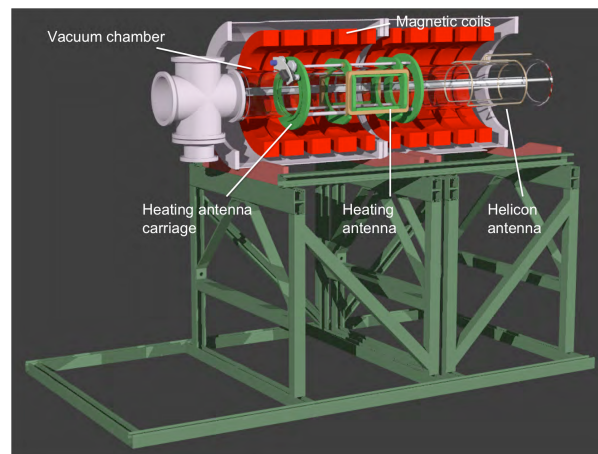


Ion acceleration by beating electrostatic waves



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Princeton University

Seminar at the Michigan Institute for Plasma Science and Engineering
The University of Michigan
November 17, 2010

Acknowledgements

Ben Jorns (GS)
Slava Spektor (GS)

Avi Ziskind (UG, Physics)
Julia Ling (UG, Physics)
Buddy Gardineer (UG, MAE)

Bob Sorenson (Tech Staff)

Supported by AFOSR, DOE, NSF and NASA

Can nature inspire us a new plasma
propulsion concept?

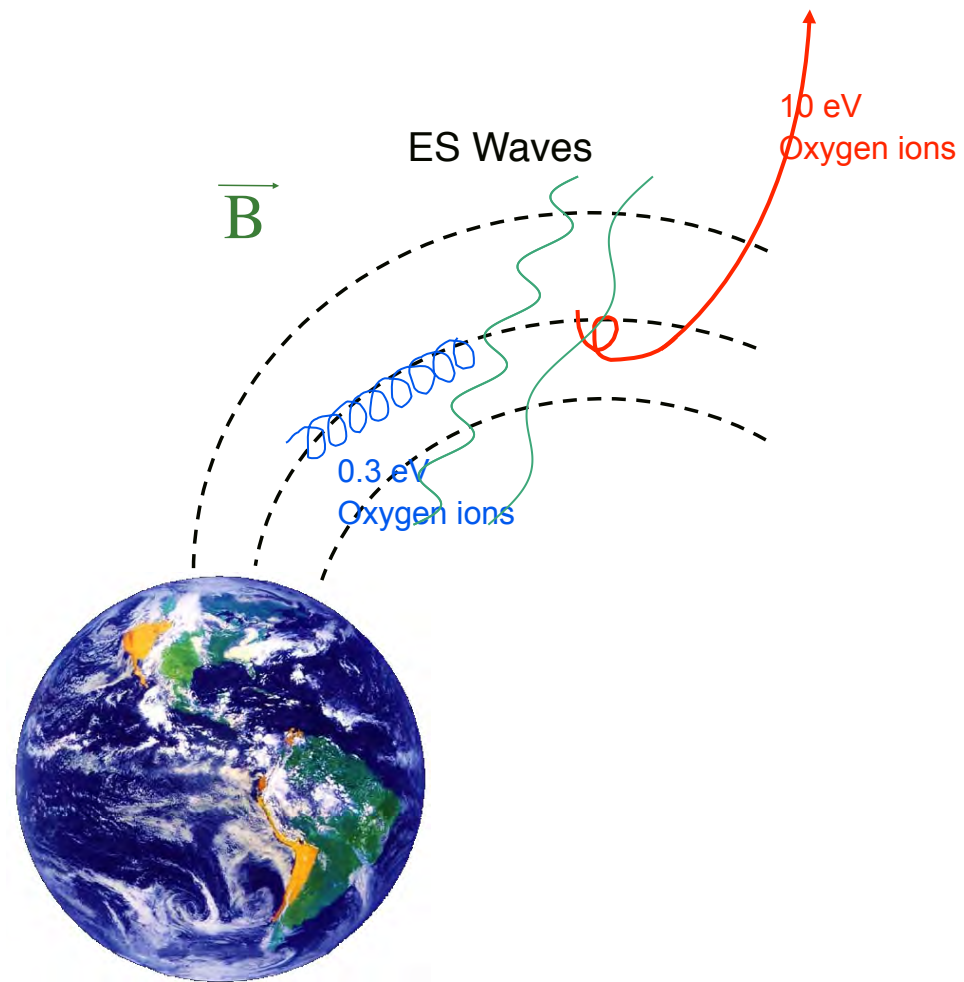
Why is it promising?

Under what conditions can it occur?

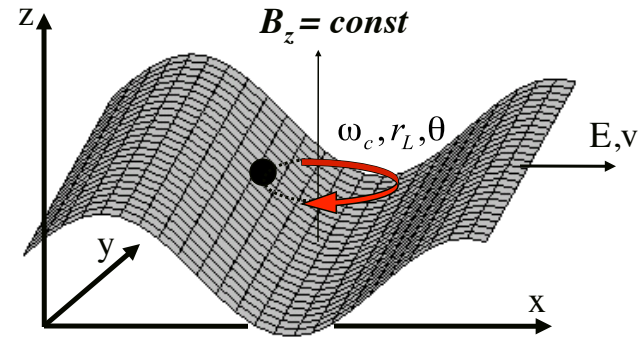
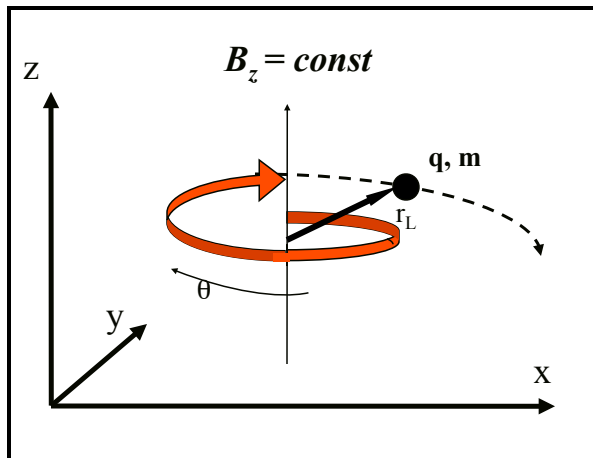
How does it work?

Can it be replicated in the lab?

Can nature inspire us a new plasma
propulsion concept?

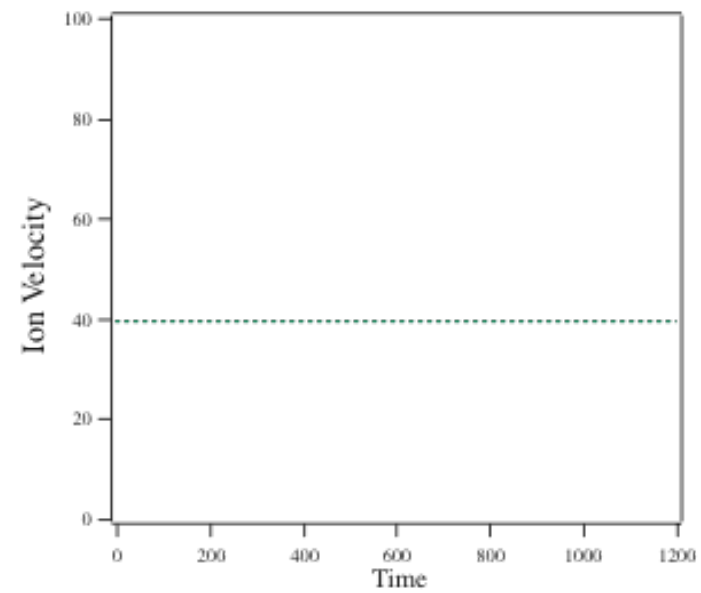
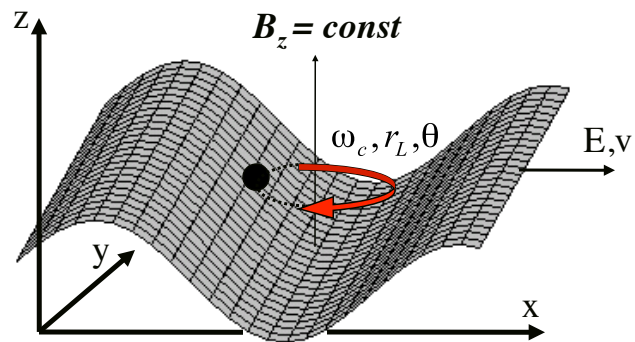


Ion Acceleration by ES Waves

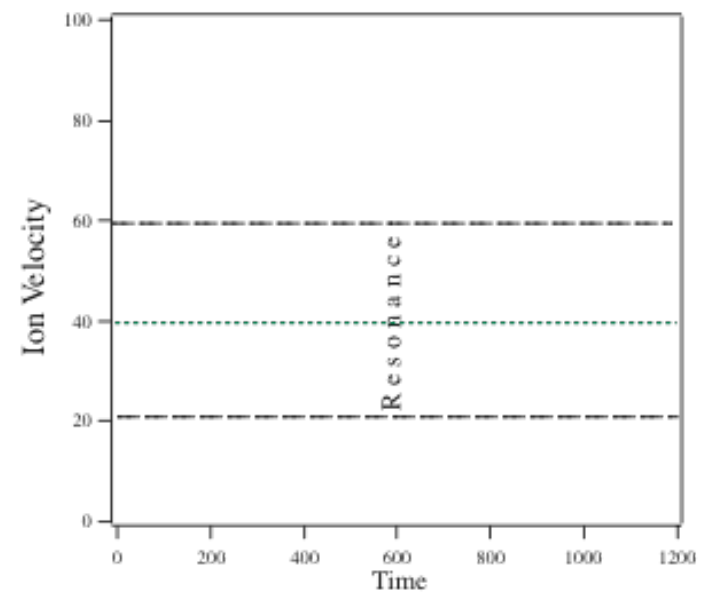
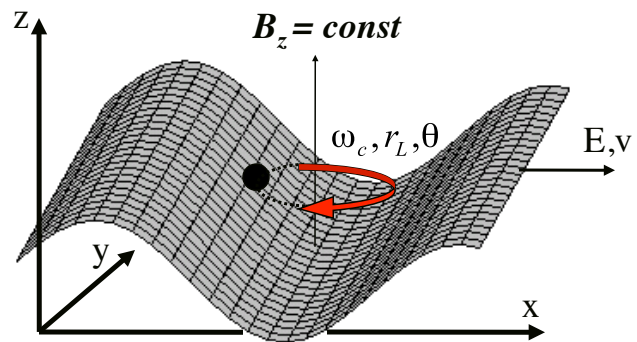


