MIPSE Seminar
December 8 (Wed), 2021

Dynamics of Low Temperature Magnetized Plasmas: Self-Organization and Anomalous Electron Transport

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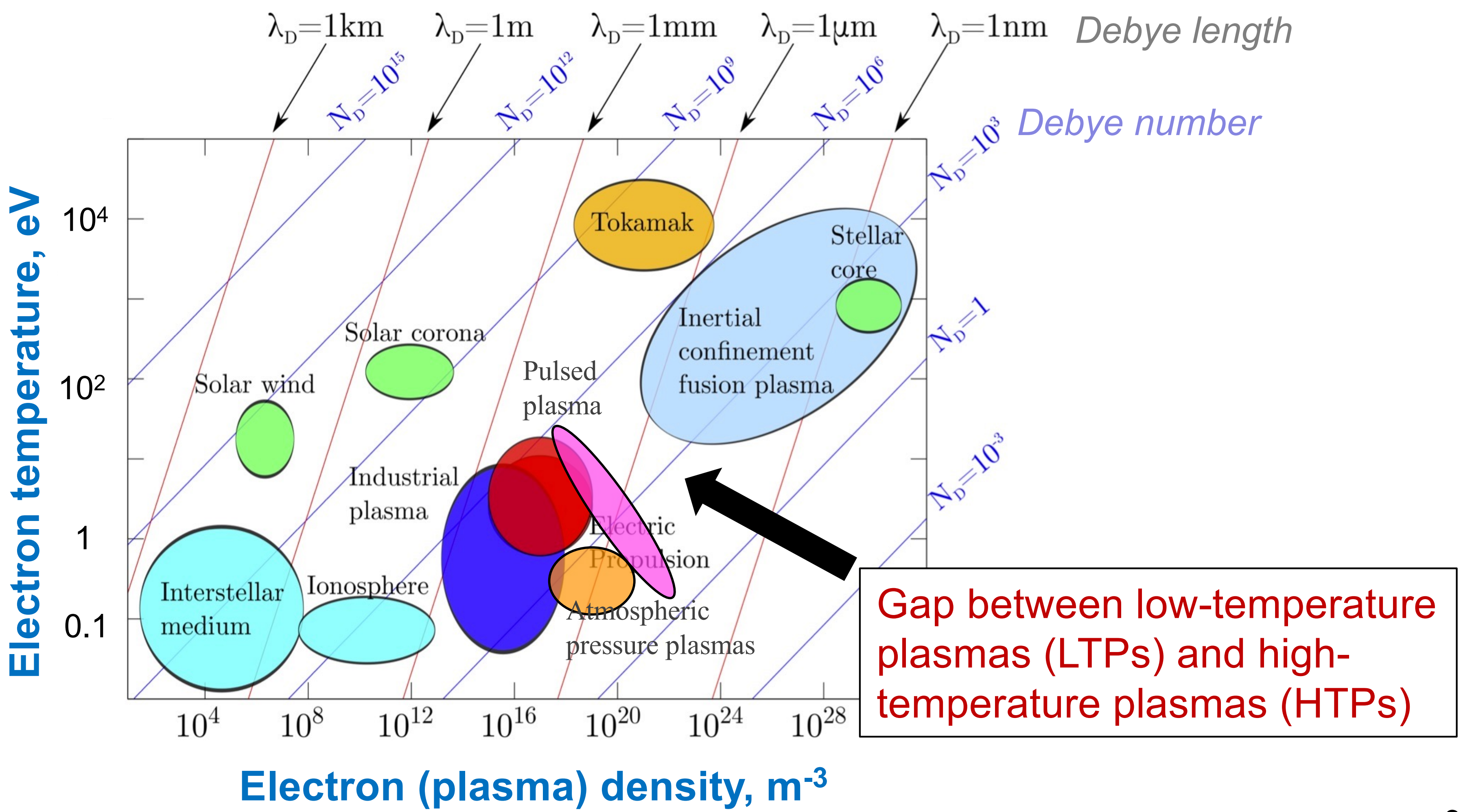
Outline

1. **Introduction: Low temperature magnetized plasmas**
2. **Computational plasma modeling**
3. **Physics and modeling of low-temperature magnetized plasmas**
   – Kinetic theory/modeling of plasma instabilities and turbulence
   – Fluid modeling of low-temperature magnetized plasmas
   – Data-driven *online* estimation for plasma chemistry
4. **Conclusion**

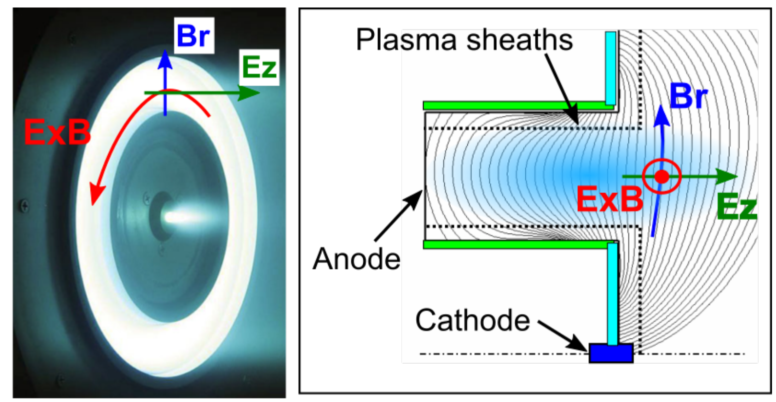


λD=1km
λD=1m
λD=1mm
λD=1μm
λD=1nm
Debye length
ND=10^15
ND=10^12
ND=10^9
ND=10^6
ND=10^3
Debye number
ND=1
ND=10^-3
Tokamak
Stellar core
Inertial confinement fusion plasma
Solar corona
Solar wind
Pulsed plasma
Industrial plasma
Electric Propulsion
Interstellar medium
Ionosphere
Atmospheric pressure plasmas
Gap between low-temperature plasmas (LTPs) and high-temperature plasmas (HTPs)
Electron temperature, eV
10^4
10^2
1
0.1
10^4
10^8
10^12
10^16
10^20
10^24
10^28
Electron (plasma) density, m^-3

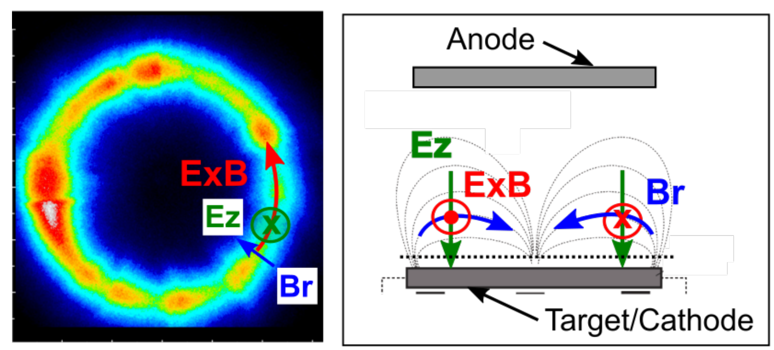
Low temperature magnetized plasmas

Using applied magnetic fields leads to higher electron temperature and density

- **Multiscale**
  - High-frequency: Instabilities, turbulence
  - Low-frequency: Self-organization
- **Multiphysics**
  - Collisional (intermolecular, walls)
  - Collisionless (non-Maxwellian, kinetic, waves, gyromotion)
- **Multispecies**
  - Electrons, ions (singly, multiply charged)
  - Excited state

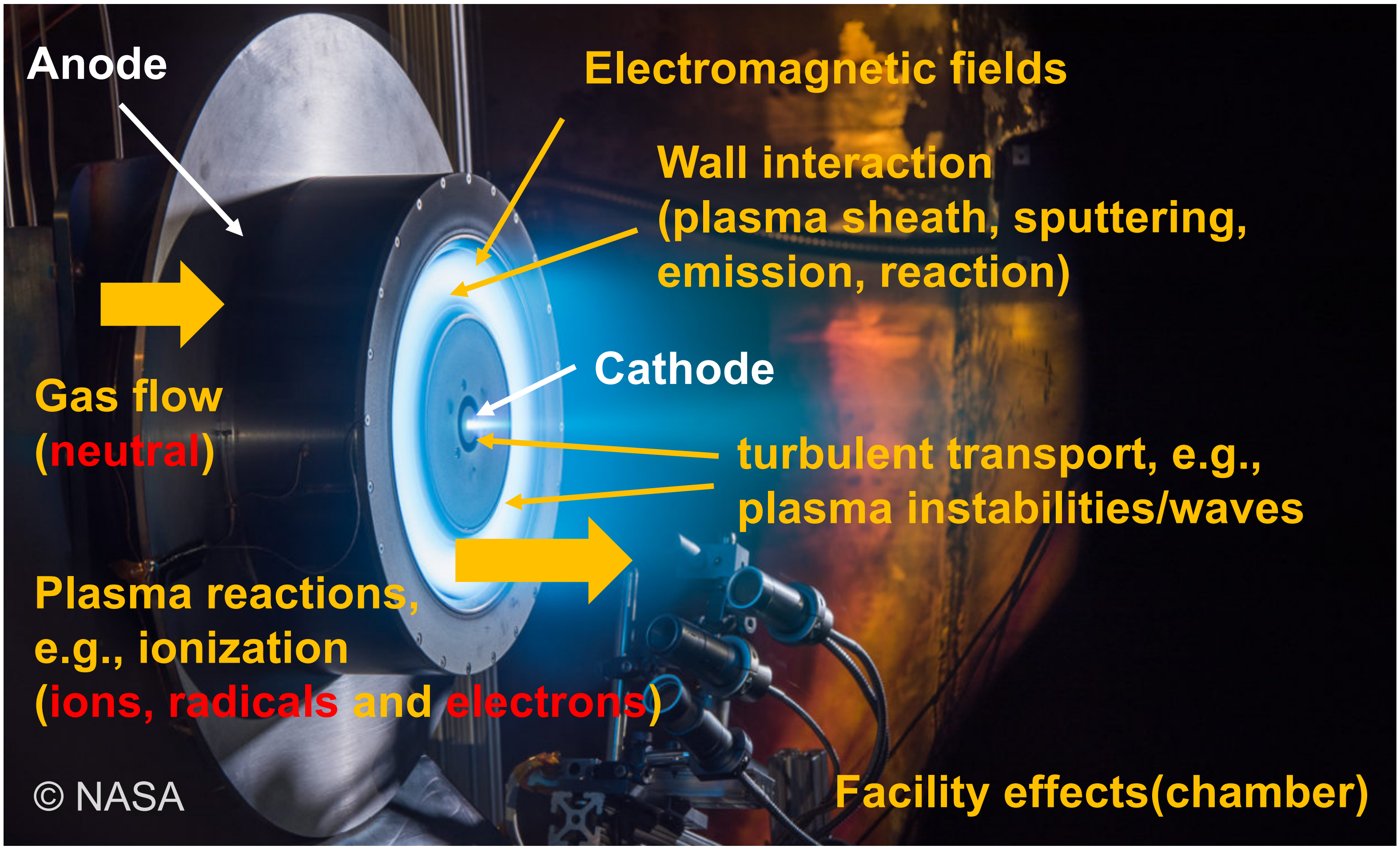


(a) Hall Effect Thruster



(b) Magnetron Discharge

Nonlinear coupling of dynamic plasma processes



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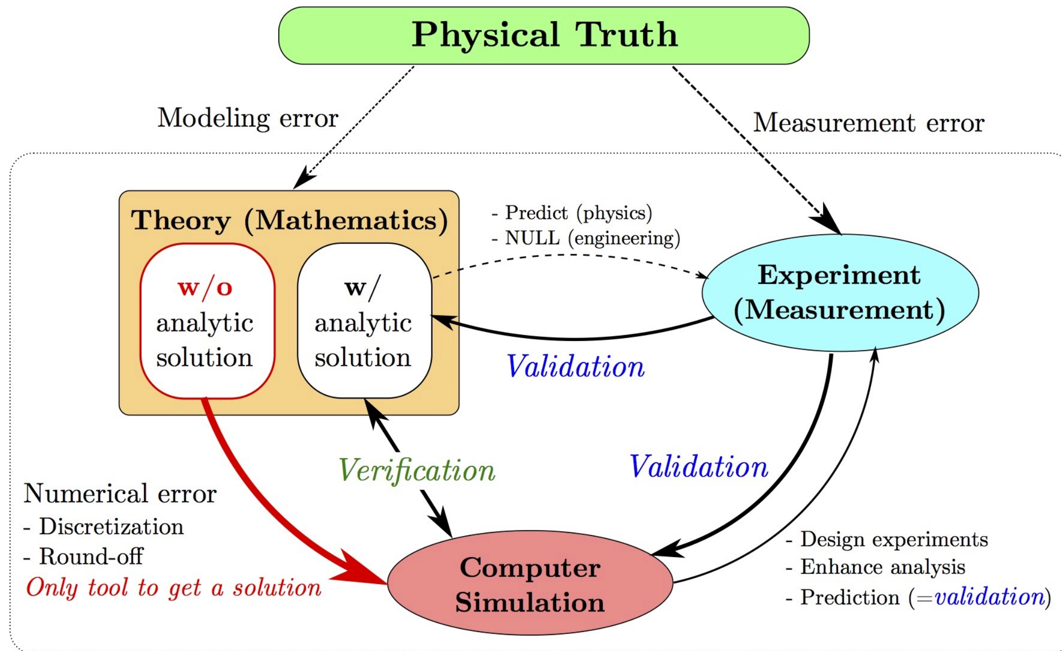
Why computational modeling?

