Effects of Random Circuit Fabrication Errors on the Mean and Standard Deviation of Small Signal Gain and Phase in a Traveling Wave Tube

Ian M. Rittersdorf[1], Thomas M. Antonsen, Jr.[2], David Chernin[3], and Y. Y. Lau[1,4]

1) University of Michigan, Ann Arbor, MI 48109, USA
2) University of Maryland, College Park, MD 20742, USA
3) Science Applications International Corporation, McLean, VA 22102, USA
4) Naval Research Laboratory, Washington, DC 20375

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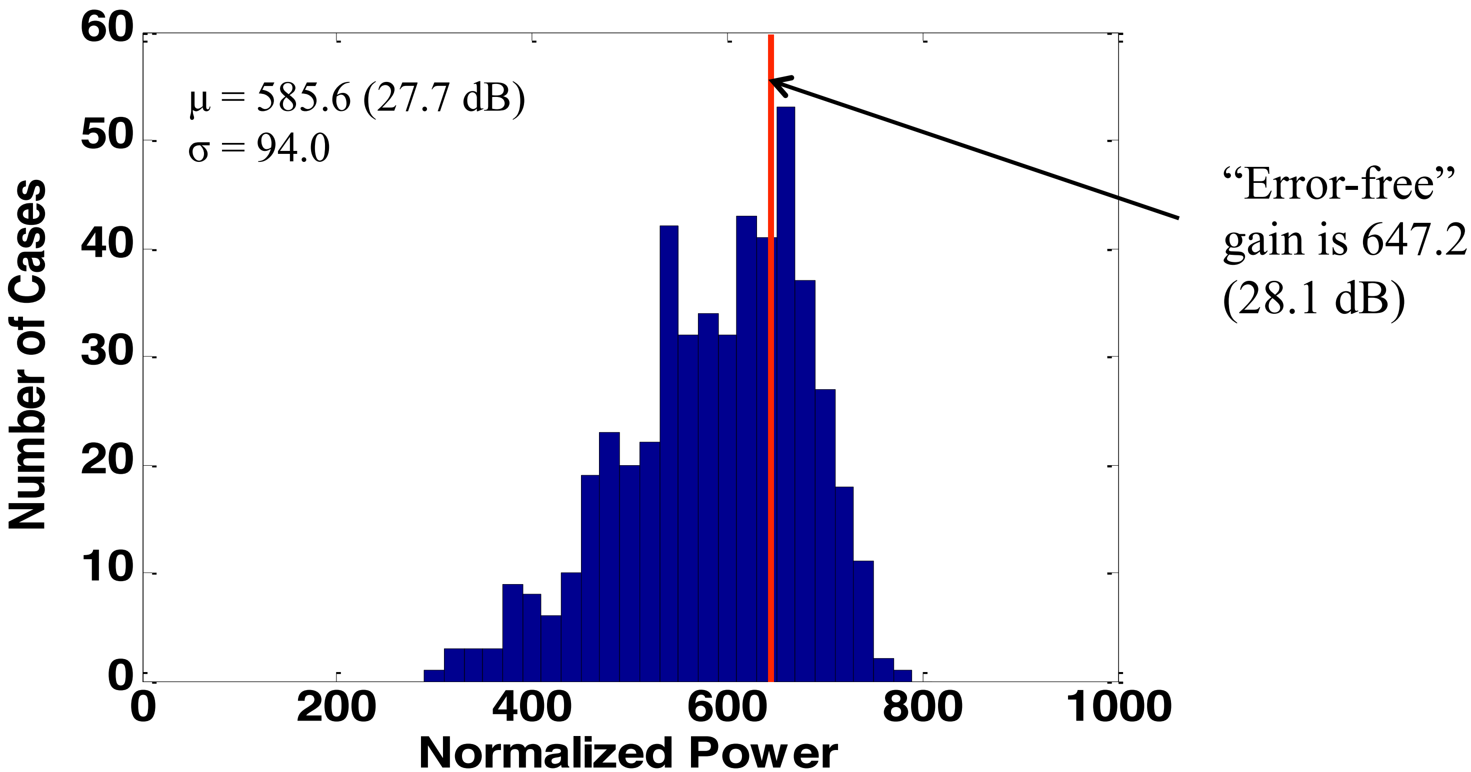
Motivations

- Random circuit errors affect TWT performance, manufacturing yield, and cost. This problem is increasingly serious at millimeter and sub-millimeter wavelengths
- The phase velocity of the circuit will be altered due to the random manufacturing errors
- Seek to derive scaling laws for the ensemble-averaged gain and phase for TWT with random axial variations in circuit parameters
- To extend existing work into regimes with non-synchronous beam velocities and to include the effects of the Pierce "space-charge" term

