# Energy transfer in compressible MHD turbulence

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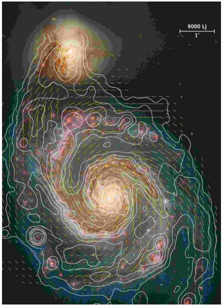
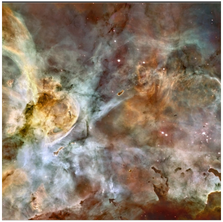
# Motivation

- In many astrophysical systems
  - turbulence
  - compressibility
  - magnetic fields

  e.g. dynamos, accretion disks, cosmic rays, star formation
- How do they interact?

$\Rightarrow$ Study energy transfer



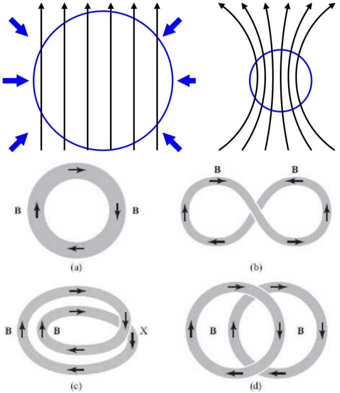



[Image credit top: MPIfR and Newcastle University]

[Image credit bottom: HST]

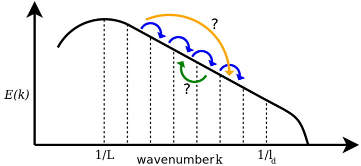
# More on energy transfer

- Total energy in an ideal MHD system is conserved
- Individual energy (and scale) budgets are not
- Different budgets and different interactions




[Dynamo image credit: Vainshtein & Zel'dovich '72]

- Regarding turbulence
  - Energy cascade
  - Inverse transfer
  - Nonlocal transfer



# Formal description of energy transfer

Nonlinear term in real space, e.g.

$$\mathbf{B}(\mathbf{x}) \cdot (\mathbf{u}(\mathbf{x}) \cdot \nabla)\, \mathbf{B}(\mathbf{x})$$

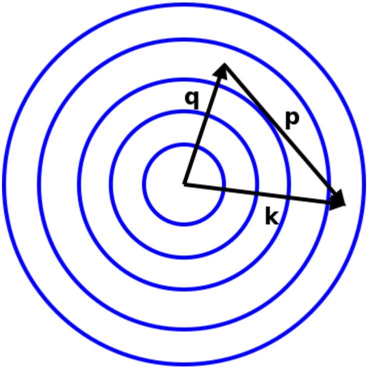
involves three wave vectors in spectral space, e.g.

$$\rightsquigarrow \left(\widehat{\mathbf{B}}(\mathbf{k}) \cdot \widehat{\mathbf{B}}(\mathbf{q})\right) (\hat{\mathbf{u}}(\mathbf{p}) \cdot \mathbf{p})$$

that must form a triangle

$$\mathbf{k} + \mathbf{q} + \mathbf{p} = \mathbf{0}$$

$\Rightarrow$ **triad** interactions

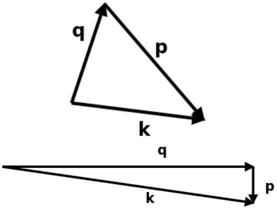



Shells in spectral space

# Interpretation of triad interactions

$$\left(\widehat{\mathbf{B}}(\mathbf{k})\cdot\widehat{\mathbf{B}}(\mathbf{q})\right)\left(\widehat{\mathbf{u}}(\mathbf{p})\cdot\mathbf{p}\right)$$

- Field $\widehat{\mathbf{B}}$ at scale $\mathbf{q}$ gives energy to field $\widehat{\mathbf{B}}$ at scale $\mathbf{k}$ by an interaction of type $\widehat{\mathbf{u}}$ at scale $\mathbf{p}$
- Here, magnetic to magnetic via kinetic advection
- Using shells rather than individual modes $\Rightarrow$ shell-to-shell transfer