- **Verification / Benchmarking**
  - Theoretical understanding
  - Numerical understanding
- **Validation**
  - vs. Experiments
  - Modeling of Diagnostics
  - Understanding of physical phenomena

- **Designing tool**
  - Predictive modeling
  - System analysis
  - Product designing



Why simulation? [with Professor Bram Van Leer (UM, Aero)]

Low temperature plasma (LTP) models

IOP PUBLISHING JOURNAL OF PHYSICS D: APPLIED PHYSICS
J. Phys. D: Appl. Phys. 42 (2009) 194013 (20pp) doi:10.1088/0022-3727/42/19/194013

REVIEW ARTICLE

Hybrid modelling of low temperature plasmas for fundamental investigations and equipment design

Mark J Kushner

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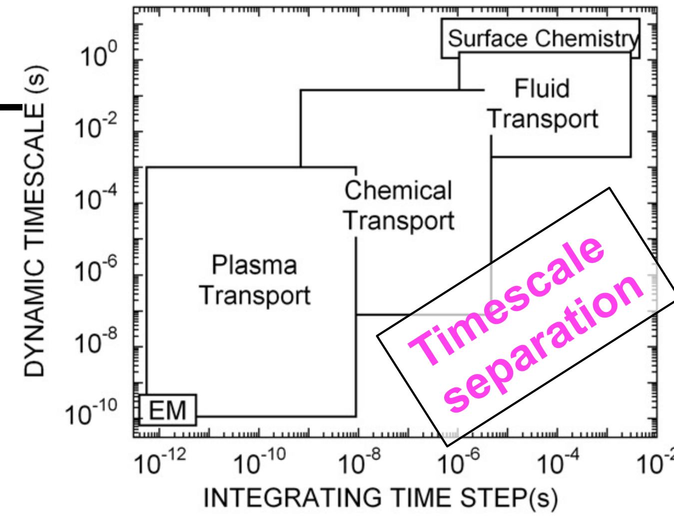
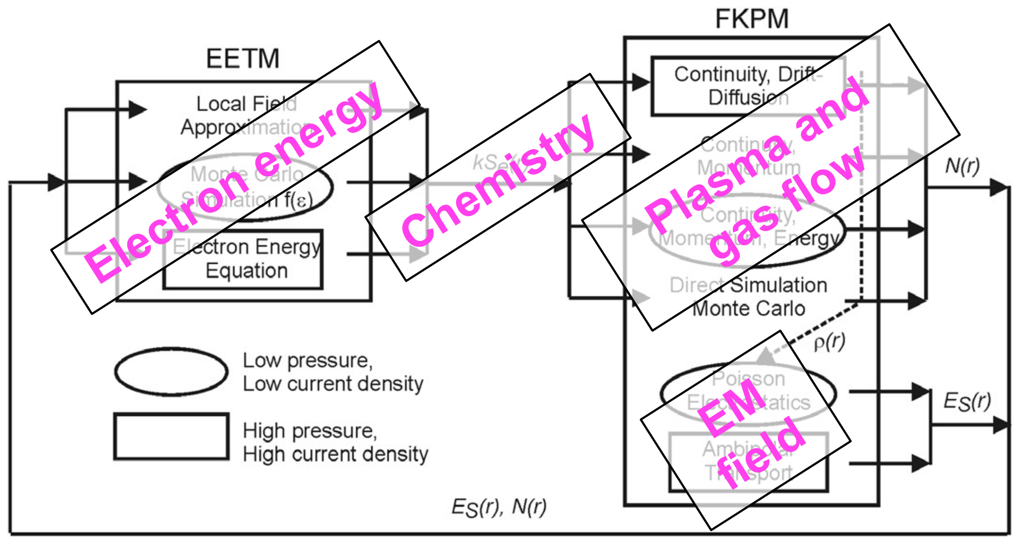


- **Multiscale nature**
- **Multiphysics**
  - Electrons
  - Heavy species
  - Wall interaction (boundary condition)



Physics-based modeling techniques for plasma flows

(a) Fluid (continuum) models

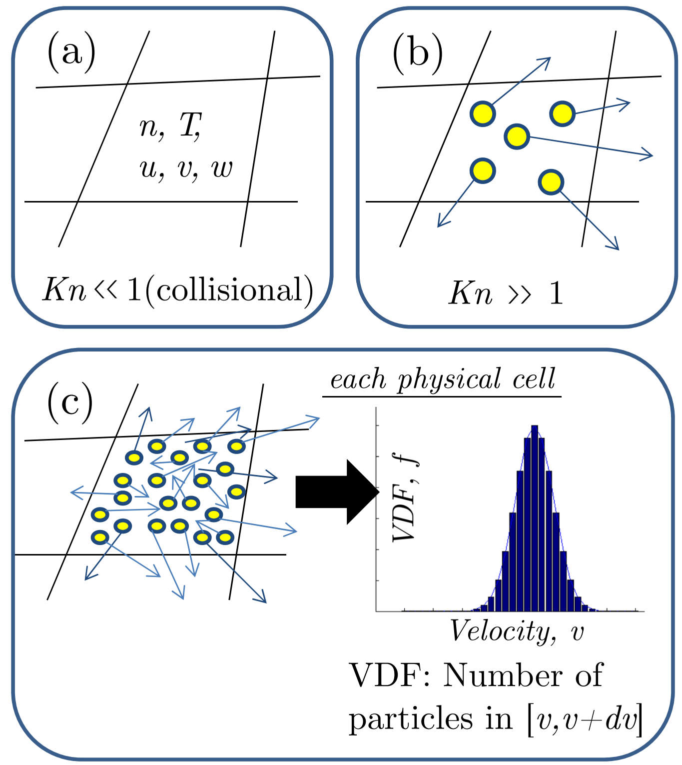
- Drift-diffusion model
- Euler/Navier-Stokes/MHD/Two-fluid
- Numerically inexpensive

(b) Particle-based kinetic methods

- 1 macroparticle ≈ 10^5-10^8 real particles
- Particle-in-cell (PIC), DSMC, MCC

(c) Grid-based direct kinetic (DK) methods

- Solve kinetic equations directly in discretized phase space
- No statistical noise vs. particle method



Cf.) Knudsen number: Kn = λ/L

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Classical vs anomalous electron transport

Chapter 5 in Chen's book

(Using single-fluid resistive MHD,) the flux associated with diffusion is

$$\Gamma = n\boldsymbol{u}_\perp = -D_\perp \nabla n. \qquad [5.98]$$

Where

$$D_\perp = \frac{2m\nu_{ei}k_BT}{e^2B^2} = 2\,\frac{\nu_{ei}}{\omega_{ce}}\frac{k_BT}{eB}. \qquad [5.99]$$

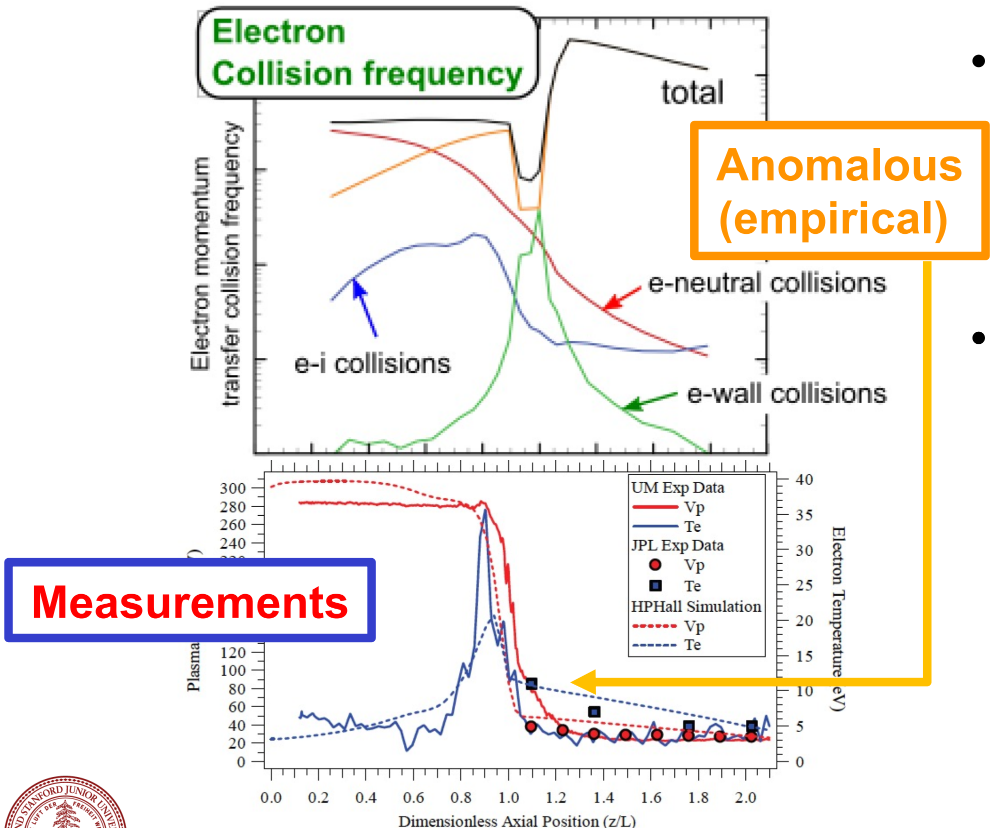
Classical

… In almost all previous experiments, $D_\perp$ scaled as B^{-1}, rather than B^{-2}, ….. Furthermore, the absolute value of $D_\perp$ was far larger than that given by Eq. [5.99]. This anomalously poor magnetic confinement was first noted in 1946 by Bohm, Burhop, and Massey, …. Bohm gave the semiempirical formula

$$D_\perp = \frac{1}{16}\frac{k_BT}{eB} = D_B. \qquad [5.111]$$

Anomalous (cf. Bohm)

The way we *thought* about cross-field electron transport in low-temperature magnetized plasmas



HPHall vs experiment [Hofer AIAA 2008]

- **Drift-diffusion flux** (e.g., Ohm's law)

$$\mathbf{\Gamma} \equiv n\mathbf{u} = \underbrace{\pm n\bar{\bar{\mu}} \cdot \mathbf{E}}_{\text{drift}} - \underbrace{\bar{\bar{D}} \cdot \nabla n}_{\text{diffusion}}$$

- **Electron mobility (transport coefficient)**

Measurement

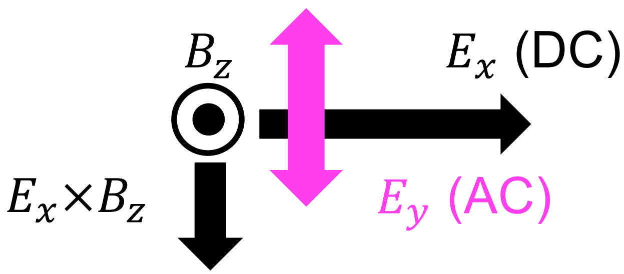
$$\mu_e = \frac{\Gamma_e}{-n_e\left(E + \frac{1}{e}\nabla p_e\right)}$$

$$= \mu_e^{\text{collisional}} + \mu_e^{\text{turbulent}}$$

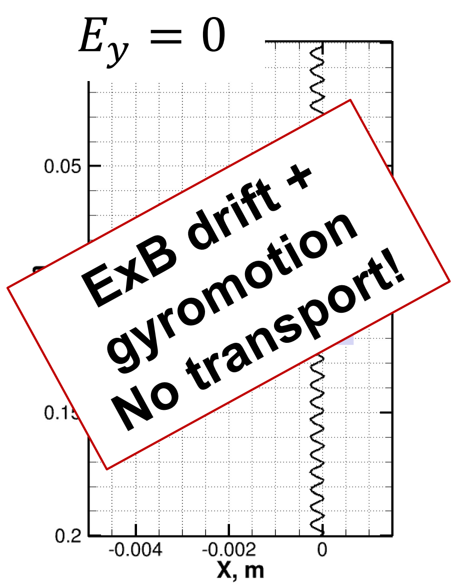
Collisionless cross-field electron transport induced by fluctuations (i.e., plasma waves)

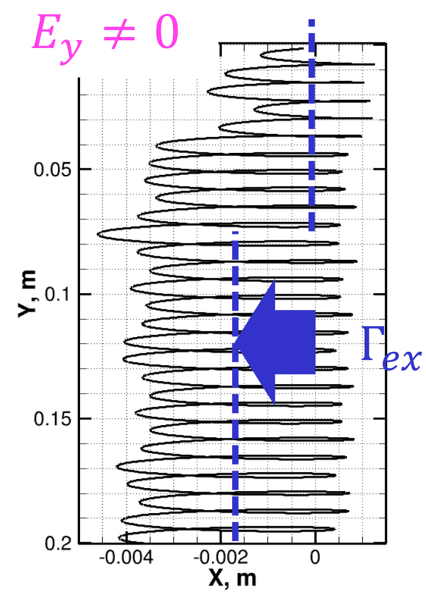
Classical theory: electrons are trapped by the magnetic field

$$\frac{d\vec{x}}{dt} = \vec{v}; \quad \frac{d\vec{v}}{dt} = \frac{q}{m}\left[\vec{E}(\vec{x}) + \vec{v}\times\vec{B}\right]$$



What if there is a plasma wave (i.e., fluctuation in the electric field)?

