$J_{CL} \propto \frac{V^{3/2}}{L^2}$ $J_{MG} \propto \frac{V^2}{L^3}$	7. Quantum regime CL scaling (PRL 2003, PRL2007 – in NTU), experimental verified: $J_{CL} \propto \frac{V^{1/2}}{L^4}$ 8. New MG scaling to relativistic (2D materials) (PRB 2017) $J_{MG} \propto (V/L)^a, a = 1.5\ to\ 2$

A good book for electron emission for bulk materials

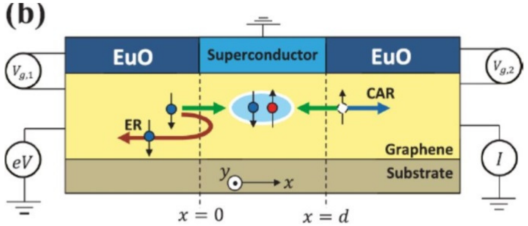
Wiley (2018)
Dr. Kevin Jensen
NRL



Research Topic : Design of 2D material-devices – new scaling laws and applications



Graphene-Semiconductor diode: *IEDM paper (2016)*



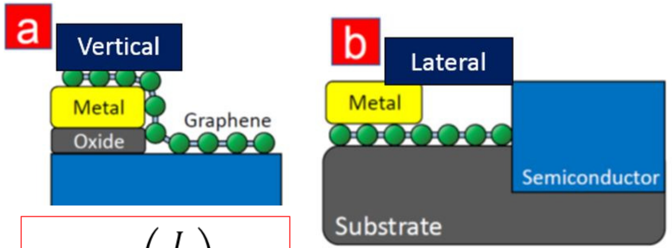
Nonlocal transistor using graphene/superconductor

PRB (rapid communication) 93, 041422 (2016)

New scaling and universal law of thermionic emission for 2D-materials cathode and Schottky diode

Phys. Rev Lett. (editor suggestion) – Aug 2018 issue

Phys Rev Appl 6, 034013 (2016); Phy Rev. Appl. 3, 014002 (2015)

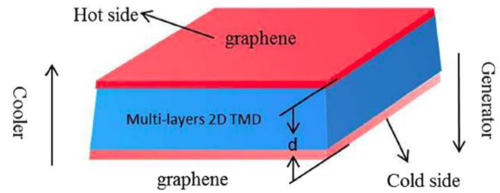


$$\log\left(\frac{J}{T^{\beta}}\right) = A - \frac{B}{T}$$

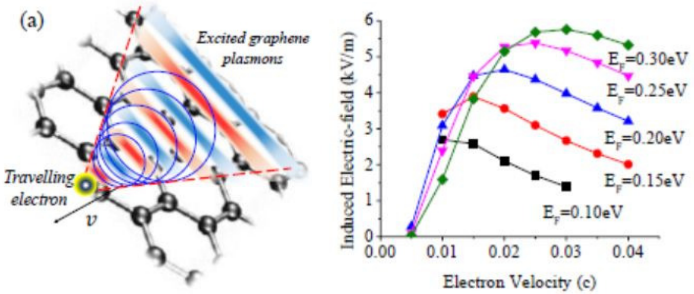
$$\beta = 1, \frac{3}{2}, 2, 3$$

Thermionic energy convertor

Sci Report 7, 46211 (2017)



Heat source	Design	efficiency
900K	Gr-air-LaB6	45%
400K	Gr-MoS2-Gr	3.15%
400K	Gr-MoSe2-Gr	7.28%
400K	Gr-WSe2-Gr	8.56%

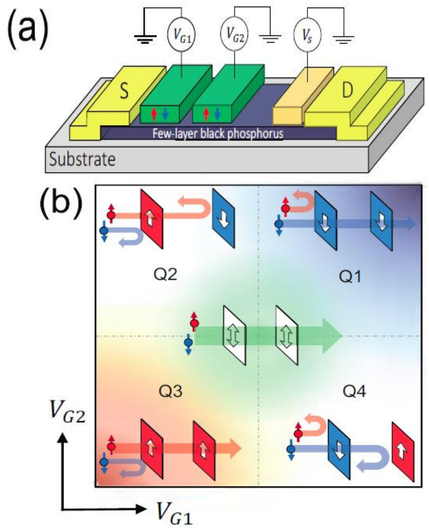


Graphene plasmon in THz (emitter) regime

Optics Express, 25, 20477 (2017)

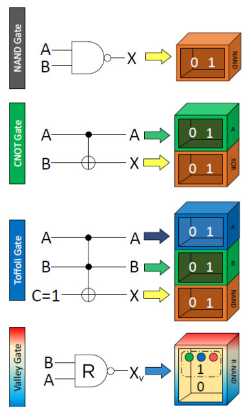
IEEE Journal of Quantum Electronics, 23, 1 (2016)

PR Applied 3, 054001 (2015)



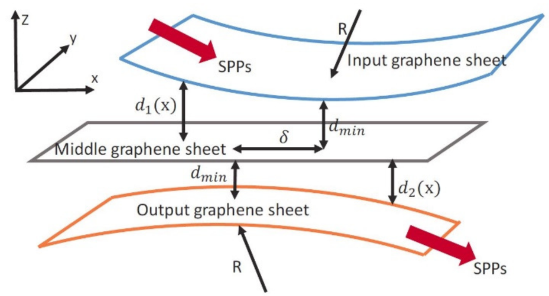
Reversible logic gate

PRB 96, 245410, 2017

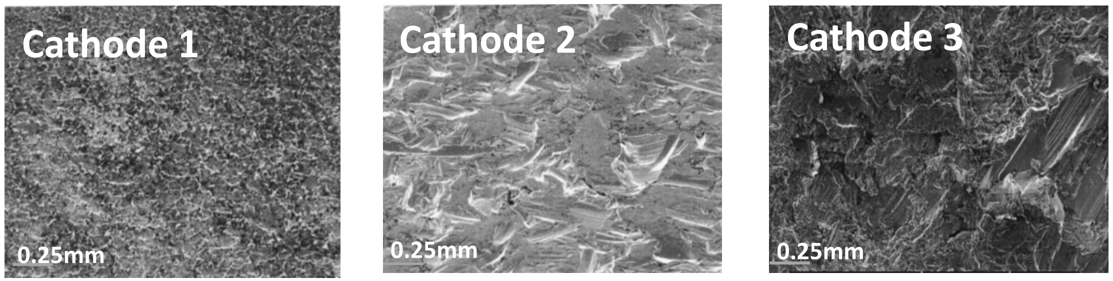


Adiabatic control of SPP on 3 layers curved graphene electrodes

Carbon 127, 187 (2018) – very robust design



Research Topic (since 2016): Testing of Fractional modeling – MSU seminar

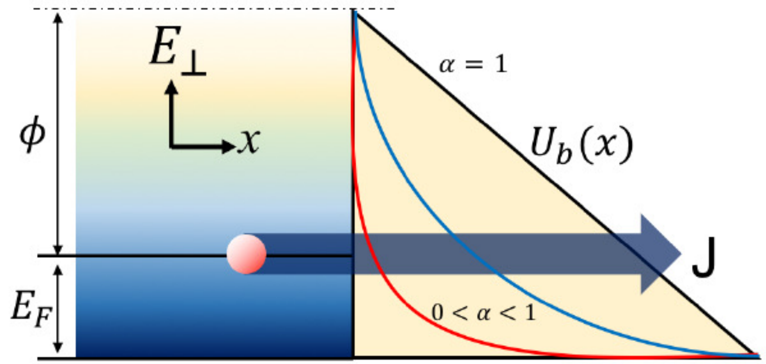


High current electron (SCL) emission from rough cathode

M. Zubair, L. K. Ang, Phys. Plasmas 23, 073118 (2016)

Light absorption in a fractal surface

M. Zubair, Y. S. Ang, K. Ooi, L. K. Ang, JAP (under review)



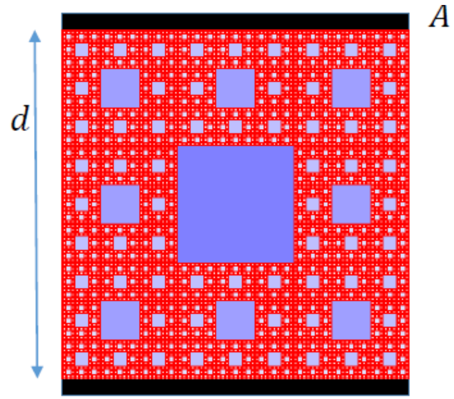
Fractional FN law of a rough surface

M. Zubair, Y. S. Ang, L. K. Ang, IEEE TED 65 2089 (2018)

$$C = \frac{k\varepsilon_0 A(\alpha, n)}{d^{f(\alpha,n)}}$$

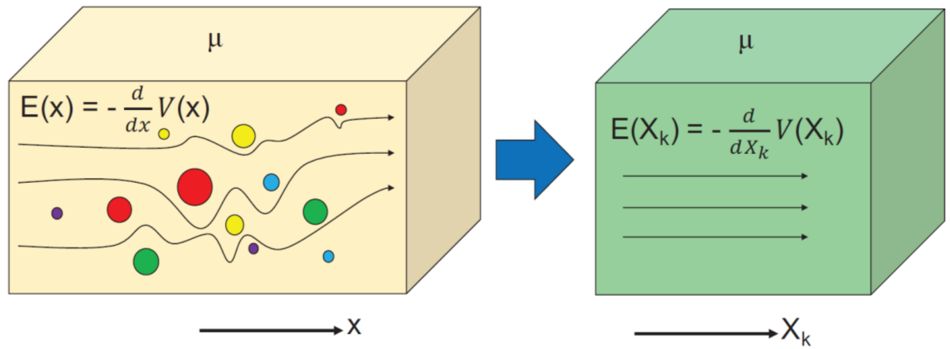
α=fractal dimension of dielectric

n=Dimension of embedding Euclidean Space



Capacitance for a fractal composite

M. Zubair, Y. Samuel, S. Athalye, L. K. Ang (in prep)



Current Transport in spatial disordered semiconductor

M. Zubair, Y. S. Ang, L. K. Ang IEEE TED (in press, 2018)

$$J_{RLD}(T) = A_{RLD} T^2 \exp\left(-\frac{\Phi}{k_B T}\right)$$

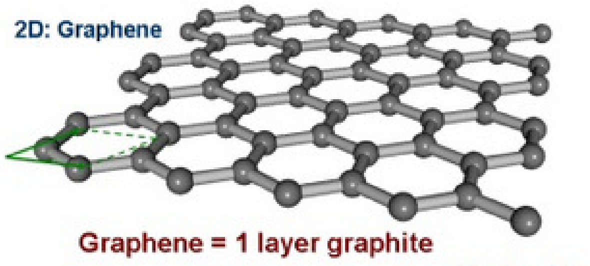
New scaling of thermionic emission for 2D materials (graphene) and its applications in Schottky contact

Postdoc: Dr. Yee Sin Ang
Former PhD student: Dr. Shijun Liang (Asst Prof in Nanjing University, Physics)

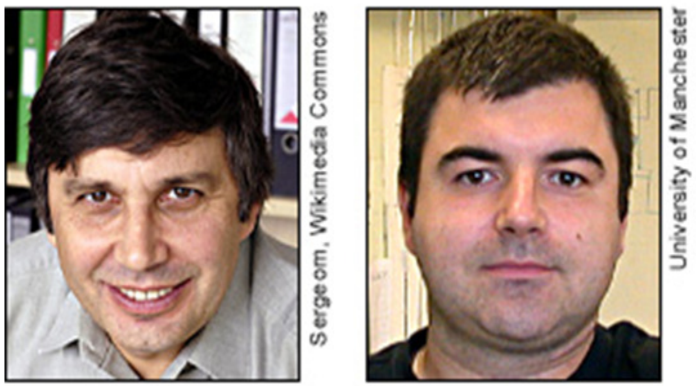
S. J. Liang, and L. K. Ang, Phys. Rev. Applied 3, 014002 (2015)
Y. S. Ang, and L. K. Ang, Phys. Rev. Applied 6, 034103 (2016)
Y. S. Ang, and S. J. Liang, MRS Bulletin (invited) 42, 505 (2017)
Y. S. Ang, H. Y. Yang, and L. K. Ang, Phys. Rev. Lett. (in press 8/2018)

2D materials, like Graphene

- Two-dimensional mono-layer 2D material

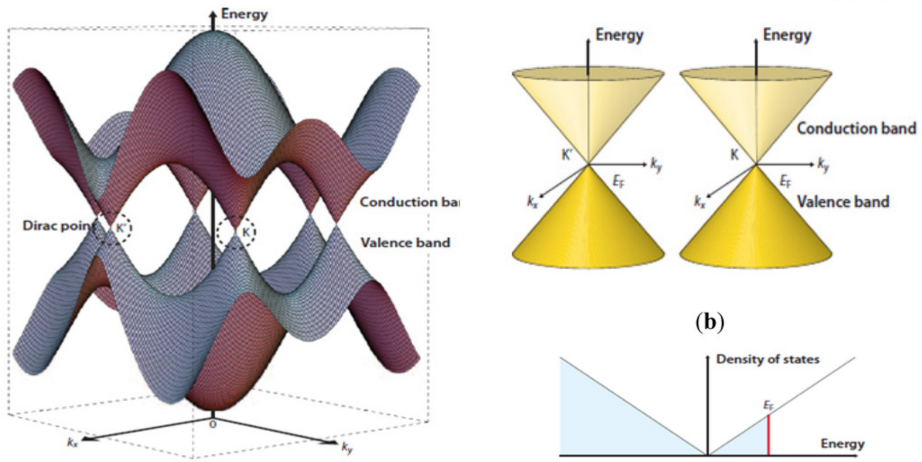


2010 Nobel Prize in Physics



Andre Geim Konstantin Novoselov

(Photo credit: Nathaniel Safron)



(a) (c)

Characteristics:

Electron in graphene mimics massless Dirac fermion (different from traditional semiconductor or metal) - Ultrafast Fermi velocity $10^6 m/s$.