Previous Works

Power distribution at $x = 100$ for $b_0 = d = 0$, $C = 0.05$
$\sigma_b = 1.7$

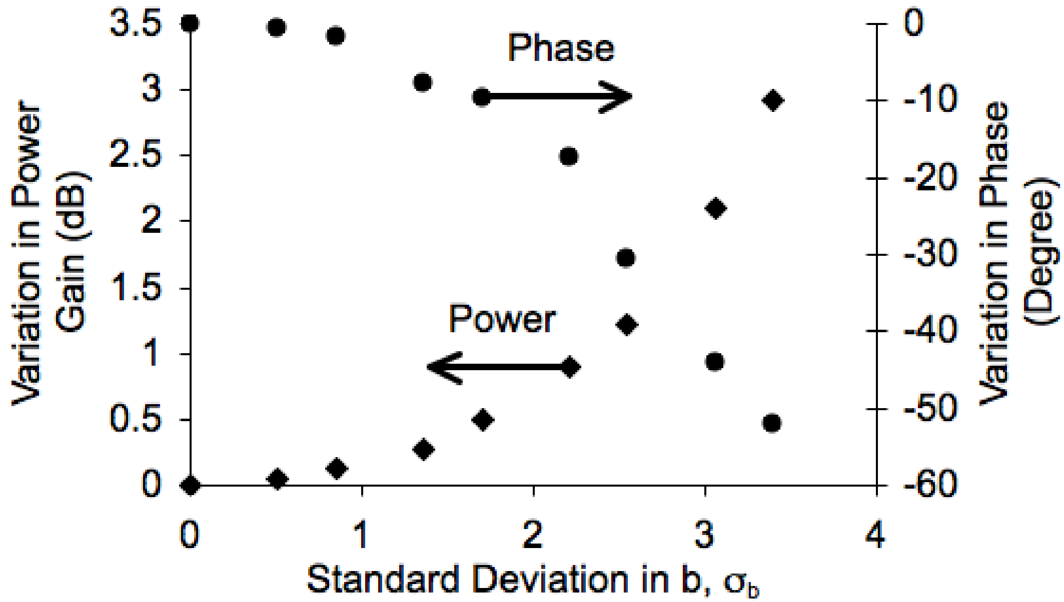


Note: In significant number of cases the gain is actually higher (!) than the error-free case.

[1] Pengvanich, Chernin, Lau, Luginsland, Gilgenbach, *IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

Previous Works (cont'd)

Variations at $x = 100$ for $b_0 = d = 0$, $C = 0.05$



The variations in gain and phase appear quadratic in σ_b (b = Pierce's detune parameter)

[1] Pengvanich, Chernin, Lau, Luginsland, Gilgenbach, *IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

Pierce Theory

Four Wave

$$\left(\frac{\partial}{\partial z}+j\frac{\omega}{v_b}\right)^2 s=\varepsilon$$ Force Law (2 beam modes)

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\omega^2}{v_p^2}\right)\varepsilon=-2\frac{\omega}{v_p}\left(\frac{\omega}{v_b}C\right)^3 s$$ Circuit Law (2 circuit modes)

Assuming $e^{j\omega t-jk_z z}$ dependence

s = electron displacement

ε = axial circuit field

ω = signal frequency

v_p = cold-circuit phase velocity

v_b = DC beam velocity

$C^3 \propto I_b$ is the Pierce gain parameter

Pierce Theory of Error Free Tube

Neglecting the backward wave gives the Pierce dispersion relation for three waves with $e^{j\omega t - j\beta z}$ dependence

$$\left(\delta^2 + 4QC\right)\left(\delta + jb + d\right) = -j$$

where

δ = is like k_z (spatial exponentiation rate)

$b = \frac{1}{C}\left(\frac{v_b - v_p}{v_p}\right)$ is the Pierce velocity parameter

d is the dimensionless Pierce loss parameter

$4QC$ is the beam space charge effects

Continuum Model of TWT

- When b, C, or d are allowed to vary axially, the Pierce dispersion relation is no longer valid
- Governing third-order differential equation in $x = \omega z/v_b$ by combining force law and circuit equation

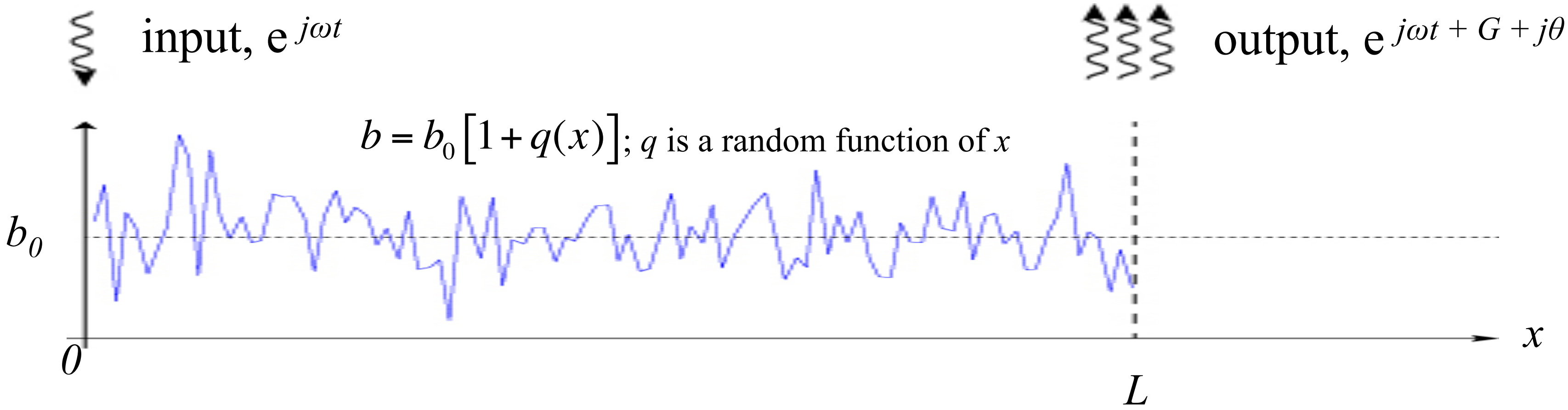
$$\frac{d^3 f(x)}{dx^3} + jC\left(b - jd\right)\frac{d^2 f(x)}{dx^2} + 4QC^3 \frac{df(x)}{dx} + jC\left(4QC^3\left(b - jd\right) + C^2\right)f(x) = 0$$