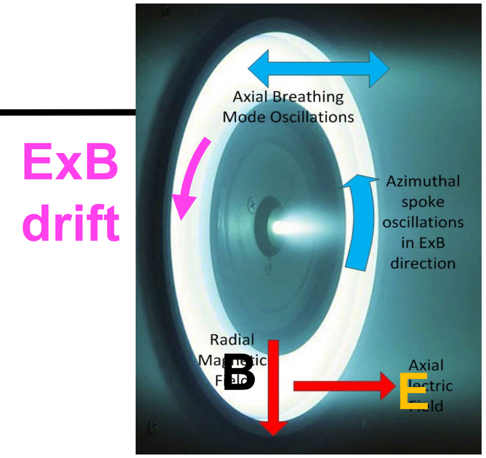


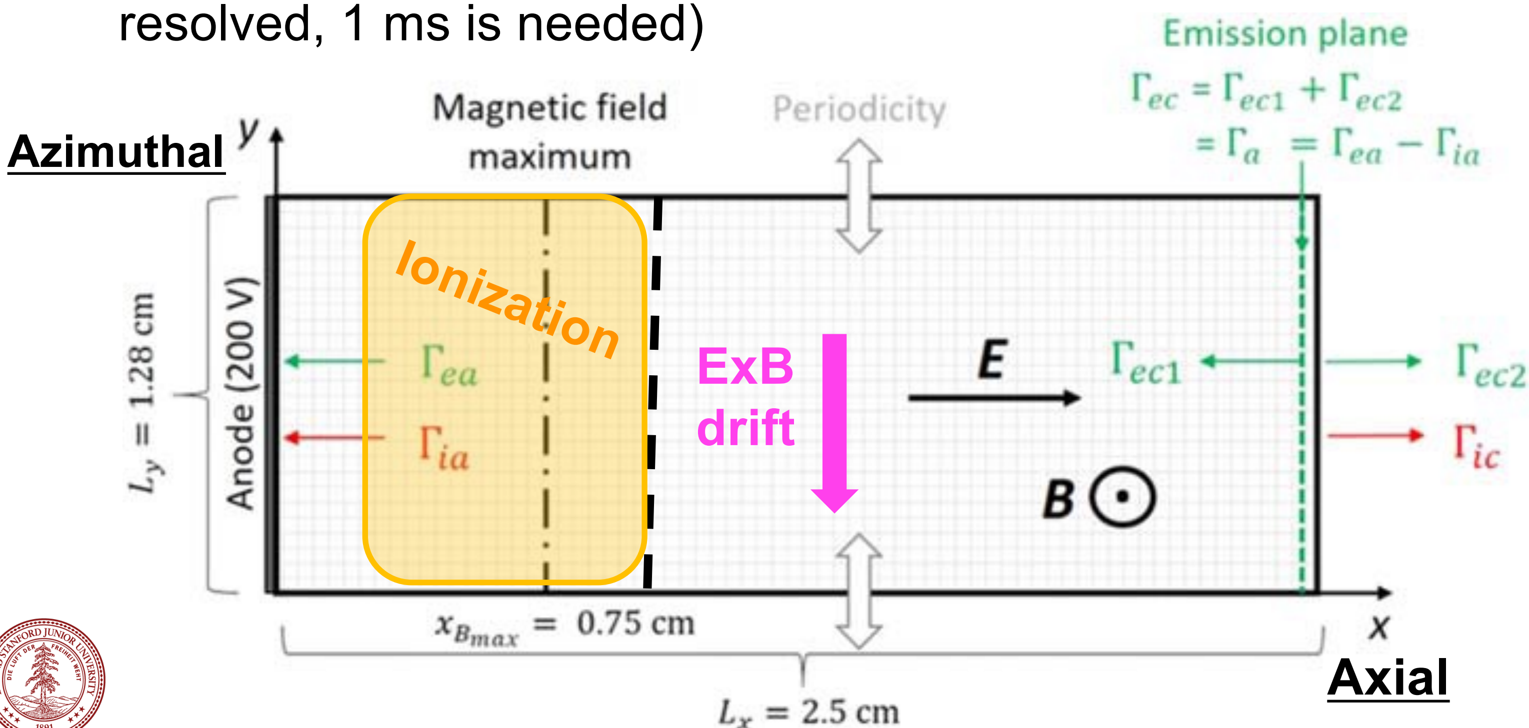


Turbulence: *Electron transport enhanced by plasma wave (<u>1D, 2D, 3D?</u>)*

2D z-theta benchmark testcase

- *Test case proposed by Boeuf and Garrigues 2018*
- BC: anode + electron injection
- **Fixed ionization rate**: run time ~30 μs (If neutrals are resolved, 1 ms is needed)





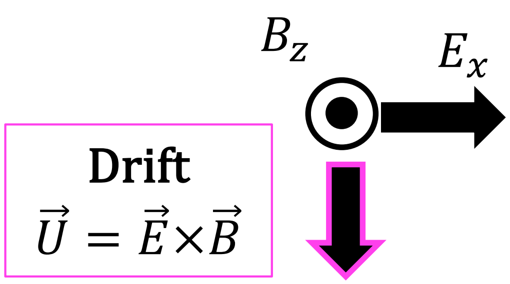
- 500x256 cells
- ~250 ppc
- 200 V
- 0.01 T peak
- *j_i =400 A/m^2*
- **Xe^+**
- 32 CPUs
- 1-2 weeks

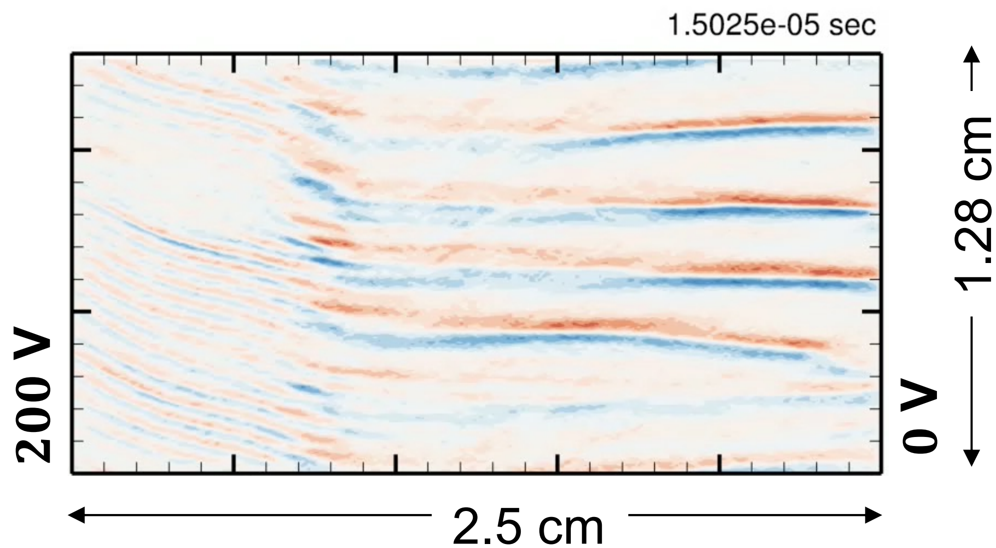
Azimuthal plasma wave initiated by electron cyclotron drift instability (ECDI)

Movie

- ExB drift in the y-direction causes the plasma wave formation in the y-direction.
- Upstream: small scale fluctuation (dominant mode: ~1 mm)
- Fluctuating E_y perturbs and detraps the electrons from the magnetic field lines

- **Singly charge ions: Xe^+**
- **Plasma wave (1D in y)**



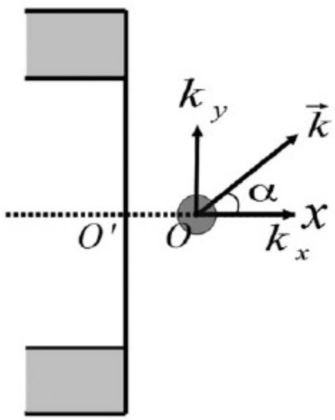


Azimuthal electric field, E_y

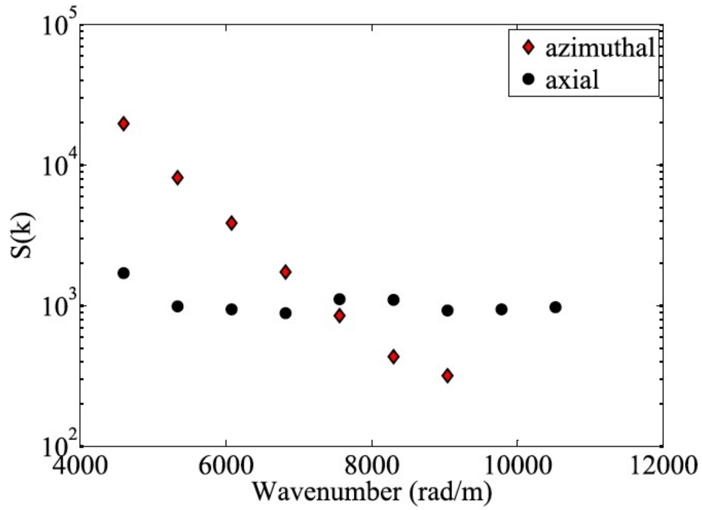
[Charoy T. et al. PSST **28**, 105010 (2019)]
[Villafana W. et al. PSST **30**, 075002 (2021)]

Experimental evidence of axial (cross-field) plasma wave due to ion-ion two-stream instability (IITSI)

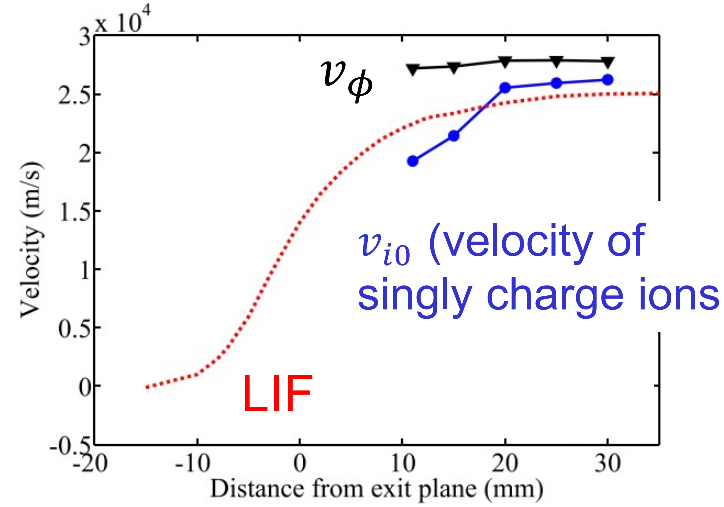
- Coherent Thomson scattering (CTS) detected signature of unambiguous **axial** plasma wave in the plume of a cross-field discharge (with **azimuthal** wave).
- Phase velocity of axial wave (v_ϕ) is *faster* than Xe^+ ion velocity (v_{i0})



$x \in [11, 30]$ **mm**



Density fluctuation amplitude



Axial plasma wave

[Tsikata S, Cavalier J, Héron A, et al. Phys. Plasmas **21**, 072116 (2014)]

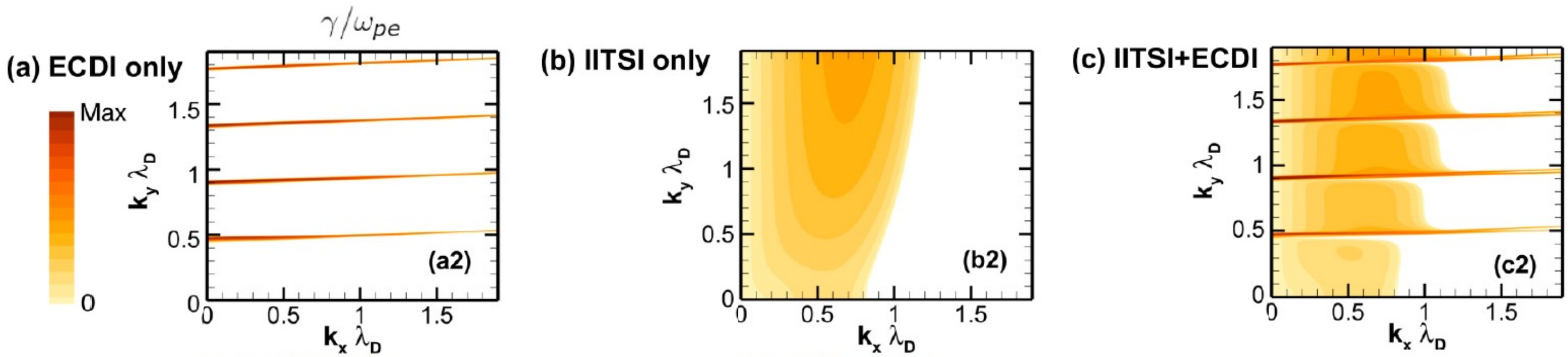
IITSI + ECDI dispersion theory

The CTS measurement suggested that the streams of singly-charged ions (Xe^{+}) and doubly-charged ions (Xe^{2+}) cause ion-ion two-stream instability (IITSI)