**Local** interactions: wavevectors with "similar" magnitudes

**Nonlocal** interactions: wavevectors with "dissimilar" magnitude

$\Rightarrow$ Locality is an important question in MHD, e.g. background fields

# Energy budgets in incompressible MHD

$$E_u(K) = \sum_Q \int - \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{u} \cdot \nabla) \mathbf{w}^{\mathrm{Q}}}_{\text{advection (kinetic cascade)}} + \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{v}_{\mathrm{A}} \cdot \nabla) \mathbf{B}^{\mathrm{Q}}}_{\text{magnetic tension}} + \cdots \mathrm{d}\mathbf{x}$$

$$E_b(K) = \sum_Q \int - \underbrace{\mathbf{B}^{\mathrm{K}} \cdot (\mathbf{u} \cdot \nabla) \mathbf{B}^{\mathrm{Q}}}_{\text{advection (magnetic cascade)}} + \underbrace{\mathbf{B}^{\mathrm{K}} \cdot \nabla \cdot \left(\mathbf{v}_{\mathrm{A}} \otimes \mathbf{w}^{\mathrm{Q}}\right)}_{\text{magnetic tension}} + \cdots \mathrm{d}\mathbf{x}$$

# Energy budgets in compressible MHD

[Grete+ PoP 2017]

$$E_u(K) = \sum_Q \int - \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{u} \cdot \nabla)\, \mathbf{w}^{\mathrm{Q}}}_{\text{advection (kinetic cascade)}} - \underbrace{\frac{1}{2}\mathbf{w}^{\mathrm{K}} \cdot \mathbf{w}^{\mathrm{Q}} \nabla \cdot \mathbf{u}}_{\text{compression}}$$

$$+ \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{v}_{\mathrm{A}} \cdot \nabla)\, \mathbf{B}^{\mathrm{Q}}}_{\text{magnetic tension}} - \underbrace{\frac{\mathbf{w}^{\mathrm{K}}}{2\sqrt{\rho}} \cdot \nabla \left(\mathbf{B} \cdot \mathbf{B}^{\mathrm{Q}}\right)}_{\text{magnetic pressure}} + \cdots \mathrm{d}\mathbf{x}$$

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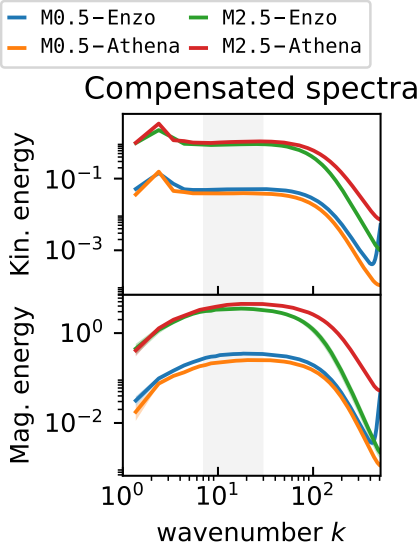
# Driven turbulence in a box ($1024^3$)

[Grete+ PoP 2017]

- Isothermal, isotropic, homogeneous
- Two codes: Enzo and Athena
- Two regimes: subsonic $\mathrm{M_s} \approx 0.5$ and supersonic $\mathrm{M_s} \approx 2.5$
- Analyzed stationary regime (30 snapshots over 3 turnover times $T$)

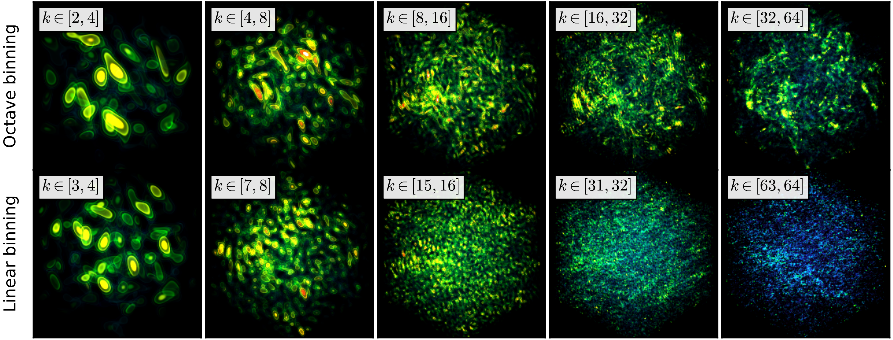


# Eddies at rest

[Grete+ PoP 2017]

How to define shells (eddies)

- Linear (thin) binning $\Rightarrow$ volume filling wave-like structures
- ✓ Logarithmic binning $\Rightarrow$ localized in real and spectral space



# Eddies in motion

[Grete+ PoP 2017]

