# Effects of Random Circuit Fabrication Errors on Small Signal Gain in a Traveling Wave Tube

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**This work was supported by AFOSR, ONR, L-3 Communications Electron Devices and Northrop Grumman Corporation.**

# Abstract

Random fabrication errors due to fabrication tolerances in the manufacturing of slow wave circuits may have detrimental effects on the performance of traveling-wave tubes (TWTs) of all types. Such errors will pose an increasingly serious problem as TWTs are designed and built to operate in the sub-millimeter wavelength regime and beyond. Previous studies have shown numerical results on the expected degradation of the small signal gain of a TWT when small random, axially varying perturbations are present in the circuit phase velocity [1]. We present a new scaling law for the reduction in the ensemble-average gain that is derived from the third order ordinary differential equation with randomly varying coefficients that governs the beam-wave interaction in a TWT in the presence of small errors. Analytical results for the average gain reduction compare favorably over a broad range of Pierce parameter values with results from numerical integrations of the differential equation. The effects of random errors on the output phase will also be discussed in our presentation.



[1] Pengvenich, Chernin, Lau, Luginsland, Gilgenbach, *IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

# Motivations

– Random circuit errors affect TWT performance, manufacturing yield, and cost. This problem is increasingly serious at millimeter and sub-millimeter wavelengths.

– Seek to derive scaling laws for the ensemble-averaged gain and phase for TWT with random axial variations in circuit parameters.

# Continuum Model of TWT

- Governing third-order differential equation in $x = \omega z/v_b$ according to Pierce's 3-wave theory

$$\frac{d^3 f(x)}{dx^3} + jC\left(b - jd\right)\frac{d^2 f(x)}{dx^2} + jC^3 f(x) = 0$$

where

$f(x) = e^{jx} E_{rf}(x)$ is Pierce's 3-wave solution;

$b$ is the mismatch between beam and circuit phase velocities;

$C$ is Pierce's gain parameter, assumed to be a constant;

$d$ is the cold tube circuit loss rate, assumed zero here;

$QC = 0$.

Concentrate mainly on random variations in $b(x)$

# Random Circuit Fabrication Errors

- Assume circuit phase velocity, $v_p$, with a random error represented by $q(x)$,

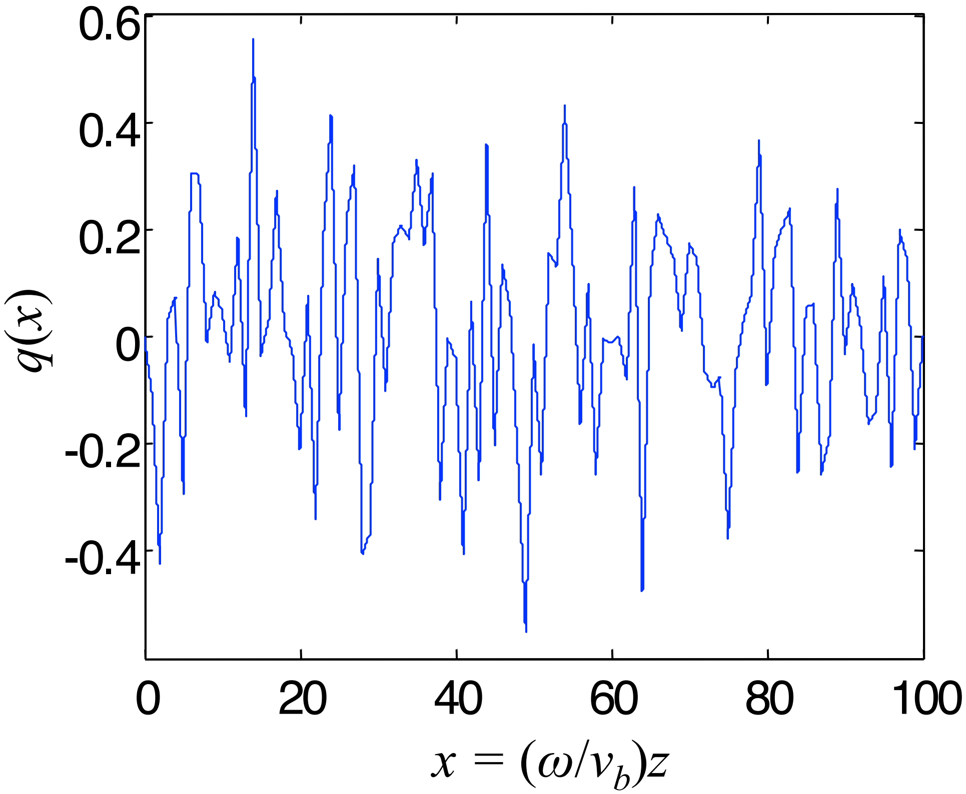
$$v_p = v_{p0}\left[1 + q(x)\right]$$

- $q(x)$ is a random function of $x$ with zero mean, standard deviation $\sigma_q$, and

$$b(x) = \frac{1}{C}\left[\frac{Cb_0 - q(x)}{1 + q(x)}\right]$$

# Random Error Profile: Piecewise Continuous Gaussian

- Sample piecewise linear Gaussian random function with zero mean; $\sigma_q = 0.2548$



# Modifications in Output Gain and Phase

$$\frac{f(x)}{f_0(x)} = e^{G_1 + j\theta_1}$$

$f_0(x)$ = error-free, 3-wave solution

$G_1$ and $\theta_1$ depend on:

tube length ($x$)