Doubly charged ion fraction: $\alpha = \frac{2n_i^{2+}}{n_e}$

$$(k_\perp \lambda_D)^2 \left[1 - \frac{(1-\alpha)\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^{+})^2} - \frac{\alpha\omega_{pi}^2}{\left(\omega - \mathbf{k}\cdot\mathbf{U}_i^{2+}\right)^2} \right] + 1 - I_0(b)\exp(-b) + \sum_{n=1}^{\infty} \frac{2\omega^2 I_n(b)\exp(-b)}{(n\omega_B)^2 - \omega^2} = 0$$

Xe^{+} Xe^{2+} *Magnetized electrons*

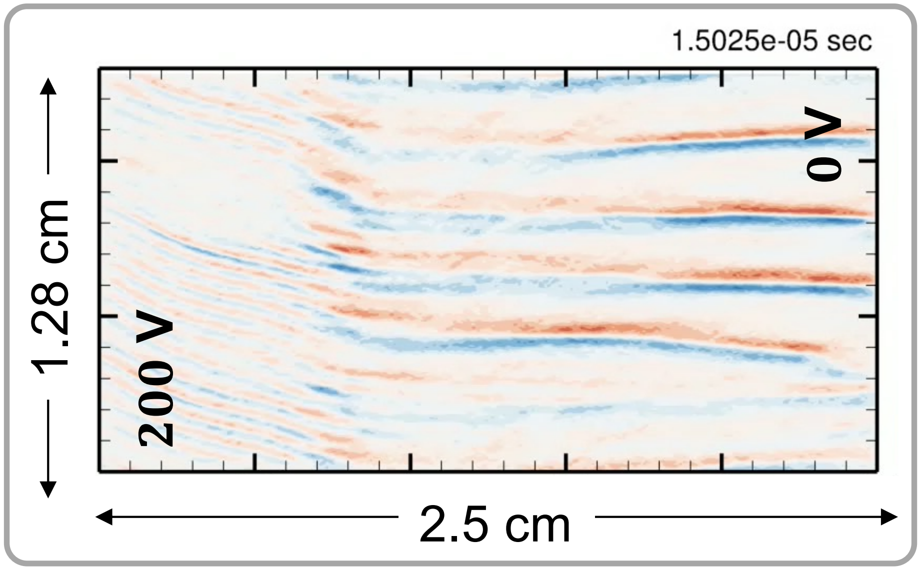


[Hara K and Tsikata S, Phys Rev E **102**, 023202 (2020)]

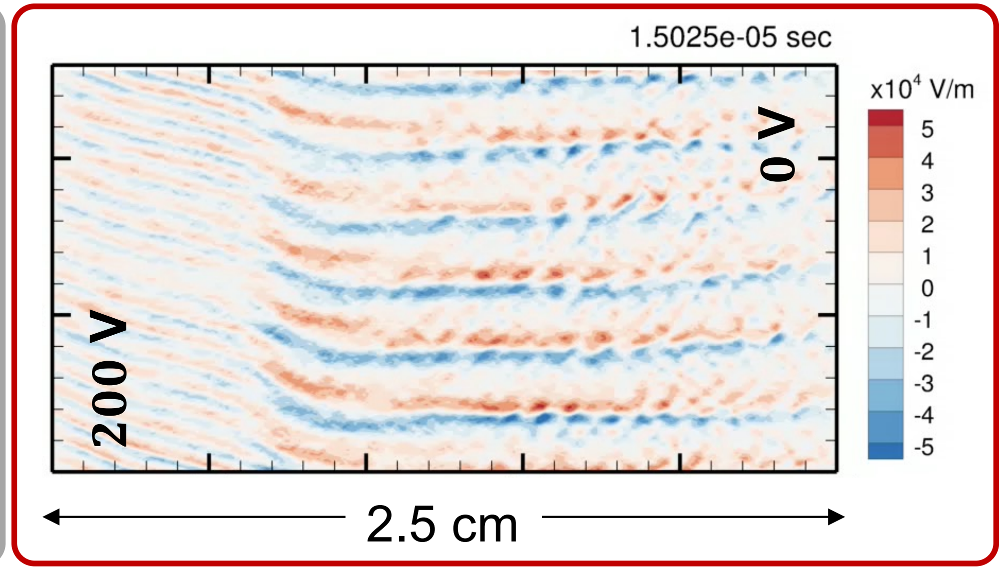
Multidimensional plasma microturbulence

Movie

Multidimensional (2D) plasma wave is observed due to the coupling of (i) **axial** oscillation via IITSI and (ii) **azimuthal** oscillation via ECDI



100% Xe^{+} (ECDI only)



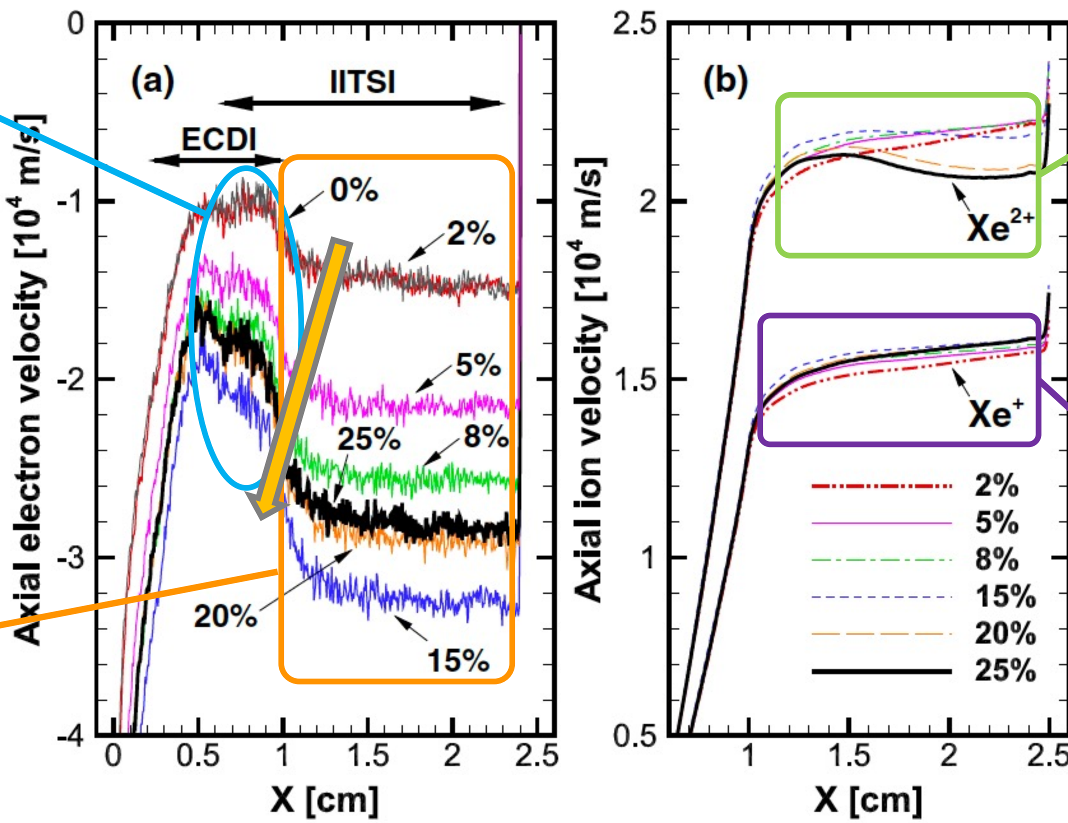
80% Xe^{+}, 20% Xe^{2+} (ECDI+IITSI)

[Hara K and Tsikata S, Phys Rev E **102**, 023202 (2020)]

Cross-field transport: electrons and ions

Nonlinear coupling of ECDI and IITSI

$|u_{xe}| \approx \mu_{\perp} E_x$ **increases** with more Xe^{2+}



Deceleration of doubly charged ions

Acceleration of singly charged ions

[Hara K and Tsikata S, Phys Rev E **102**, 023202 (2020)]

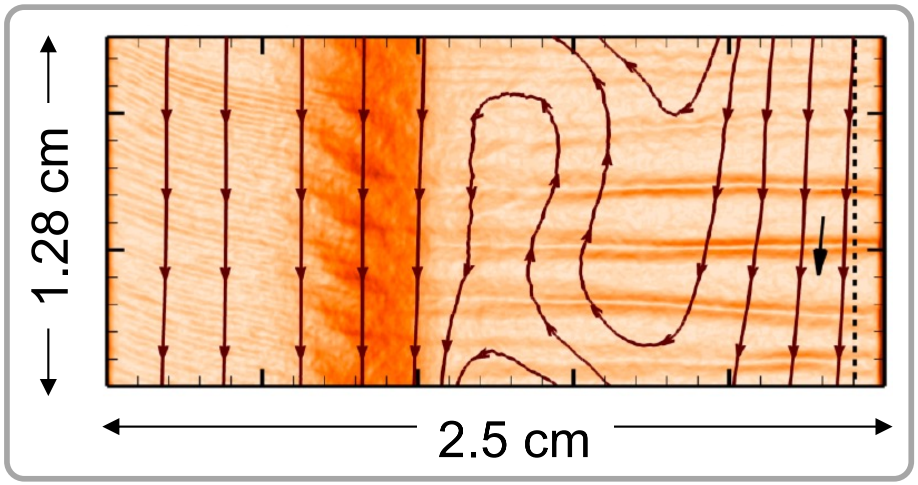
Time-averaged electron streamline

Fluctuation-based electron transport
[Waltz PoF 1982, Liewer Nucl Fusion 1985]

$$\langle \Gamma_{ex} \rangle = \frac{\langle n'_e E'_y \rangle}{B_z} \qquad \langle \Gamma_{ey} \rangle = -\frac{n_{e0} E_{x0}}{B_z} - \frac{\langle n'_e E'_x \rangle}{B_z}$$

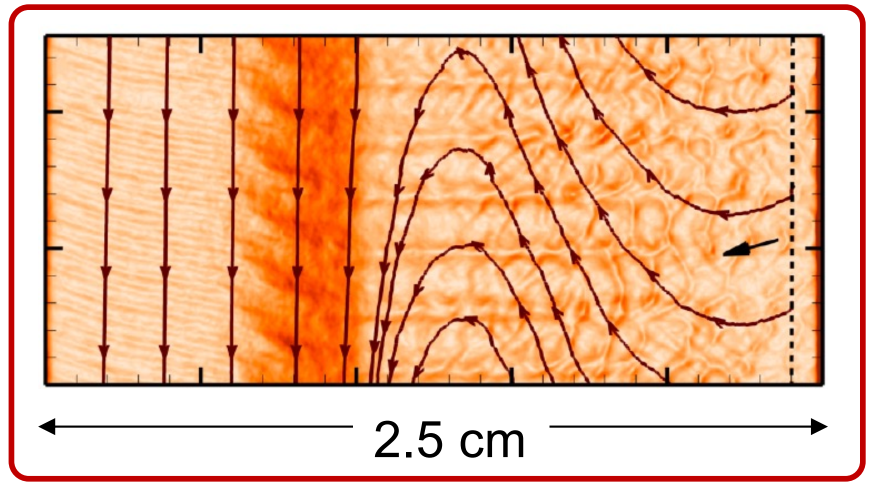
100% Xe^{+}
(ECDI only)

$|\langle \Gamma_{ex} \rangle| < |\langle \Gamma_{ey} \rangle|$



80% Xe^{+}, 20% Xe^{2+}
(ECDI+IITSI)

$|\langle \Gamma_{ex} \rangle| > |\langle \Gamma_{ey} \rangle|$

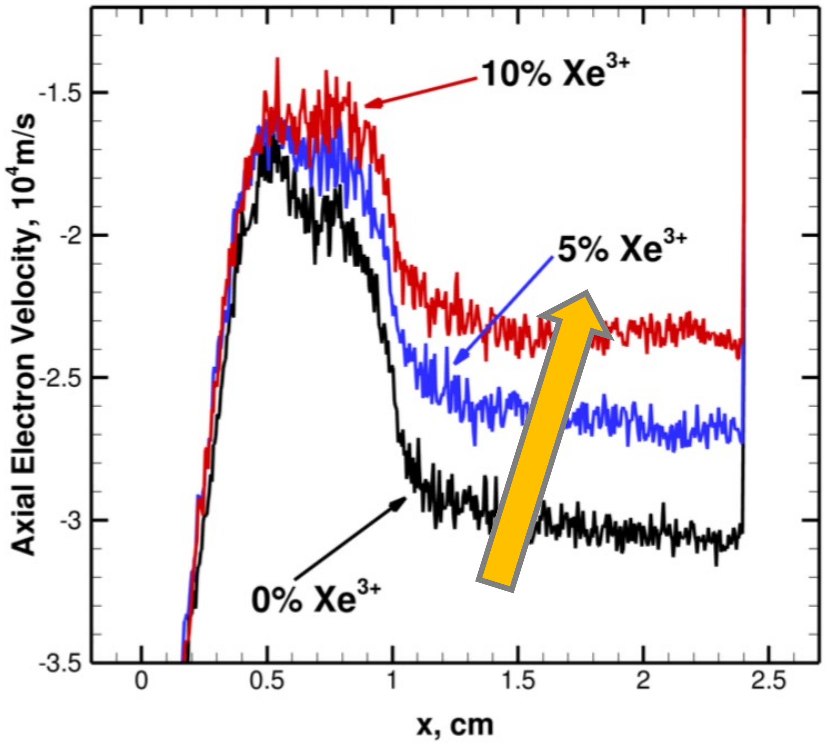


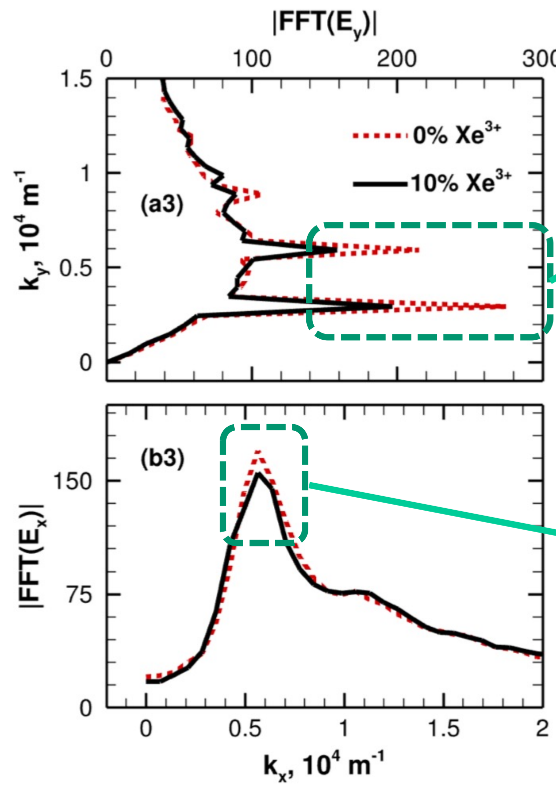
Color: amplitude of electric field. Line: electron streamlines.