$$\ddot{x} + \omega_{ci}^2 x = \frac{q}{m} \sum_i E_i \cos(k_i x - \omega_i t)$$

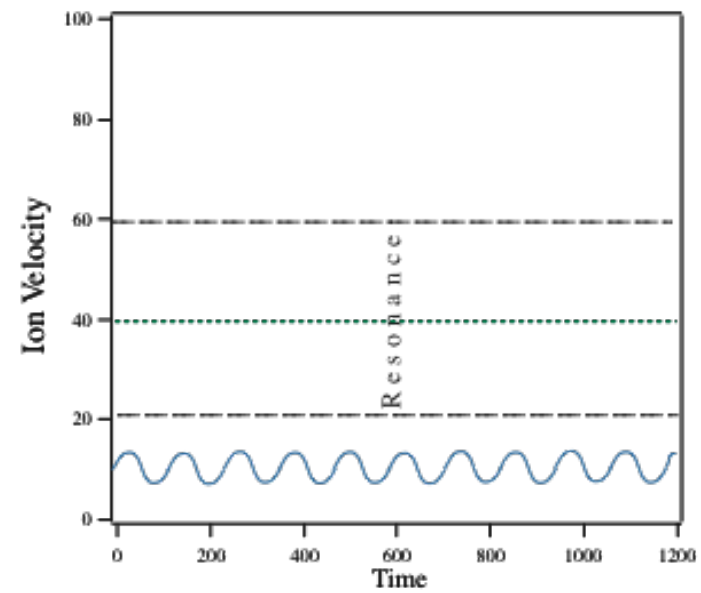
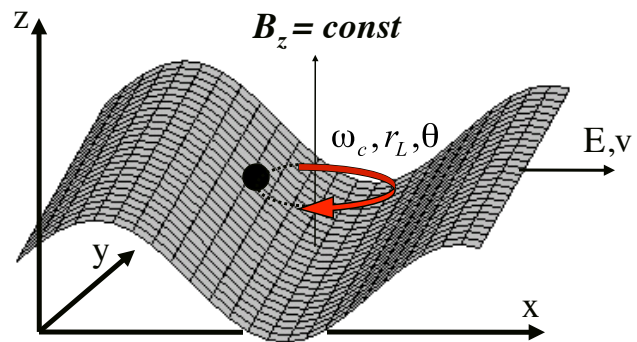
Ion Acceleration by ES Waves



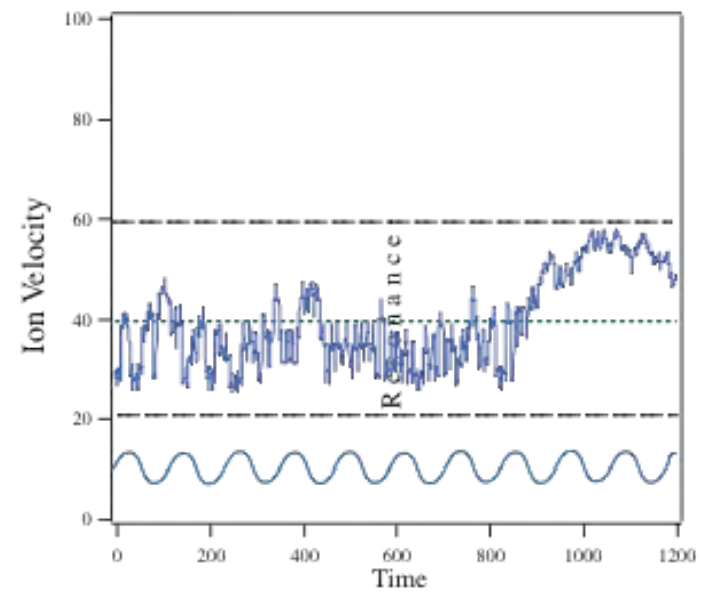
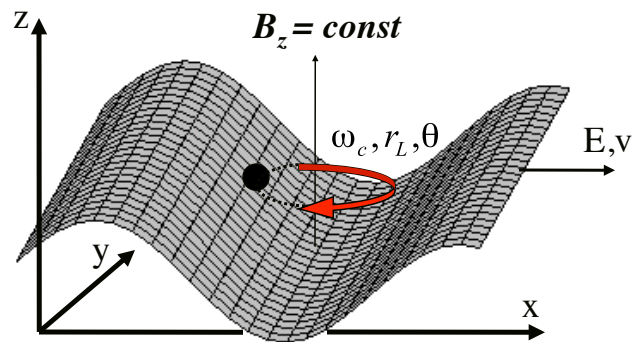
Ion Acceleration by ES Waves



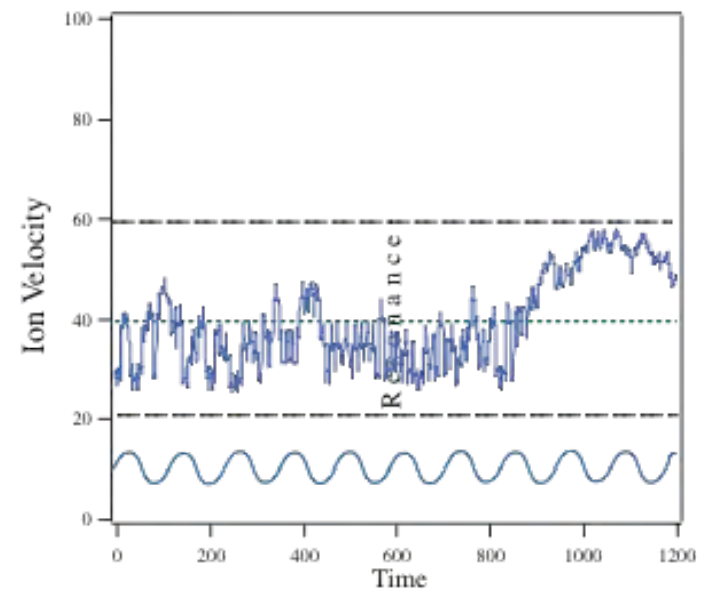
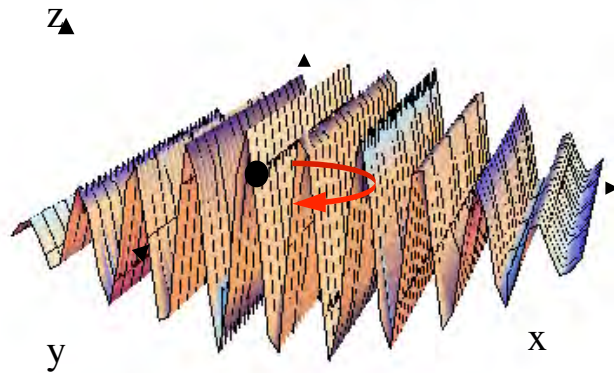
Ion Acceleration by ES Waves



Ion Acceleration by ES Waves

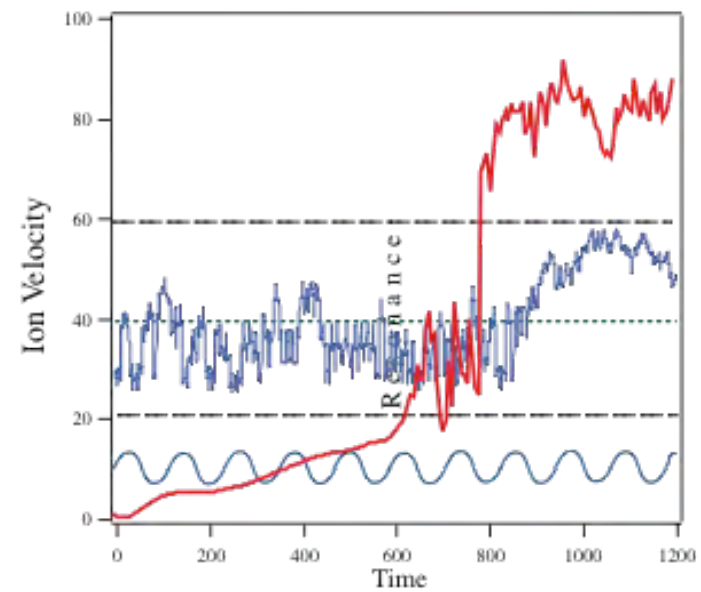
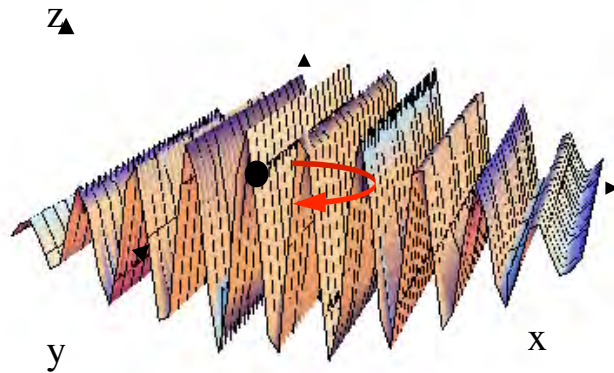


Two Beating waves: $\omega_1 - \omega_2 = n\omega_c$

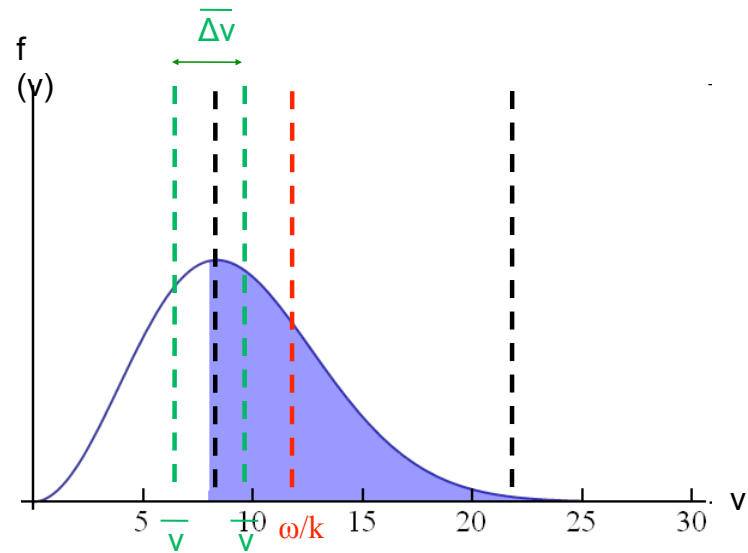


Why is it promising?