where $f(x) = e^{jx}s(x)$ is Pierce's 3-wave solution

- Random variations can be introduced into the parameters b, C, or d
  - Previous work has shown that variations in b produce greatest effect on the output
  - This work will concentrate only on random variations in $b(x)$

Random Circuit Fabrication Errors

- Assume velocity mismatch, *b*, with a random error represented by $q(x)$.



Numeric Approach

- Numerical Analysis. Solve

$$\frac{d^3 f(x)}{dx^3} + jCb(x)\frac{d^2 f(x)}{dx^2} + 4QC^3 \frac{df(x)}{dx} + jC\left(4QC^3 b(x) + C^2\right) f(x) = 0$$

5000 times

  - Each with a different random $b(x)$ profile
  - Assuming no losses, i.e. $d = 0$
  - Initial conditions: $f(0) = 0, f'(0) = 0, f''(0) = 1$
- Calculate the mean and standard deviation for the gain and phase

Effects of QC was also numerically analyzed recently by Professor John Booske's group [2]

[2] S. Sengele, M. L. Barsanti, T. A. Hargrave, C. M. Armstrong, J. H. Booske, and Y. Y. Lau, J. Appl. Phys., vol. 113, 074905 (2013).

Analytic Approaches

- Perturbative analysis. Linear theory carried to second order in $b(x)$, for all *three waves*

$$\left\langle G_1(x) + j\theta_1(x) \right\rangle = -\frac{1}{2}\sigma_b^2 \Delta \int_0^x ds\, P(x,s)$$

$P(x,s)$ depends only on error-free, 3-wave solution.

- Riccati analysis*. Nonlinear formulation of wavenumber, for a *single wave*

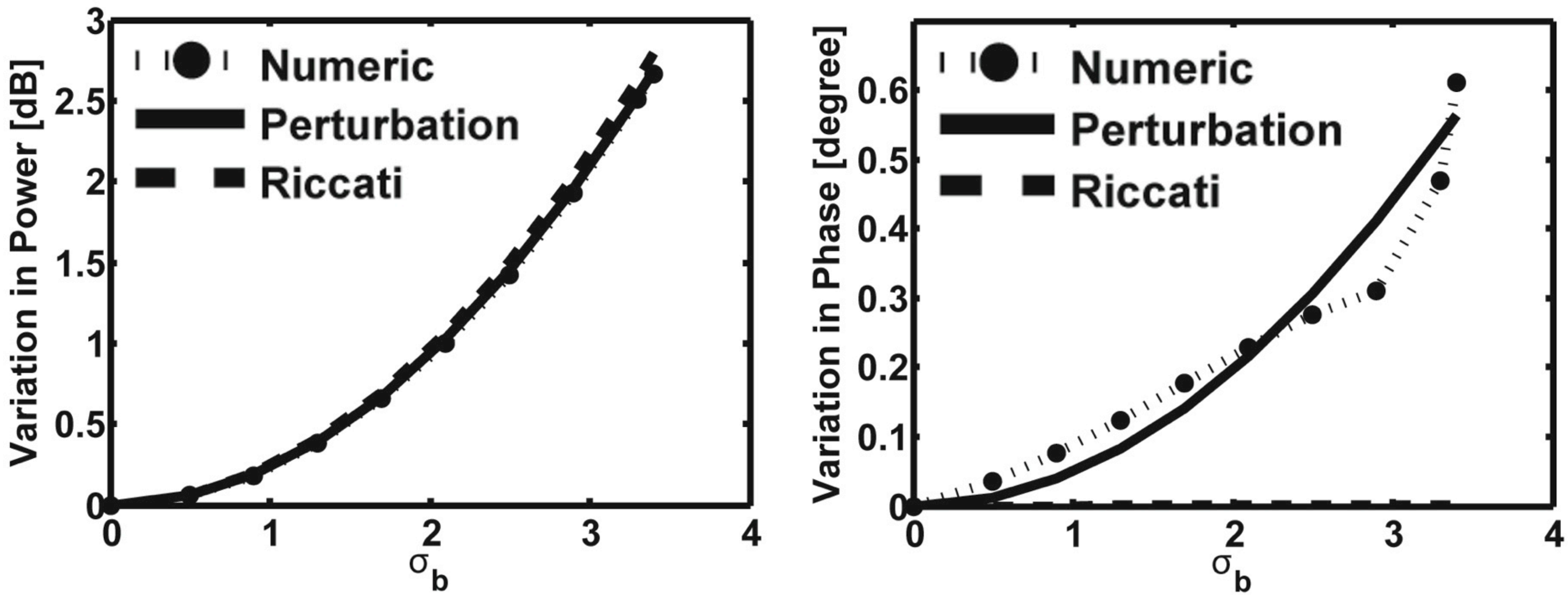
$$\left\langle G_1(x) + j\theta_1(x) \right\rangle = -\frac{\lambda}{2}\left(\frac{C}{1+Cb_0}\right)^2 x\sigma_b^2 \Delta$$

where λ is a complex constant that depends on the velocity mismatch parameter, b_0.