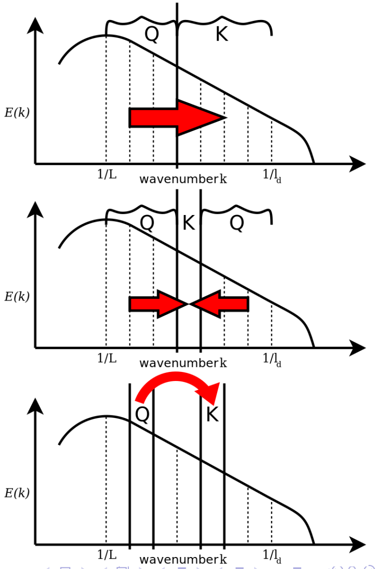
*movie plays here. . . maybe*

# What can we learn from the transfer functions $\mathcal{T}_{\mathrm{XYZ}}(Q,K)$?

- Cross-scale transfer: $\sum_{Q\leq k}\sum_{K>k}\mathcal{T}$
  e.g. relevant for subgrid-scale turbulence modeling

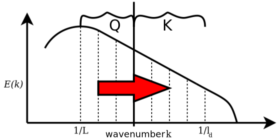
- Total transfer: $\sum_{Q}\mathcal{T}$
  e.g. relevant for the net effects cf. dynamos

- Shell-to-shell transfer: $\mathcal{T}$
  helps to explain ~~everything~~ a lot

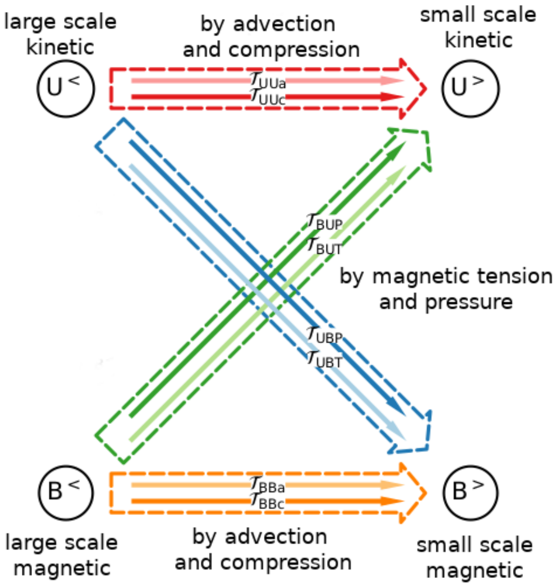


# Cross-scale transfer overview

[Grete+ PoP 2017]

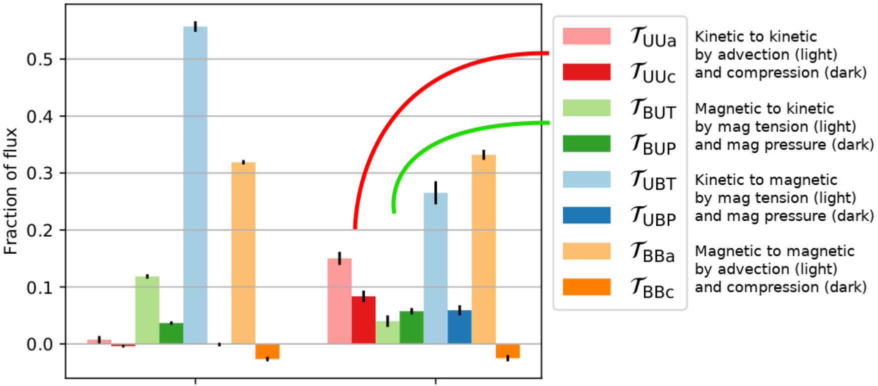



- Not only one energy reservoir
- Transfer within and between



# Mean cross-scale flux in the inertial range

[Grete+ PoP 2017]