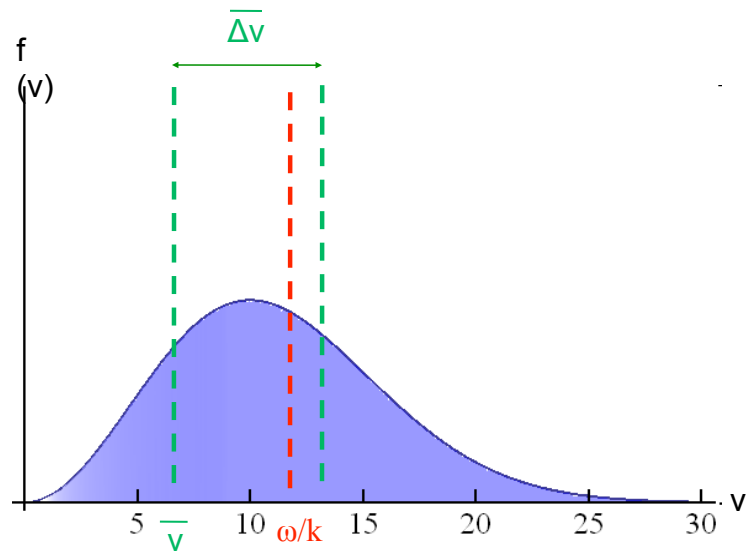
Two Beating waves: $\omega_1 - \omega_2 = n\omega_c$



SEW

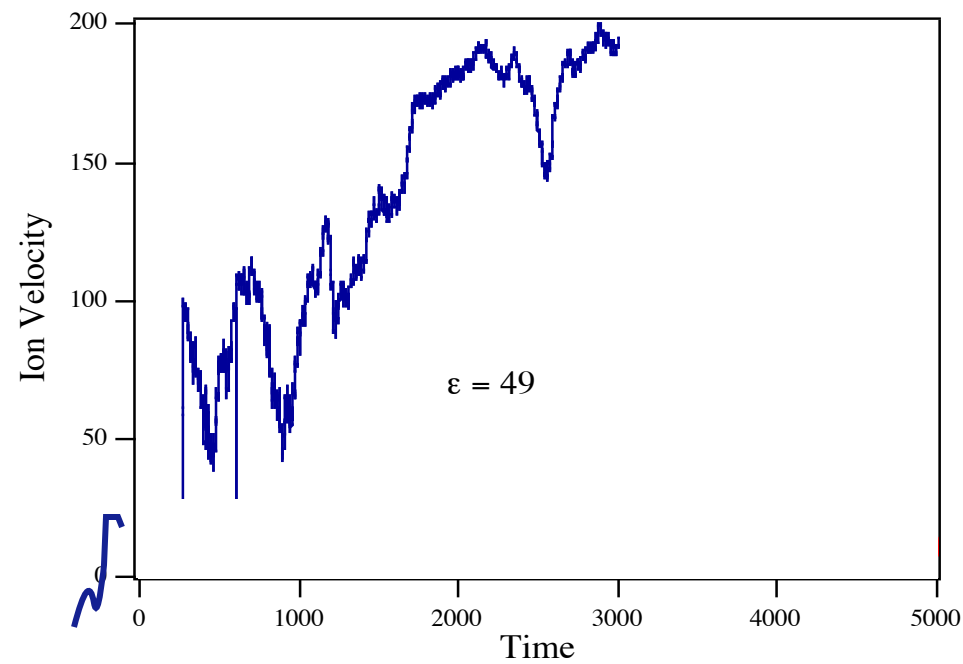


BEW



Under what conditions can it occur?

Does it always occur?



No

Are there Necessary and Sufficient Criteria for Acceleration?

yes

Spektor & Choueiri, Physical Review E, March 2004

Approach



Hamiltonian

$$H = \frac{\rho^2}{2} + \sum_i \frac{\varepsilon_i}{\kappa_i} \cos(\kappa_i \rho \sin \theta - \nu_i \tau)$$

Numerical Solution:
Symplectic Integration
Algorithm

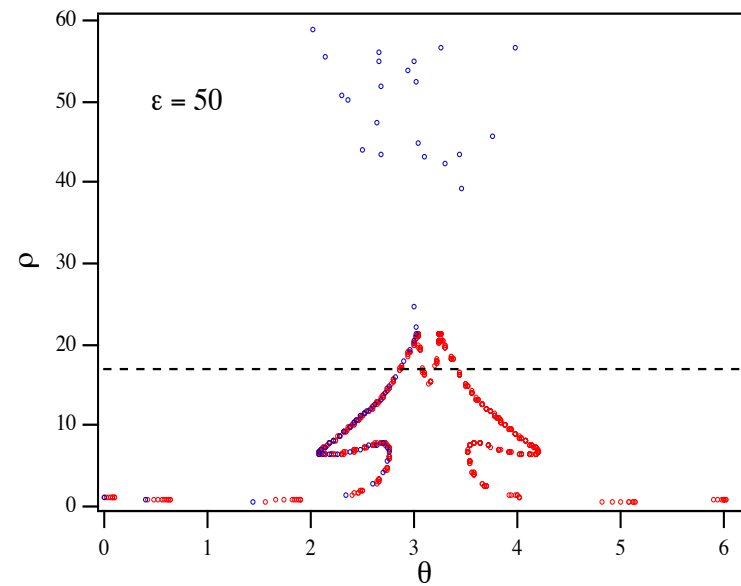
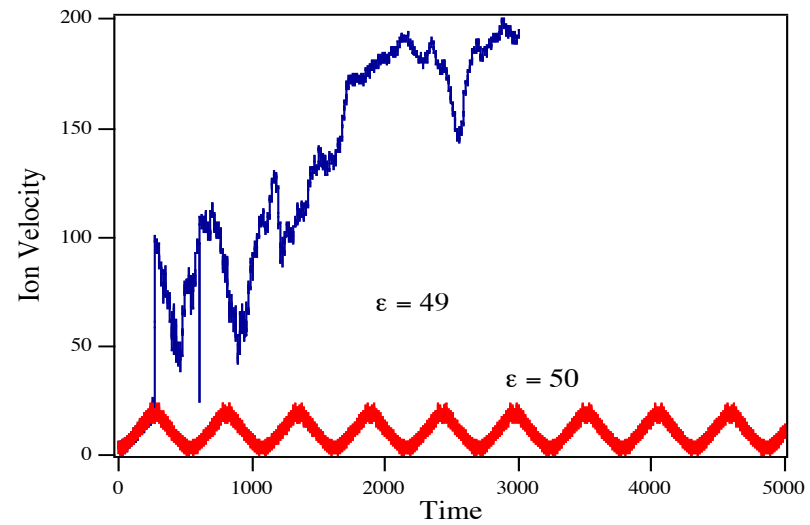
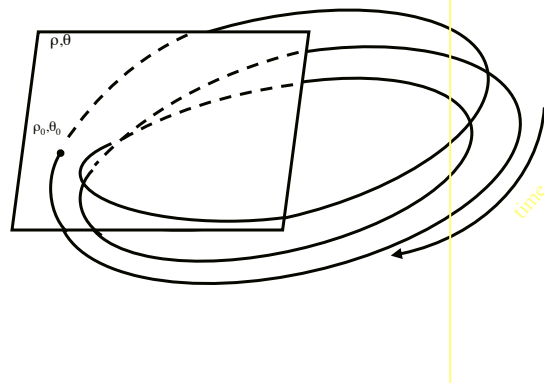
J. Comp. Phys. **92** 230
(1991)

Analytical Solution:
2nd order perturbation theory +
Lie transformations

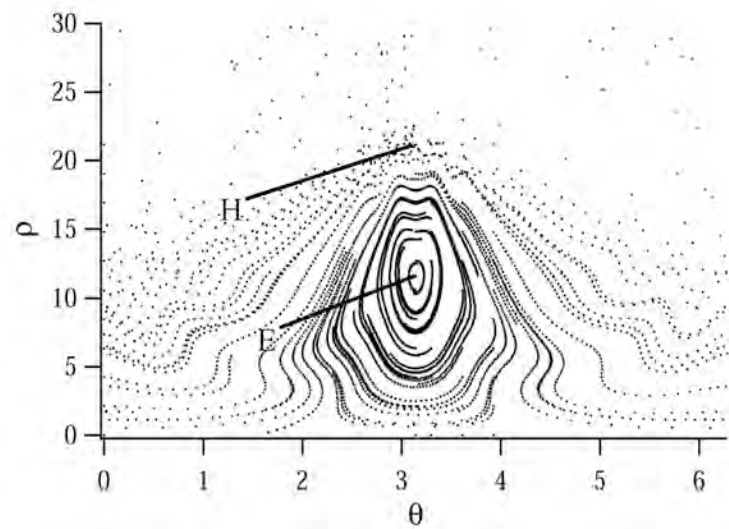
Phys. Plasmas **3**(5), May
1996, 1545

Poincare surface of section
(Reduced phase space
diagram)

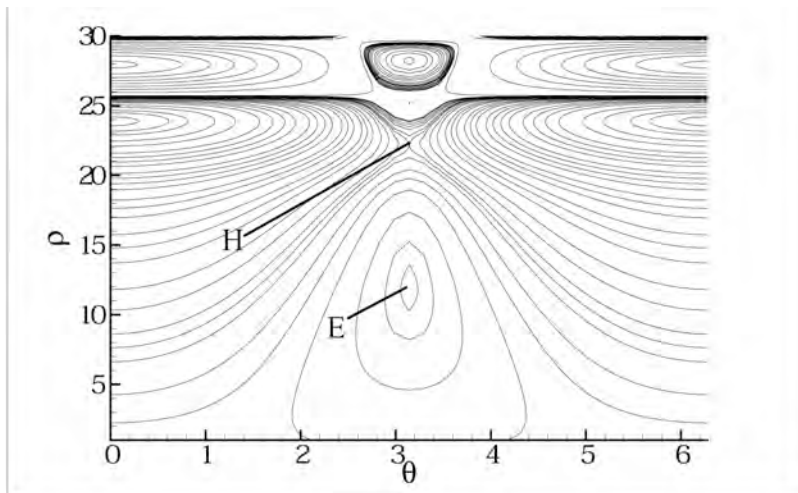
Poincare Cross Section

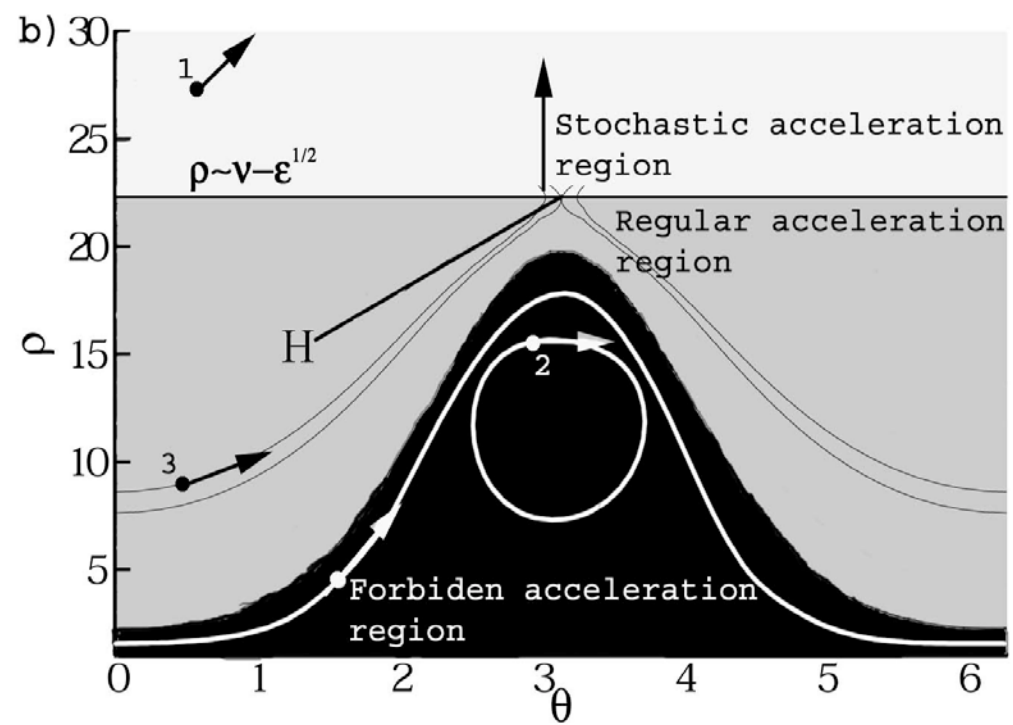


Numerical Simulation



2nd-order Perturbation Theory





Hamilton's Eqns

$$\begin{aligned}\dot{\rho} &= \frac{\partial H}{\partial \theta} = \varepsilon \{ \nu_i J_{\nu_i}(\rho) \sin(\nu_i \theta) \\ &+ \nu_j J_{\nu_j}(\rho) \sin(\nu_j \theta) \} \\ &+ \varepsilon^2 (\nu_i - \nu_j) S_6^{\nu_i, \nu_j}(\rho) \sin[(\nu_j - \nu_i) \theta] = 0, \\ \dot{\theta} &= -\frac{\partial H}{\partial \rho} = \varepsilon \{ J'_{\nu_i}(\rho) \cos(\nu_i \theta) \\ &+ J'_{\nu_j}(\rho) \cos(\nu_j \theta) \} \\ &+ \varepsilon^2 \{ S_1^{\nu_i}(\rho) + S_1^{\nu_j}(\rho) \\ &+ S_6^{\nu_i, \nu_j}(\rho) \cos[(\nu_i - \nu_j) \theta] \} = 0.\end{aligned}$$

$$\begin{aligned}H &= \frac{\varepsilon^2 \pi}{8 \sin \nu \pi} \left(1 + \frac{\rho}{\nu} \cos \theta \right) \\ &\left[- J_{\nu-1}(\rho) J_{-(\nu-1)}(\rho) + J_{\nu}(\rho) J_{-\nu}(\rho) \right. \\ &\quad \left. + J_{\nu+1}(\rho) J_{-(\nu+1)}(\rho) - J_{\nu+2}(\rho) J_{-(\nu+2)}(\rho) \right]\end{aligned}$$