[Hara K and Tsikata S, Phys Rev E **102**, 023202 (2020)]

Wave damping due to addition of third species (Xe^{3+}) leading to *reduced* cross-field electron diffusion

$|u_{xe}| \approx \mu_\perp E_x$ **decreases** with more Xe^{3+} due to damping of plasma waves





E_y (azimuthal) **30% reduction**

E_x (axial) **8% reduction**

[Kumar P, Tsikata S, and Hara K, J. Appl. Phys. **30**, 173307 (2021)]

Azimuthal IVDF : Nonlinear trapping

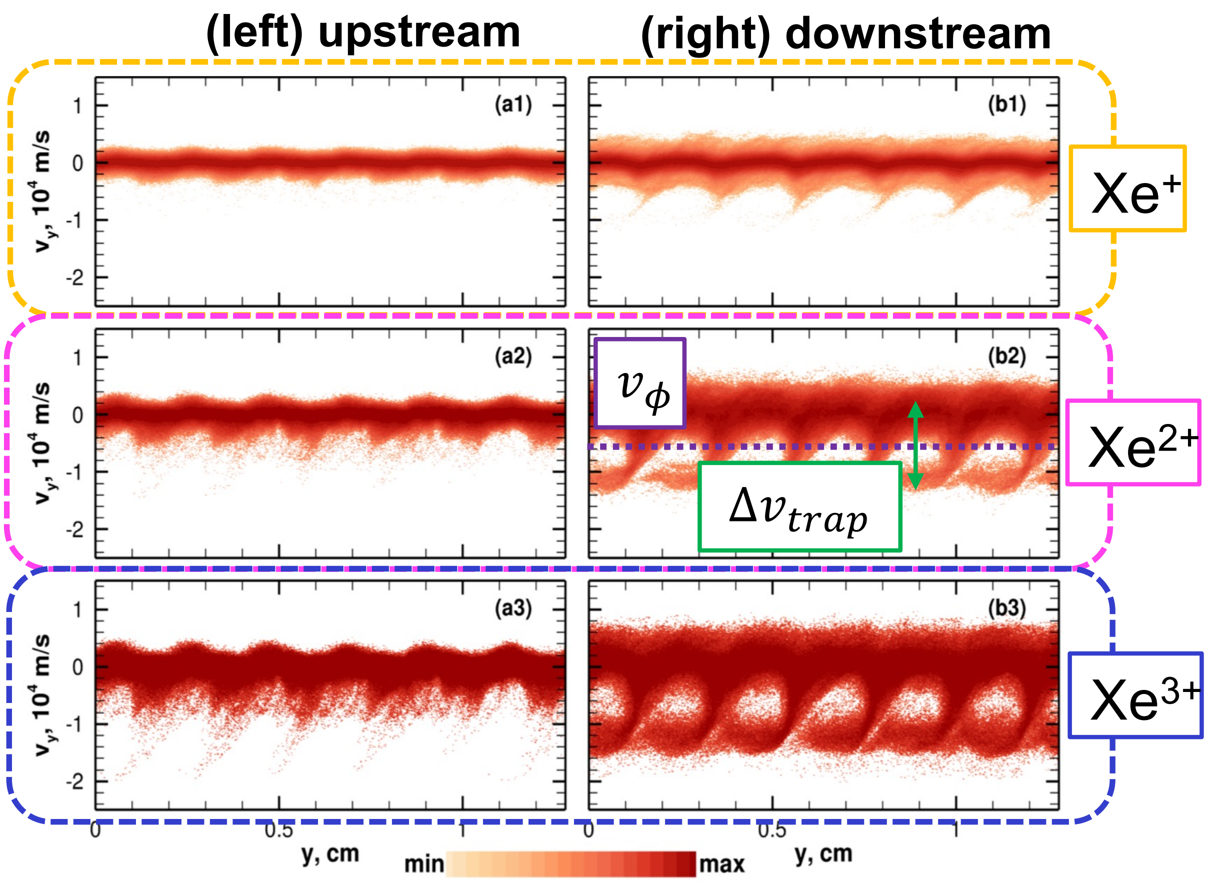
Phase velocity θ-wave

$$v_\phi = 6\times10^3\ \mathrm{m/s}$$

Trapping velocity

$$\Delta v_{trap} = \left(\frac{2Ze\phi_0}{m_i}\right)^{1/2}$$

ϕ_0: potential amplitude



[Kumar P, Tsikata S, and Hara K, J. Appl. Phys. **30**, 173307 (2021)]

Summary: Part 1 (anomalous electron transport)

- Coupling of ECDI (**azimuthal**) and IITSI (**axial**) modes is studied using a 2D parallel PIC code. Analyzed plasma waves and cross-field electron transport.
- The results adding Xe^{2+} and Xe^{3+} suggest that cross-field electron transport/diffusion is affected by the ***amplitude*** of the multidimensional (2D) plasma wave.
- Recent PIC simulations show the effects of the ***wavelength*** of the azimuthal plasma wave on cross-field electron transport. [Kumar, Sewell, Hara, AIAA SciTech 2022]

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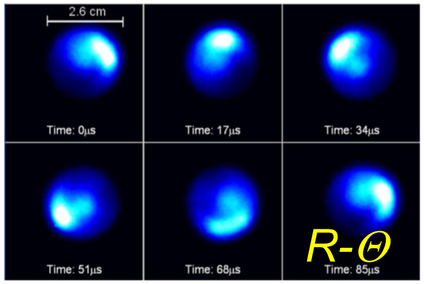
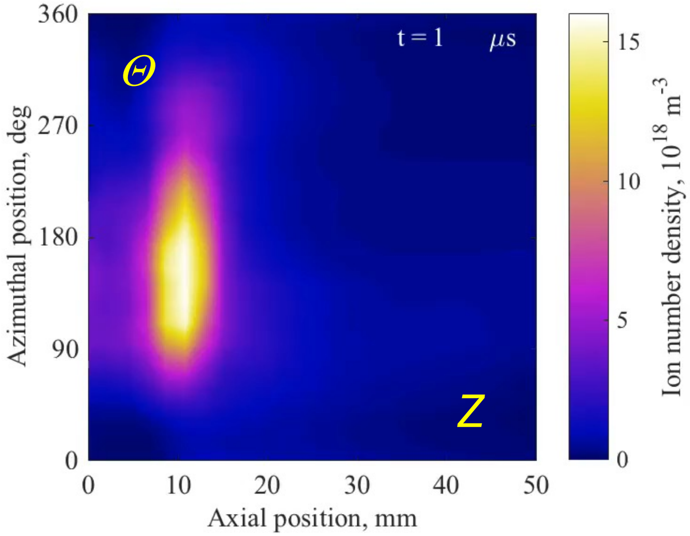
Low-frequency (10-30 kHz) plasma oscillations

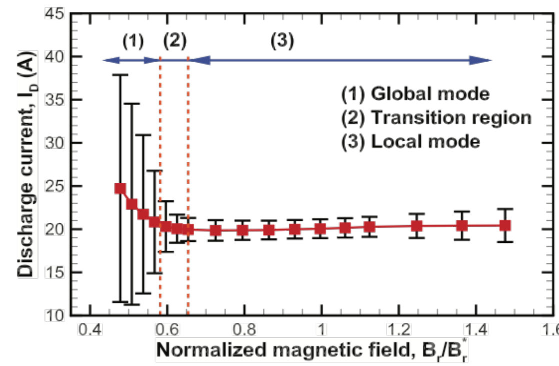
Movie



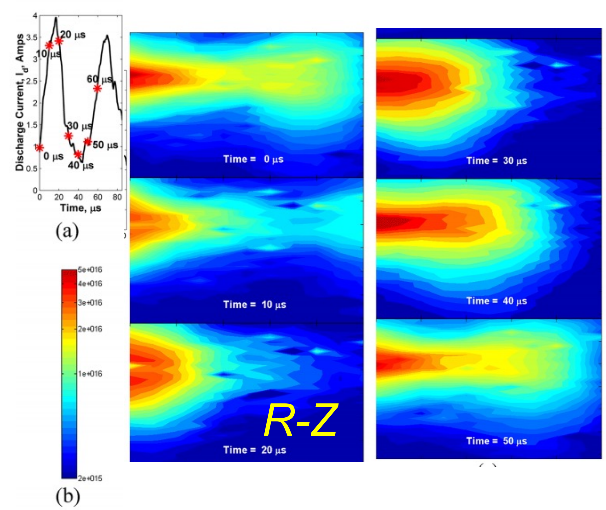
Azimuthally rotating spokes [Ellison 2011]



Gradient drift instability [Kawashima & Hara 2018]



Mode transition: ionization oscillation [Sekerak PhD 2014]



Breathing mode: high-speed probe [Lobbia PhD 2010]

Theory: fluid moment equations

Kinetic equation $$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\delta f}{\delta t}\right)_c$$

f: Velocity distribution function (VDF)

Fluid equations

Anisotropic pressure tensor

$$p_{ij} = mn\iiint (v_i - u_i)(v_j - u_j)\hat{f}(\mathbf{v})d^3\mathbf{v}$$

$$\begin{cases} \dfrac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0 \\ \dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \dfrac{1}{mn}\nabla\cdot\bar{\bar{p}} - \dfrac{q}{m}(\mathbf{E}+\mathbf{u}\times\mathbf{B}) = \mathbf{F} \\ \dfrac{\partial (n\epsilon)}{\partial t} + \nabla\cdot(n\epsilon\mathbf{u} + \bar{\bar{p}}\cdot\mathbf{u} + \mathbf{q}) = \mathbf{j}\cdot\mathbf{E} + S \end{cases}$$

Collision terms ($\mathbf{F}$, S)

Heat flux ($\mathbf{q}$)

Simulation: plasma fluid modeling strategies

Drift-diffusion (DD) flux models

$$\nabla \cdot \left(\vec{\Gamma}_i - \vec{\Gamma}_e\right) = 0 \Rightarrow \nabla \cdot \left(n_e \overline{\overline{\mu_e}} \cdot \nabla\phi\right) = f\left(p_e, \vec{\Gamma}_i\right)$$

- **Quasineutral** $(n_i \approx n_e)$ **DD model**:

  [HPHall (Fife, Martinez-Sanchez, 1998), HPHall-2/3 (Ahedo 2006, Hofer 2008), Hall2De (Mikellides 2009), Detailed fluid model (Choi and Boyd 2008), other models (Boeuf and Garrigues 1998, Barral 2003, Hara 2014)]

- **Non-neutral** $(n_i \neq n_e)$ **DD model**:

  $$\frac{\partial n_e}{\partial t} + \nabla \cdot \left[-\overline{\overline{\mu_e}} \cdot \left(n_e \vec{E} + \nabla p_e\right)\right] = \dot{n}_e$$

  $$\& \; \nabla^2\phi = -\frac{e}{\epsilon_0}(n_i - n_e)$$

  Scharfetter-Gummel scheme, Dielectric relaxation time

  Used in LTP models [Kushner 2009]

Full fluid moment (FFM) model

Need non-oscillatory schemes for the nonlinear inertia term (cf. hyperbolic PDE)

Used in other fields, e.g., CFD and HTP [Hakim, Hammett, Srinivasan, Shumlak]

$$\frac{\partial U}{\partial t} + \nabla \cdot \vec{\Gamma} = S \text{ , where } U = [n, n\vec{u}, n\varepsilon] \;\; \& \;\; \nabla^2\phi = -\frac{e}{\epsilon_0}(n_i - n_e)$$

Fluid equations for magnetized plasmas

Total energy (TE) $\frac{\partial(n\epsilon)}{\partial t} + \nabla\cdot\left(n\epsilon\mathbf{u} + \mathbf{u}\cdot\bar{\bar{P}}\right) = \mathbf{j}\cdot\mathbf{E} + S,$ $S_{heat} = \boldsymbol{j}\cdot(\boldsymbol{E} + \boldsymbol{u}\times\boldsymbol{B})$

Internal energy (IE) $\frac{\partial(ne)}{\partial t} + \nabla\cdot[(ne+p)\mathbf{u}] = S_{\mathrm{trans}} + S,$ $S_{trans} = -\boldsymbol{u}\cdot(\boldsymbol{R} - \nabla p),$

$\epsilon = e$ *(internal energy)* $+K$ *(kinetic/drift energy)*

(i) If **non-magnetized** $(K \to 0)$, $S_{heat} = S_{trans}$
- Total energy input $(\boldsymbol{j}\cdot\boldsymbol{E})$ goes only to the internal energy.
- Drift-diffusion (DD) approximation can be *exactly* recovered.