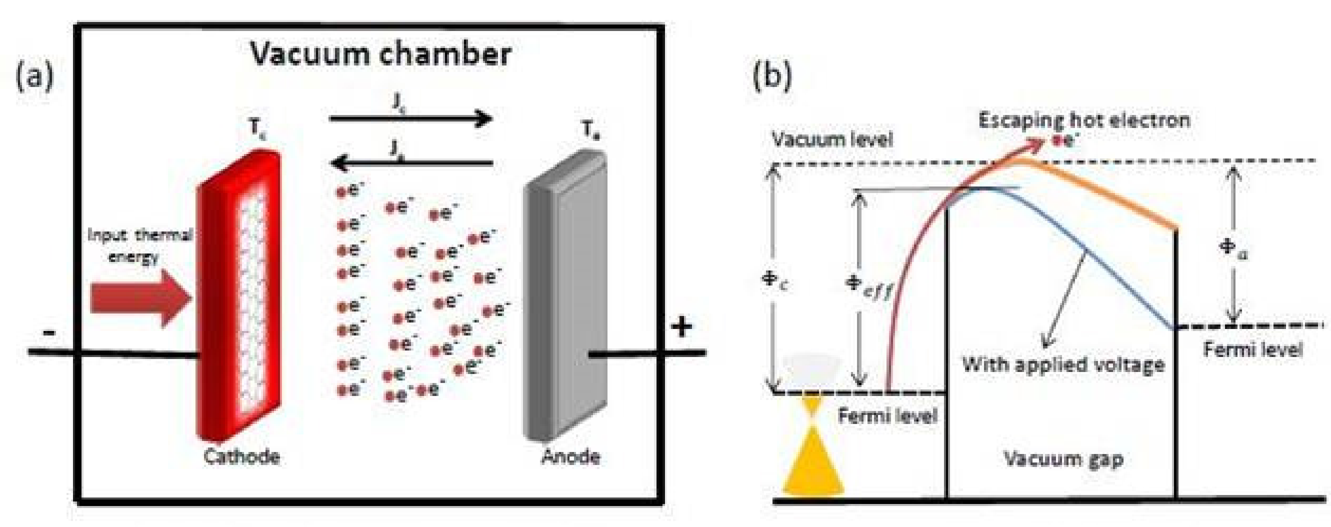
Tunable Fermi level in the graphene via chemical and electrostatic doping.

Finite Density of States (DOS)

Linear energy dispersion: E ~ k

NPG Asia Materials (2009)

Graphene Thermionic Emission new scaling of T^2 due to linear dispersion



Model for thermionic emission

Energy diagram

- A scaling of T^3 as opposed to the T^2 scaling by the traditional Richardson-Dushman (RD) law

$$J = AT^2 \exp\left[-\frac{\Phi}{k_B T}\right].$$

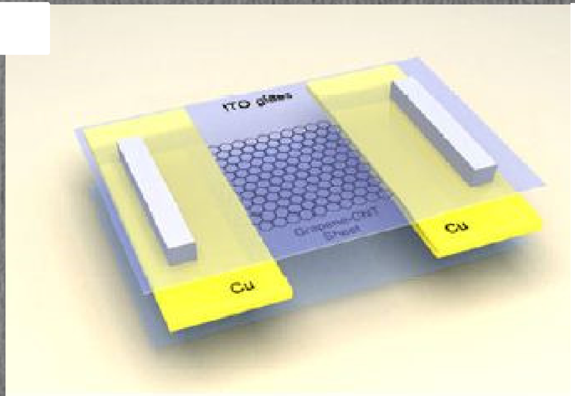
$$A = 4\pi q m^* k_B^2 / h^3$$

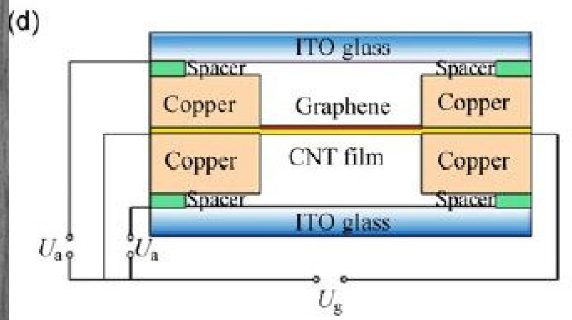
A is a parameter depends on electron mass !!

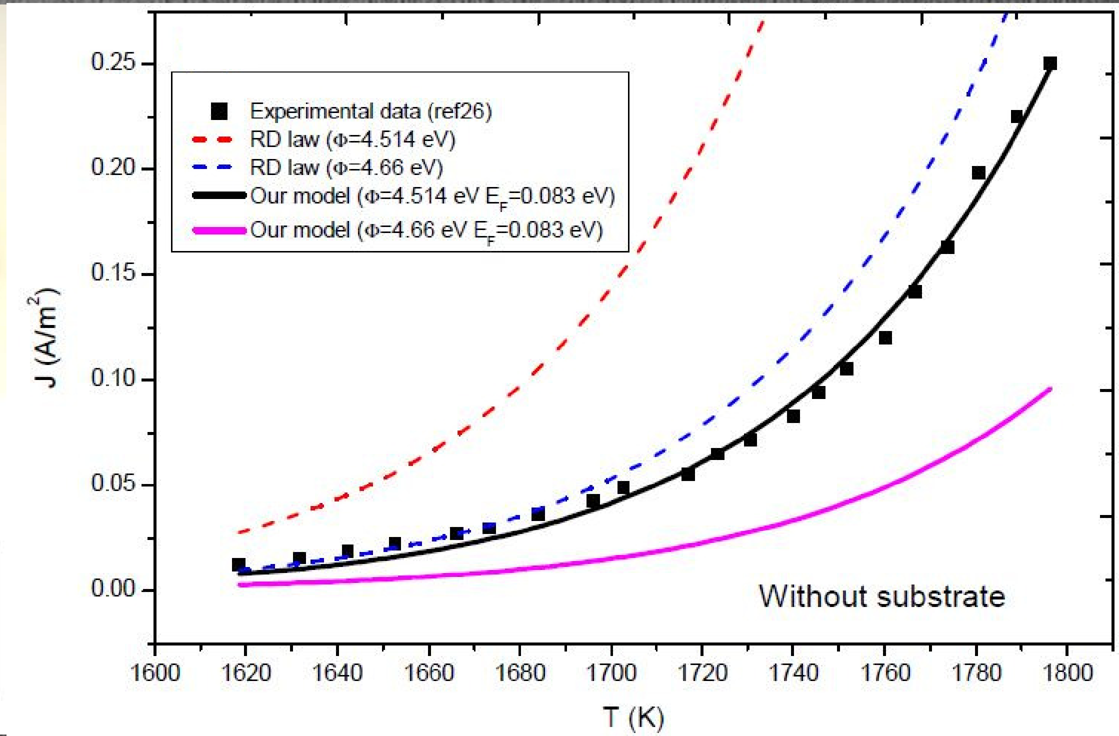
$$J(E_F, T) = \frac{e k_B^3 T^3}{\pi \hbar^3 v_f^2} \exp\left[-\frac{\Phi - E_F}{k_B T}\right]$$

S. J. Liang, and L. K. Ang,
Phys. Rev. Applied 3, 014002 (2015)

Verification with experiment







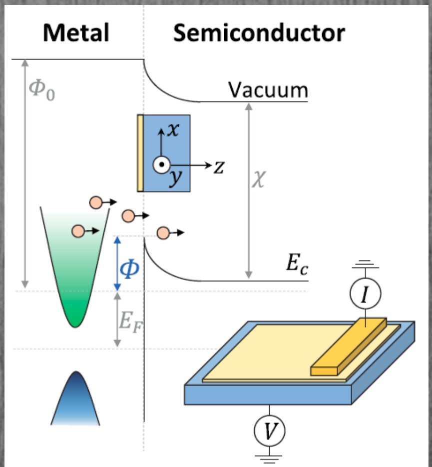
Work function of a suspended graphene is predicted to be 4.514 eV (same as DFT and photoemission measurement)

* Theory: S. J. Liang, and L. K. Ang, Phys. Rev. Applied 3, 014002 (2015)

*Experiment results: Kaili Jiang group: Nano Research , 553 (2014) (from Prof. Kailin Jiang)

Schottky diode equation for 2D materials based electronics and smooth transition between T^2 and T^3

$$
\begin{aligned}
J &= J_S[\exp(qV/\eta k_B T) - 1], \\
J_S &= A \times T^2 \exp(-\phi_{Bn}/k_B T).
\end{aligned}
$$

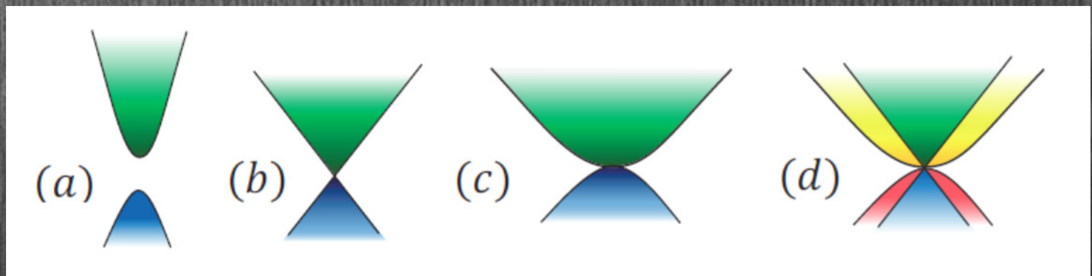


$$A = 4\pi q m^* k_B^2 / h^3$$

$$J(E_F, T) = \frac{e k_B^3 T^3}{\pi \hbar^3 v_f^2} \exp\left[-\frac{\phi_{Bn} - E_F}{k_B T}\right]$$

$$J_{Kane} = \frac{g_{s,v} e m k_B^2}{4\pi^2 \hbar^3} \left(T^2 + 2\gamma k_B T^3\right) e^{-\frac{\Phi}{k_B T}} \left(e^{\frac{eV}{\eta k_B T}} - 1\right)$$

$$J_{Kane} = \begin{cases} \mathcal{A} T^2 e^{-\frac{\Phi}{k_B T}} & , \gamma \to 0 \\ \mathcal{B} T^3 e^{-\frac{\Phi}{k_B T}} & , \gamma \to \infty \end{cases}$$



Y. S. Ang, and L. K. Ang, Phys. Rev. Applied 6, 034013 (2016)

Lateral momentum conservation for thermionic emission from 2D materials-based contact

PHYSICAL REVIEW APPLIED **3**, 014002 (2015)

Electron Thermionic Emission from Graphene and a Thermionic Energy Converter

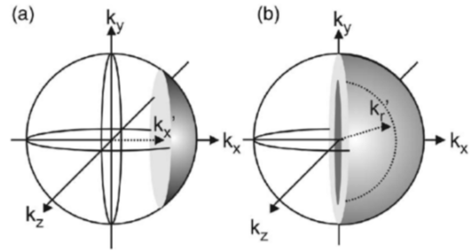
Shi-Jun Liang and L. K. Ang*

$$\boxed{J(E_F,T)=\frac{ek_B^3T^3}{\pi\hbar^3v_f^2}\exp\left[-\frac{\Phi-E_F}{k_BT}\right]}$$

Liang-Ang Model

Assumptions

- Continuous cross-plane dispersion
- In-plane electron momentum strictly conserved



Momentum non-conserving thermionic model

D. Vashaee & A. Shakouri, Phys. Rev. Lett. 92, 106103 (2004).
D. Vashaee & A. Shakouri, J. Appl. Phys. 95, 1233 (2004).
M. F. O'Dwyer, R. A. Lewis, C. Zhang, & T. E. Humphrey, Phys. Rev. B 72, 205330 (2005).

Graphene/Insulator/Graphene thermionic model

J. F. Rodriguez-Nieva, Nano Lett. 16, 6036 (2016)
J. F. Rodriguez-Nieva, Nano Lett. 15, 1451 (2015)

Tunneling of electrons from a two-dimensional channel into the bulk

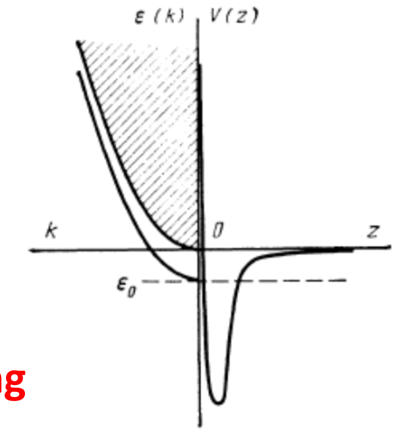
S. V. Meshkov

Institute of Solid State Physics, Academy of Sciences of the USSR, Moscow

(Submitted 23 June 1986)
Zh. Eksp. Teor. Fiz. **91**, 2252–2262 (December 1986)

$$\Psi \propto \exp\left\{-\hbar^{-1}\int^{z}[2m(V(z)-\varepsilon_0)]^{1/2}\,dz\right\}.$$

$$|\Psi|^2 \propto \exp\left\{-2\hbar^{-1}\int^{z}[2m(V(z)-\varepsilon)]^{1/2}\,dz\right\}.$$



Scattering-induced momentum-non-conserving 2D-to-3D electron tunneling

Universal Arrhenius law for thermionic emission

Universal Arrhenius law – A **Universality** arising from thermionic transport

$$\boxed{\log\left(\frac{J}{T^{\beta}}\right)} = A - \frac{B}{T}$$

Reversed saturation current in the most general form

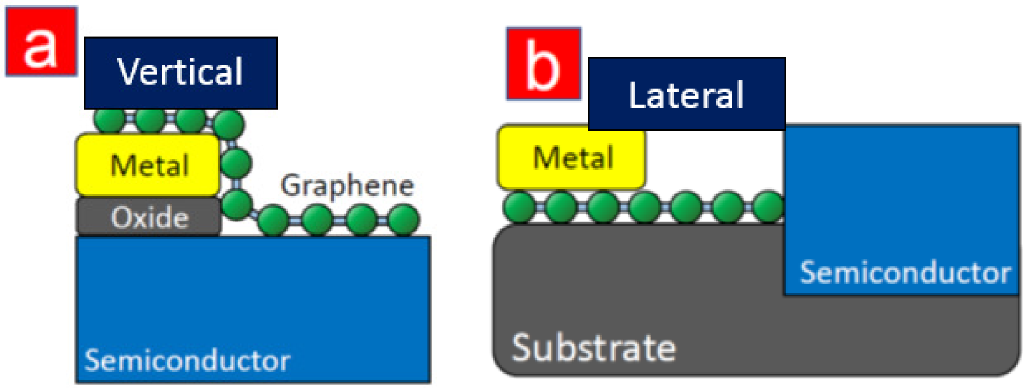
Current practices in literatures of all types of materials

$$\beta = 1, \frac{3}{2}, 2, 3$$

What is the right β in 2D materials?

