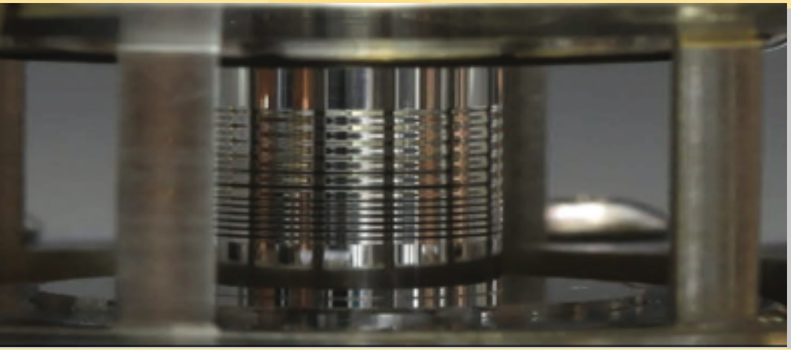
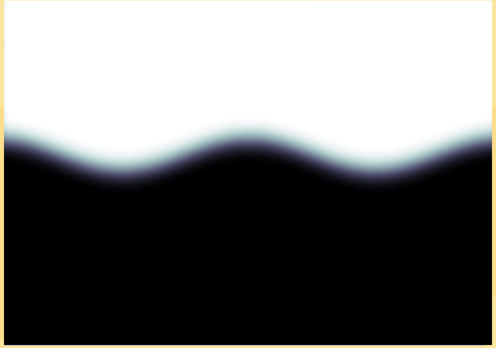
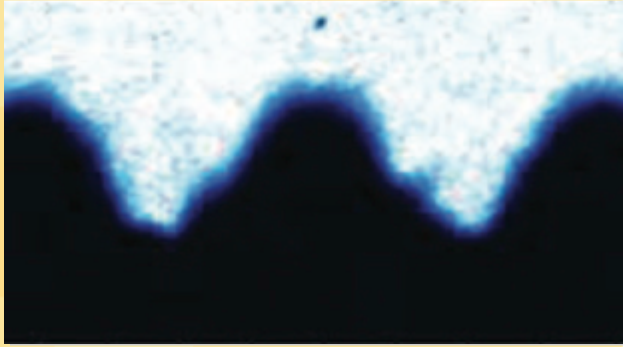




Magneto-Rayleigh-Taylor growth and feedthrough in cylindrical liners

M. R. Weis, Y.Y. Lau, R.M. Gilgenbach,
Plasma, Pulsed Power and Microwave Laboratory,
Nuclear Engineering and Radiological Sciences Dept.,
University of Michigan
Ann Arbor, MI 48109-2104 USA

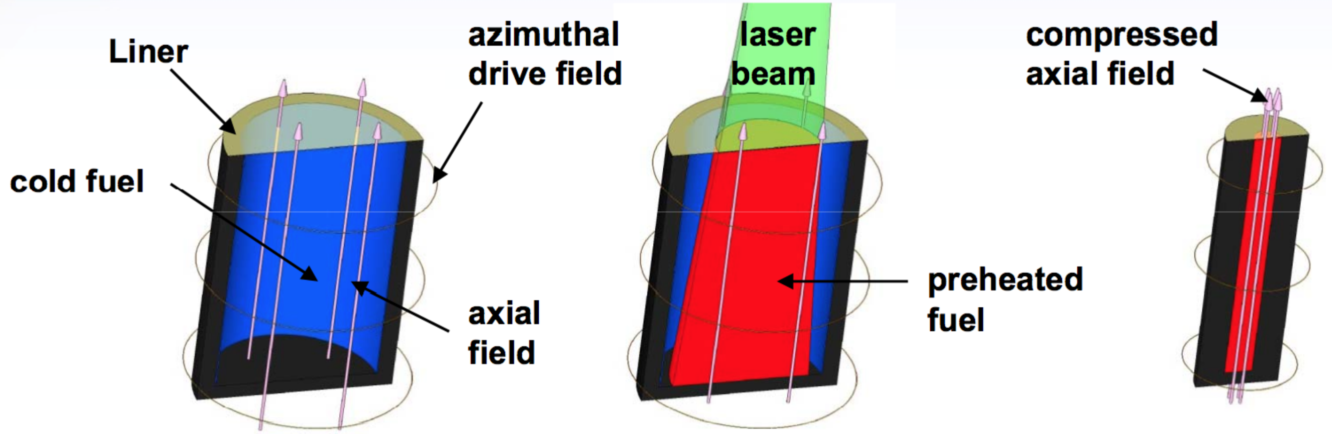
M. Hess, C. Nakhleh
Sandia National Laboratories
Albuquerque, NM 87185 USA