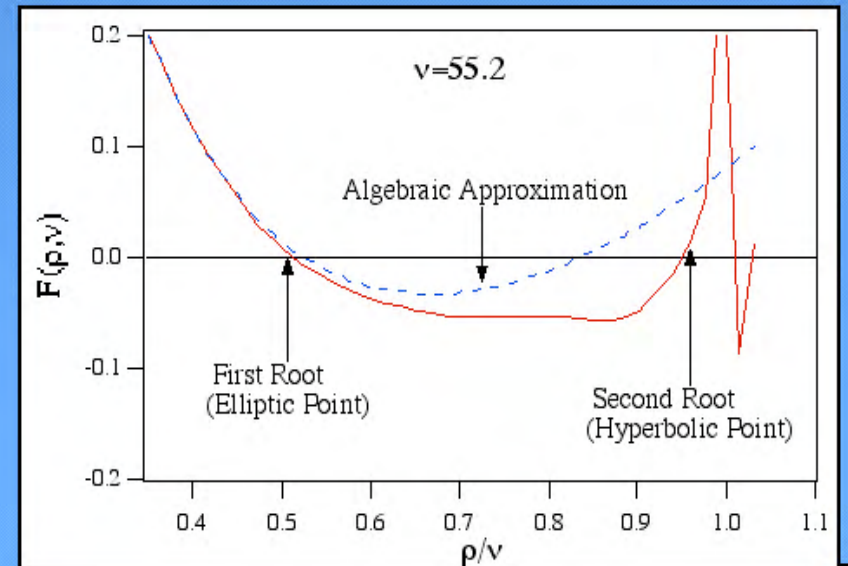
Hamiltonian from 2nd order perturbation theory + Lie Transforms

Spektor & Choueiri, Physical Review, E, 69(4):Article No. 046402, 2004.

Bessel functions properties

$$\begin{aligned}F(\rho, \nu) &\simeq \frac{\rho}{\nu} - 1 + \frac{2}{3} \left(f_{\nu-1} + f_{-(\nu-1)} + f_{\nu+2} + f_{-(\nu+2)} \right) = 0 \\ \text{where } f_{\nu} &= f_{\nu}(\rho) \equiv J'_{\nu}/J_{\nu}\end{aligned}$$

Simplified Eqn. for locating critical points



Necessary and Sufficient Criteria

Benisti's beat criterion:

$$\omega_2 - \omega_1 = n\omega_c$$

Additional criterion:

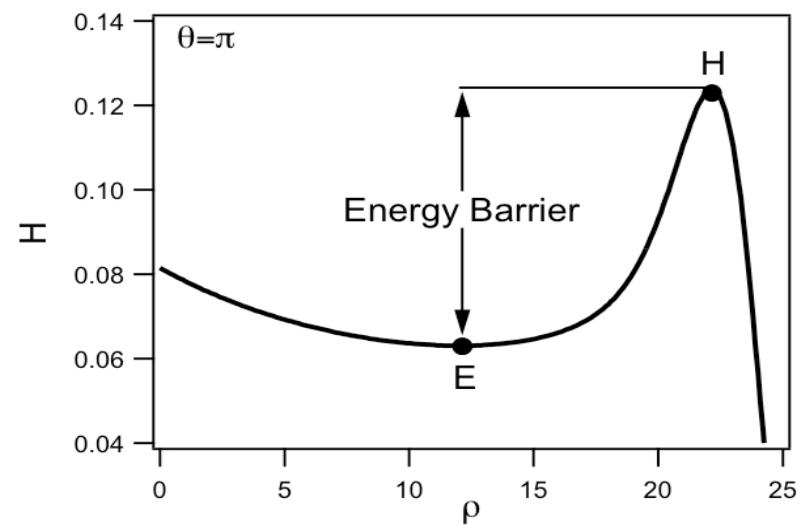
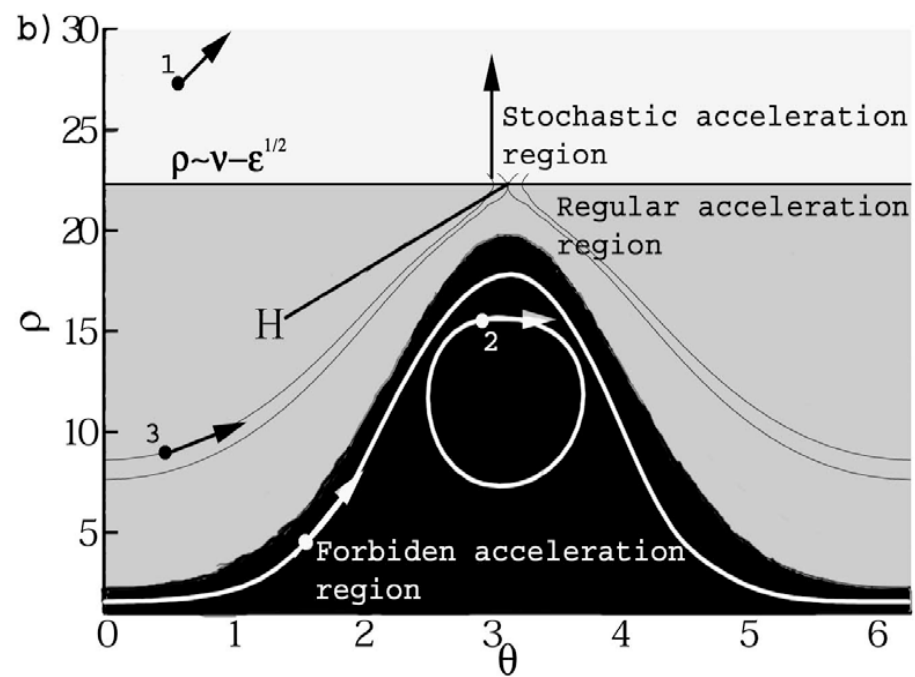
$$H_H < H(\rho_0, \theta_0) < H_E$$