Problem 1: Contact-geometry-dependence: **Lateral vs. Vertical Schottky heterostructure**

Problem 2: **Momentum-non-conservation** model for vertical Schottky heterostructure

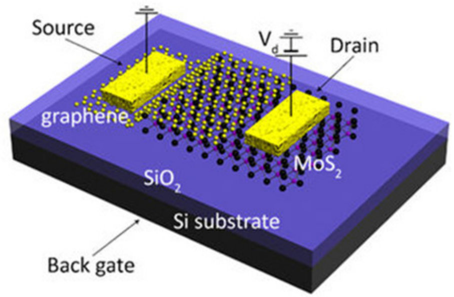


nature materials | REVIEW ARTICLE

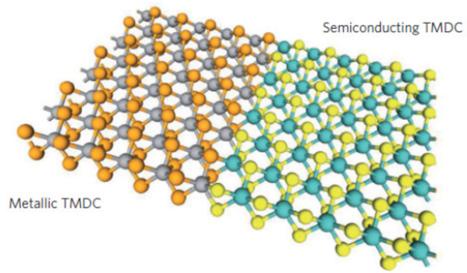
PUBLISHED ONLINE: 20 NOVEMBER 2015 | DOI: 10.1038/NMAT4452

Electrical contacts to two-dimensional semiconductors

Adrien Allain[1], Jiahao Kang[2], Kaustav Banerjee[2]* and Andras Kis[1]*



Vertical contact



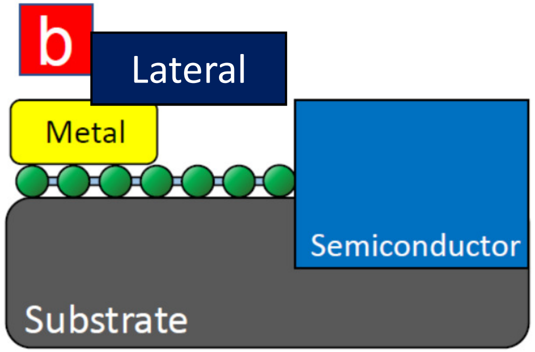
Lateral contact

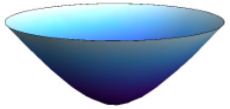
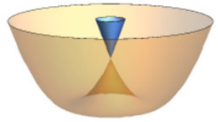
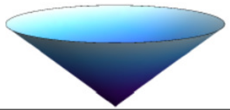
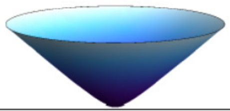
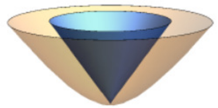
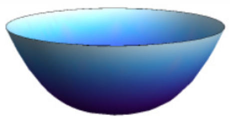
Lateral Schottky contact with universal $T^{3/2}$ scaling

- **Thermionic current density:** $$\mathcal{J}(k_F, T) = \frac{g_{s,v} e}{(2\pi)^2} \sum_{k_\perp^{(i)}} \int \mathrm{d}^2\mathbf{k}_\parallel v_x(k_\parallel) f(\mathbf{k}_\parallel, k_F) \mathcal{T}(k_x, \Phi_{B0})$$ $\boldsymbol{k}_\parallel$: in-plane electron momentum

$\log(J / T^{\beta}) =$
$A - B / T$

$\beta = 3/2$



2D system	Energy dispersion, $\varepsilon_\parallel(\mathbf{k}_\parallel)$	Reversed saturation current density
γ-2DEG	 $\frac{\sqrt{1+2\gamma\hbar^2\lvert\mathbf{k}_\parallel\rvert^2/2m^*}-1}{2\gamma}$	$\mathcal{J}_{\mathrm{NP}} \simeq \frac{g_{s,v} e m^{1/2}}{\hbar^2} (2\gamma\Phi_{B0}+1) \left(\frac{k_B T}{2\pi}\right)^{3/2} \exp\left(-\frac{\Phi_{B0}}{k_B T}\right)$
R-2DEG	 $\frac{\hbar^2\lvert\mathbf{k}_\parallel\rvert^2}{2m^*} + s\alpha_R\lvert\mathbf{k}_\parallel\rvert$	$\mathcal{J}_{\mathrm{Rashba}} \simeq \frac{e m^{1/2}}{\hbar^2} (\Lambda_+ + \Lambda_-) \left(\frac{k_B T}{2\pi}\right)^{3/2} \exp\left(-\frac{\Phi_{B0}}{k_B T}\right)$
Gapless Dirac	 $\hbar v_F\lvert\mathbf{k}_\parallel\rvert$	$\mathcal{J}_{\mathrm{Gr}} \simeq \frac{g_{s,v} e \Phi_B^{1/2}}{\hbar^2 v_F} \left(\frac{k_B T}{2\pi}\right)^{3/2} \exp\left(-\frac{\Phi_{B0}}{k_B T}\right)$
Gapped Dirac	 $\sqrt{\hbar^2 v_F^2\lvert\mathbf{k}_\parallel\rvert^2 + \Delta^2}$	$\mathcal{J}_{\Delta} \simeq \frac{g_{s,v} e \Phi_B^{1/2}}{\hbar^2 v_F} \left(\frac{k_B T}{2\pi}\right)^{3/2} \exp\left(-\frac{\Phi_{B0}}{k_B T}\right)$
ABA-FLG	 $\sum_{n=1}^{N}\left(\alpha_{N,n} + \sqrt{\hbar^2 v_F^2\lvert\mathbf{k}_\parallel\rvert^2 + \alpha_{N,n}^2}\right)$	$\mathcal{J}_{\mathrm{ABA}}^{(N)} \simeq \frac{g_{s,v} e \Phi_{B0}^{1/2}}{\hbar^2 v_F} \times \left(\frac{k_B T}{2\pi}\right)^{3/2} \exp\left(-\frac{\Phi_{B0}}{k_B T}\right) \times \sum_{n=1}^{N}\left[\Theta(\Phi_{B0} - 2\alpha_n^{(N)})\sqrt{\frac{\Phi_{B0} - 2\alpha_n^{(N)}}{\Phi_{B0} - \alpha_n^{(N)}}}\right]$
ABC-FLG	 $\frac{(\hbar v_F\lvert\mathbf{k}_\parallel\rvert)^N}{t_\perp^{N-1}}$	$\mathcal{J}_{\mathrm{ABC}}^{(N)} \simeq \frac{g_{s,v} e t_\perp^{1-1/N} \Phi_{B0}^{1/N-1/2}}{\sqrt{N}\hbar^2 v_F} \left(\frac{k_B T}{2\pi}\right)^{3/2} \exp\left(-\frac{\Phi_{B0}}{k_B T}\right)$

Y. S. Ang, H. Y. Yang, and L. K. Ang, arXiv:1803.01771 (PRL, under review)

ADVANCED MATERIALS
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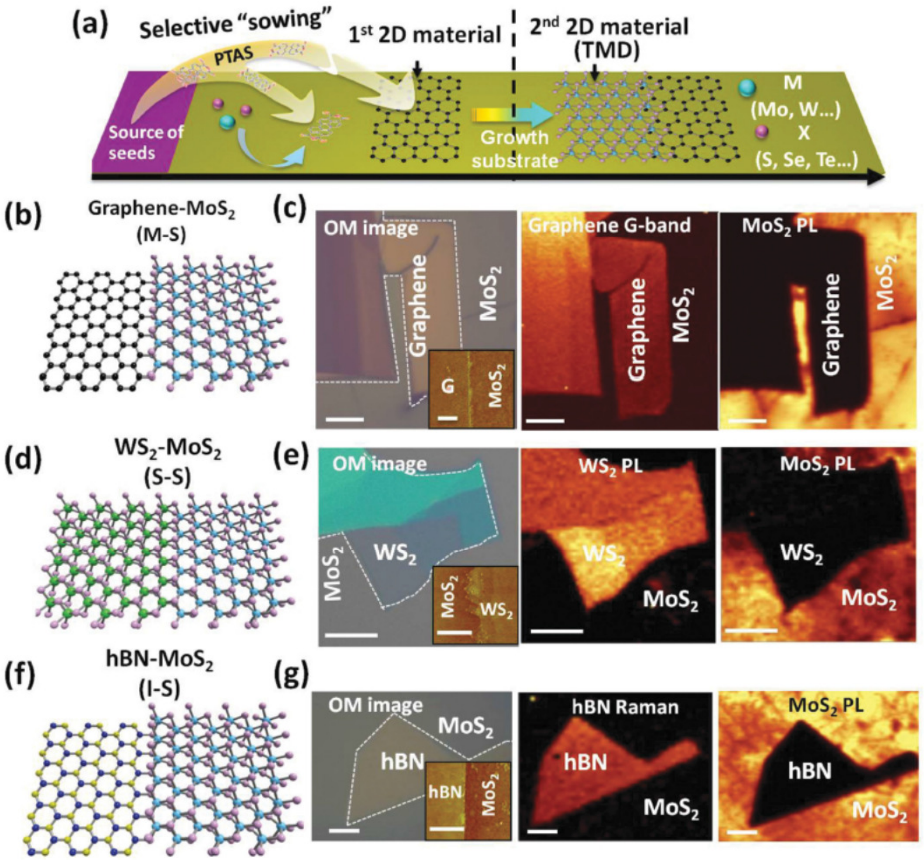
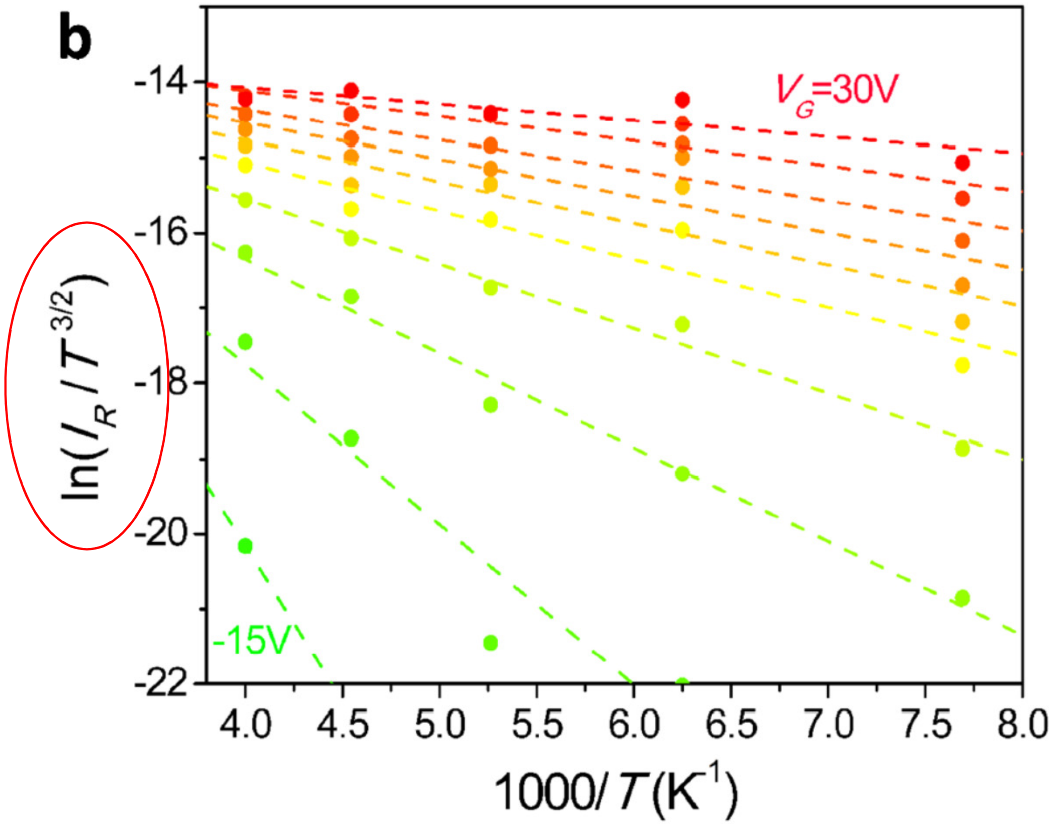
Parallel Stitching of 2D Materials

Xi Ling, Yuxuan Lin, Qiong Ma, Ziqiang Wang, Yi Song, Lili Yu, Shengxi Huang, Wenjing Fang, Xu Zhang, Allen L. Hsu, Yaqing Bie, Yi-Hsien Lee, Yimei Zhu, Lijun Wu, Ju Li, Pablo Jarillo-Herrero, Mildred Dresselhaus, Tomás Palacios, and Jing Kong**

Experimental data fitted with **3/2**-Arrhenius plot





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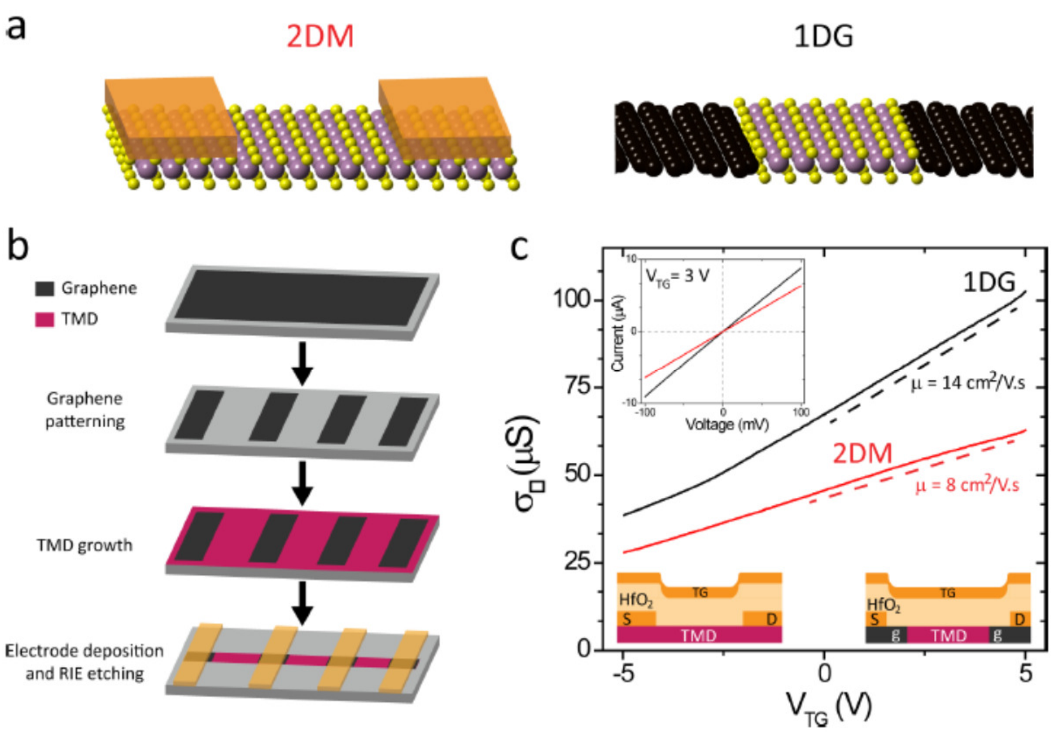
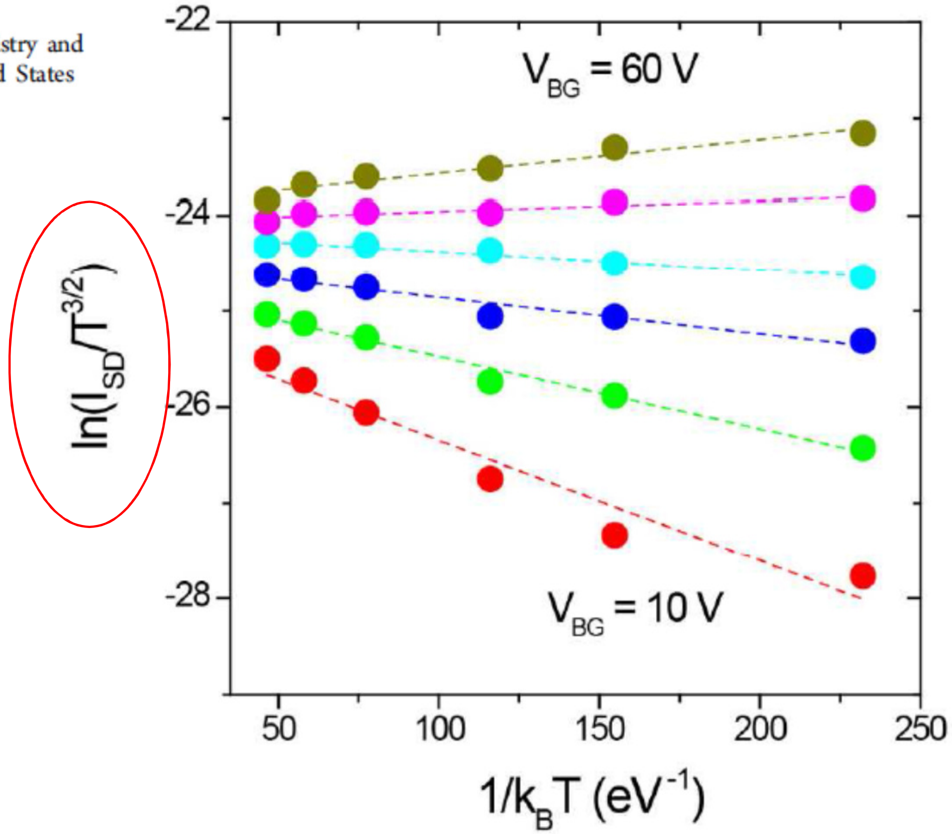
Atomically Thin Ohmic Edge Contacts Between Two-Dimensional Materials