* This work done in collaboration with Tom Antonsen of the University of Maryland, College Park MD

Results: Synchronous Circuit Velocity

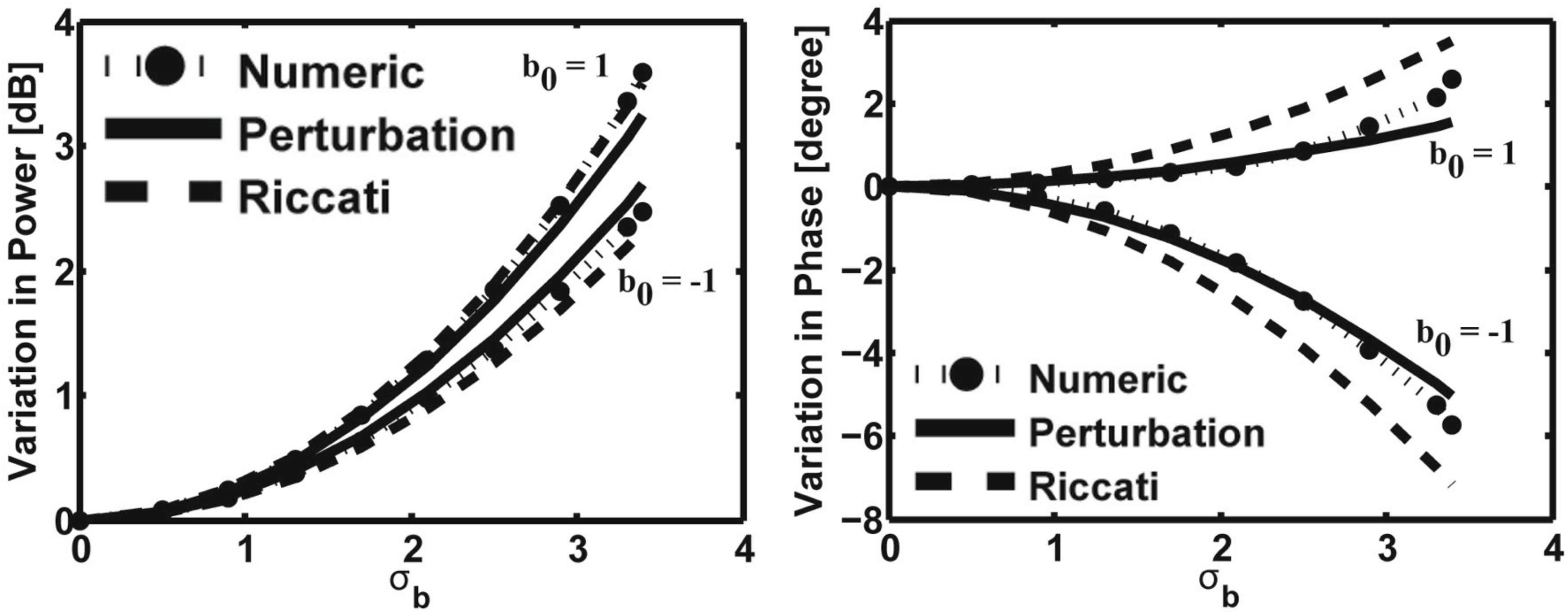
$x = 100$, $C = 0.05$, $\Delta = 1$, $QC = 0$, $b_0 = 0$



- Perturbation analysis shows good agreement with numerical solution for gain over a wide range of parameters.
- Perturbation analysis yields phase variation close to zero, similar to the Riccati analysis. Numerical solutions also show small variations in phase.

Results: Nonsynchronous Circuit Velocity

$x = 100$, $C = 0.05$, $\Delta = 1$, $QC = 0$, $b_0 = \pm 1$



- All three methods are in agreement for non-synchronous beam velocities
- Perturbation analysis more accurate than Riccati analysis as expected due to Riccati analysis only considering a single wave

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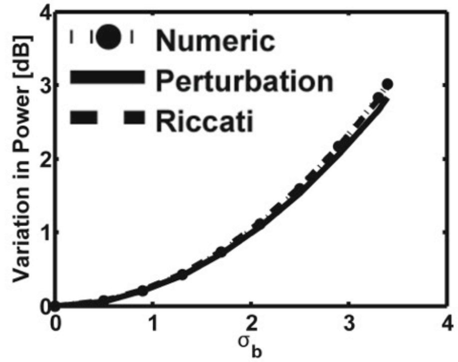
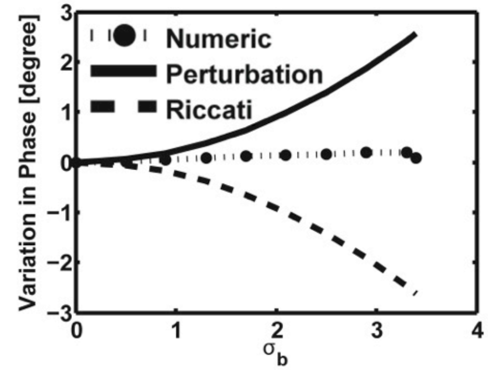
Results: Synchronous Circuit Velocity, QC ≠ 0