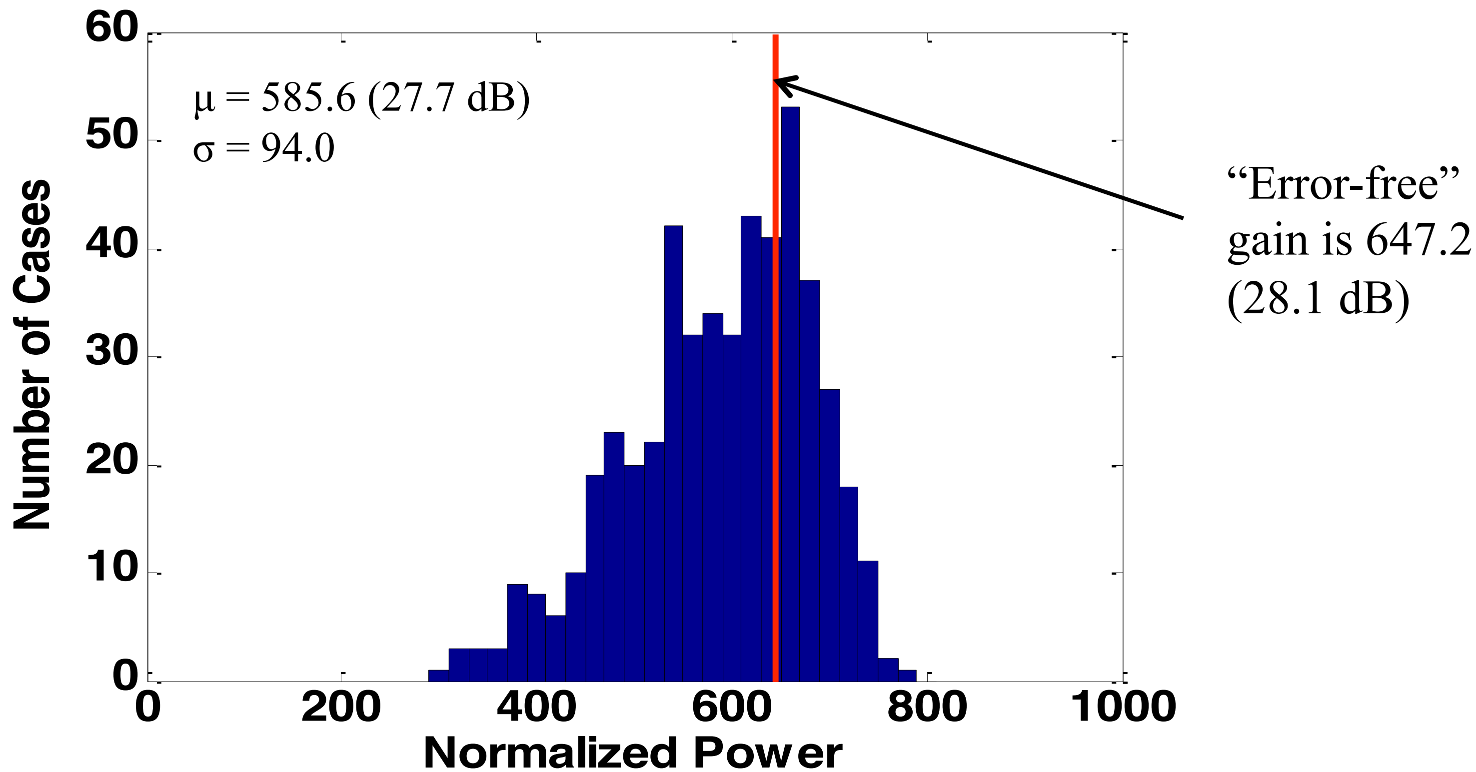
correlation length ($\Delta = x/N$)

standard deviation in $b$ ($\sigma_b$)

# Statistics of 500 Runs

Power distribution at $x$ = 100 for $b_0 = d = 0$, $C = 0.05$

$\sigma_b = 1.7$ (corresponds to $\sigma_q = 0.085$)



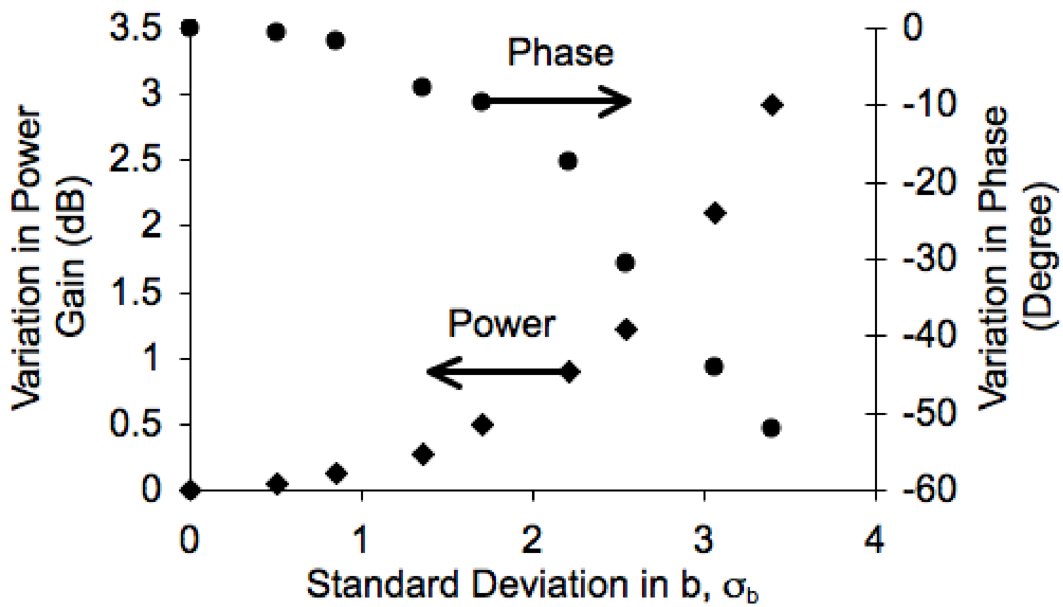

Note: In significant number of cases the gain is actually higher (!) than the "error-free" case.