Marcos H. D. Guimarães,[†,‡,⊥] Hui Gao,[⊥,§] Yimo Han,[∥] Kibum Kang,[§] Saien Xie,[∥] Cheol-Joo Kim,[§] David A. Muller,[†,∥] Daniel C. Ralph,[†,‡] and Jiwoong Park*[,†,§]

[†]Kavli Institute at Cornell for Nanoscale Science, [‡]Laboratory of Atomic and Solid State Physics, [§]Department of Chemistry and Chemical Biology, and [∥]School of Applied and Engineering Physics, Cornell University, Ithaca, New York 14853, United States



Experimental data fitted with **3/2**-Arrhenius plot

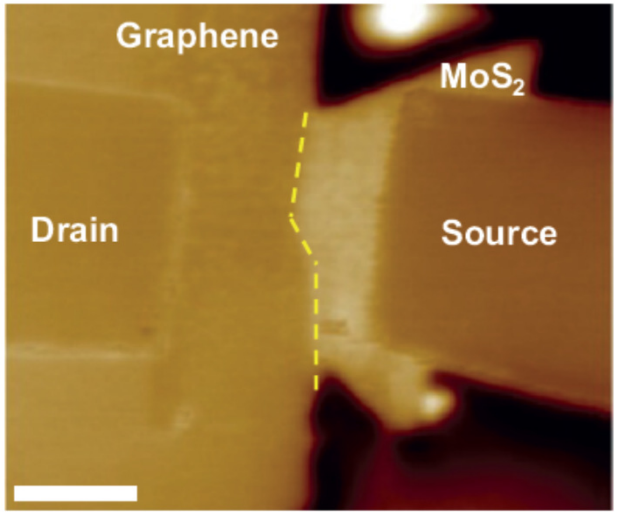


full papers

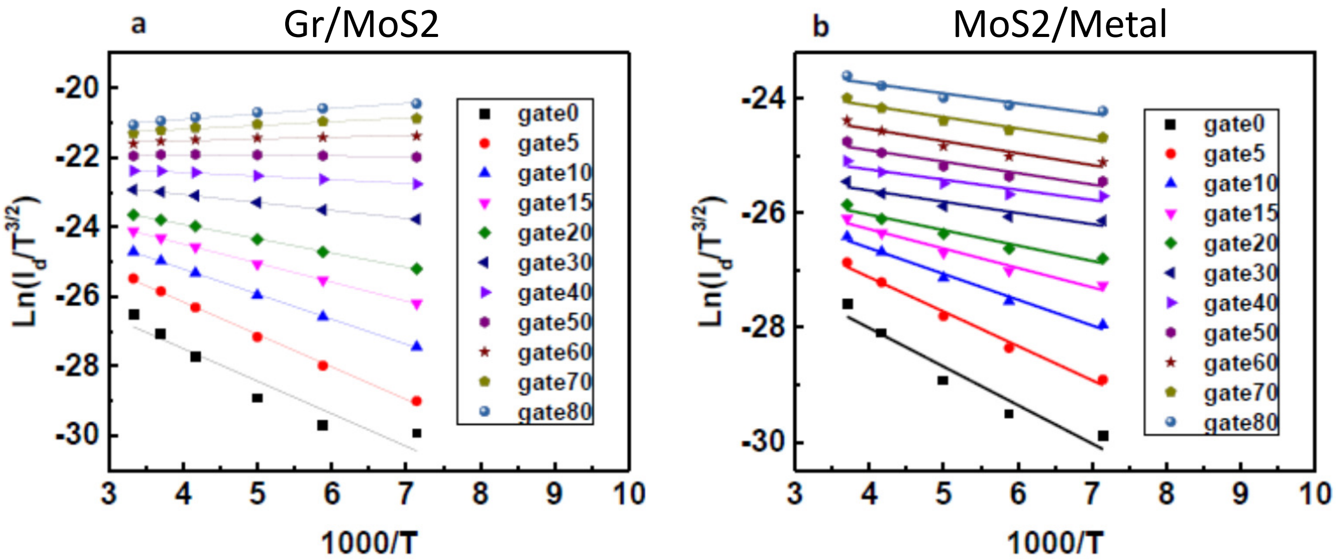
ADVANCED SCIENCE NEWS
www.advancedsciencenews.com
Electronic Devices

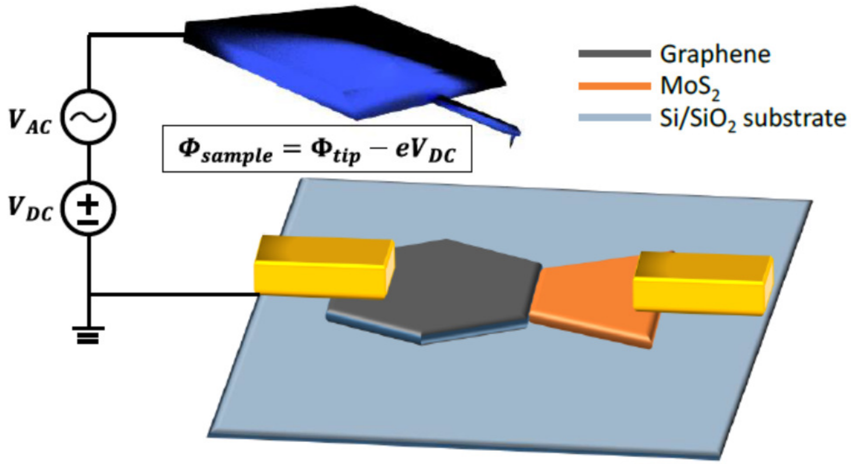
Direct Growth of High Mobility and Low-Noise Lateral MoS_2–Graphene Heterostructure Electronics

Amirhossein Behranginia, Poya Yasaei, Arnab K. Majee, Vinod K. Sangwan, Fei Long, Cameron J. Foss, Tara Foroozan, Shadi Fuladi, Mohammad Reza Hantehzadeh, Reza Shahbazian-Yassar, Mark C. Hersam, Zlatan Aksamija, and Amin Salehi-Khojin**



Experimental data fitted with **3/2**-Arrhenius plot





Vertical contact: universal T scaling (non-conserved momentum)

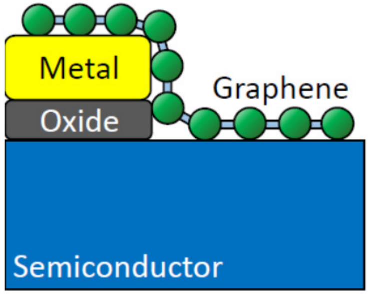
- **Thermionic current density:**

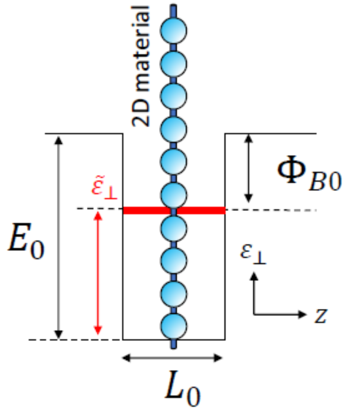
$$\mathcal{J}_\star^{(j)}(k_F,T)=\frac{g_{s,v}e}{(2\pi)^2}\int \mathrm{d}^2\mathbf{k}_\parallel$$

$$\times\left[\frac{\lambda j}{L_\perp}\sum_{k_\perp^{(i)}} v_\perp\left[\varepsilon_\perp^{(i)}(k_z^{(i)})\right] f(\mathbf{k},k_F)\mathcal{T}(\mathbf{k}_\parallel,k_\perp^{(i)})\right.$$ Bound state

$$\left.+\frac{1}{2\pi}\int \mathrm{d}k_\perp v_\perp\left[\varepsilon_\perp(k_\perp)\right] f(\mathbf{k},k_F)\mathcal{T}(j\mathbf{k}_\parallel,k_\perp)\right]$$ Continuum state

Vertical





$j=0,1$ to denote $\boldsymbol{k}_\parallel$-(non)conservation

$$\mathcal{J}_\star^{(j=0)}=\frac{\lambda g_{s,v}ek_B}{2\pi}T\xi_T\int_0^\infty k_\parallel dk_\parallel e^{-\frac{\varepsilon_\parallel}{k_BT}}$$

Continuum state

$$\mathcal{J}_\star^{(j=1)}=\frac{\lambda g_{s,v}e}{2\pi}\left(\frac{\tilde{v}_\perp}{L_\perp}+\cancel{\frac{k_BT}{2\pi\hbar}}\right)\int_{\Phi_{B0}}^\infty k_\parallel dk_\parallel e^{-\frac{\varepsilon_\parallel-\varepsilon_F}{k_BT}}$$

Bound state

Φ_B-limited

Universal scaling law for vertical Schottky heterostructure

$$\boxed{\mathcal{J}_\star^{(j=1)}\propto T\left[1+(2l-1)\frac{k_BT}{\Phi_{B0}}\right]\exp\left(-\frac{\Phi_B}{k_BT}\right)}$$

log(J / T) = A – B / T

$\boldsymbol{\beta}=\mathbf{1}$, a new **universality** in vertical Schottky heterostructure

Vertical Schottky transport (T universal scaling)

2D system	Conserving lateral momentum ($j=0$)	Non-conserving lateral momentum ($j=1$)
	Non-universal	**Universal**
γ-2DEG	$\mathcal{J}_{\star\mathrm{NP}}^{(j=0)} = \frac{g_{s,v} e m^* k_B^2}{4\pi^2\hbar^3}\left[T^2 + 2\gamma k_B T^3\right]\xi_T$ (see Ref. [11])	$\mathcal{J}_{\star\mathrm{NP}}^{(j=1)} = \frac{\lambda g_{s,v} e m^* v_\perp}{2\pi\hbar^2 L_\perp}\left[2\gamma\left(k_BT\right)^2\left(1+\frac{\Phi_{B0}}{k_BT}\right)+k_BT\right]\xi_T$
R-2DEG	$\mathcal{J}_{\star\mathrm{Rashba}}^{(j=0)} = \frac{g_{s,v} e m^*}{4\pi^2\hbar^3}\left(k_BT\right)^2 \times \left[1+\sqrt{\frac{\pi\varepsilon_R}{k_BT}}\mathrm{erf}\left(\sqrt{\frac{\varepsilon_R}{k_BT}}\right)\right]\xi_T$	$\mathcal{J}_{\star\mathrm{Rashba}}^{(j=1)} = \frac{\lambda e m^* v_\perp}{\pi\hbar^2 L_\perp} k_BT\xi_T$
Gapless Dirac	$\mathcal{J}_{\star\mathrm{Gr}}^{(j=0)} = \frac{g_{s,v} e}{4\pi^2\hbar^3 v_F^2}\left(k_BT\right)^3\xi_T$ (see Ref. [9])	$\mathcal{J}_{\star\mathrm{Gr}}^{(j=1)} = \frac{\lambda g_{s,v} e v_\perp}{2\pi\hbar^2 v_F^2 L_\perp}\left(k_BT\right)^2\left(1+\frac{\Phi_{B0}}{k_BT}\right)\xi_T$ (see Ref. [8])
Gapped Dirac	$\mathcal{J}_{\star\Delta}^{(j=0)} = \frac{g_{s,v} e}{4\pi^2\hbar^3 v_F^2}\left(k_BT\right)^3\xi_T$	$\mathcal{J}_{\star\Delta}^{(j=1)} = \frac{\lambda g_{s,v} e v_\perp}{2\pi\hbar^2 v_F^2 L_\perp}\left(k_BT\right)^2\left(1+\frac{\Phi_{B0}}{k_BT}\right)\xi_T$
ABA-FLG	$\mathcal{J}_{\star\mathrm{ABA}}^{(N,j=0)} = N\frac{g_{s,v} e}{4\pi^2\hbar^3 v_F^2}\left(k_BT\right)^3\xi_T$ (see Ref. [11])	$\mathcal{J}_{\star\mathrm{ABA}}^{(N,j=1)} = N\frac{\lambda g_{s,v} e v_\perp}{2\pi\hbar^2 v_F^2 L_\perp}\left(k_BT\right)^2\left(1+\frac{\Phi_{B0}}{k_BT}\right)\xi_T$
ABC-FLG	$\mathcal{J}_{\star\mathrm{ABC}}^{(N,j=0)} = \frac{\Gamma(2/N)}{N}\frac{g_{s,v} e t_\perp^{2-2/N}}{4\pi^2\hbar^3 v_F^2}\left(k_BT\right)^{2/N+1}\xi_T$ (see Ref. [11])	$\mathcal{J}_{\star\mathrm{ABC}}^{(N,j=1)} = \frac{1}{N}\frac{\lambda g_{s.v} e t_\perp^{2-2/N} v_\perp}{2\pi\hbar^2 v_F^2 L_\perp}\left(k_BT\right)^{2/N}\Gamma\left(\frac{2}{N},\frac{\Phi_{B0}}{k_BT}\right)e^{\frac{\Phi_{B0}}{k_BT}}\xi_T \simeq \frac{1}{N}\frac{\lambda g_{s.v} e v_\perp}{2\pi\hbar^2 v_F^2 L_\perp}\left(\frac{\Phi_{B0}}{t_\perp}\right)^{2/N-2}\left(k_BT\right)^2\left(1+\frac{\Phi_{B0}}{k_BT}\right)\xi_T$