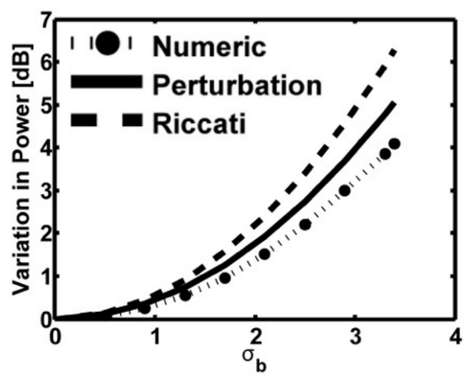
$x = 100$, $C = 0.05$, $\Delta = 1$, $b_0 = 0$

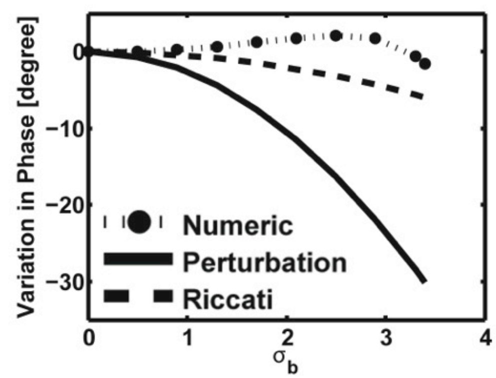
QC = 0.15





QC = 0.35





- It is possible that nonzero *QC* enlarges the range of *b* in which the amplifying wave would have reduced or zero gain.
  - In this case all three waves have comparable amplitudes
  - This violates the basic assumption behind the Riccati approach

Standard Deviation Analysis Extension

$$\sigma_{Gb} = S_{Gb}\sigma_b, \quad \sigma_{\theta b} = S_{\theta b}\sigma_b$$

Note: Standard deviation is first order in σ_b, much larger than deviation from the mean, which is second order in σ_b.

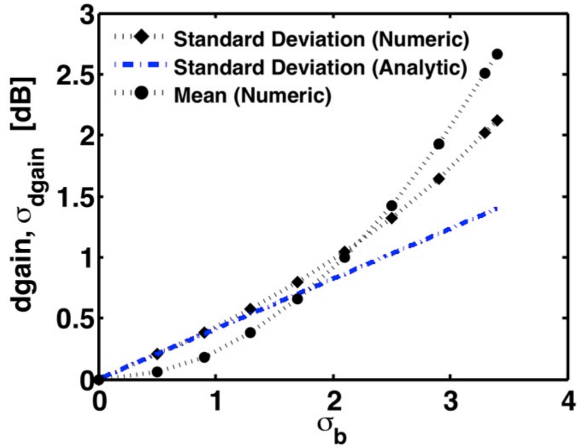
- S_{Gb}, $S_{\theta b}$ are relatively simple functions of x
- Analysis extended from that of [1] to include the Pierce space charge term, $4QC$.

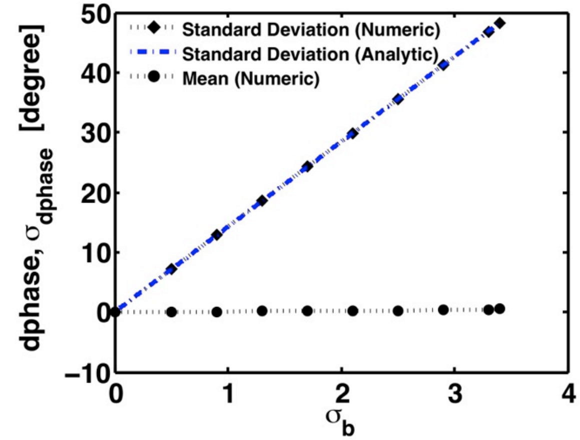
[1] Pengvanich, Chernin, Lau, Luginsland, Gilgenbach, *IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

Standard Deviation Results: Synchronous

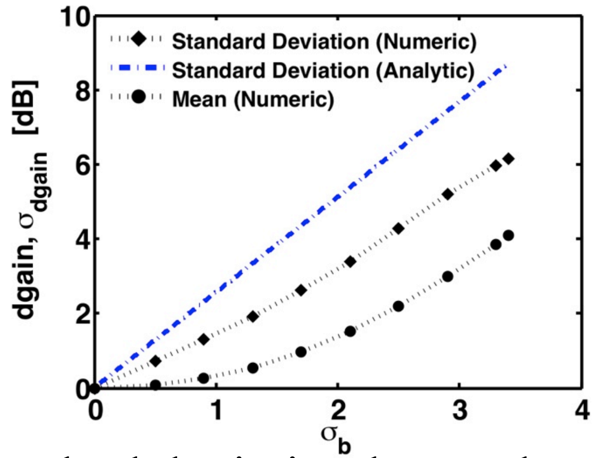
$x = 100, C = 0.05, \Delta = 1, b_0 = 0$