Pengvenich *et. al.*, *IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

# Statistics of 500 runs (cont' d)

Variations at $x = 100$ for $b_0 = d = 0$, $C = 0.05$




The variations in gain and phase appear quadratic in $\sigma_b$. Scaling law is to be derived (in this paper).

Pengvenich *et. al., IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

# Two Analytic Approaches

- Perturbative analysis. Linear theory carried to second order in $q(x)$

$$\left\langle G_1(x) + j\theta_1(x) \right\rangle = -\frac{1}{2}\sigma_b^2 \Delta \int_0^x ds P(x,s)$$

$P(x,s)$ depends only on error-free, 3-wave solution.

- Riccati analysis. Nonlinear formulation of wavenumber, for a single wave

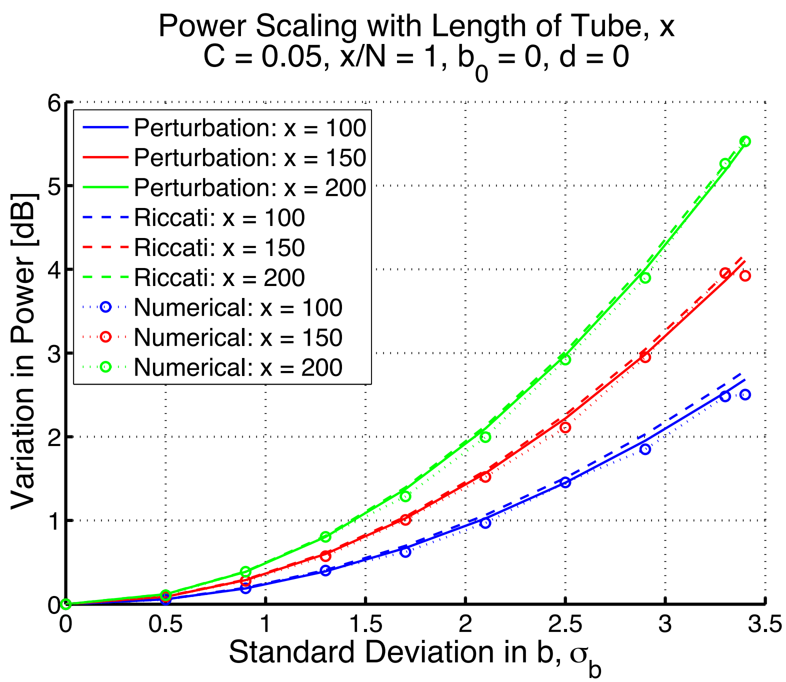
$$\left\langle G_1(x) + j\theta_1(x) \right\rangle = -\frac{\lambda}{2}\left(\frac{C}{1+Cb_0}\right)^2 x\sigma_b^2 \Delta$$

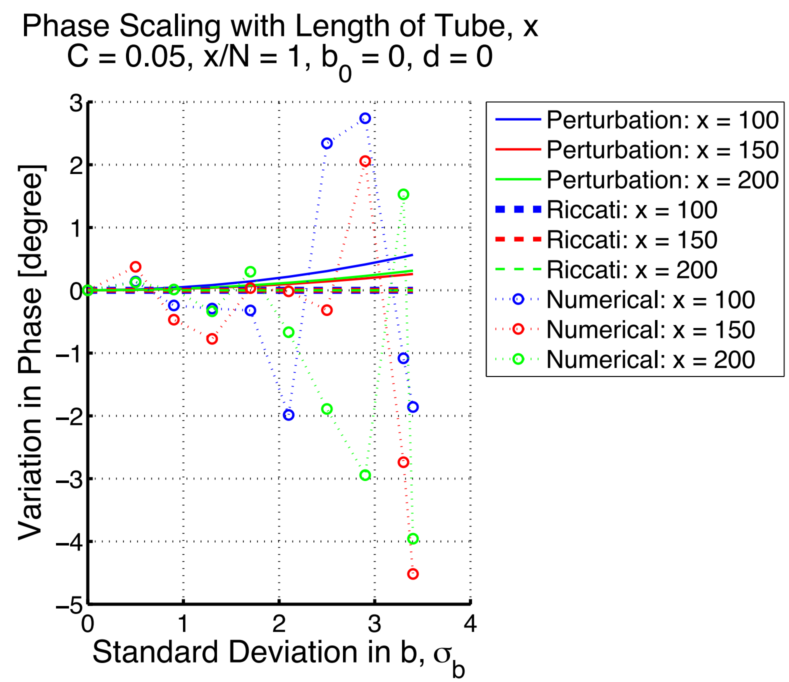
where λ is a complex constant that depends on the velocity mismatch parameter, $b_0$.