Y. S. Ang, H. Y. Yang, and L. K. Ang, arXiv:1803.01771 (PRL, in press 8/2018)

$$J_{FN}(F) = A_{FN}F^2 \exp\left(-\frac{B\Phi^{3/2}}{F}\right)$$

Field emission: new scaling of FN law [1] and fractional FN law [2]

Postdoc: Dr. Yee Sin Ang

Former postdoc: Dr. Muhammad Zubair (now Asst Professor in Pakistan)

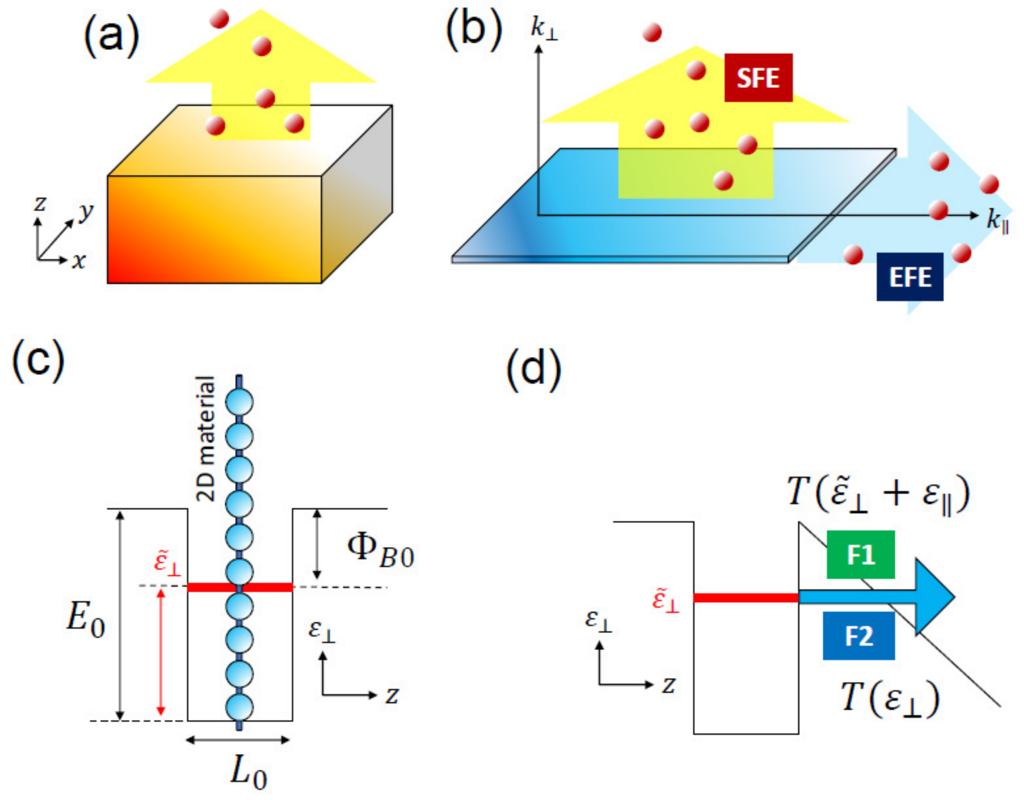
[1] Y. S. Ang, M. Zubar, K. Ooi and L. K. Ang, (under review, 2018)

[2] M. Zubar, Y. S. Ang, and L. K. Ang, IEEE TED 65, 2089 (2018)

Similarity between thermionic and field emission

$$\mathscr{J}_{\mathrm{RD}} = A_{\mathrm{RD}} T^2 \exp\left[-\frac{\Phi_B}{k_B T}\right],$$

$$\mathcal{J}_{\mathrm{FN}} = A_{\mathrm{FN}} F^2 \exp\left[-B_{\mathrm{FN}} \frac{\Phi_B^{3/2}}{F}\right],$$



Problem 1: Direction of field emission for 2D materials EFE (Edge field emission) and SFE (surface field emission) – energy dispersion in lateral direction is linear.

Problem 2: For SFE, we have discrete bound state as the material thickness is very thin (only 1 monolayer of atoms)

Problem 3: For SFE, we have both non-conserving and conserving lateral momentum (NCLM): F1 and (CLM): F2 field emission process

Problem 4: Fermi energy of 2D materials is small

Problem 5 – space charge effects and temperature

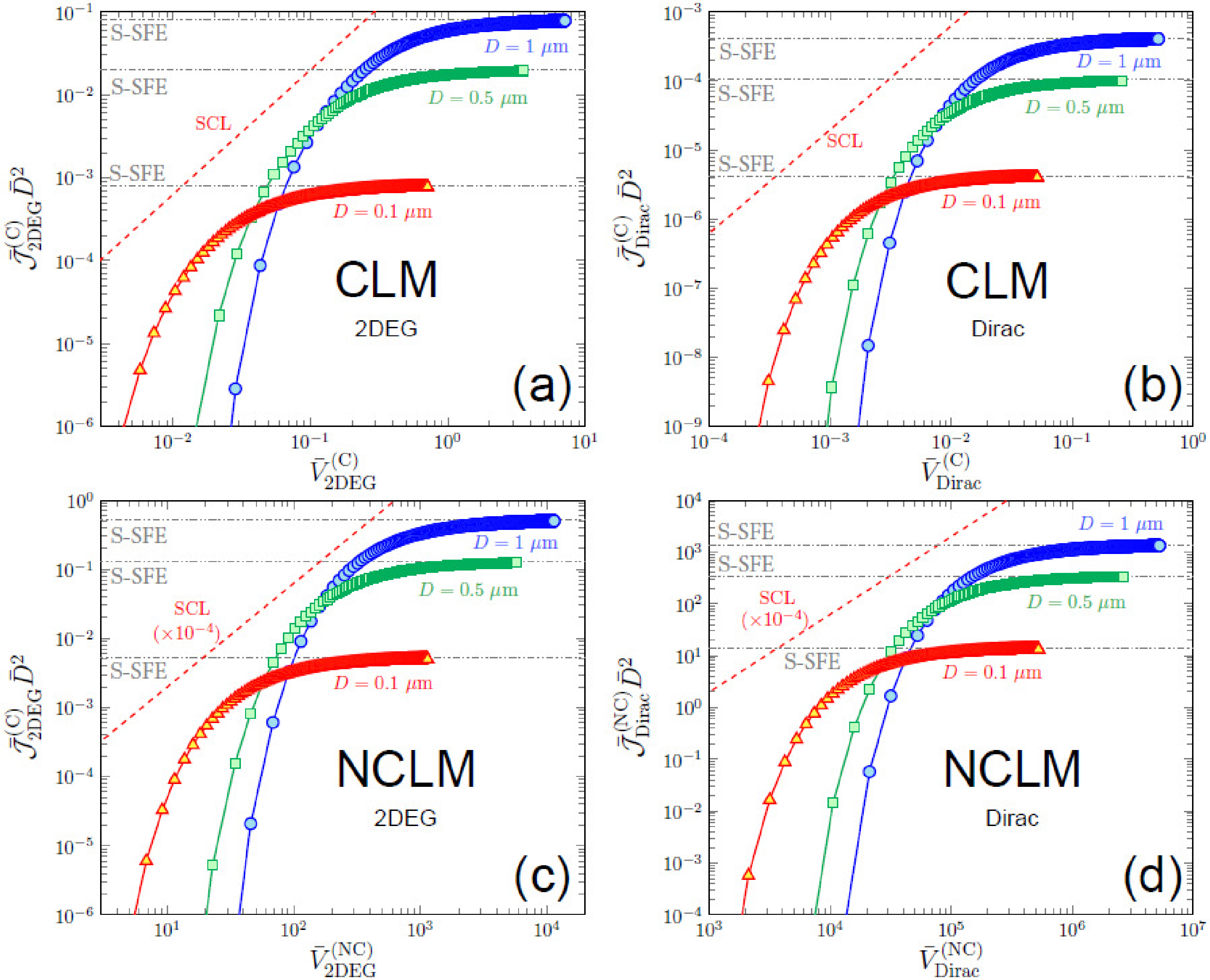
Y. S. Ang, M. Zubar, K. Ooi and L. K. Ang, (under review, 2018)

Different scaling law for 2D FN law (surface emission)

Model	Lateral energy dispersion ($\varepsilon_{\parallel}$)	Empirical form	Full expression
FN Law	Parabolic	$\mathcal{J}_{\mathrm{FN}} = \mathcal{C}F^2 \exp\left(-\frac{\mathcal{B}}{F}\right)$ Classical FN law	Eq. (1b)
CLM	parabolic Reduced dimension	$\mathcal{J}^{(\mathrm{C})}_{\mathrm{2DEG}} = \mathcal{C} \exp\left(-\frac{\mathcal{B}}{F}\right)$ Type A	Eq. (14a)
	linear Discrete energy	$\mathcal{J}^{(\mathrm{C})}_{\mathrm{Dirac}} = \mathcal{C} \exp\left(-\frac{\mathcal{B}}{F}\right)$	Eq. (14b)
NCLM	Parabolic	$\mathcal{J}^{(\mathrm{NC})}_{\mathrm{2DEG}} = \mathcal{C}F\left[1 - \exp\left(-\frac{\mathcal{A}}{F}\right)\right] \exp\left(-\frac{\mathcal{B}}{F}\right)$	Eq. (17a)
	Linear Due to some scattering. the parallel momentum is no conserved, and that transmission depends on parallel energy	$\mathcal{J}^{(\mathrm{NC})}_{\mathrm{Dirac}} = \mathcal{C}F^2\left[\frac{\mathcal{A}}{F} - 1 + \exp\left(-\frac{\mathcal{A}}{F}\right)\right] \exp\left(-\frac{\mathcal{B}}{F}\right)$	Eq. (17b)
(Low-field regime)	Parabolic	$\mathcal{J}^{(\mathrm{NC})}_{\mathrm{2DEG}} = \mathcal{C}F \exp\left(-\frac{\mathcal{B}}{F}\right)$ Type B	Eq. (18a)
	Linear	$\mathcal{J}^{(\mathrm{NC})}_{\mathrm{Dirac}} = \mathcal{C}F \exp\left(-\frac{\mathcal{B}}{F}\right)$	Eq. (18b)
(High-field regime)	Parabolic	$\mathcal{J}^{(\mathrm{NC})}_{\mathrm{2DEG}} = \mathcal{C} \exp\left(-\frac{\mathcal{B}}{F}\right)$ Type A	Eq. (20a)
	Linear	$\mathcal{J}^{(\mathrm{NC})}_{\mathrm{Dirac}} = \mathcal{C} \exp\left(-\frac{\mathcal{B}}{F}\right)$	Eq. (20b)

- Under CLM, when the material is very thin (not necessary 2D materials), due to discrete bound states, it shows a new scaling of Type A
- Under NCLM, high and low field will two different scaling of Type A and Type B

Saturated Surface field emission (S-SFE) at high F



$$\mathcal{J}_{2D} \propto \exp\left(-B_{FN}\frac{\Phi_B^{3/2}}{F}\right)$$

At sufficient large F, the exp. term = 1, where the J is constant and it becomes source limited – this is independent of CLM or NLM; parabolic or linear, types of materials – as long as we have reduced in dimensions (discrete energy)

Not due to space charge effect

Fractional FN law for rough cathode

Fractional Calculus: Applications in physics

- Fractional calculus – to study the behavior of physical systems that are determined by
  - integrations of non-integer (fractional) orders,
  - differentiation of non-integer (fractional) orders.

$$\frac{d^n f}{dt^n} \; ?$$

What if the order will be n=1/2 ?

$$\frac{d^n f}{dx^n} \quad \begin{cases} \dfrac{d f}{dx} & \text{Ordinary differentiation: Integer } n=1 \\[2ex] \dfrac{d^{1/2} f}{dx^{1/2}} = ? & \text{Non-integer } n \text{ Fractional differentiation} \end{cases}$$

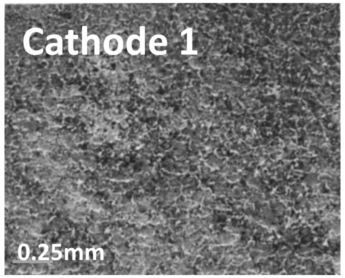
Leibniz (1646-1716)

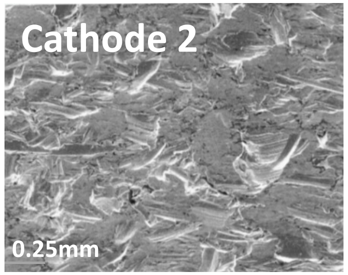
It will lead to a paradox from which one day useful consequences will be drawn

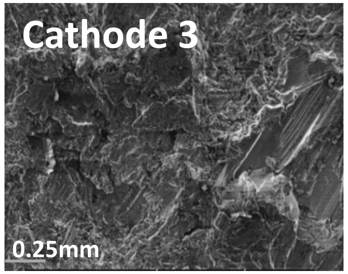
L'Hopital (1661-1704)

- Initial works by Leibniz, Liouville, Grunwald, Letnikov and Riemann
- Fractional vector calculus is relatively new in past 10 years (example: Tarasov from Russia)
- Equations with fractional-order calculus are used to describe objects with:
  - power-law nonlocality, disordered, roughness, fractal objects