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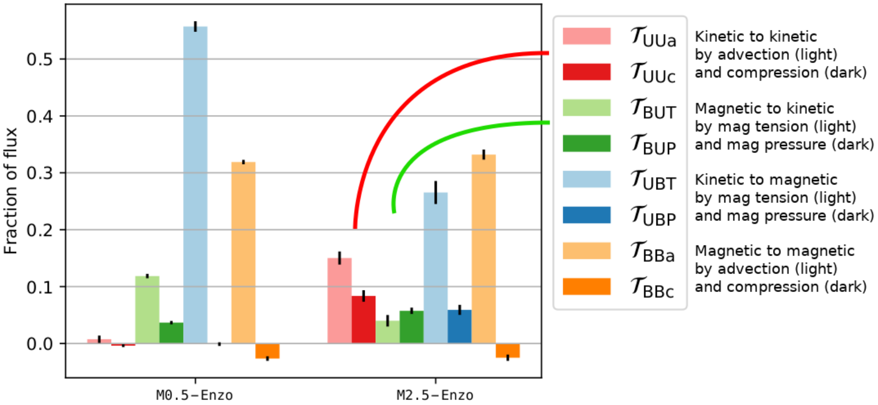
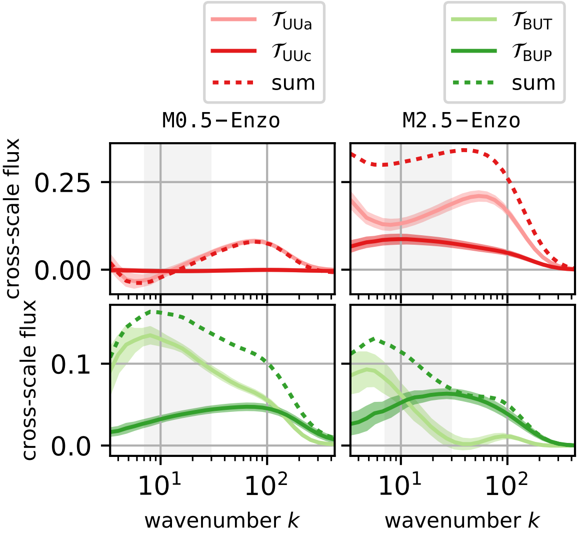



- Subsonic transfers match results of spectral code [Debliquy+ PoP 2011]
- Supersonic transfers are more dynamic

# Cross-scale transfer versus scales

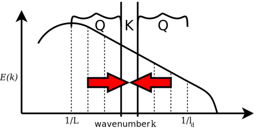
[Grete+ PoP 2017]

- Individual fluxes are not constant
- Fluxes between regimes
  - are similar (shape)
  - vary in magnitude
- Total flux (all terms) is constant

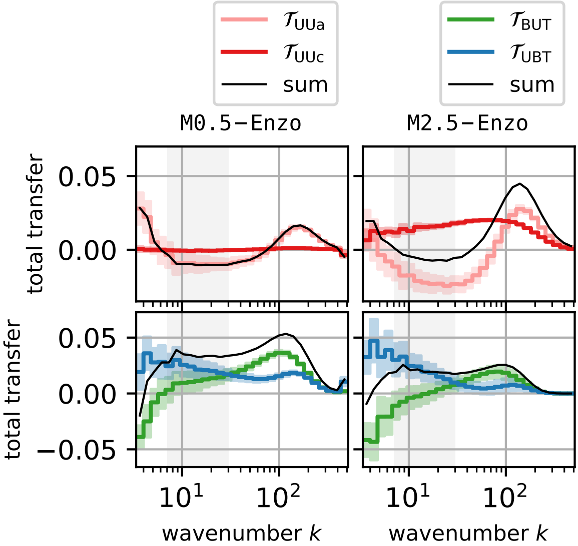


# Total transfer in (or out) a shell

[Grete+ PoP 2017]

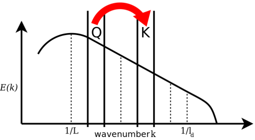



- Advection and compression work against each other
- Magnetic tension transfers energy to most scales



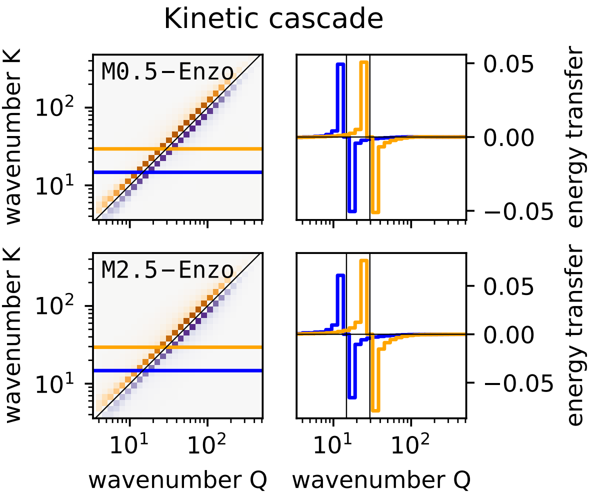
# The energy cascades

[Grete+ PoP 2017]