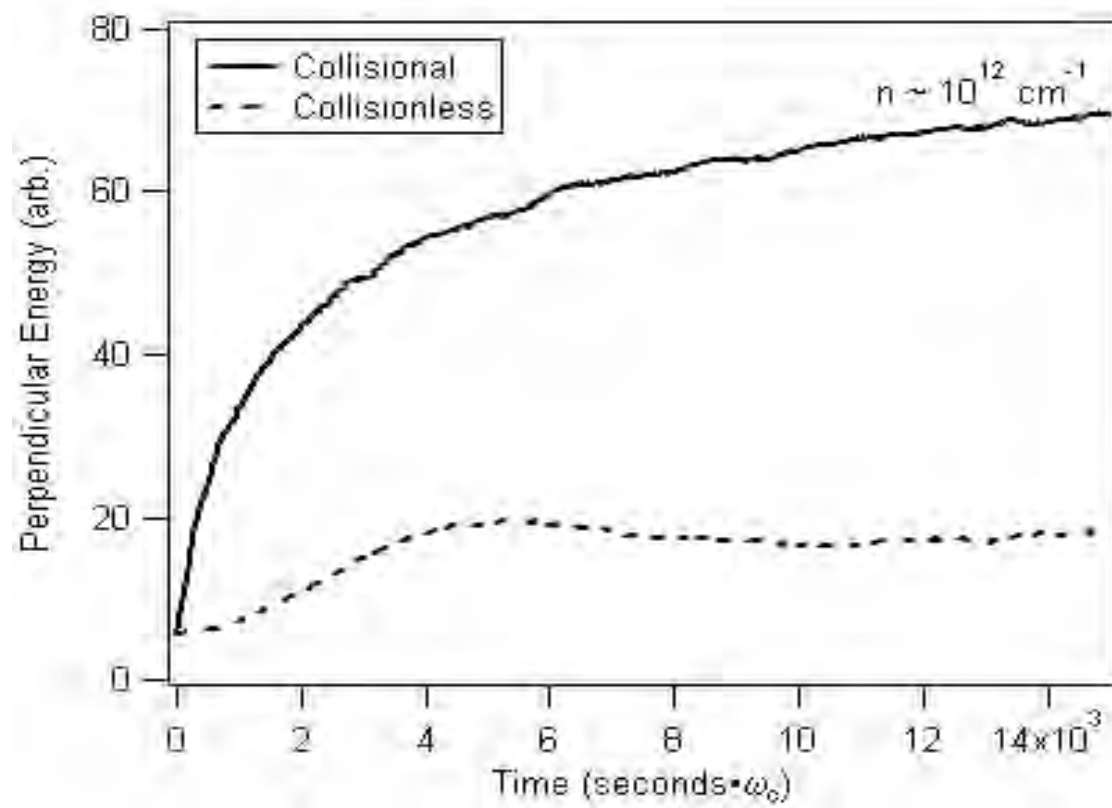

$$H_H = H(\rho \simeq \nu - \sqrt{\epsilon}; \theta = \pi).$$


$$H_H \simeq H(v \simeq \frac{\omega}{k}; \theta = \pi).$$

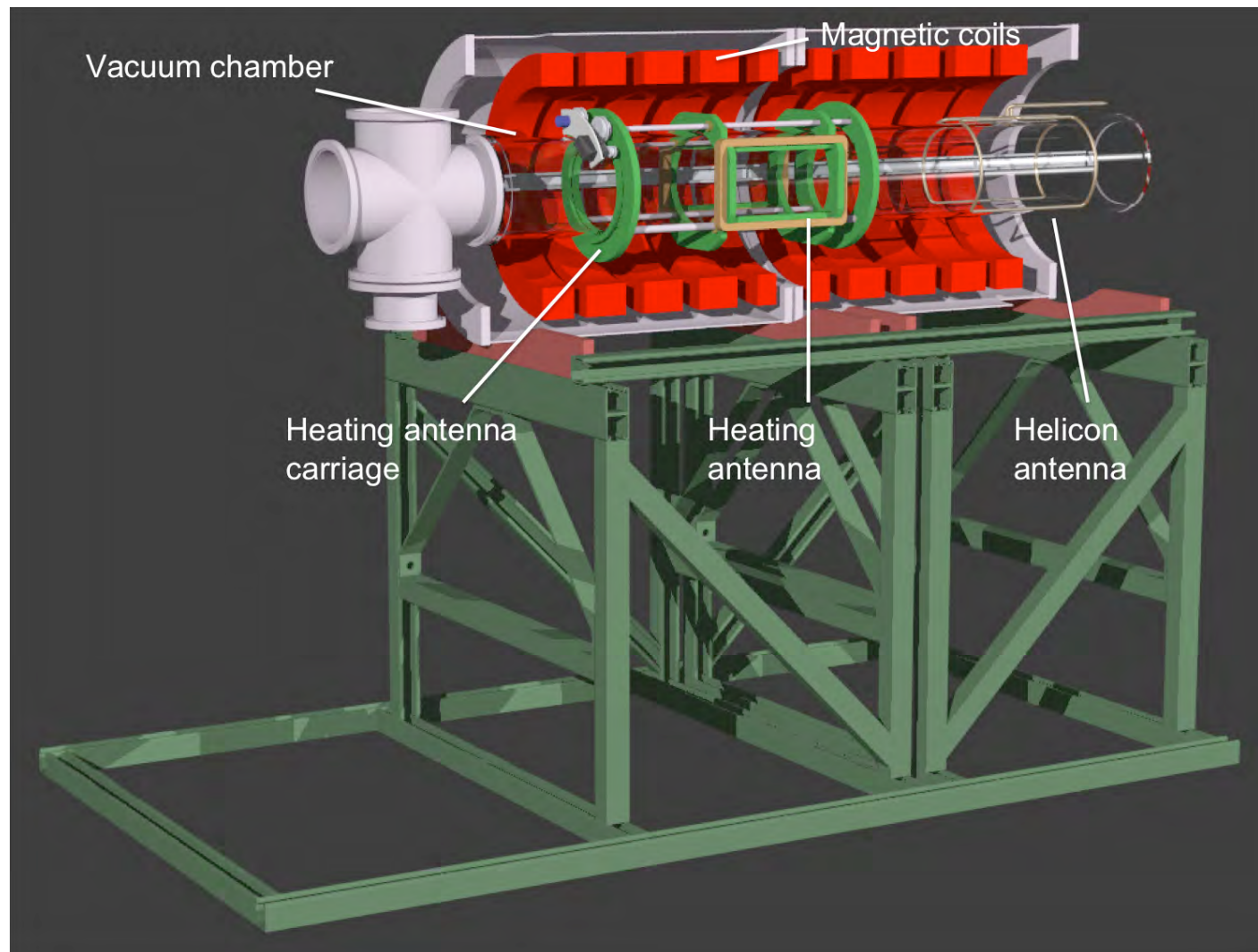
$$H_E = H(\rho \simeq \frac{\nu - \sqrt{\epsilon}}{2}; \theta = \pi).$$


$$H_E \simeq H(v \simeq \frac{\omega}{2k}; \theta = \pi).$$

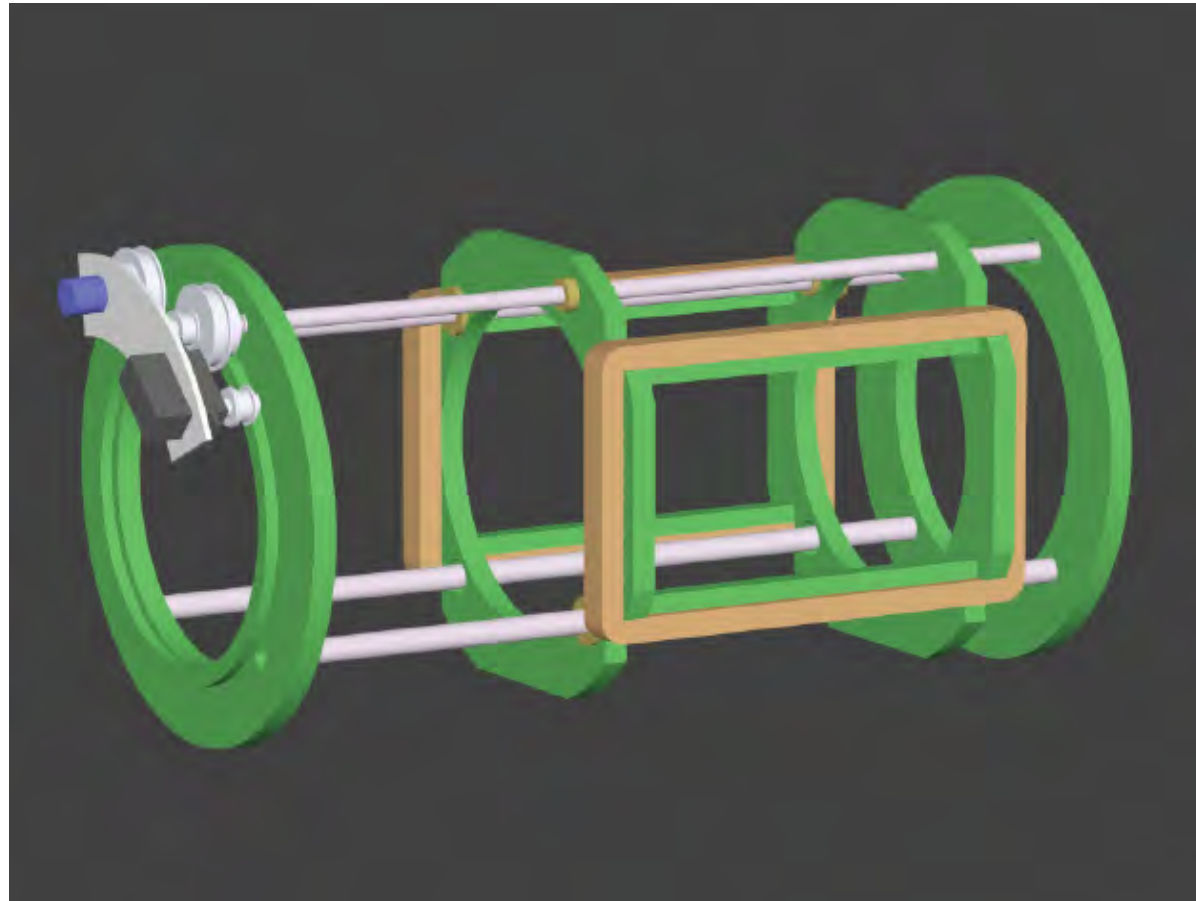


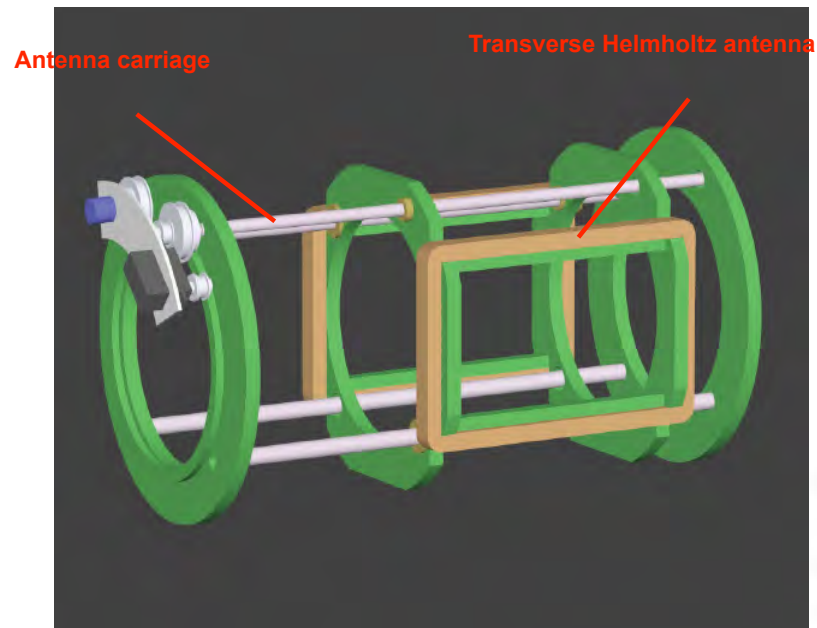


Can it be replicated/verified in the lab?