Research Area 3 (since 2016): Testing of Fractional modeling – MSU seminar





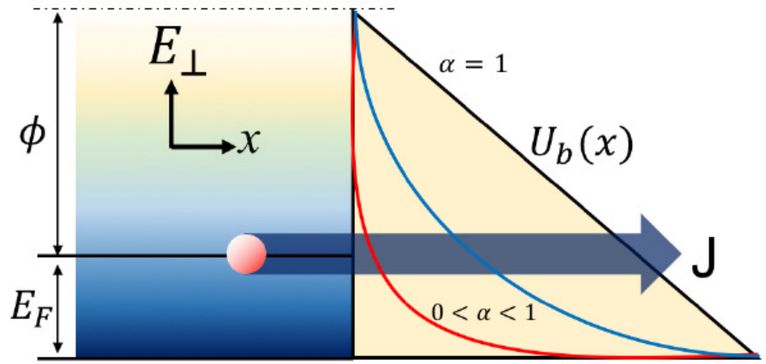


High current electron (SCL) emission from rough cathode

M. Zubair, L. K. Ang, Phys. Plasmas 23, 073118 (2016)

Light absorption in a fractal surface

M. Zubair, Y. S. Ang, K. Ooi, L. K. Ang (JAP under review)

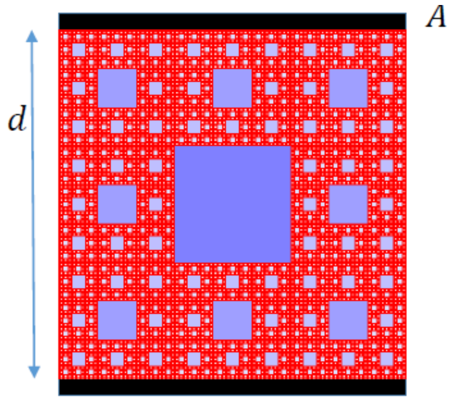


Fractional FN law of a rough surface

M. Zubair, Y. S. Ang, L. K. Ang, IEEE TED 65, 2089 (2018)

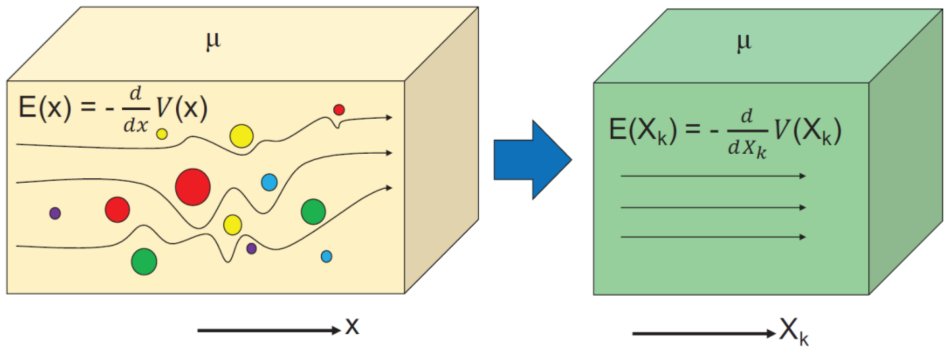
$$C = \frac{k\varepsilon_0 A(\alpha, n)}{d^{f(\alpha,n)}}$$

α=fractal dimension of dielectric
n=Dimension of embedding Euclidean Space



Capacitance for a fractal composite

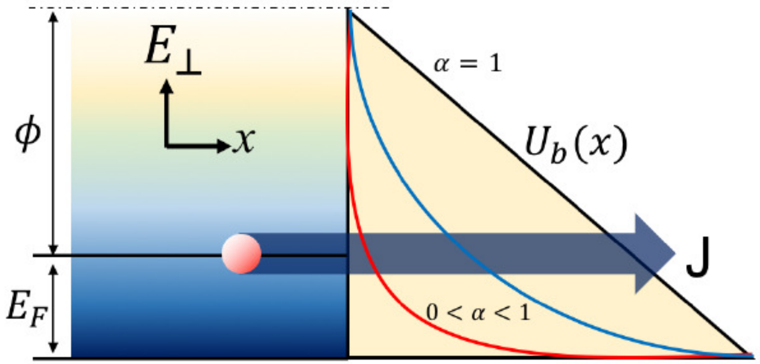
M. Zubair, Y. Samuel, S. Athalye, L. K. Ang (in prep)



Current Transport in spatial disordered semiconductor

M. Zubair, Y. S. Ang, L. K. Ang, IEEE TED (in press, 2018)

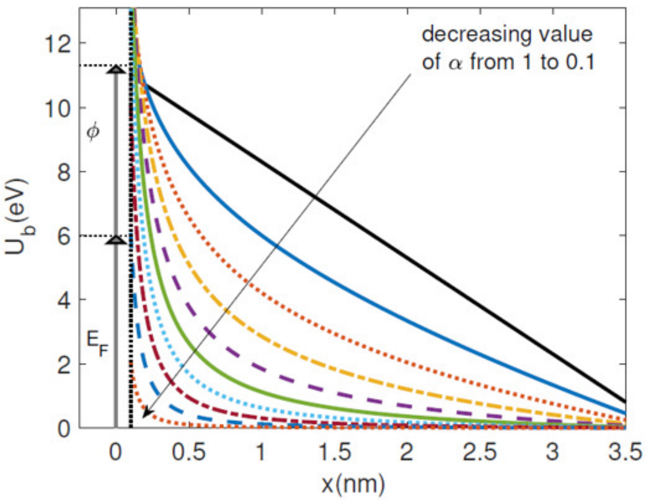
New Fractional FN law for rough surface



Approach is to solve the Schrodinger Equation and WKBJ in fractional mathematical form

$$J_{FN\alpha} = a_{FN\alpha} \frac{F^{2\alpha}}{\phi^{2\alpha-1}} \exp\left(-\frac{b_{FN\alpha}\phi^{\alpha+1/2}}{F^{\alpha}}\right)$$

$\alpha = 1$, it becomes regular FN law

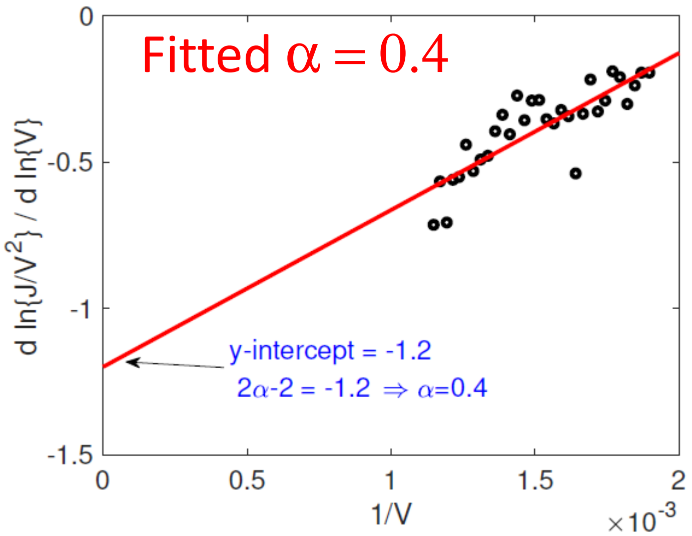


To obtain α:

$$\ln(i/V^2) = \ln(C) + (2\alpha - 2)\ln(V) - B/V^{\alpha},$$

$$\frac{d\ln(i/V^2)}{d\ln(V)} = 2\alpha - 2 + \alpha B/V^{\alpha}.$$

$$lim_{\frac{1}{V}\to\min} \frac{d\ln(i/V^2)}{d\ln(V)} = 2\alpha - 2.$$



M. Zubar, Y. S. Ang, and L. K. Ang, IEEE TED (June 2018)

Photoemission at ultrafast time scale, plasmonic effect & 2D materials

L. Wu, and L. K. Ang, Physical Review B 78, 224112 (2008).

M. Pant, and L. K. Ang, Phys. Rev. B 86, 045423 (2012).

L. K. Ang, and M. Pant, Physics of Plasmas 20, 056705 (2013).

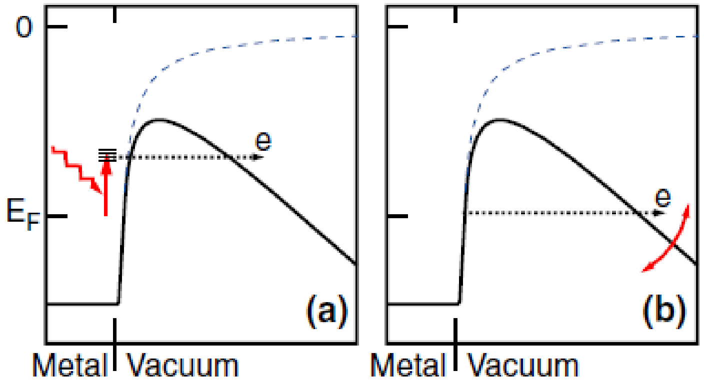
M. Pant, and L. K. Ang, Phys. Rev. B 88, 195434 (2013).

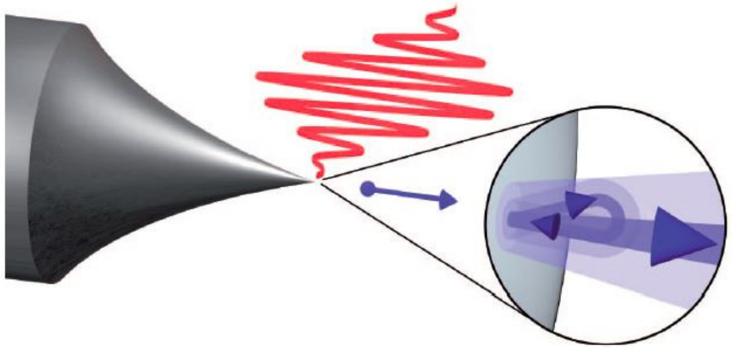
S. J. Liang, and L. K. Ang, Carbon, 61, 291 (2013)

S. J. Liang, and L. K. Ang, IEEE TED 61, 1764 (2014)

L. K. Ang, and Y. B. Zhu, - plasmonic effect, SPIE conference (2018)

Ultrafast laser induced electron emission





Keldysh parameter $\gamma \propto \dfrac{\sqrt{\Phi}}{\lambda F}$

γ >> 1 (multiphoton)

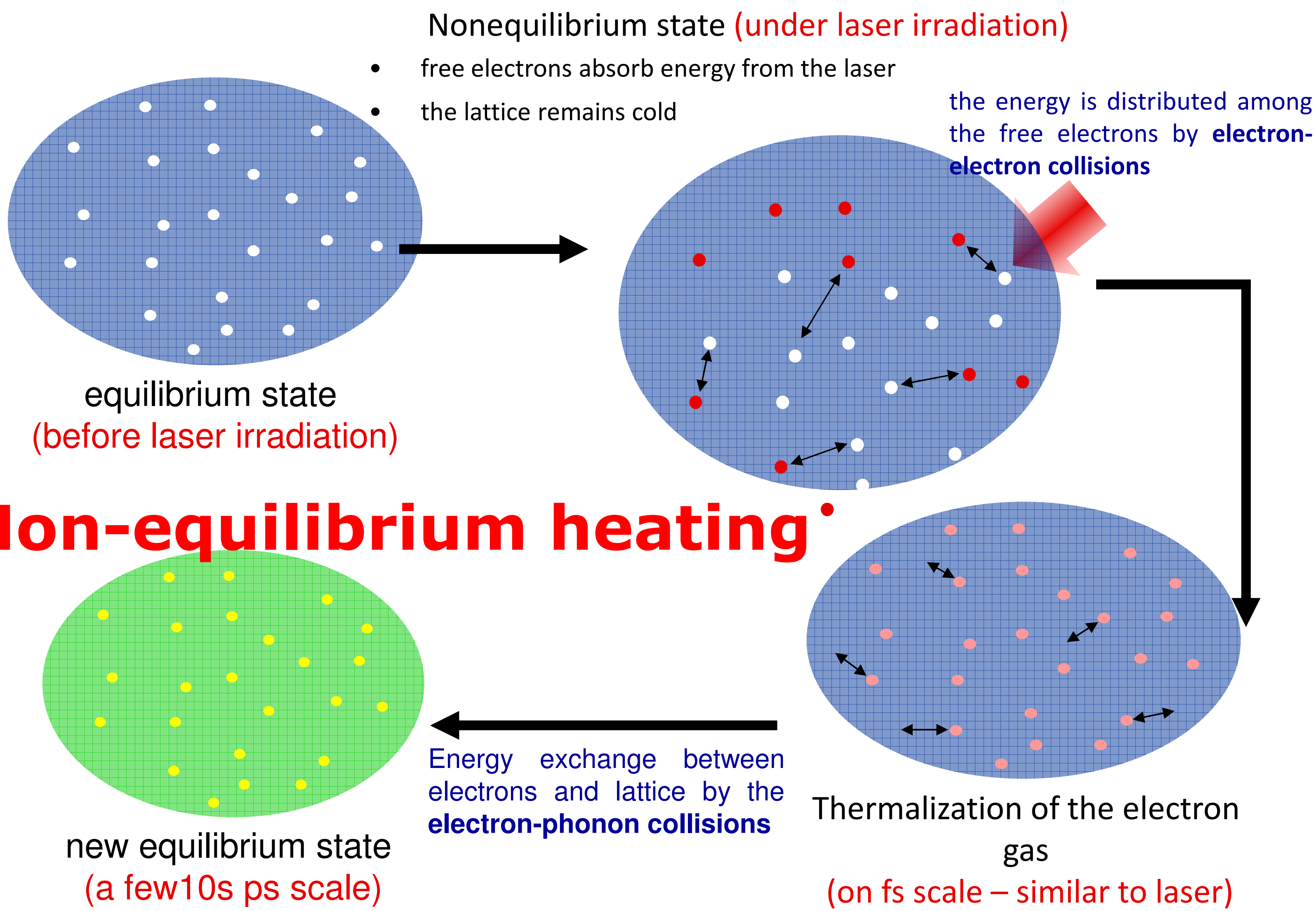
γ << 1 (optical tunneling)

γ ~ 1 is of interests recently

Femtosecond laser: 800nm, 6 – 55 fs, up to 600 mW, up to 4 x 10^{11} W/cm^2 (below damage)

- New physics under ultrafast laser
  - Many new physics: ATI, coherent beam, phase, pondermotive field, etc…
  - Non-equilibrium heating
  - Time dependent tunneling
  - Plasmonic effect
  - New materials like graphene

Non-equilibrium heating



W. Lin and L. K. Ang, PRB 78, 224112 (2008)

SUTD-MIT INTERNATIONAL DESIGN CENTRE (IDC)

Undertake the impossible, Design the unexpected

Combination of multiple energy TDSE + non-equilibrium heating

- From the non-equilibrium electron distribution, we solve time-dependent Schrodinger equation for each $E_x(t)$ using the following potential