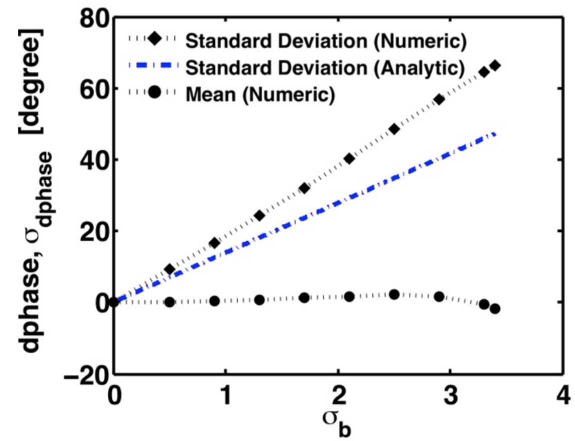
QC = 0





QC = 0.35

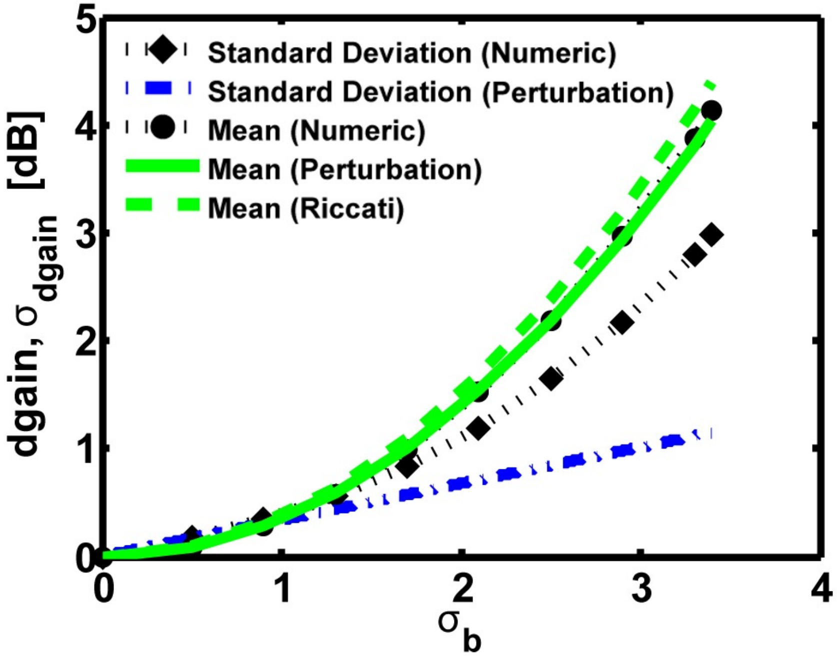


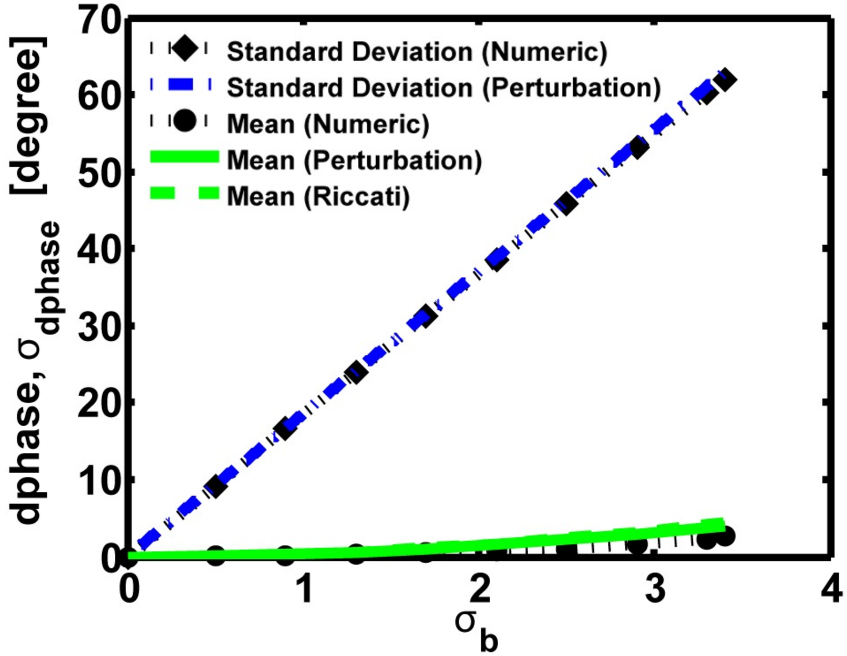


- Standard deviation larger than the mean variation leads to significant fraction of samples with gain higher than the error-free case

Three Wave: G-Band TWT Example

$x = 240$, $C = 0.0197$, $\Delta = 4$, $b_0 = 0.36$, $QC = d = 0$





$V_b = 11.7$ kV, $I_b = 120$ mA, $L = 1.17$ cm, circuit pitch = 0.02 cm

[3] Chernin, Rittersdorf, Lau, Antonsen, and Levush, IEEE Trans. Electron Devices, vol. 59, 1542 (2012).