(ii) If **magnetized** (e.g., ExB drift; $K \neq 0$), $S_{heat} \neq S_{trans}$
- $\epsilon = e + K \neq e$: TE and IE equations are NOT identical.
- DD approximation cannot be used for magnetized plasmas with drift.

[Hara, PSST **28**, 044001 (2019)]

Full-Fluid Moment (FFM) Model

Model description

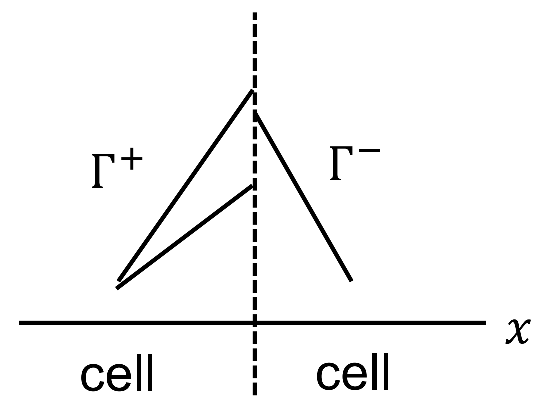
$$\frac{\partial U}{\partial t} + \nabla \cdot \vec{\Gamma} = S$$

flux source

- 1D2V/2D2V finite volume model: ions, electrons, and neutrals
- 5-moment fluid equation with Poisson equation
- Global Lax-Friedrich flux splitting with MUSCL reconstruction

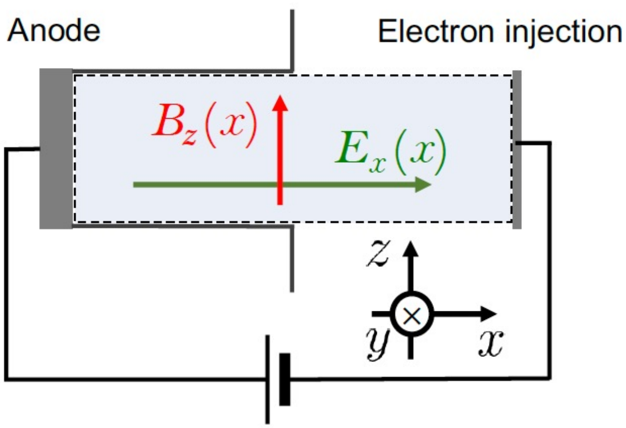
$$\Gamma = \Gamma^{+} + \Gamma^{-}$$

- Time integrator: Strong Stability Preserving Runge-Kutta (RK3-SSP)
- Boundary conditions
  - Kinetic flux treatment
  - Quasineutral electron injection at cathode for 1D
- Message Passing Interface (MPI) for 2D

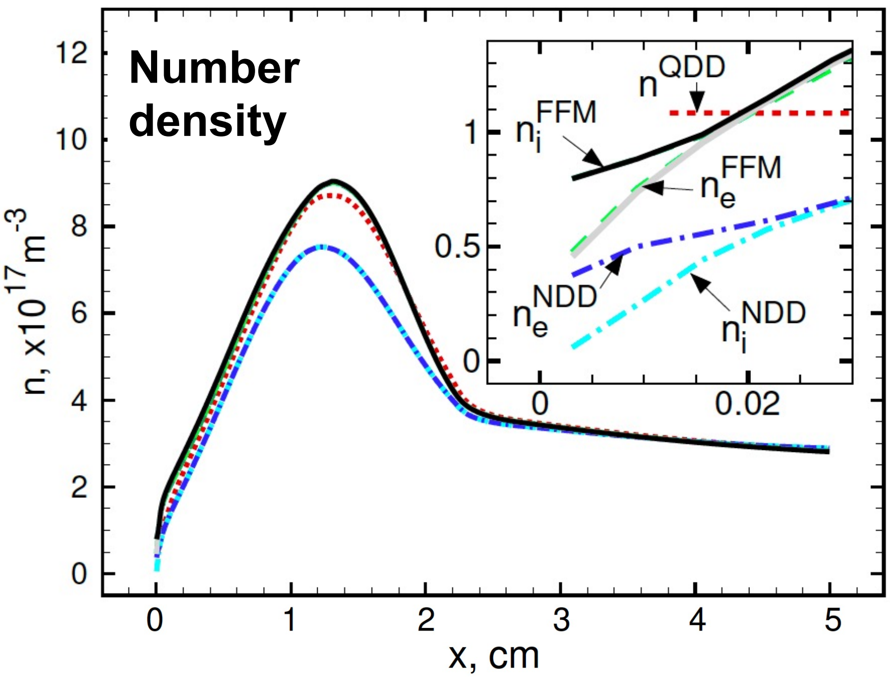


1D cross-field plasma discharge benchmark test

- Xenon mass flow from anode
- Ions: collisionless and non-magnetized
- Electrons: collisional and magnetized
- Applied potential drop of 300 V
- Applied magnetic field profile (160 G)
- Anomalous transport for steady state

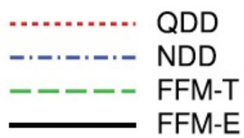


Full-fluid moment (FFM) model vs. Quasineutral drift-diffusion (QDD) and nonneutral drift-diffusion (NDD) models



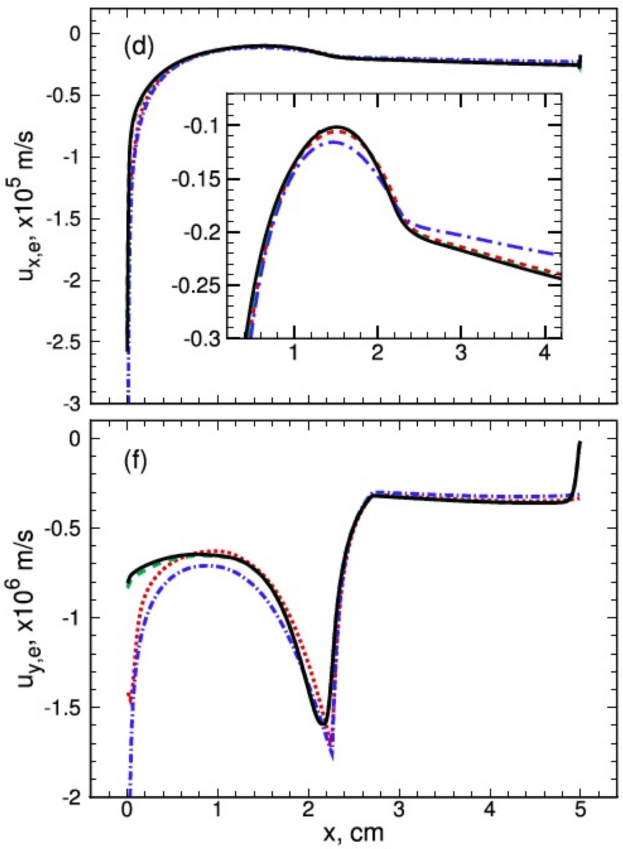
[R. Sahu, A. R. Mansour, and K. Hara, Phys. Plasmas **27,** 113505 (2020).]

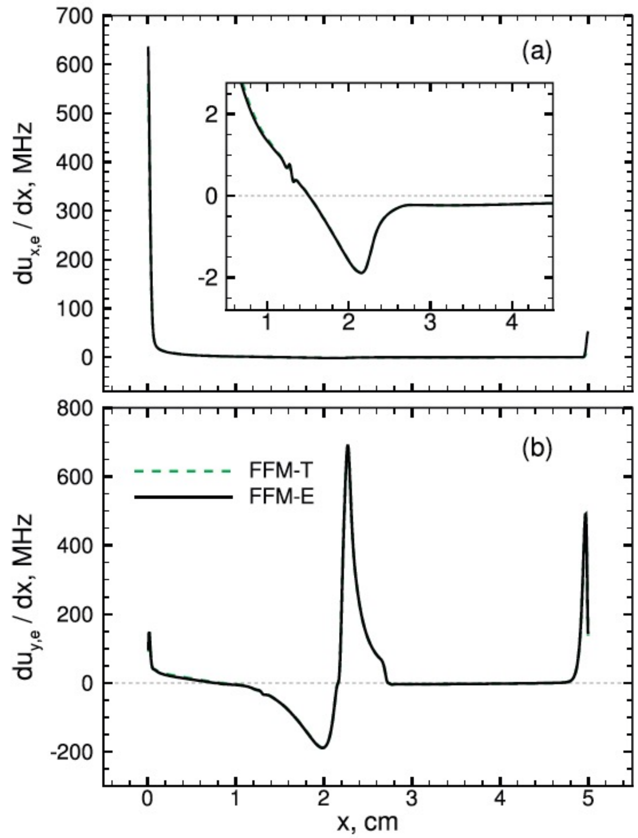
Inertia terms can be captured using the 5-moment FFM



u_{ex} **(axial)**

u_{ey} **(azimuthal)**





$\frac{\partial u_{ex}}{\partial x}$ **(axial shear)**

$\frac{\partial u_{ey}}{\partial x}$ **(azimuthal shear)**

[R. Sahu, A. R. Mansour, and K. Hara, Phys. Plasmas **27,** 113505 (2020).]

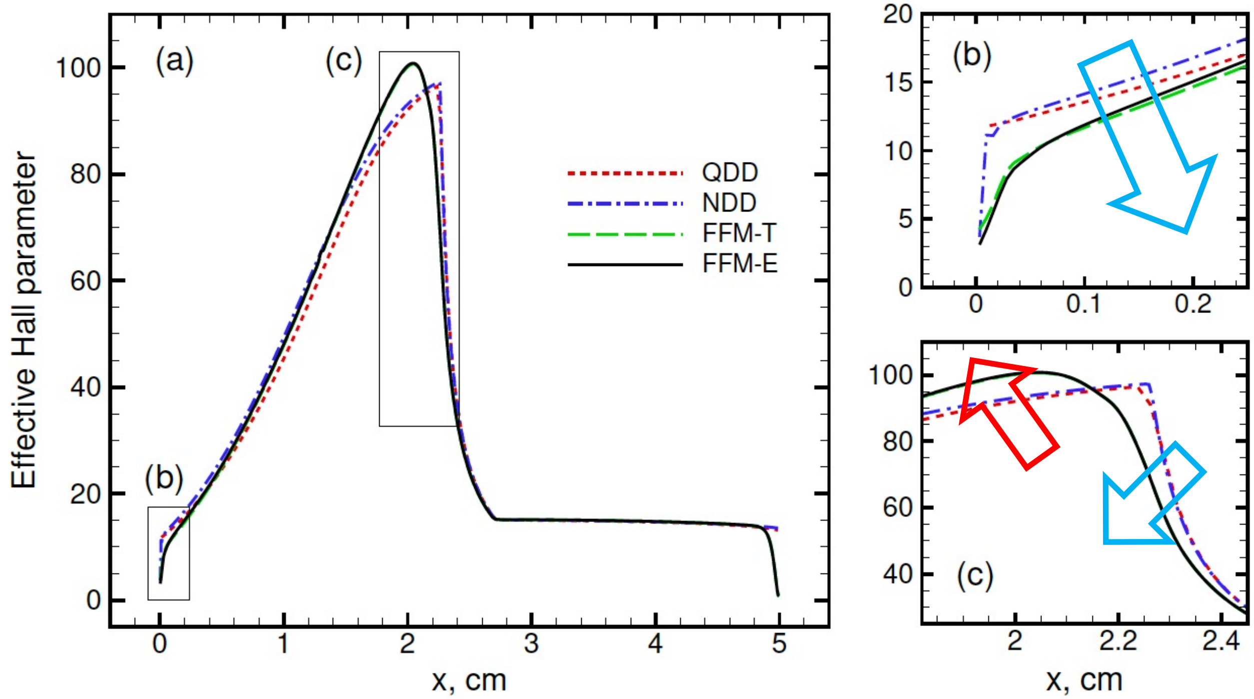
Shear-driven cross-field electron transport

- Effects of the nonlinear inertia term (e.g., shear) on electron transport are observed: **shear diamagnetic drift**.

$$\vec{u}_\perp = \frac{1}{|\vec{B}|^2}\left[-\vec{E}\times\vec{B} + \frac{1}{qn}(\nabla p \times \vec{B}) + \frac{m}{qn}\{\nabla\cdot(n\vec{u}\otimes\vec{u})\times\vec{B}\}\right]$$

Effective Hall parameter

$$\Omega_{eff} = \frac{\overline{nu_y}}{\overline{nu_x}}$$



u_x accelerates toward anode (sheath)

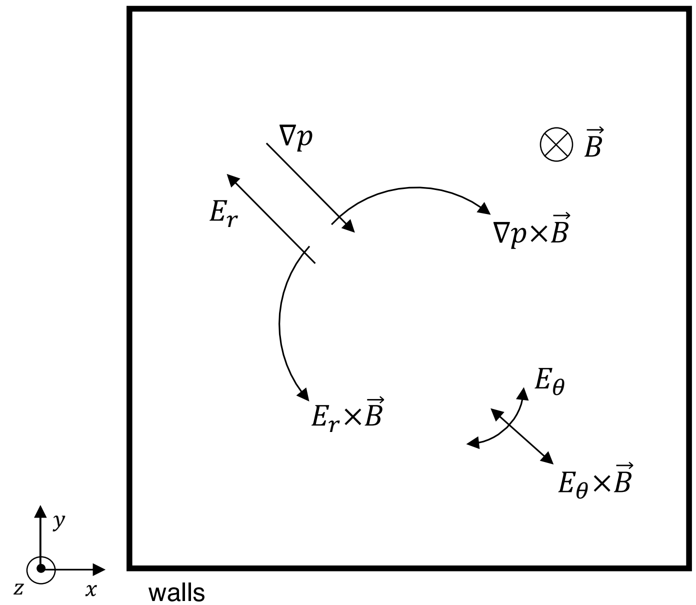
u_y (ExB drift) is maximum near channel exit

[R. Sahu, A. R. Mansour, and K. Hara, Phys. Plasmas **27,** 113505 (2020).]