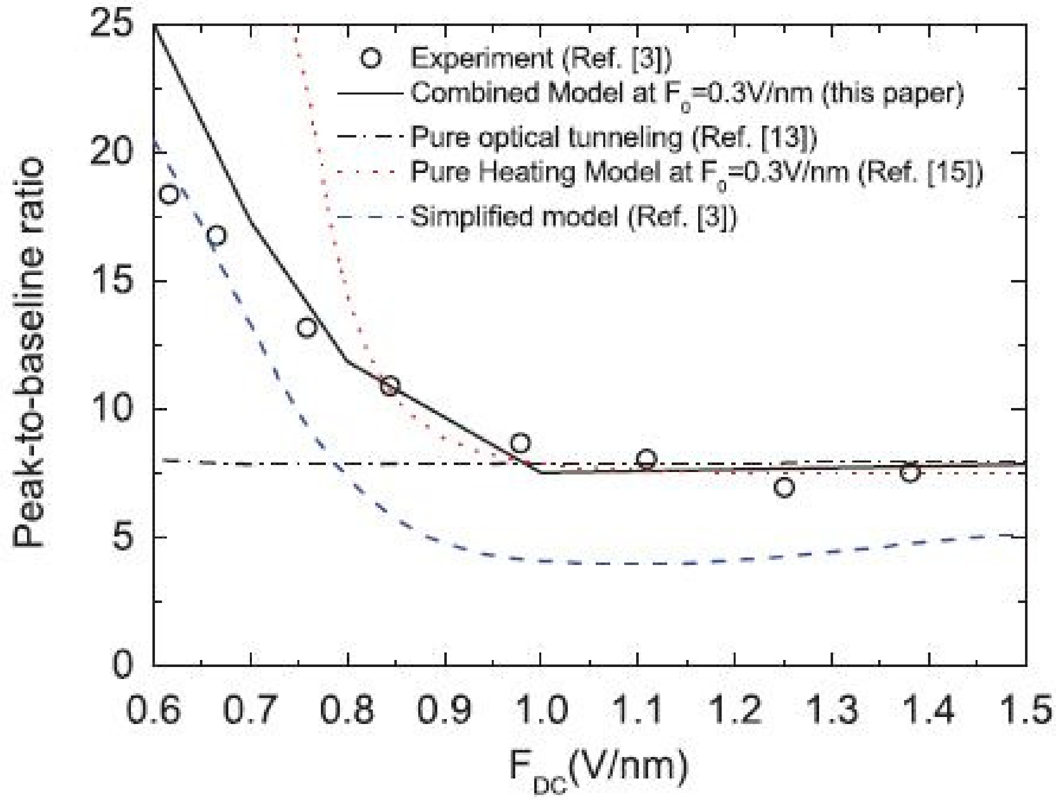
$$U(x,t) = E_{\text{vac}} - \frac{e^2}{16\pi\epsilon_0(x+x_0)} - e\left\{F_{\text{dc}}x + F_0 x \exp\left(-2\ln 2\frac{(t-t_0)^2}{\tau^2}\right) \times \cos[\omega(t-t_0)+\phi]\right\}$$

Transition between multiphoton absorption and optical tunnelling

$$\gamma_c = 1.18 \times \sqrt{\Phi_m}$$

Φ_m = 4.5 - 5 eV (gold and tungsten) γ_c = 2.5 – 2.6

Experiment condition is $\gamma \sim 2-3$

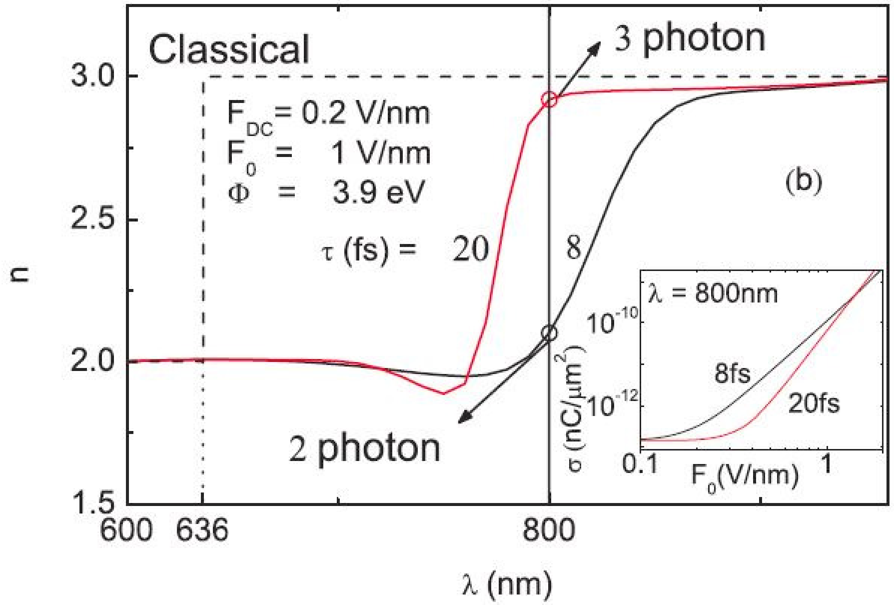


M. Pant, and L. K. Ang, Phys. Rev. B 86, 045423 (2012).

M. Pant, and L. K. Ang, Phys. Rev. B 88, 195434 (2013).

Photoelectric effect at few laser pulse

M. Pant and L. K. Ang, PRB 86, 045423 (2012)



A reduction of multiphoton emission from n =3 to n = 2 at λ = 800 nm when the laser pulse is decreased from 20 to 8 fs

The energy difference between n =3 and n = 2 is 0.4 eV

At 8 fs, **$\Delta E = h/\tau$**, 0.52 eV at 8 fs (> 0.4 eV), but only 0.21 eV at 20 fs

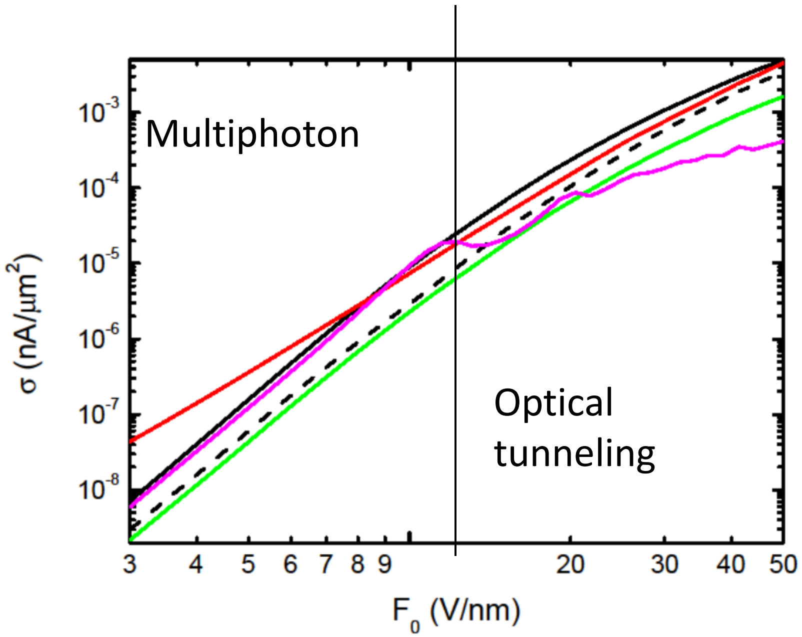
- To realize such a transition at 800 nm, we can use a material of about 3.9 eV (Hafnium)
- For 4.5 eV (gold), the transition is at 668 nm for n = 3 (20 fs) to n = 2 (8 fs pulse)

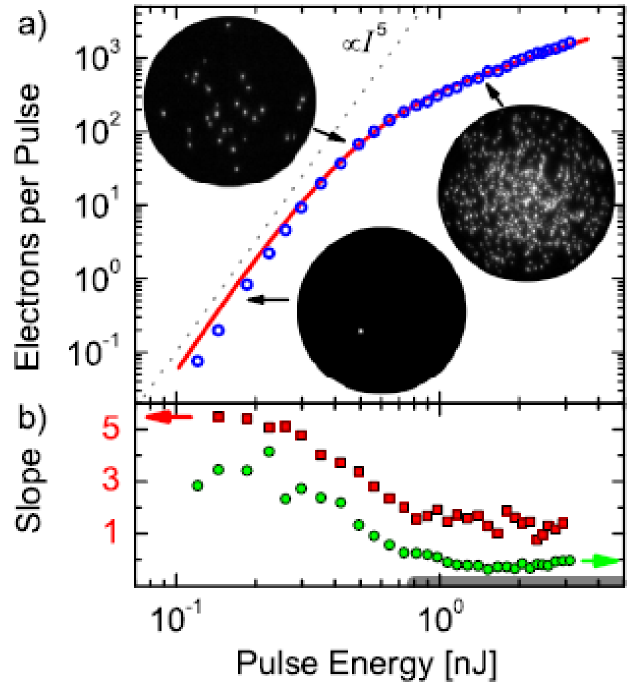
M. Pant, and L. K. Ang, Phys. Rev. B 86, 045423 (2012)

SUTD-MIT INTERNATIONAL DESIGN CENTRE (IDC)

Undertake the impossible, Design the unexpected

TDSE multi-energy + heating



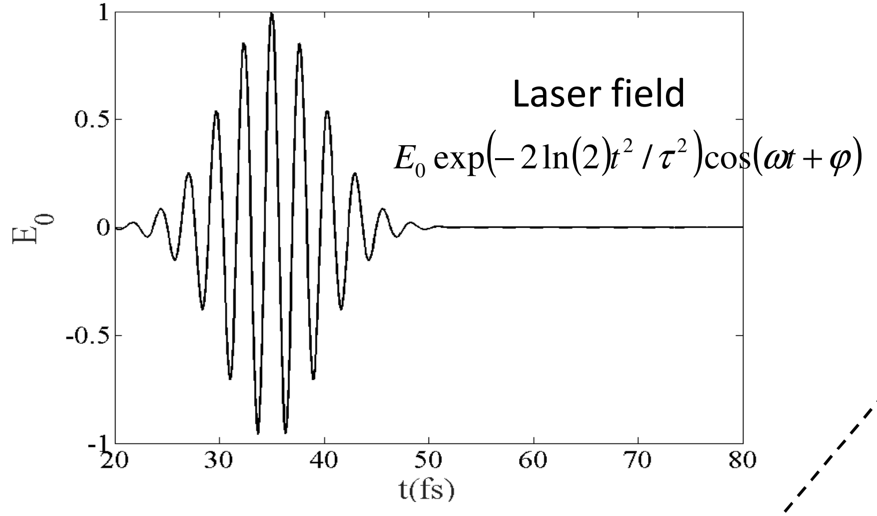


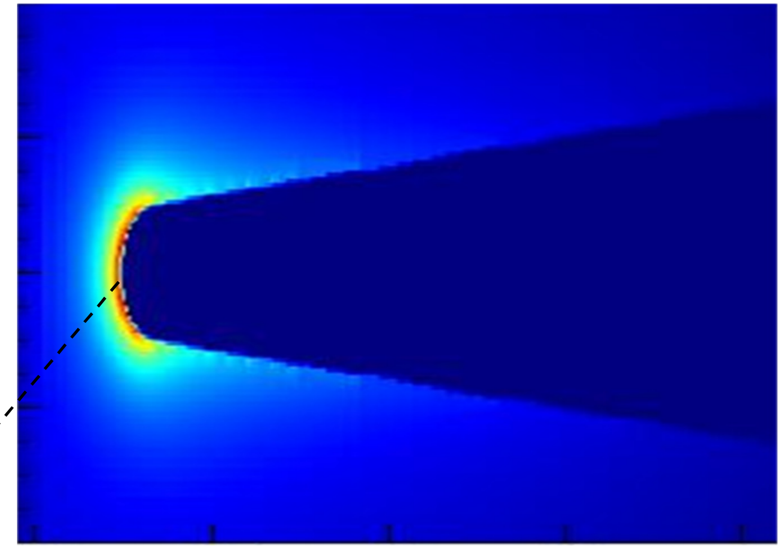
A consistent **multiple energy time-dependent quantum model with non-equilibrium heating** to show a smooth transition from multiphoton into the optical tunneling regime [RIGHT is Bosman, et al PRL 105, 147601 (2010)]

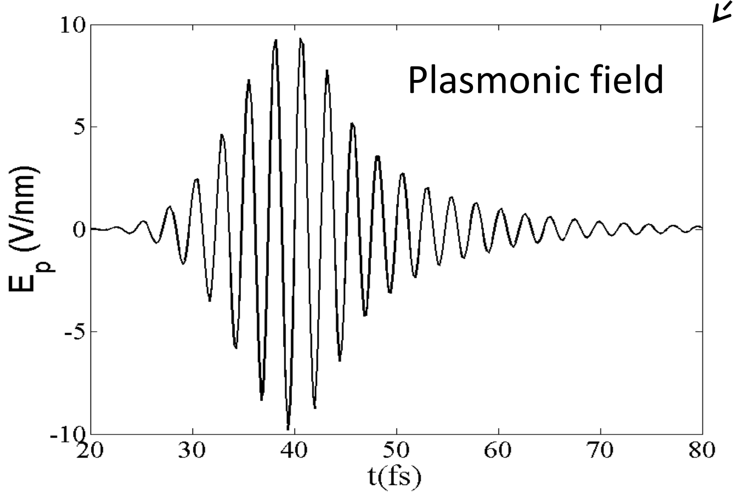
space charge is important (6 to 60 e at 10-100 nm^2) at high laser power (un-solved)

L. K. Ang, and M. Pant PHP 20, 056703 (2013) – APS-DPP invited talk

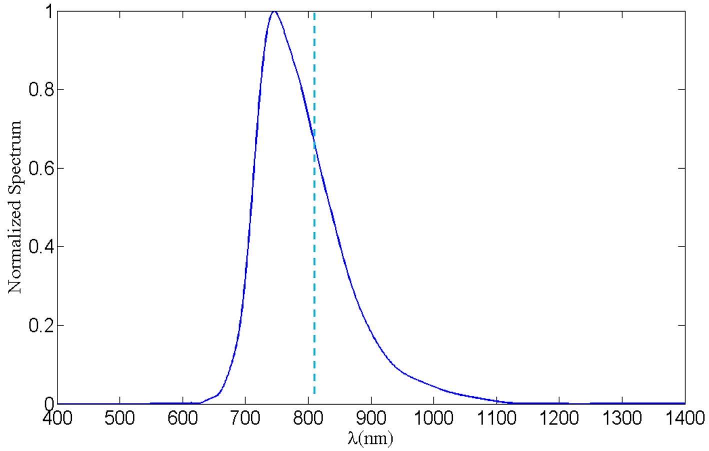
Plasmonic effect of Ultrafast laser of 8fs on metal tip using FDTD simulation