Four-Wave Analysis[3]

- Circuit equation must be modified to contain the backward wave

$$\left(\frac{\partial}{\partial z} + j\frac{\omega}{v_b}\right)^2 s = \varepsilon$$

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{v_p^2}\right)\varepsilon = -2\frac{\omega}{v_p}\left(\frac{\omega}{v_b}C\right)^3 s$$

- Again, focus on random variations in *b*

[3] Chernin, Rittersdorf, Lau, Antonsen, and Levush, IEEE Trans. Electron Devices, vol. 59, 1542 (2012).

Four-Wave Results

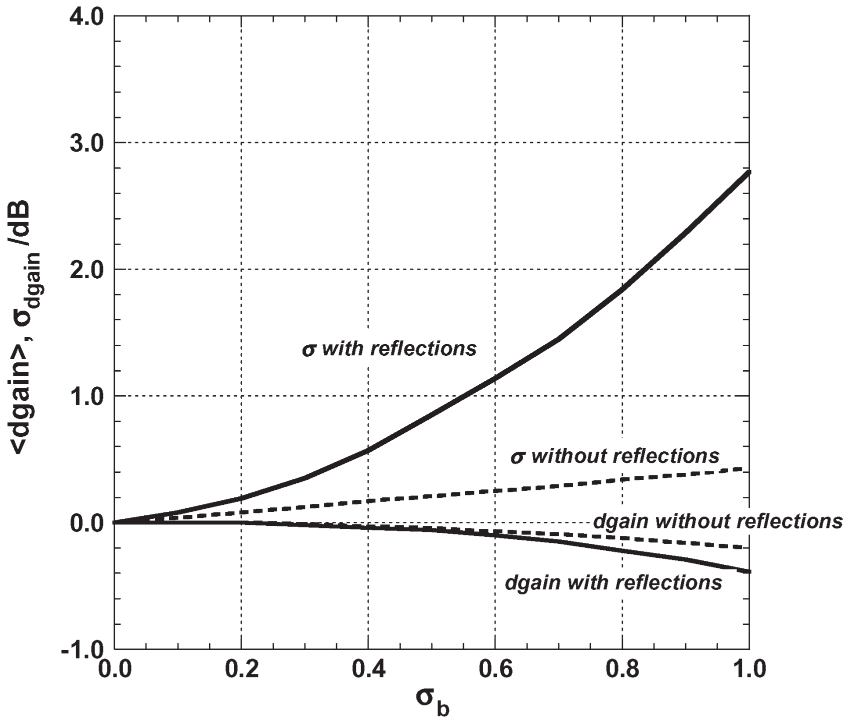


Figure 8: Departures from the error-free values of small-signal gain and the standard deviations of those departures versus σ_b. Solid lines indicate results including reflections (4^{th} model), whereas dashed lines indicate results omitting reflections (3^{rd} model). $C = 0.05$, $b = 0$, $xN = 100$, and $N = 100$.

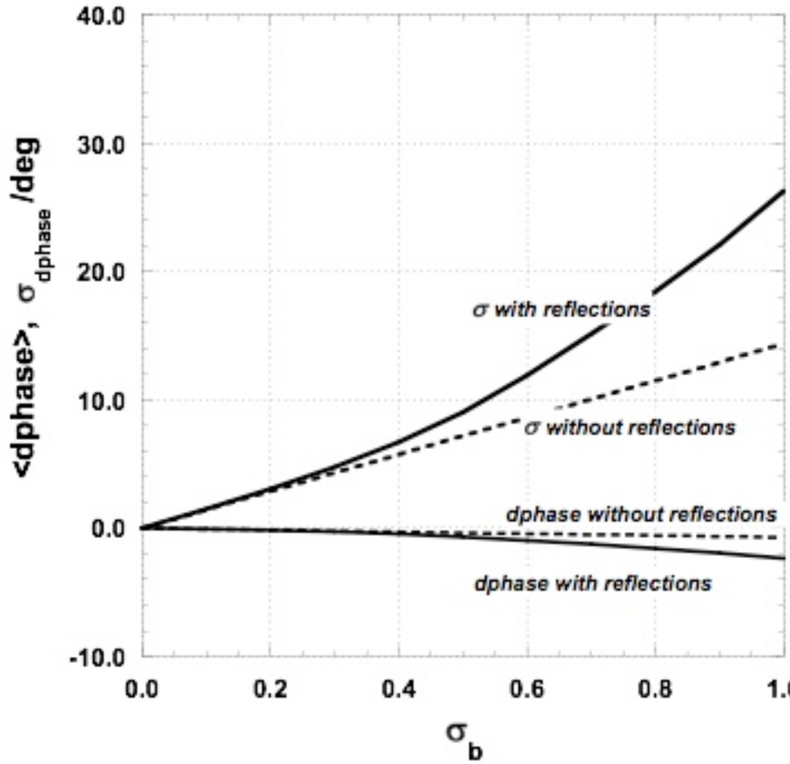


Figure 9: Departures from the error free values of small signal phase and the standard deviations of those departures vs. σ_b. Solid lines indicate results including reflections (4^{th} order model); dashed lines indicate results omitting reflections (3^{rd} order model). $C = 0.05$, $b = 0$, $x_N = 100$, $N = 100$.