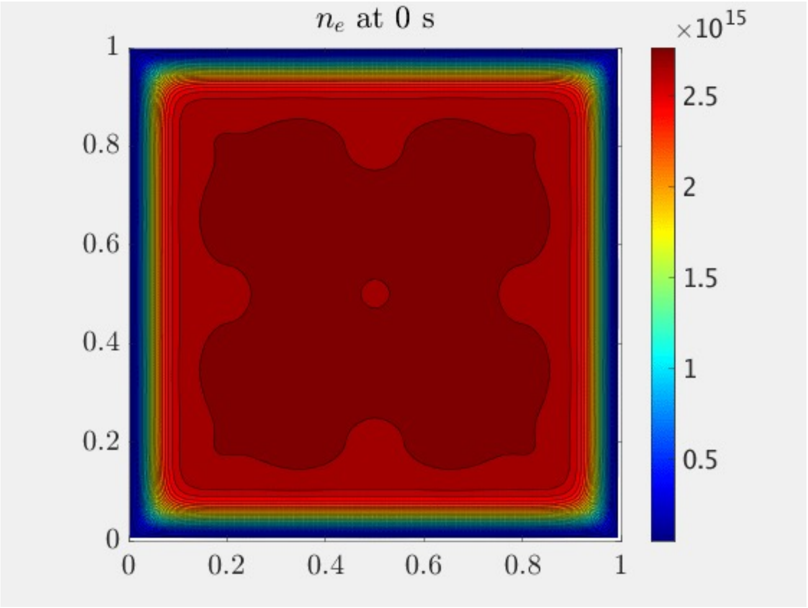
2D FFM model of Penning discharge

Movie

- Steady-state discharge is maintained with the ionization (source)
- Ambipolar diffusion sets up: electric field and density gradient



Numerical setup



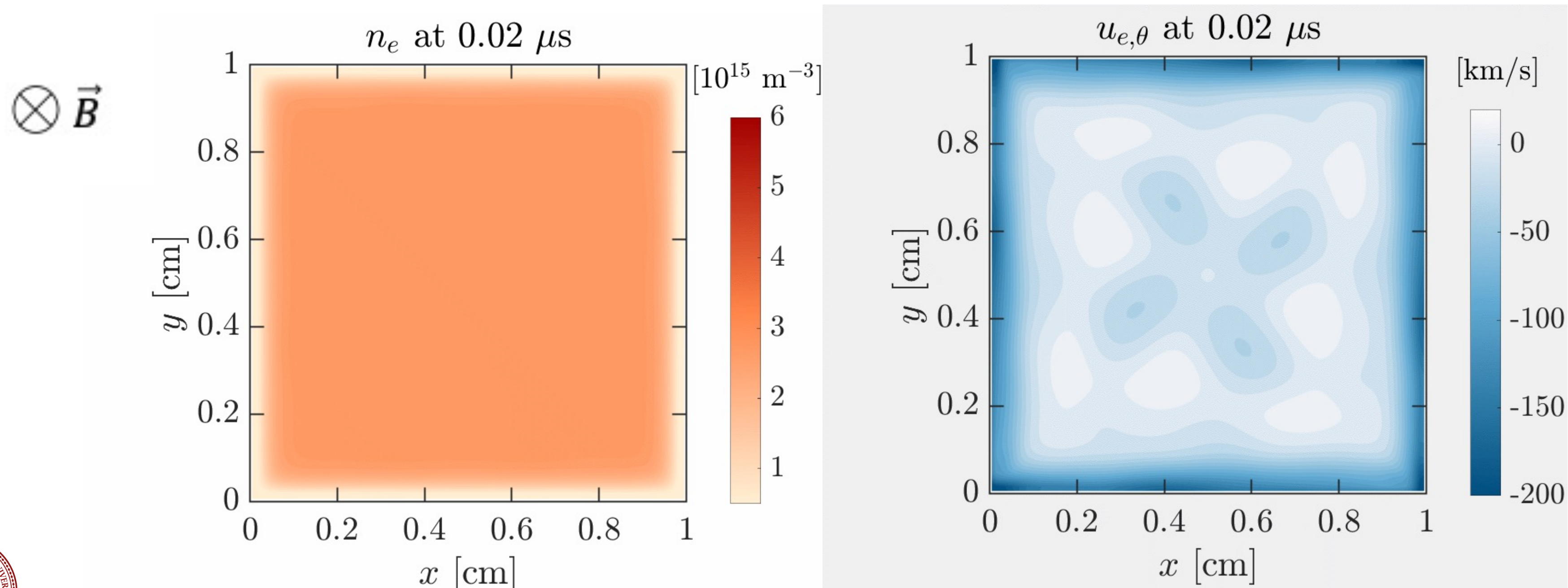
Unmagnetized case

[A. R. Mansour and K. Hara, AIAA 2021, GEC 2021; submitted]

Rotating spoke in Penning configuration

Movie

- Azimuthally rotating spokes are robustly observed using a 2D full fluid moment (FFM) model.
- Rotation is in the direction of the **diamagnetic drift** (not ExB drift)



[A. R. Mansour and K. Hara, AIAA 2021, GEC 2021; submitted]

Summary: Part 2 (fluid theory and simulation)

- We showed that drift-diffusion (DD) models can be *invalid* in magnetized plasmas where the drift is large.
- We developed a *5-moment* full-fluid moment (FFM) model
  1. 1D cross-field discharge plasma. Benchmarked against quasineutral DD (QDD) and nonneutral (NDD). **Shear-induced electron transport**.
  2. 2D self-organizing **azimuthally rotating spokes** in a Penning-type setup. Spoke rotation found to be in the direction of diamagnetic drift.
- Current development: (i) capacitively coupled plasma (CCP) sources, (ii) plasma formation in high-power microwaves (HPMs), and (iii) RF breakdown.
- We are also developing a *10-moment* FFM model [Kuldinow, Mansour, and Hara, AIAA SciTech 2022]

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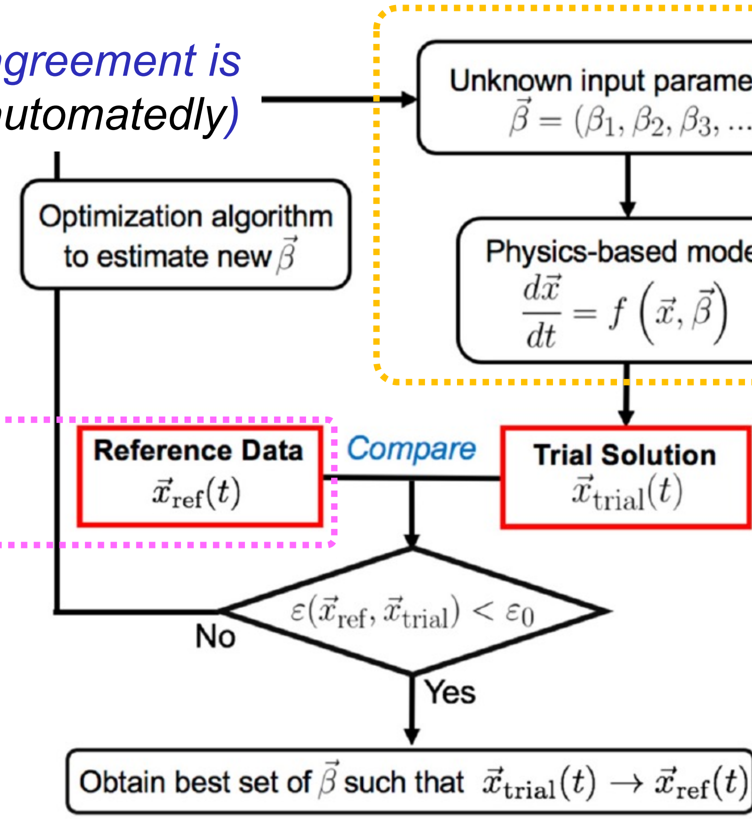
4. **Conclusion**

Data-driven modeling: offline validation

***Iterate** until agreement is obtained (manually /automatedly)*



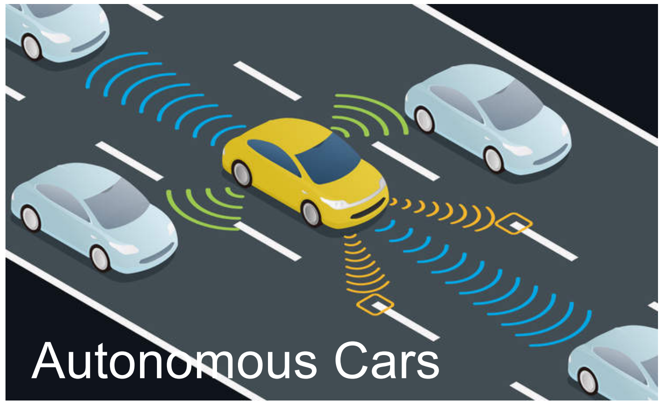
Experimental data collection

Low-fidelity plasma system model

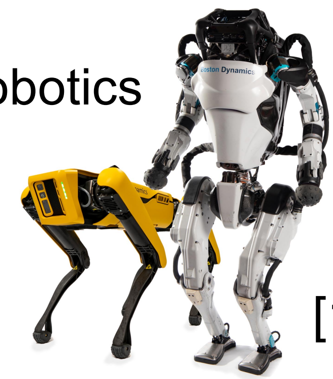
Data-driven model (model-data fusion) toward predictive modeling tool

[Greve, Hara, Koo, Eckhardt, Martin, J. Appl. Phys **125**, 244901 (2019)]

Introduction: online estimation modeling



Robotics



[1]

- *Real-time* estimation of states, parameters, and associated uncertainties
- Examples: **extended/ensemble Kalman filter** (*linearized model to predict*), **particle filter** (good for highly-nonlinear, unstable systems), **recursive least squares** (*fitting a function*), **neural network** (typically *black box; works well when training data are abundant*)

[1] Boston Dynamics

Real-time estimation of plasma oscillations

Can we estimate the unknown parameter (=Te) from measurement?

Physics-based model. 0D time-dependent predator-prey system [*]:

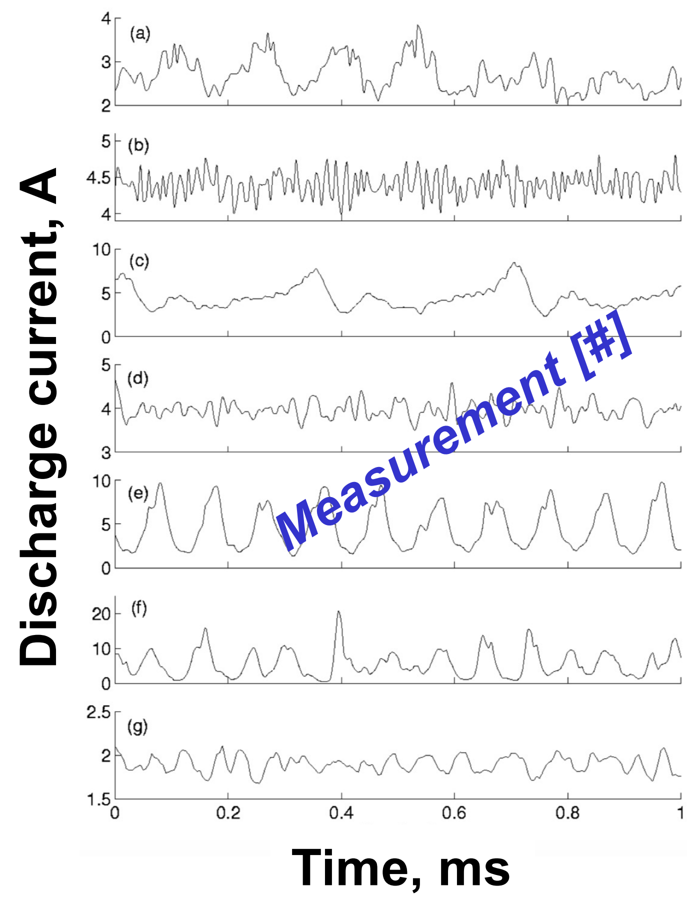
Ions: $\frac{dN_i}{dt} + \frac{N_i U_i}{L_{ch}} + \frac{2N_i U_{i,w}}{R_\Delta} = N_i N_n \xi_{ion}$

Neutrals: $\frac{dN_n}{dt} + \frac{(N_n - N_{int})U_n}{L_{ch}} = -N_i N_n \xi_{ion}$

Ionization rate coefficient:

$$\xi_{ion} \approx \left[A T_e^2 + B \exp\left(-\frac{C}{T_e}\right)\right]\left(\frac{8eT_e}{\pi m}\right)^{1/2}$$

Inverse problem



[*] K Hara, M. Sekerak, I Boyd, A. Gallimore, Phys Plasmas (2014)
[#] N Gascon and M Dudeck et al., Phys Plasmas (2003)

Physics-informed extended Kalman filter (EKF)

$$\frac{dN_i}{dt} = f_1(N_i, N_n, T_e)$$
$$\frac{dN_n}{dt} = f_2(N_i, N_n, T_e)$$
$$\frac{dT_e}{dt} = f_3(N_i, N_n, T_e)$$

Satisfy physical constraints!!