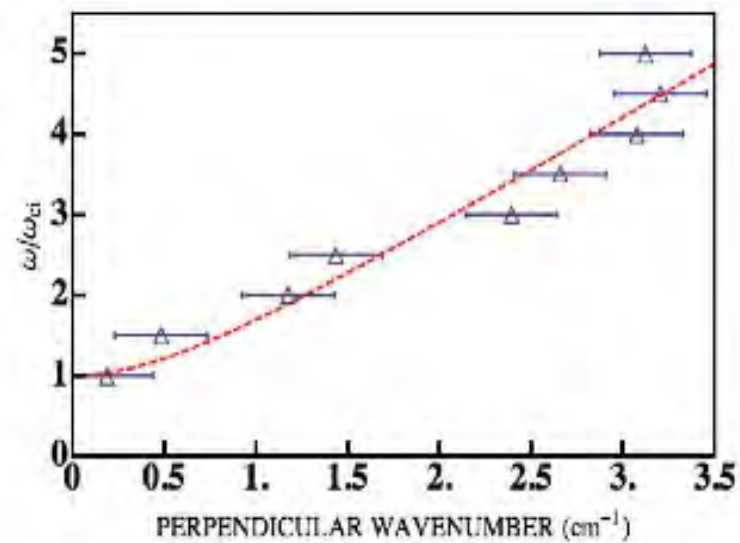
Transverse Helmholtz Antenna

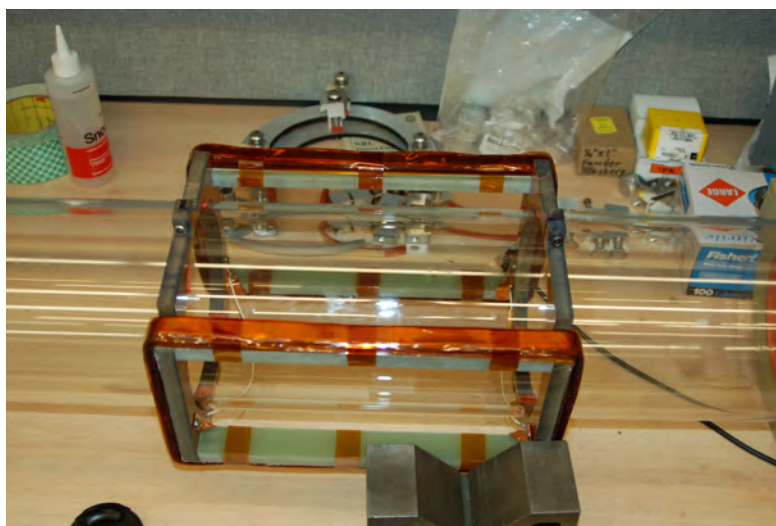




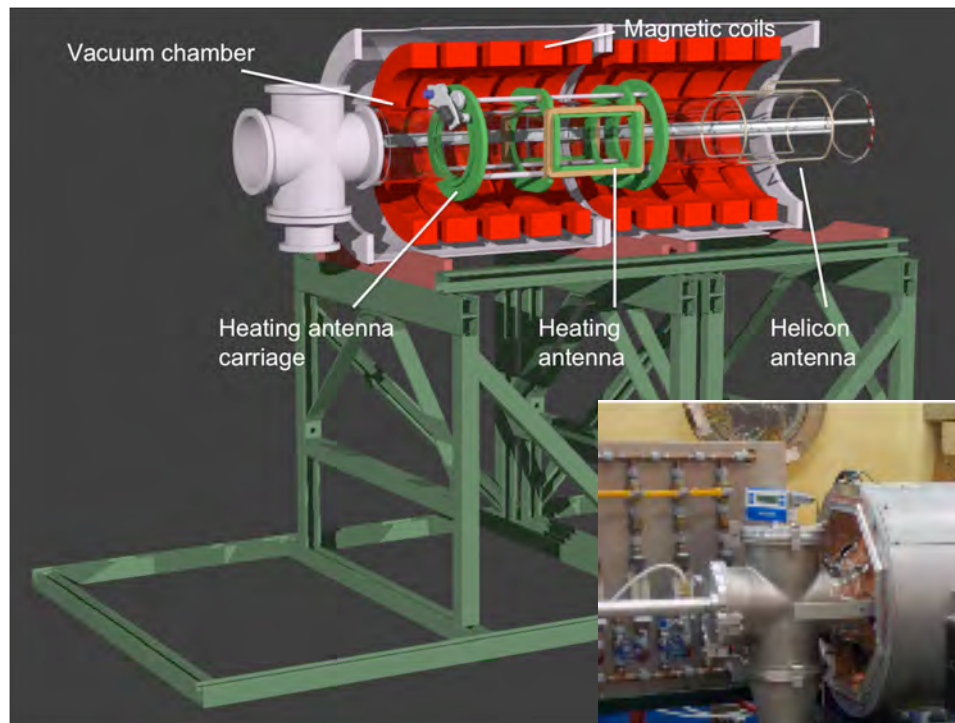
Configuration	Transverse Helmholtz*
Power Supply	100W ENI
Matching circuit	None
Frequency	$1.92f_{ci}$
Max RMS Field	15 G

$$\omega^2 = \omega_{ci}^2 + k^2 \frac{T_e}{m}$$



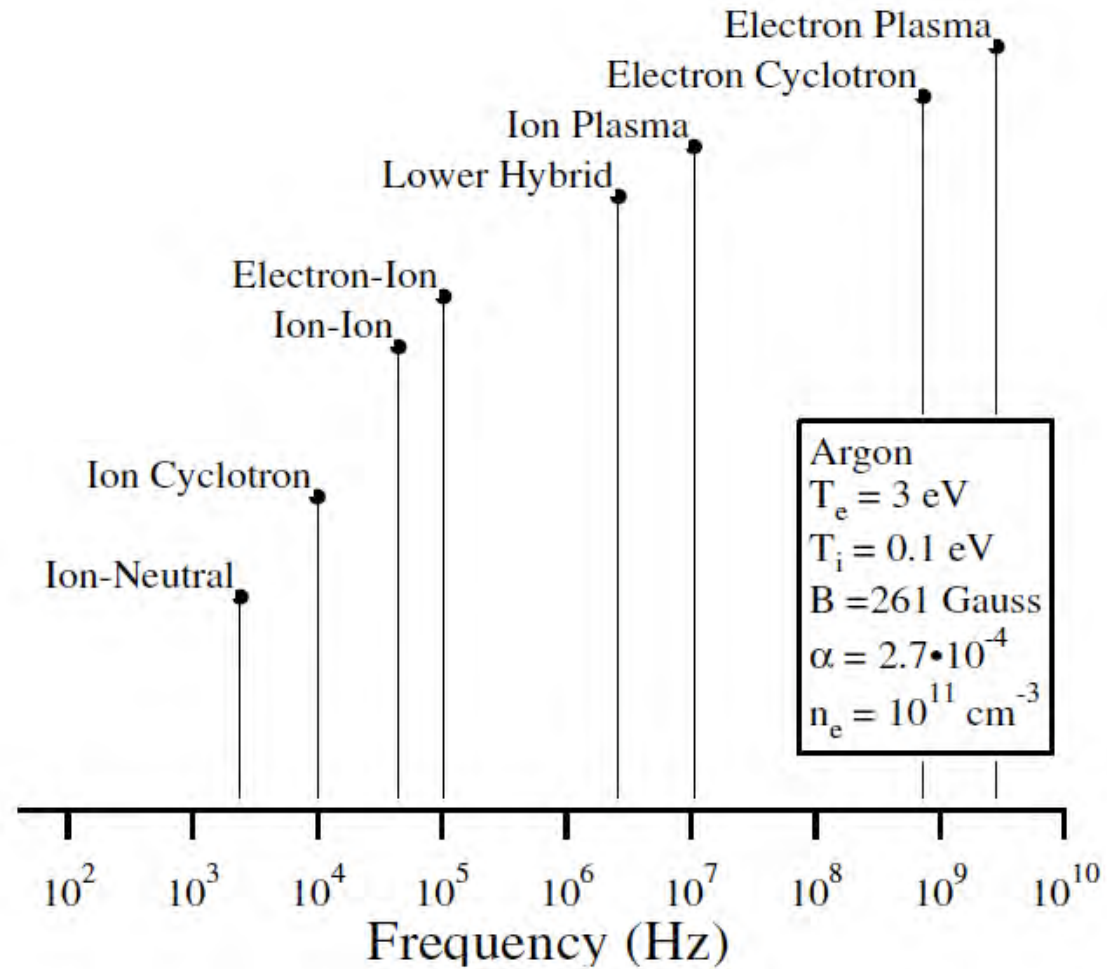




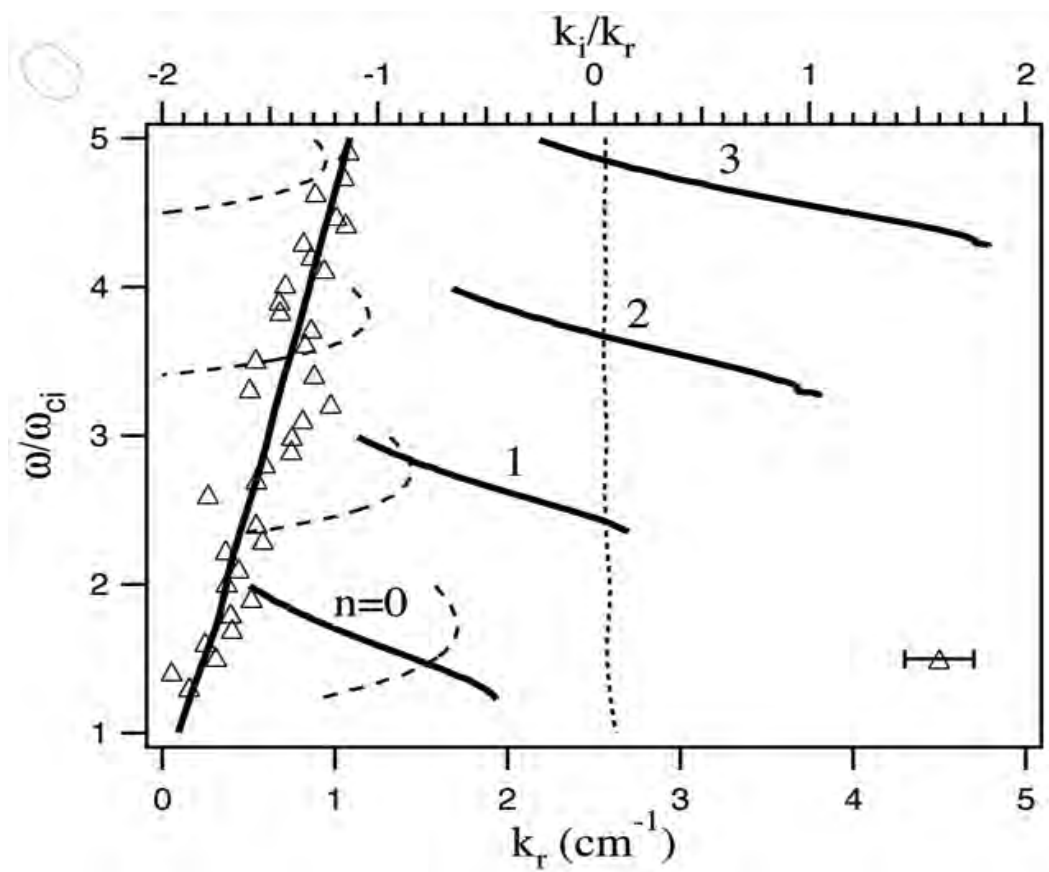


Parameters	
RF Power	250 W (13.56 MHz)
Mag. Field (B)	500 G
f_{ci}	19 kHz
P_B	~1 mT Argon
T_i	0.09 eV
P_A	30 W
n_e	10^{11} cm^{-3}

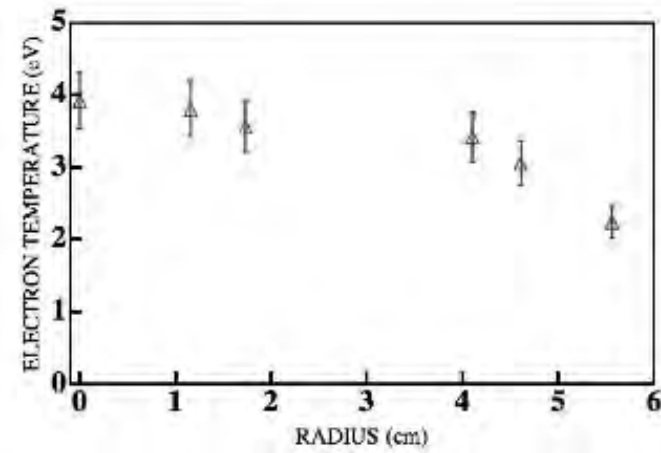
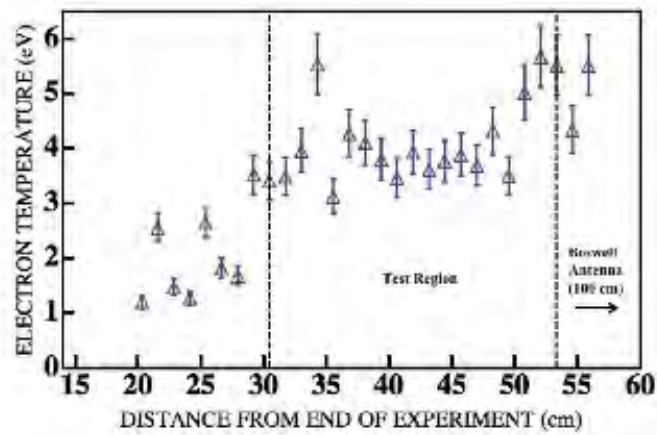
Plasma Characterization