[3] Chernin, Rittersdorf, Lau, Antonsen, and Levush, IEEE Trans. Electron Devices, vol. 59, 1542 (2012).

Four-Wave: G-Band TWT Example

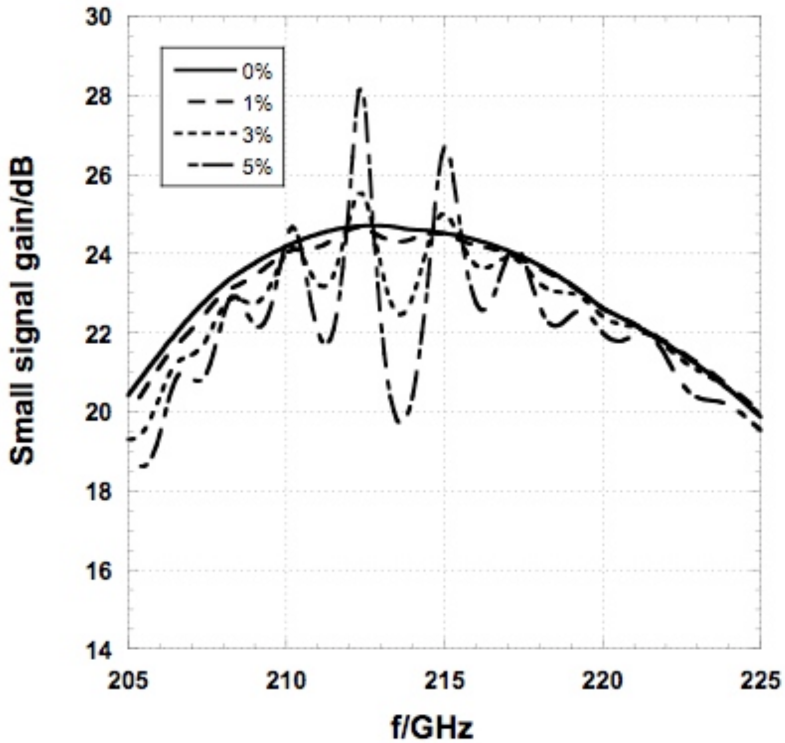


Figure 13: Small signal gain vs. frequency for different values of standard deviation of the circuit pitch distribution.

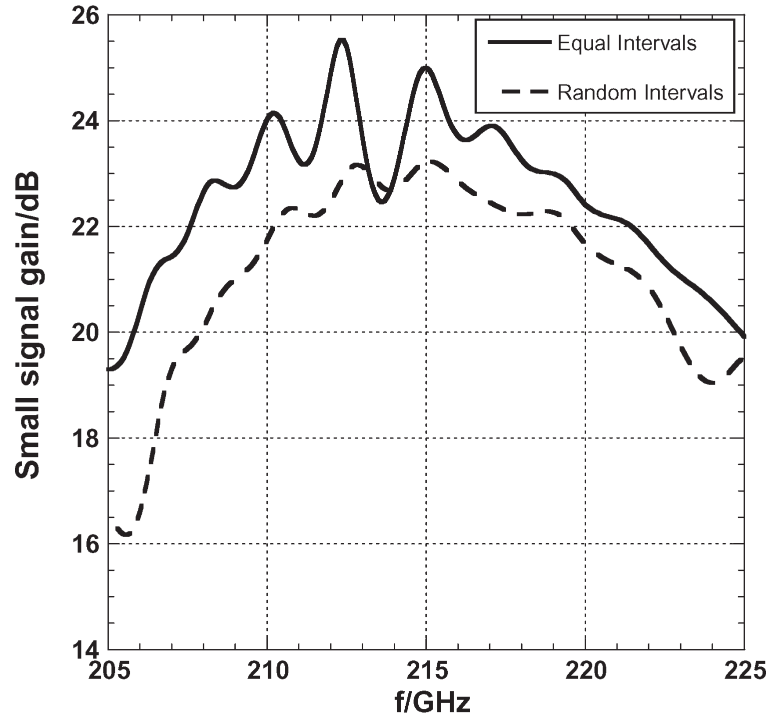


Figure 15: Small-signal gain versus frequency for equally spaced and randomly spaced joints in the presence of 3% random pitch errors.

V_b = 11.7 kV, I_b = 120 mA, L = 1.17 cm, circuit pitch = 0.02 cm

[3] Chernin, Rittersdorf, Lau, Antonsen, and Levush, IEEE Trans. Electron Devices, vol. 59, 1542 (2012).

Summary

- Mean deviation in small signal gain and phase is found to be quadratic in σ_b. It is a higher order effect than the standard deviations, statistically resulting in higher gain in a significant number of simulated TWT's.
- Good agreement is found between analytic theory (perturbative or Riccati) and numerical computation in the absence of space charge effects ($QC = 0$). Agreement was poor for nonzero QC.
- Study of the reverse propagating wave (four-wave theory) shows that its effects on gain and phase are significant
- Effects of small pitch errors in a G-Band TWT were evaluated as an example.
- Remaining problems:

  Can TWT oscillations be caused by reflected waves from random errors?

  What is the true nature of the higher gain with random errors? QC effects?