Note: $\left\langle \theta_1(x) \right\rangle = 0$ (!), $(b_0 = 0)$

# Perturbation, Riccati Analysis

## Scaling with x, $b_0 = 0$

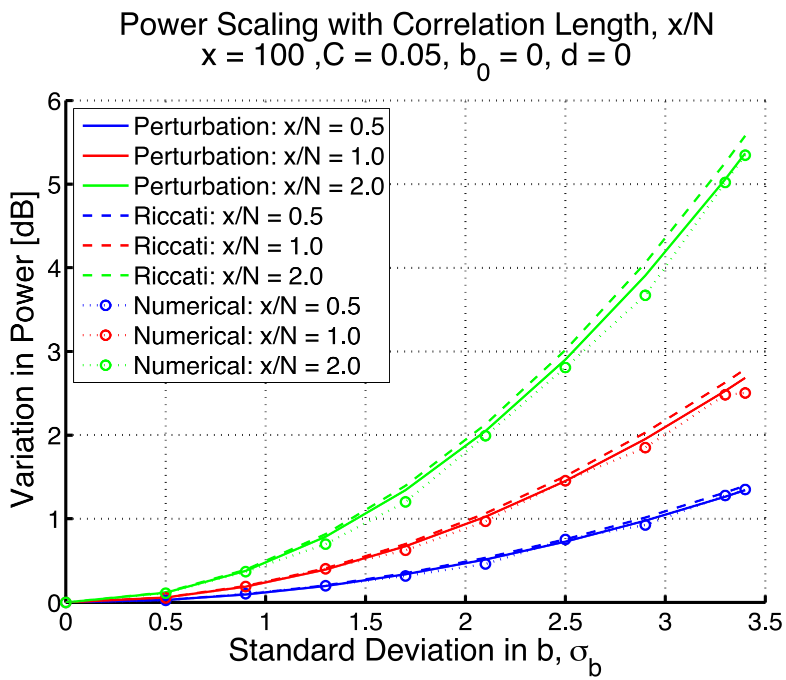


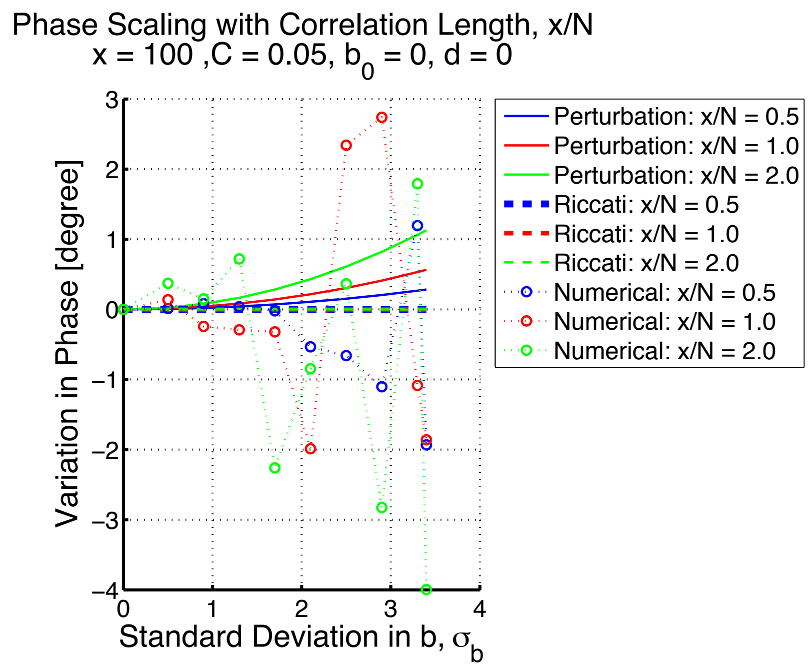




- Perturbation analysis shows good agreement with numerical solution (from 500 runs) for gain over a wide range of parameters
- Perturbation analysis yields phase variation close to zero, similar to the Ricatti analysis. Numerical solutions also show small variations in phase.