Dispersion Relation



Plasma is uniform



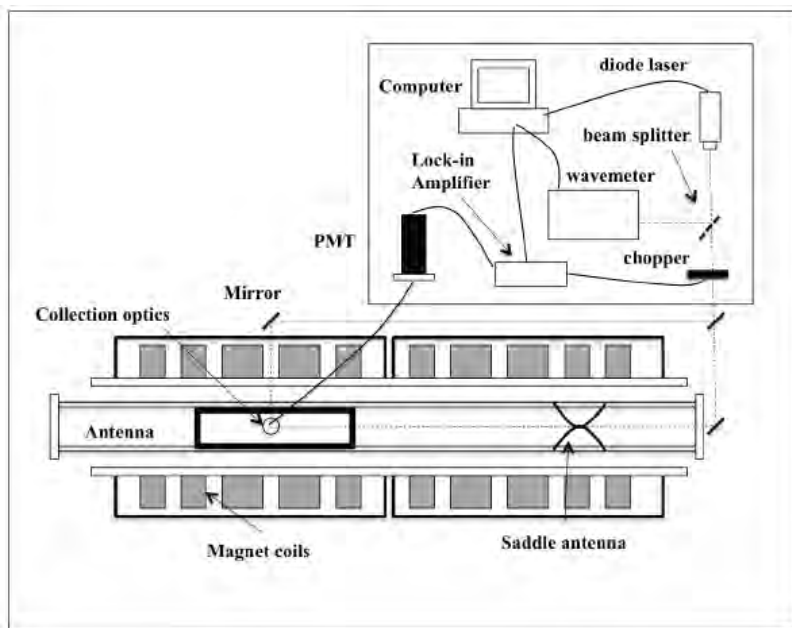
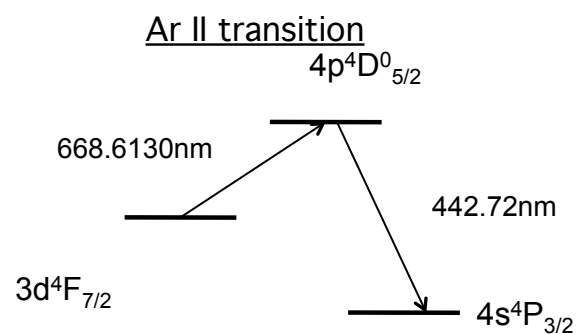
$$\frac{\nabla_z T_e}{T_e} r_i \ll 1$$

$$\frac{\nabla_z T_e}{T_e} \lambda \ll 1$$

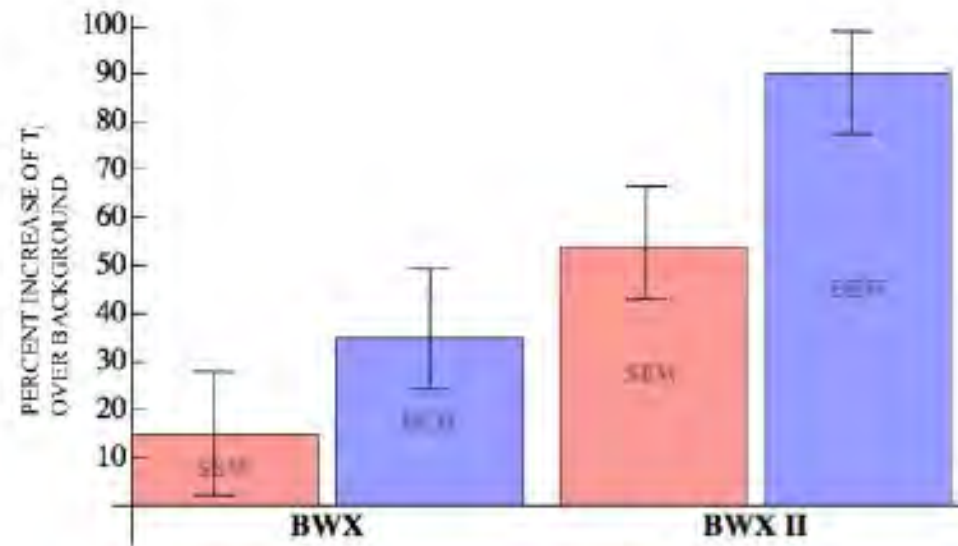
$$\frac{\nabla_r T_e}{T_e} r_i \ll 1$$

$$\frac{\nabla_r T_e}{T_e} \lambda \ll 1$$

Laser Induced Fluorescence System



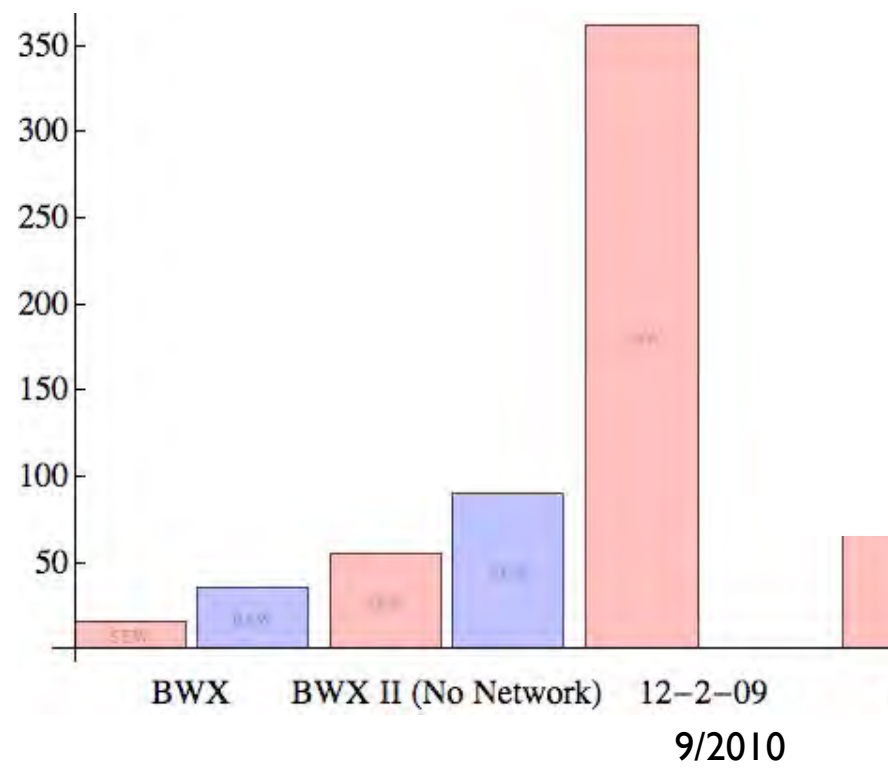
Ion Heating in BWX II

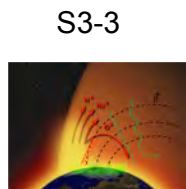


MAXIMUM HEATING	
SEW	55 ± 12%
BEW	90 ± 17%

BEW is unambiguously superior

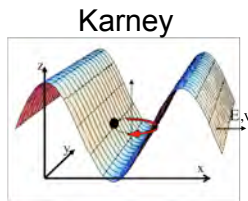
Possibility of improving result





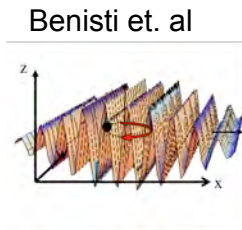
S3-3

1976



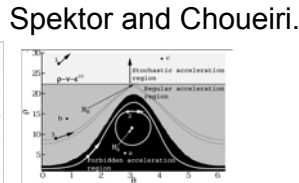
Karney

1978



Benisti et. al

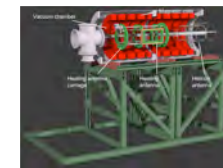
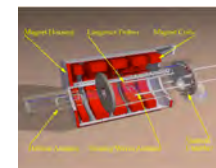
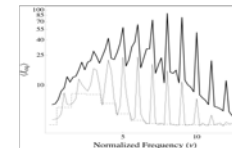
1998



Spektor and Choueiri.

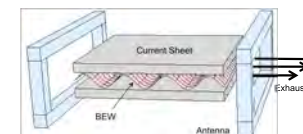
2004

BEW Heating



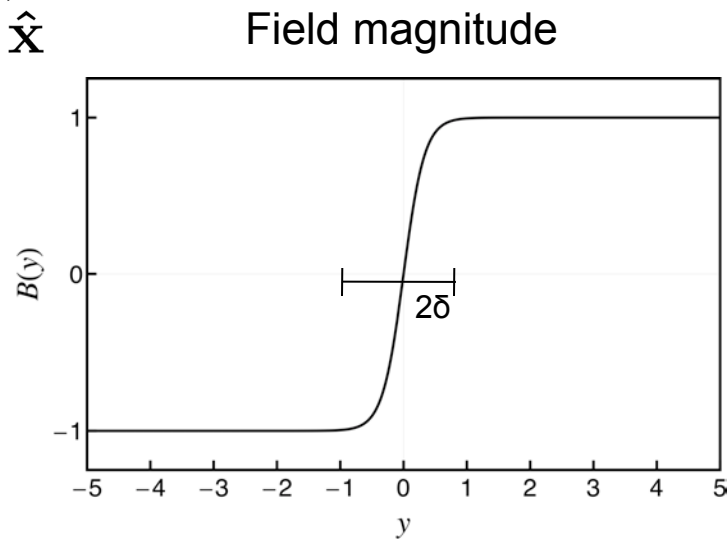
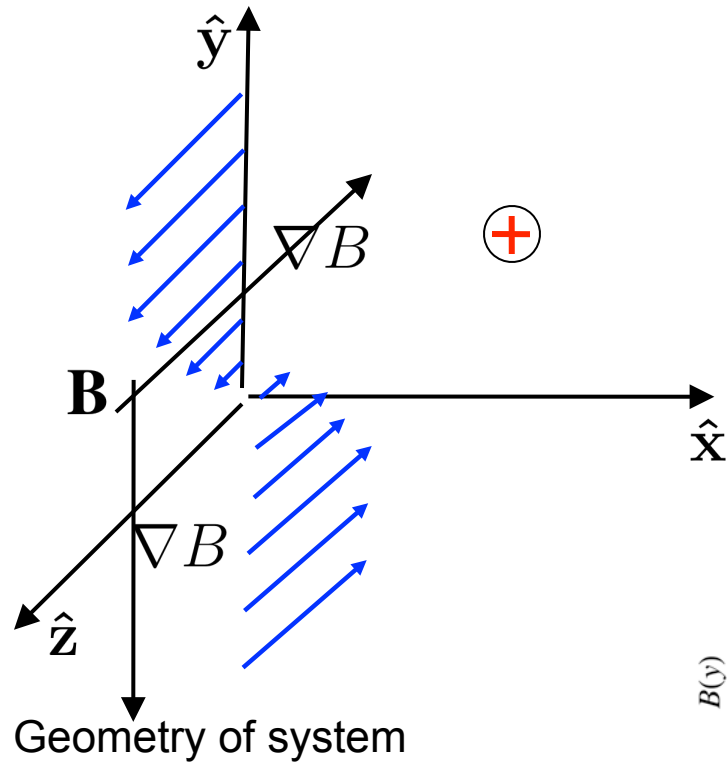
2006 2009 2010 >