Model forecast (predictor)

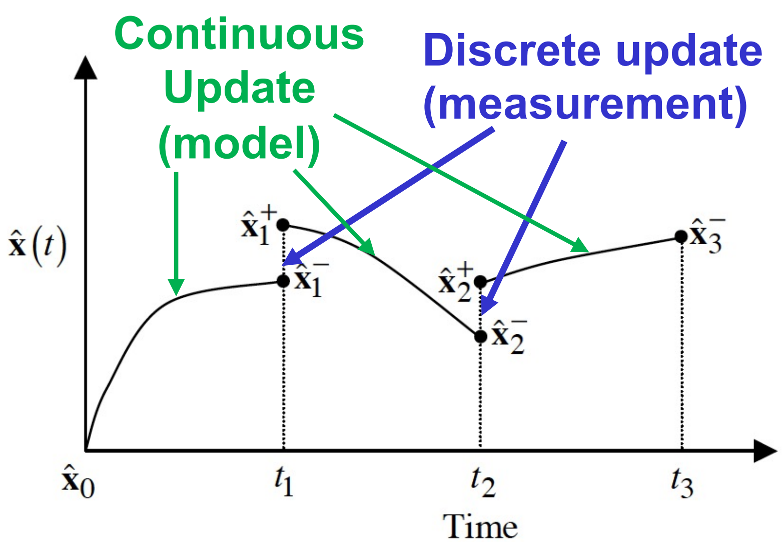
$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{f}(\hat{\boldsymbol{x}})$$

- Estimated variables: $\hat{\boldsymbol{x}}$
- Estimated covariance (error): $\boldsymbol{P}$

Data assimilation (corrector)

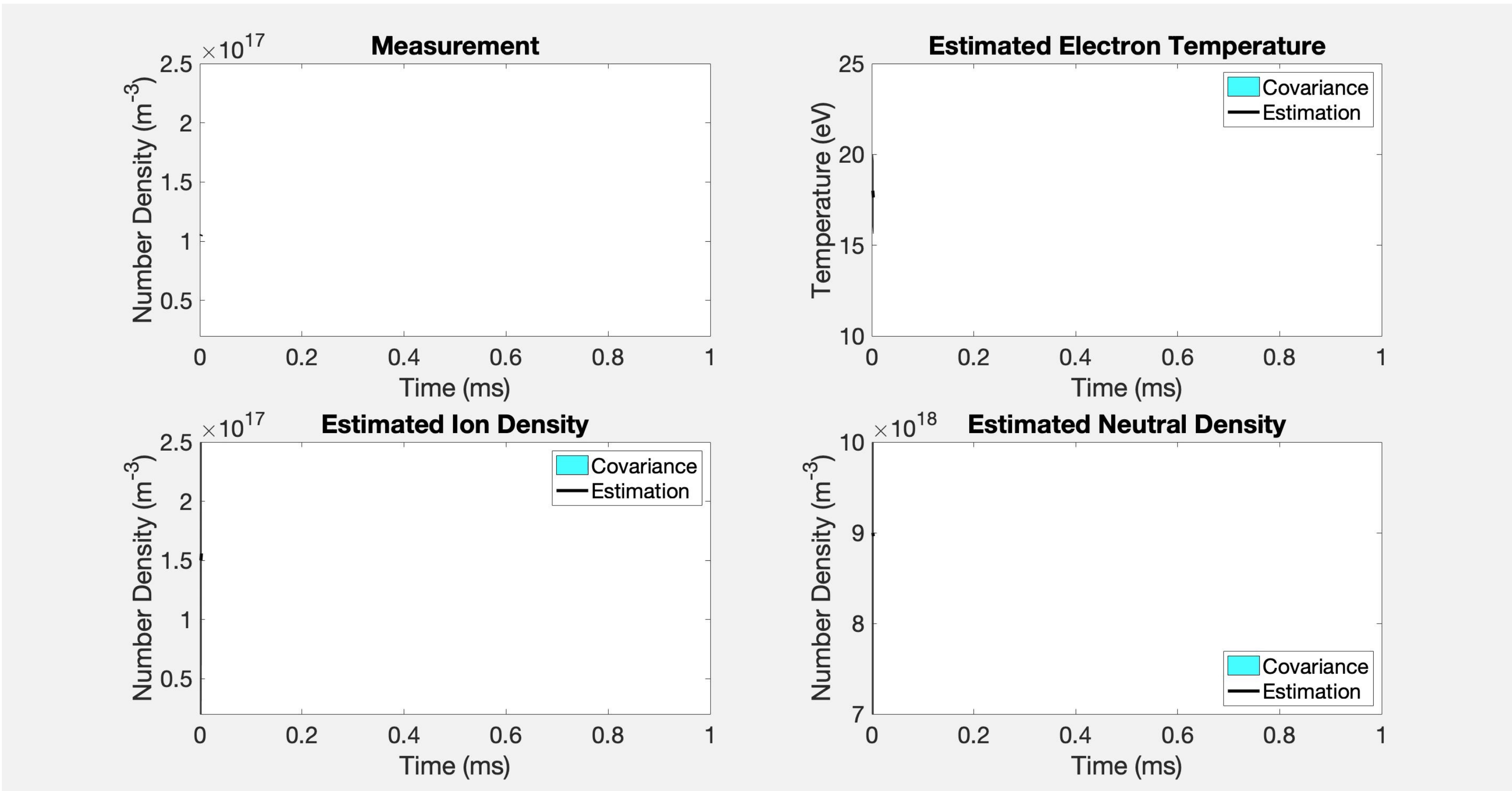
- Measurements: $\tilde{\boldsymbol{y}}$
- Measurement error: $\boldsymbol{\sigma_r}$
- Update estimated variables:

$$\hat{\boldsymbol{x}} \leftarrow \hat{\boldsymbol{x}} + \boldsymbol{K}[\tilde{\boldsymbol{y}} - \boldsymbol{h}(\hat{\boldsymbol{x}})]$$



Kalman gain: $\boldsymbol{K}$ (*a coefficient for how much the estimated quantity is* *updated/corrected* *by the measurement*)

EKF for plasma oscillations in Hall effect thrusters



[Greve, Majji, and Hara, Phys. Plasmas **28**, 093509 (2021)]

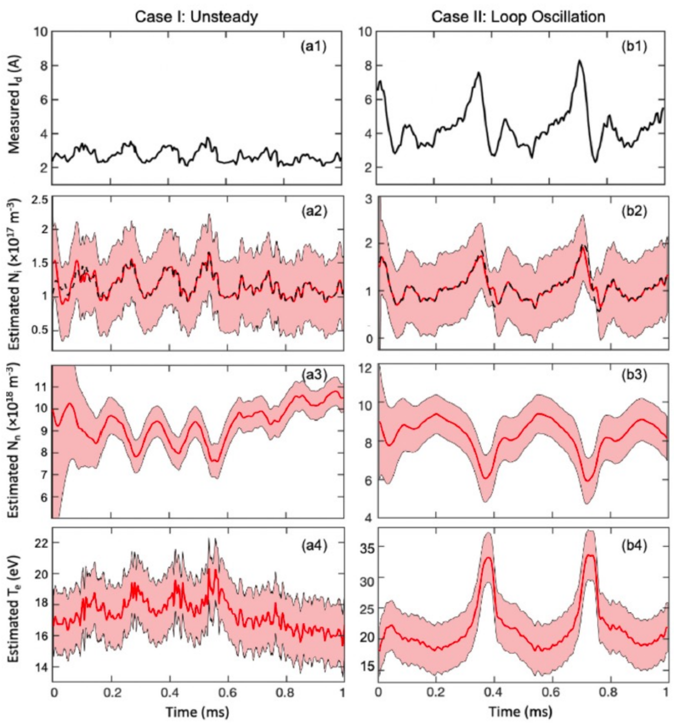
EKF for plasma chemistry and reaction

Data (I_D) [Gascon 2003]

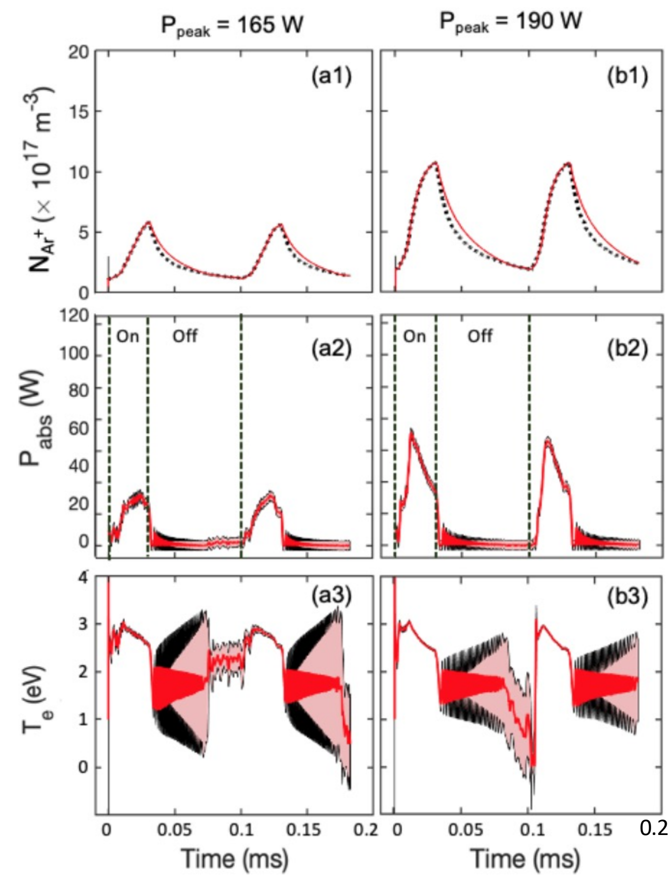
n_i

n_n

Unknown T_e



Plasma oscillation in HETs



Pulsed ICP modes

Data (plasma density) [Hebner 1997]

Unknown P_{abs}

Te

[Greve, Majji, and Hara, Phys. Plasmas **28**, 093509 (2021)]
[Greve and Hara, GEC 2021; in preparation]

Summary: Part 3 (model-data fusion)

- **Model-data fusion** is an approach to use *low-fidelity* physics-based models and augment/correct the estimates with experimental data.
  - Offline approach: correlation of datasets / data calibration
  - Online (**real-time**) approach: use incoming measurements and uncertainty propagation to estimate states and parameters

- We are currently applying the EKF model to a few applications
  - Plasma chemistry in semiconductor devices
  - 1D and 2D partial differential equations (PDEs)
  - Circuit and plasma coupling in electric propulsion systems

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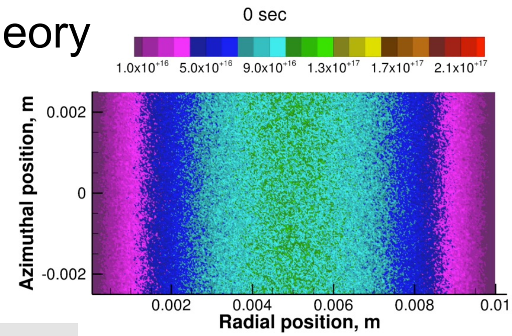
Toward a self-consistent model of electron transport in partially magnetized plasmas

GHz
(ns)

MHz
(μs)

kHz
(ms)

Verification with kinetic instability theory



Small scale physics

Nonlinear coupling

Device scale phenomena

Validation with observable experimental data (discharge, plasma, gas, materials, vacuum facility)

Self-consistent plasma model

1. A new electron **fluid** model = fluid turbulence
2. A massively parallel **kinetic** (e.g., particle-in-cell) simulation
3. Data-driven modeling, Model-data fusion

Conclusion

We are developing **physics-based** models

- **Kinetic**: particle-in-cell (PIC), Monte Carlo collision (MCC), direct simulation Monte Carlo (DSMC), grid-based direct kinetic (DK)
- **Fluid**: drift-diffusion (DD), full-fluid moment (FFM)

as well as **data-driven** models

- **Online** (real-time) state estimation: extended Kalman filter (EKF)

for a wide range of plasma applications and rarefied gas flows.

These simulations, verified with theory (e.g., linear instability), are used to understand the nonlinear saturation, i.e., turbulence, leading to cross-field plasma transport.

Special thanks to…

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- John Luginsland (AFOSR)
- Prabhat Kumar (former postdoc; LBNL)

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- MIPSE

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