# Perturbation, Riccati Analysis

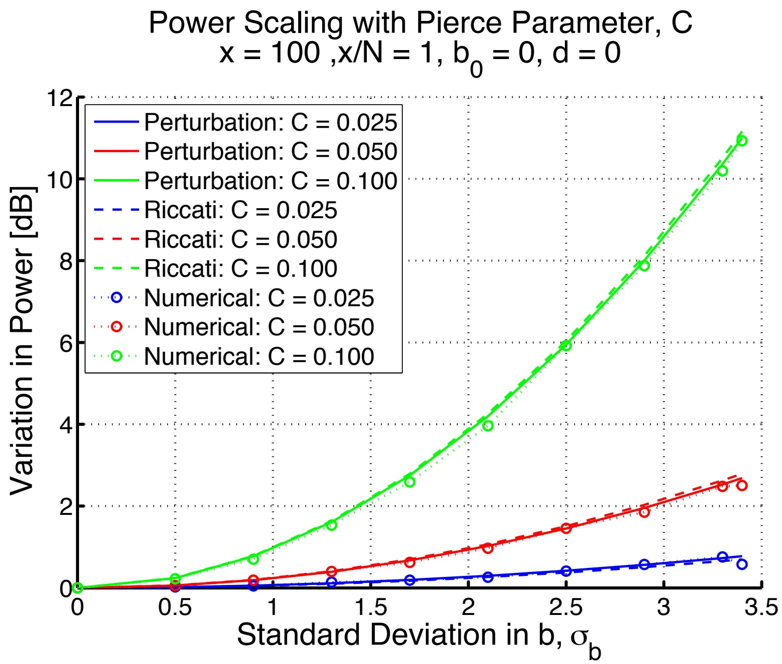
## Scaling with x/N, $b_0$ = 0

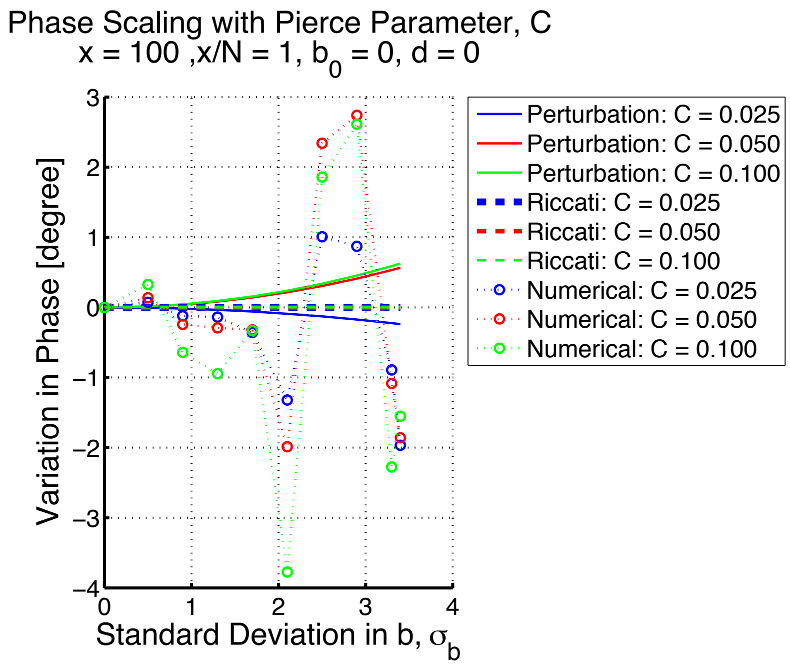


# Perturbation, Riccati Analysis

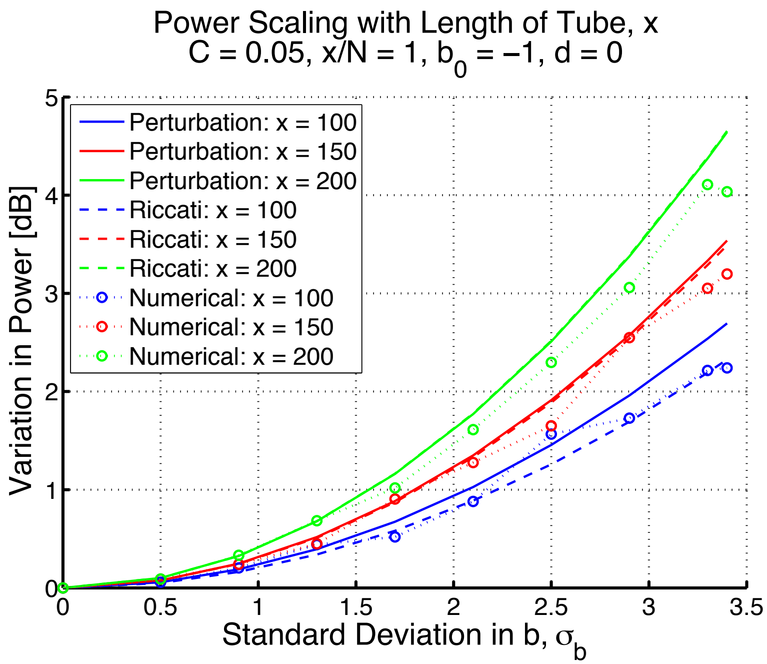
## Scaling with C, $b_0$ = 0

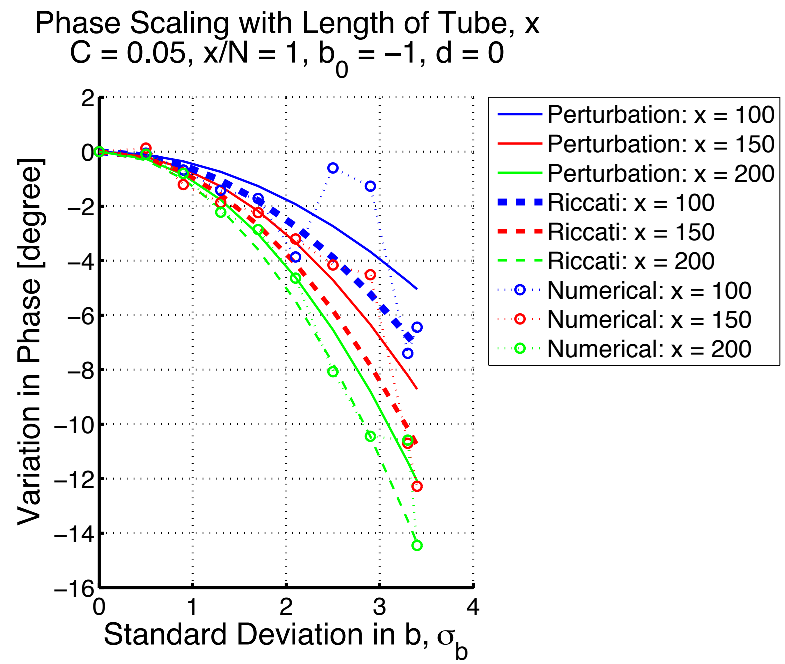


# Perturbation, Riccati Analysis

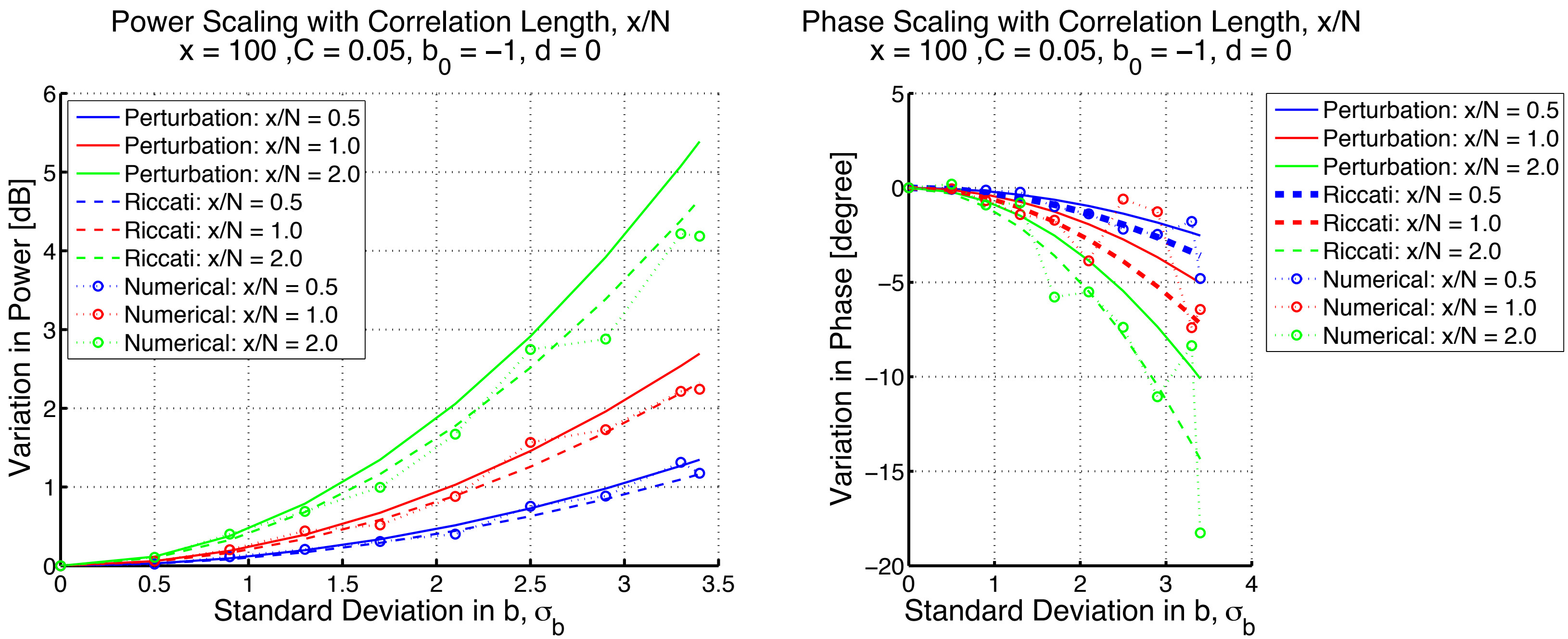