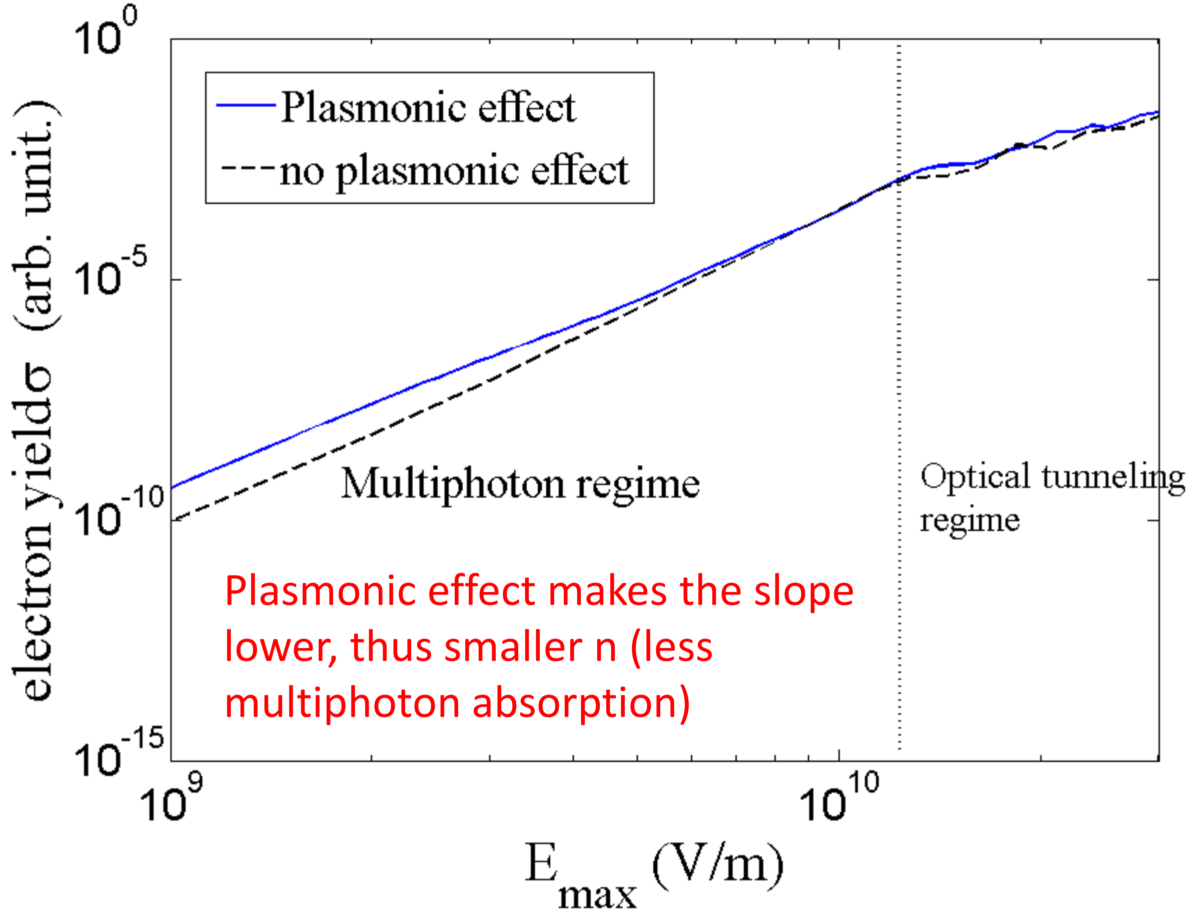
Peak is shifted from 800nm to lower λ



Time dependent plasmonic enhanced field

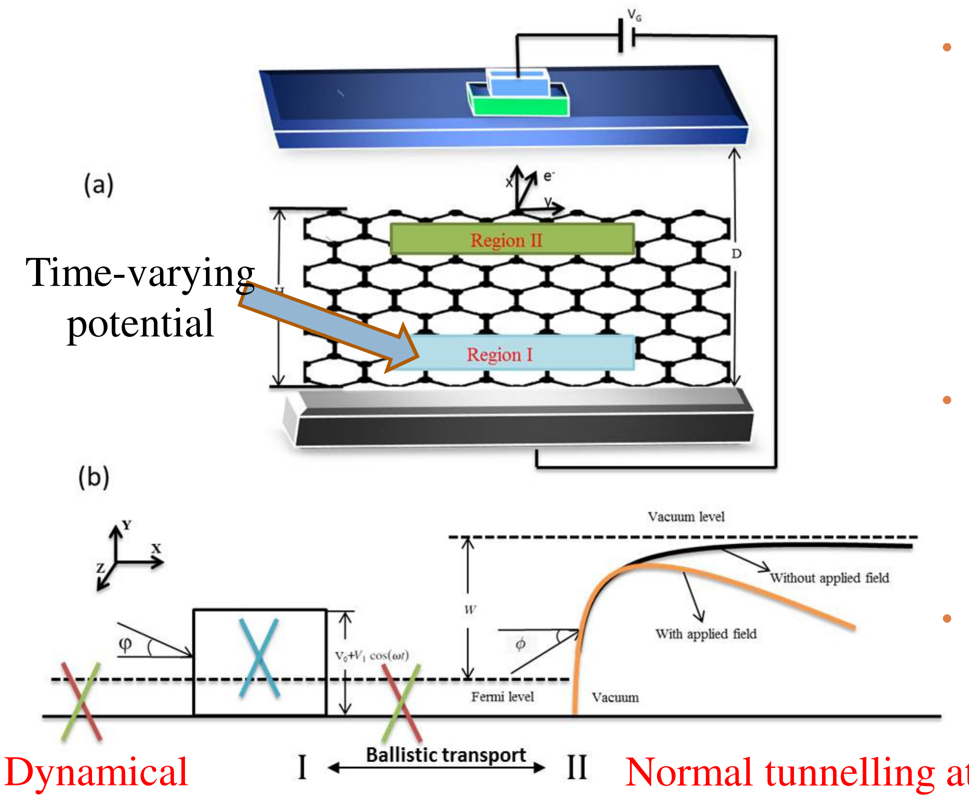
$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(x,t) + V(x,t)\psi(x,t)$$

$$V(x,t) = -\frac{e^2}{16\pi\varepsilon_0 x} - qx\left(F_{DC} + E_{laser}\right)$$



SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN
Established in collaboration with MIT

Laser assisted electron emission from graphene edge



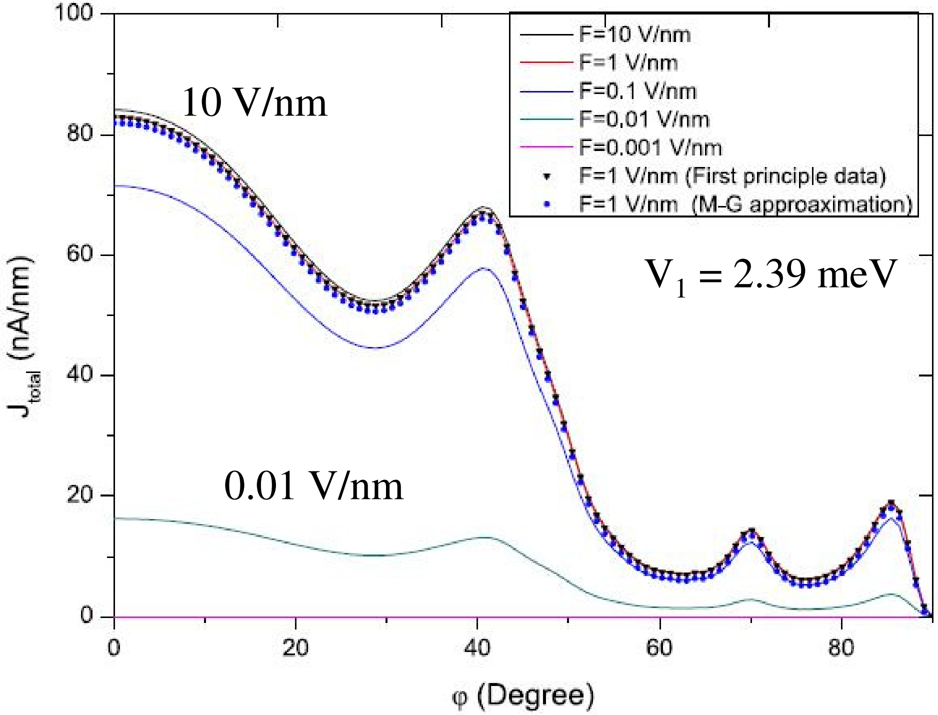
Dynamical Klein tunnelling in graphene

Normal tunnelling at edge

- In addition to DC voltage, we assume there is a small time-varying potential $V = V_0 + V_1 \cos(\omega t)$ in region I due to ultrafast laser excitation or AC modulation
- Determine the electron emission process from region I to region II, which are emitted from the edge
- 2 models: single side band and all side bands (6-bands is sufficient)

S. J. Liang and L. K. Ang, Carbon 61, 294-298, (2013) and IEEE TED 61, 1764 (2014) (Invited paper in IVEC)

High line current emission at reasonable F



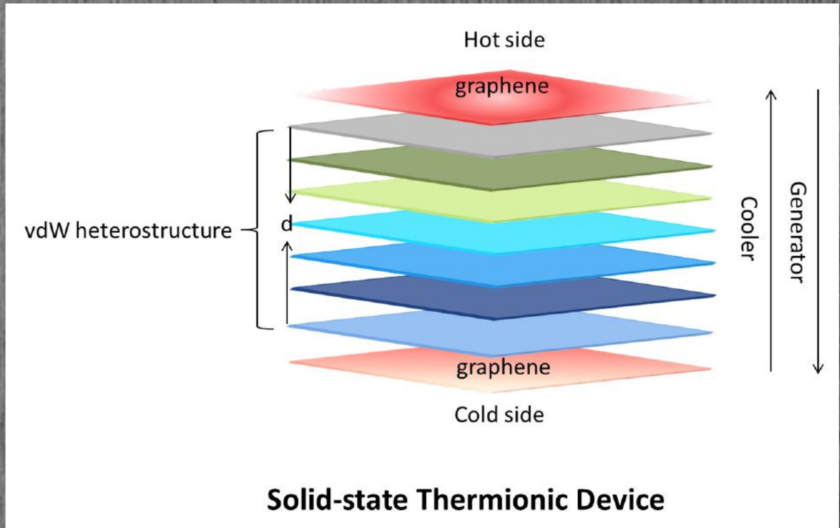
Dependence of local E-field at the edge, which has been enhanced by 100x from the applied field

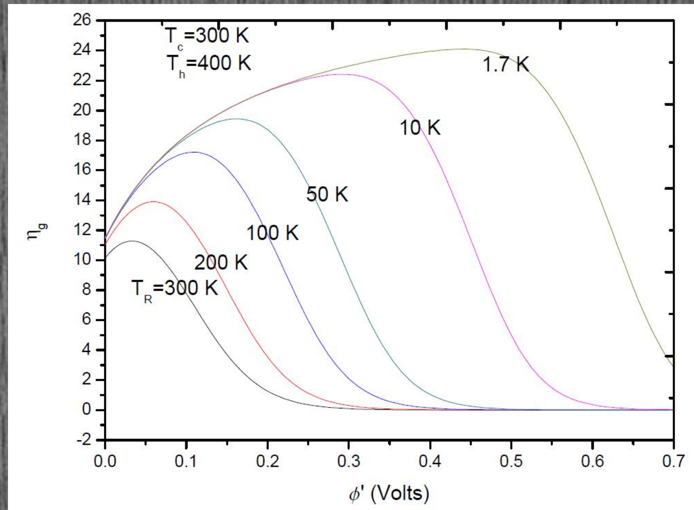
- At small F = 0.01 V/nm, it is able to have J = 15 nA/nm
- High current J up to 80 nA/nm, threshold is local F = 0.001 V/nm (1 V/μm), $J \simeq 0$
- Strong increases from F = 0.01 to 0.1 V/nm, and saturates towards 10 V/nm

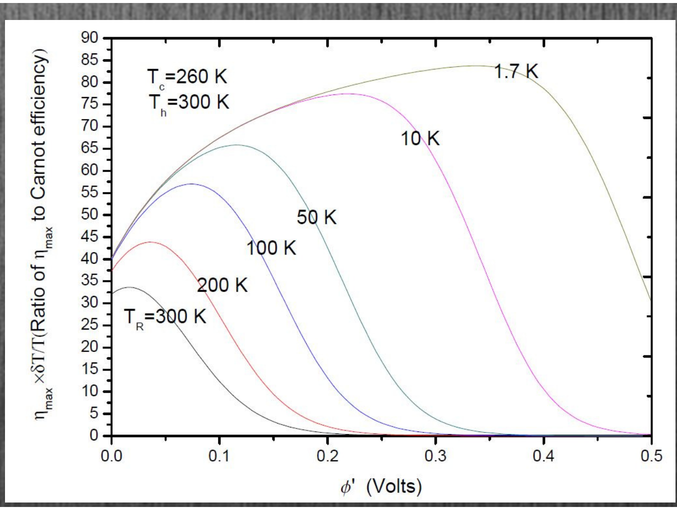
Summary

- We have shown that the traditional emission laws are no longer valid for 2D novel materials and/or ultrafast time scale
- Time dependent photo-emission model on 2D materials remains unsolved
- Using these models, we have explored various new applications by making use of the unique properties of 2D materials:
  - *Graphene-cathode thermionic energy harvesting at 900 K (PR Appl 3, 014002, 2015): 45%*
  - *Graphene-semiconductor pn junction – contact is the most important for 2D electronics*
  - *Solid-state graphene-TMDC thermionic energy harvesting at 400K (Scientific report 7:43664, 2017): 8%*
  - *Robust graphene plasmonic electron waveguide, (Carbon 127, 187, 2018)*
  - *Ultrafast graphene electron switching (Semiconductor Science and Technology 33, 035014 (2018)*
  - *Reversible computing using black phosphorous (PRB 96, 245410, 2017) – highlighted by PRB*
  - *Characterization of mobility of 2D materials using SCLC model (PRB 95, 165409, 2017)*
- Other potential applications in plasma related areas:
  - *Graphene emitters to be used plasma propulsion*
  - *Sheet-beam electron gun for THz sources using graphene cathode*
  - *Graphene electrode in plasma processing – easier to discharge ??*
  - *Fractional modelling – a good technique to model rough cathode, interface. electrical contact*

Thermionic energy convertor using vdW heterostructure as generator and cooling







Power generation at 400K (hot) to 300K (cold) can be > 10% which is better than the best theoretical TE-based of 9.6% by assuming ZT=4 material is possible

Cooling at 300 K (hot) to 260 K (cold)
About 40% to 80% of Carnot efficiency

$$\boxed{T_R[K] = 4666 \times (d/\kappa)^{-1/3}}$$

Low temperature (400K) thermal energy harvesting using vdW thermionic devices

System	Thickness d [nm]	κ for [W/m/K]	Experiment SBH [V]	Calculated T_R [K]	Optimal-SBH [V]	Maximum efficiency [%]
G/MoS_2	50	0.3	0 to 0.11	847	0	3.15
G/WS_2	50	0.2125	0 to 0.37	758	0	3.93
$G/MoSe_2$	70	0.089	0 to 0.4	501	0.005	7.28
G/WSe_2	62	0.048	0 to 0.44	428	0.02	8.56

$$\boxed{T_R[K] = 4666 \times (d/\kappa)^{-1/3}}$$

Shi-Jun Liang, Bo Liu, Wei Hu, Kun Zhou and L. K. Ang, *Thermionic Energy Conversion Based on Graphene van der Waals Heterostructures*, Sci Report 7:4621 (2017)

Thank you

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