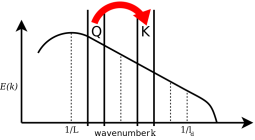

- Energy transfer is local

⇒ Shell $N$
  - receives energy from shell $N - 1$
  - transfer energy to shell $N + 1$
- Applies to (the stronger) magnetic cascade, too

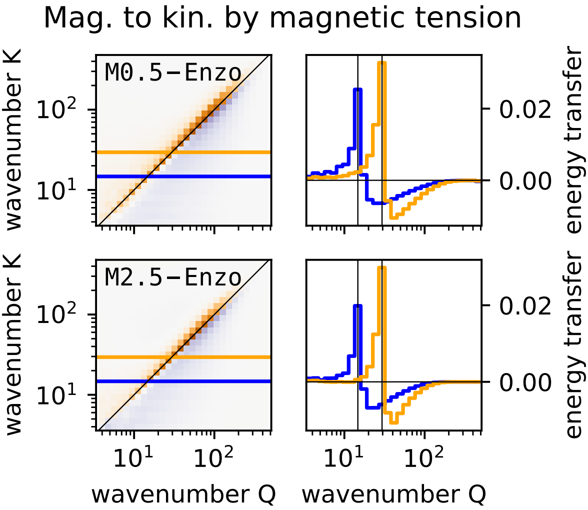


# Transfer mediated by magnetic tension

[Grete+ PoP 2017]

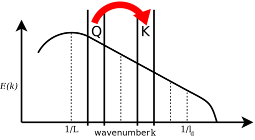



- Energy transfer is weakly local
- Velocity and magnetic field exchange most energy at $K = Q$
- Energy is received from few larger scales $Q \leq K$ and transferred to more smaller scales $Q > K$

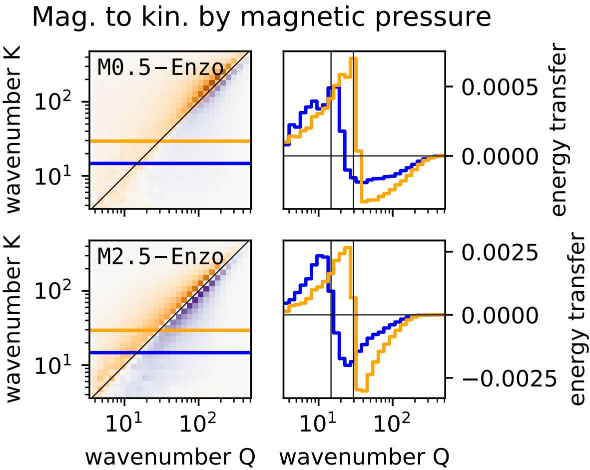


# Transfer mediated by magnetic pressure

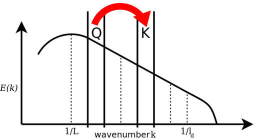
[Grete+ PoP 2017]



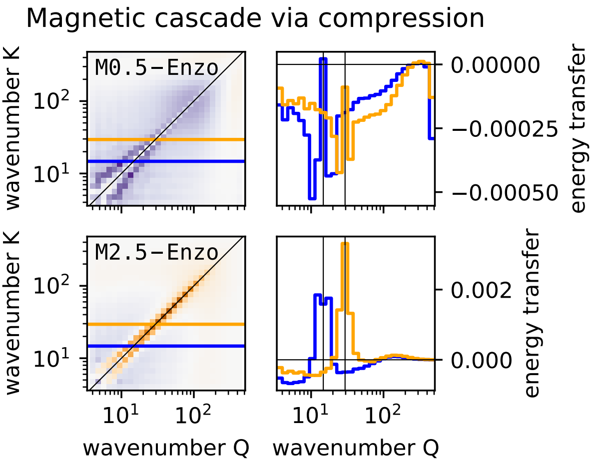

- Energy transfer is even less local
- Much stronger in the supersonic regime
- Similar shapes in both regimes



# The compressive component in the magnetic cascade




- Overall weak (few %)
- Very dissimilar between regimes



# Conclusions

[Grete+ PoP 2017]

- Established a method to analyze the compressible regime
- Underlying transfers between regimes
  - are overall similar
  - but can differ in their components and magnitudes
- Next: exploration of parameter space (in more "realistic" environments)