**Scaling with x, $b_0$ = -1**



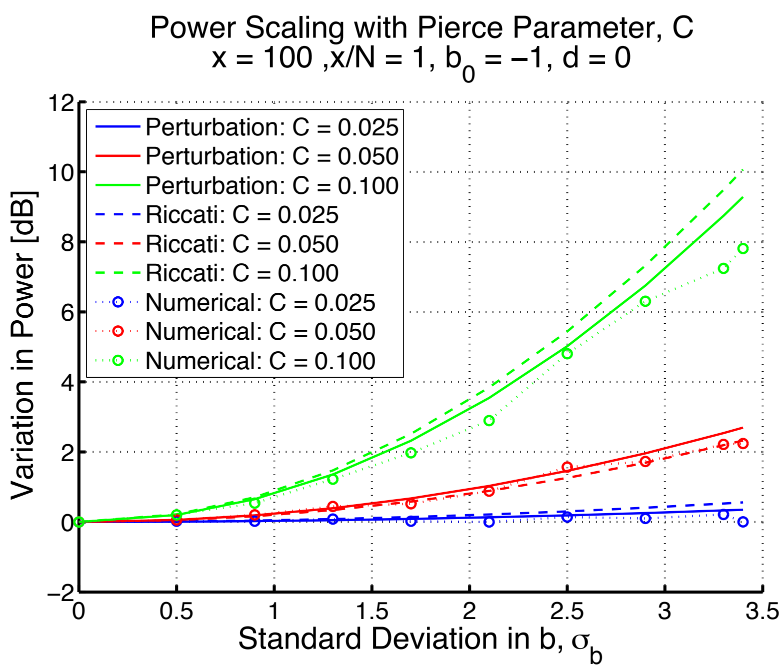
# Perturbation, Riccati Analysis

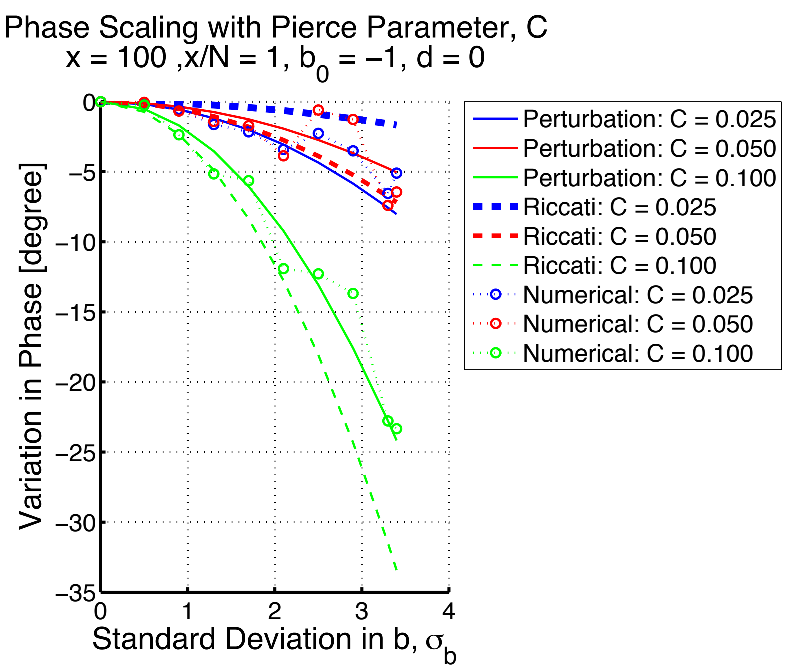
**Scaling with x/N, $b_0$ = -1**



# Perturbation, Riccati Analysis

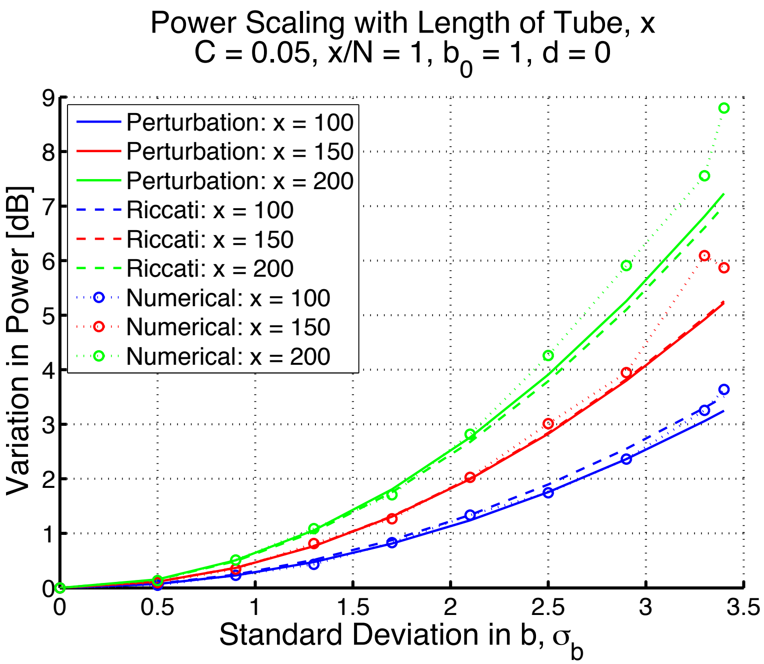
## Scaling with C, $b_0$ = -1

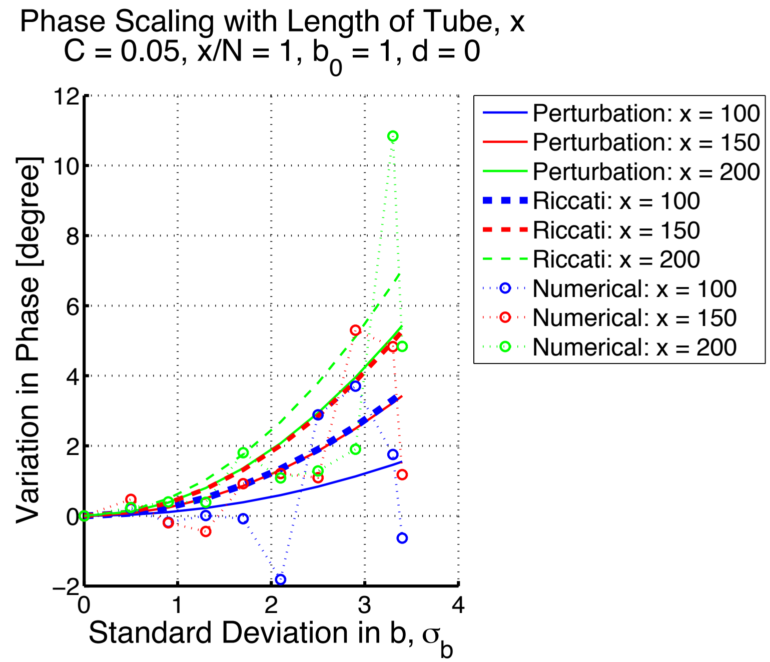