2010 >

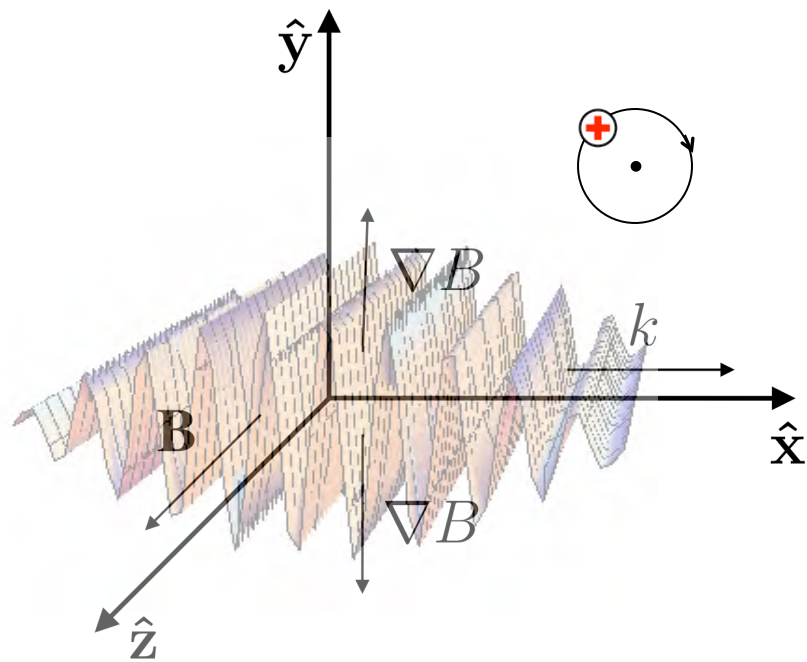


Directed ion acceleration

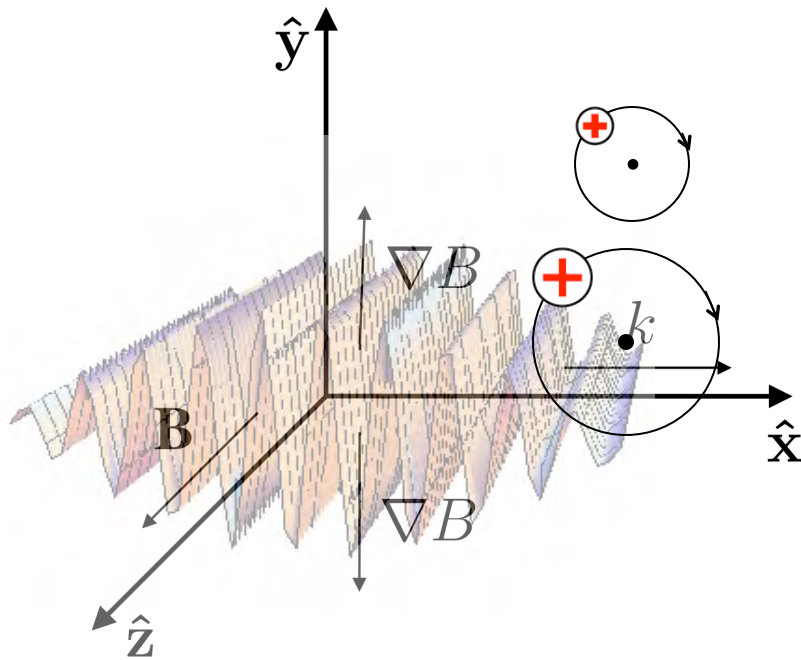
Geometry: magnetic slope



BEW in a magnetic slope

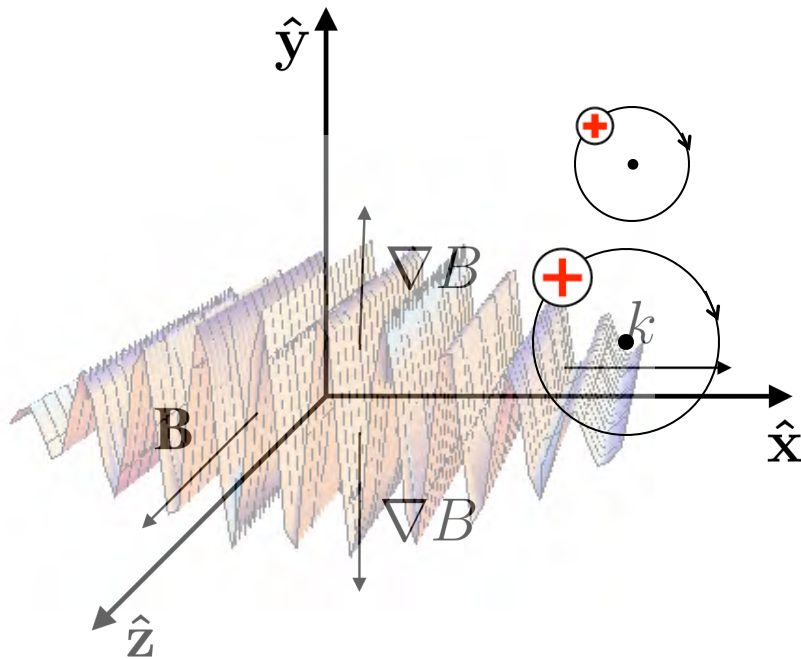


BEW in a magnetic slope



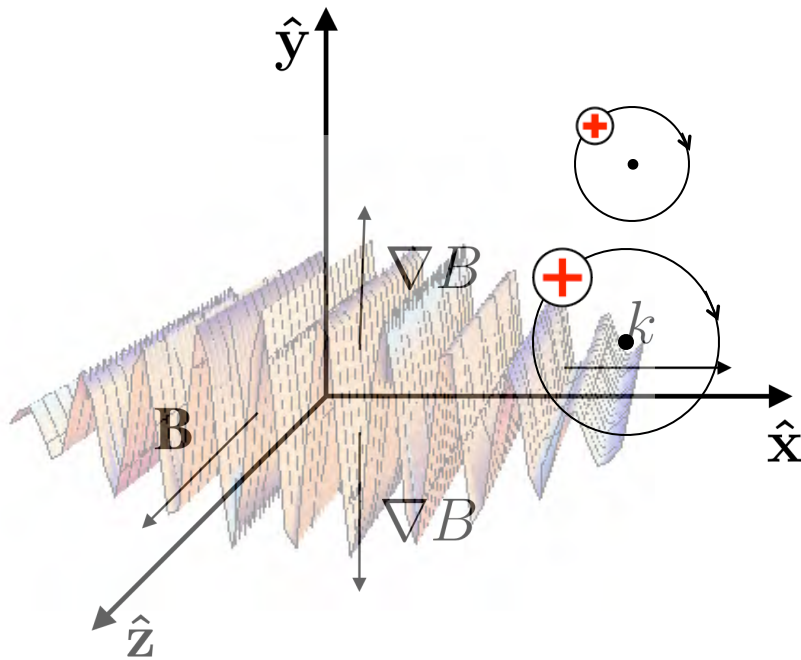
1. Guiding centers move toward magnetic null

BEW in a magnetic slope



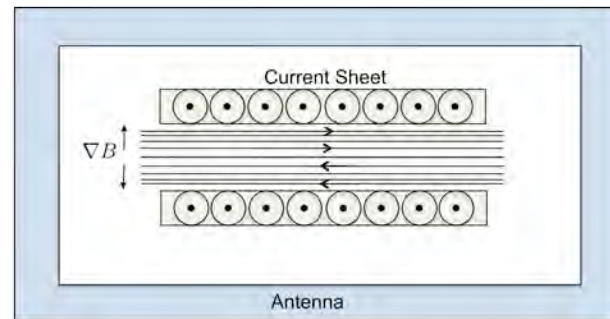
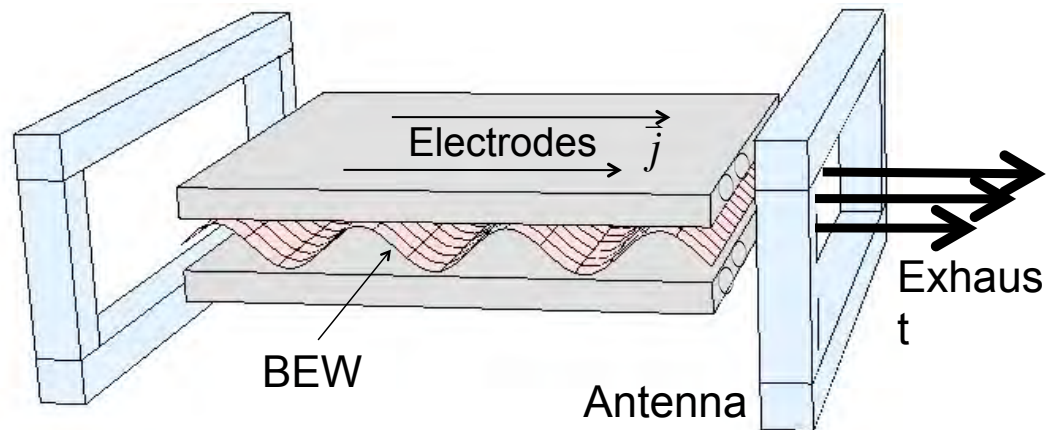
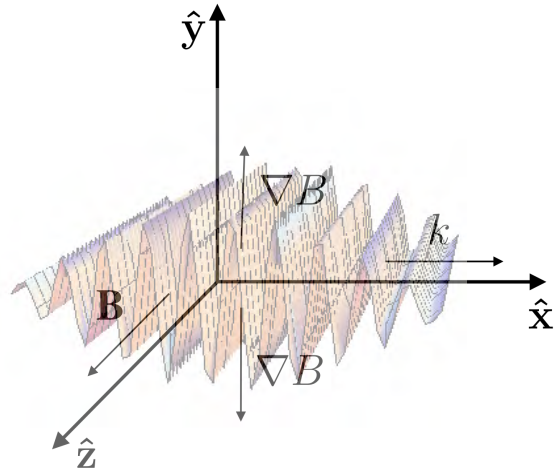
1. Guiding centers move toward magnetic null
2. Larmor radius increases

BEW in a magnetic slope

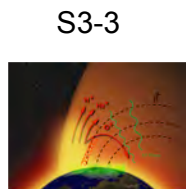


1. Guiding centers move toward magnetic null
2. Larmor radius increases
3. Trajectories pushed into magnetic slope

Thruster Proof of Concept

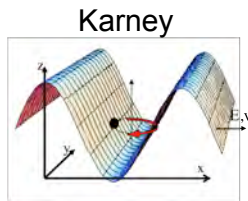


Front View



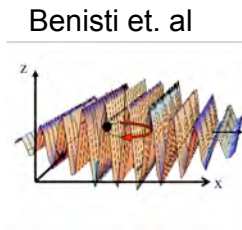
S3-3

1976



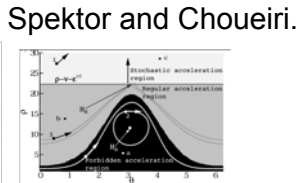
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Benisti et. al

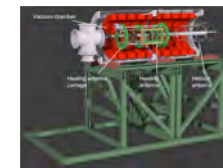
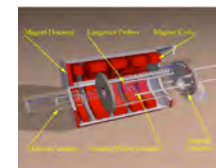
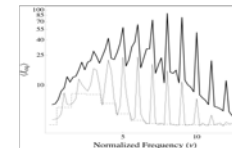
1998



Spektor and Choueiri.

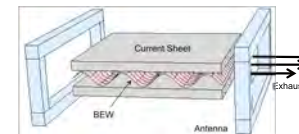
2004

BEW Heating



2006 2009 2010 >

2010 >



Directed ion acceleration

Conclusions