# Perturbation, Riccati Analysis

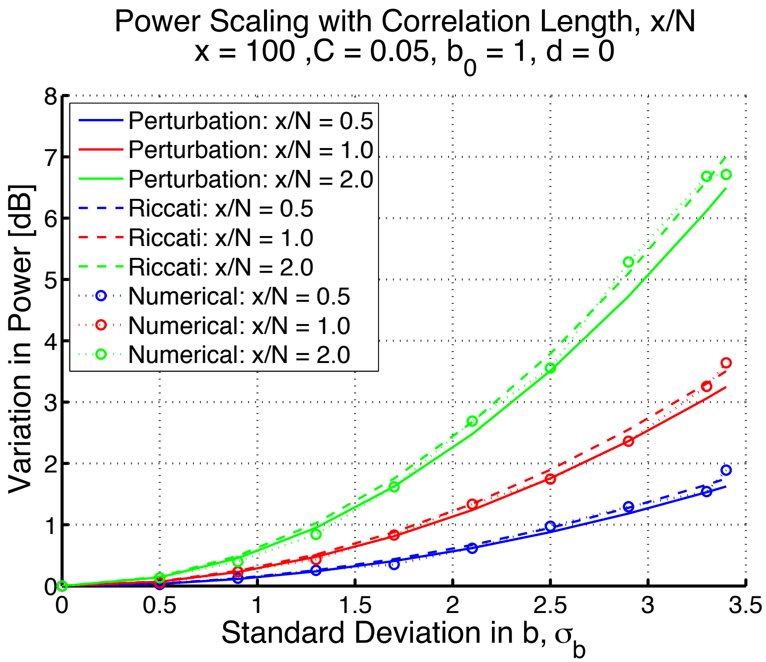
**Scaling with x, $b_0$ = 1**

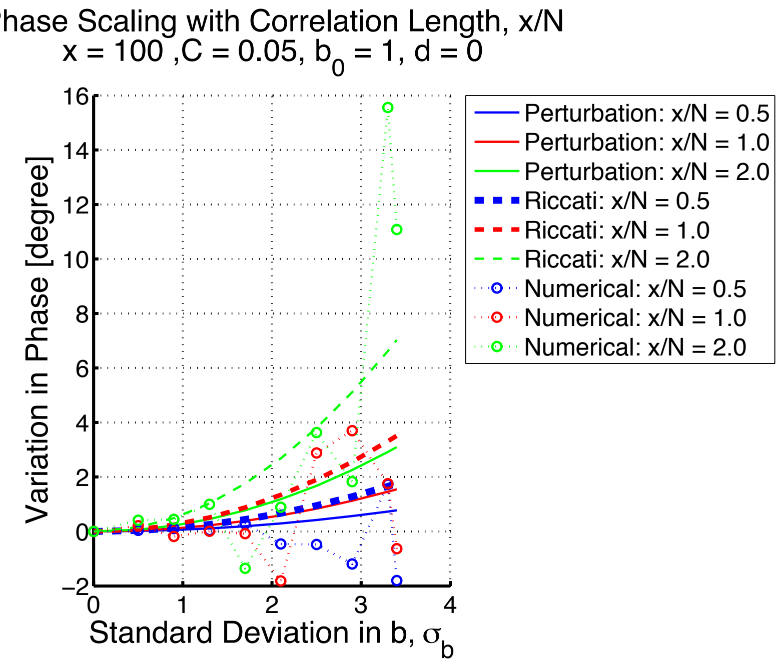


# Perturbation, Riccati Analysis

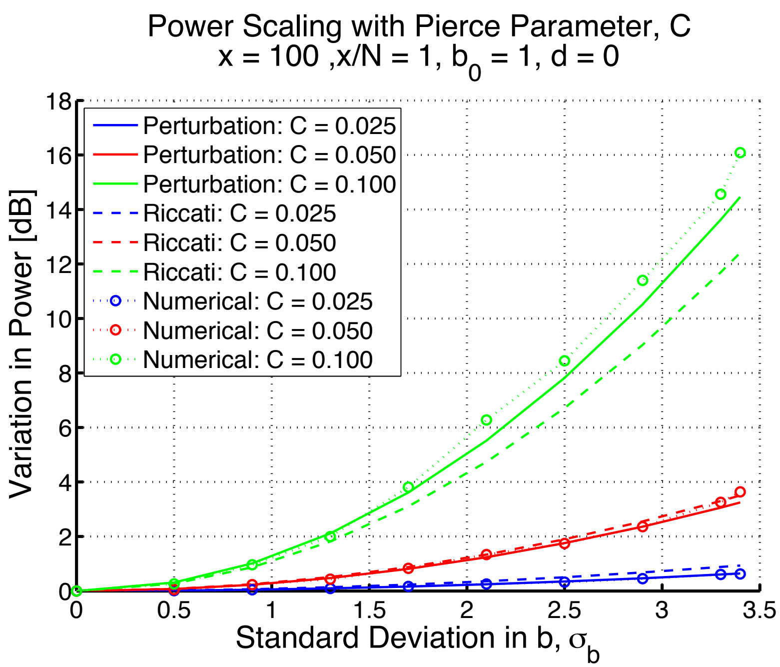
**Scaling with x/N, $b_0$ = 1**

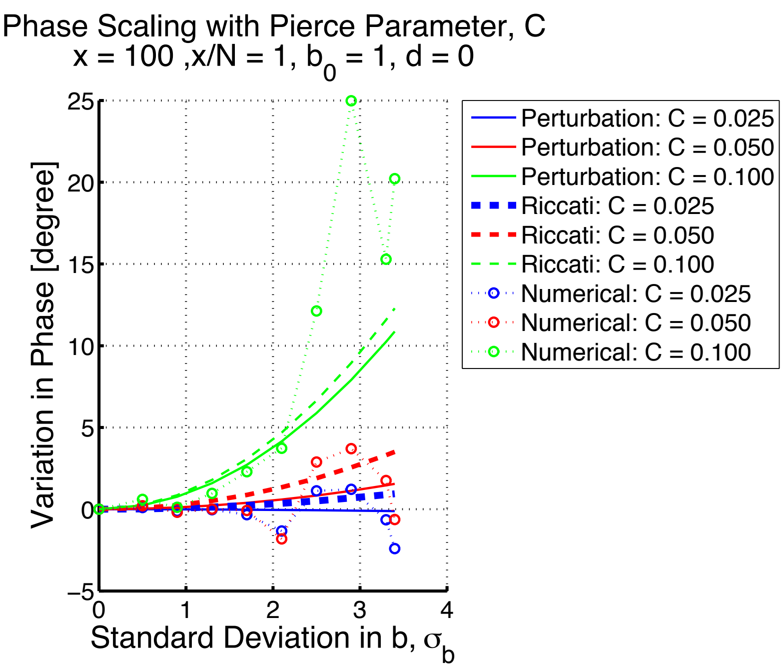


# Perturbation, Riccati Analysis

## Scaling with C, $b_0$ = 1



# Conclusion

- Deviation in small signal gain is found to be quadratic in $\sigma_b$, the standard deviation in Pierce's velocity mismatch parameter $b$. Deviation in phase is negligible for the cases of $b_0 = 0, 1$. Good agreement is found between analytic theory and numerical computation.