Motivation

- MRT is one of the greatest challenges to success of the Magnetized Liner Inertial Fusion (MagLIF) concept
  - Magnetic fields introduce additional complexity over classical RTI
- Feedthrough has an important role in the stability of the fuel/liner interface in MagLIF concept
  - Also relevant to dynamic materials experiments on Z
- Analytic results provide a fast way to analyze these problems
- Hydra, a rad-hydro-MHD code, provides another tool for modeling experiments on Z and other HEDP platforms
  - Needs benchmarking

Goal: apply these tools to a liner implosion and compare to experimental results

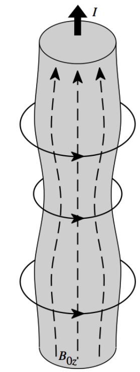
Magnetized Liner Inertial Fusion (MagLIF)*,** may be a promising path to high yields on Z, but liner integrity is critical

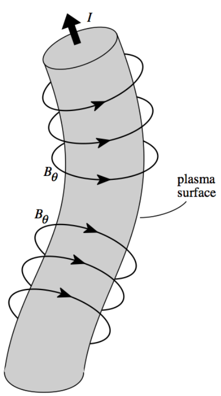


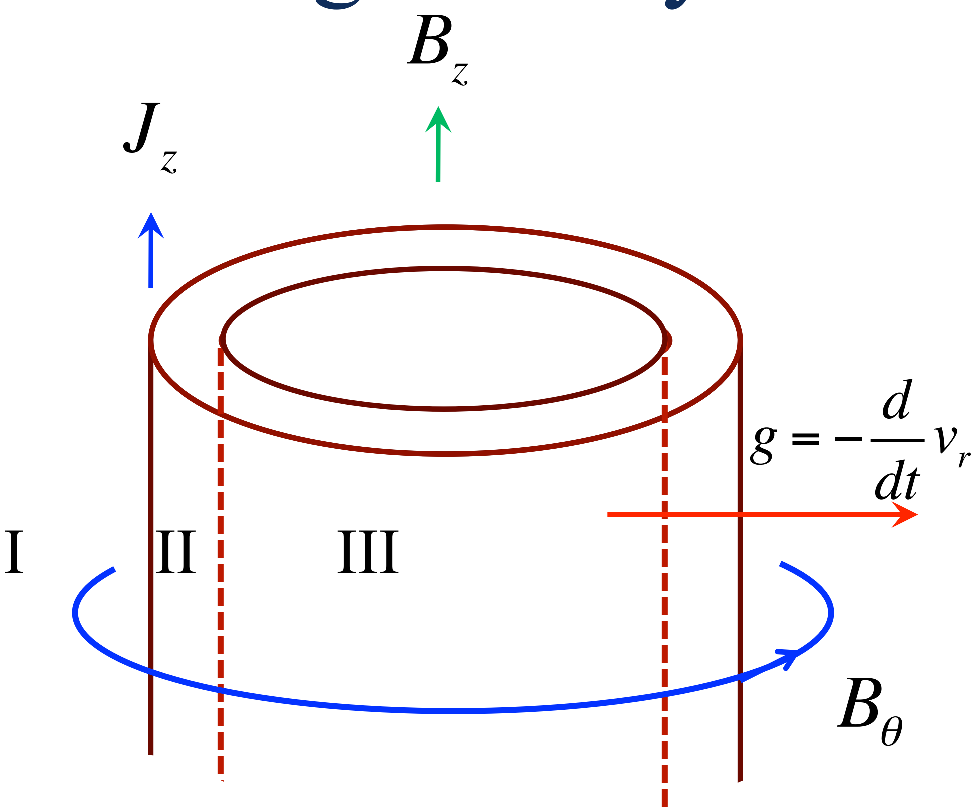
- Initial calculations suggest Z-Beamlet laser can do the preheat
  - Preheating the fuel reduces the required compression ratio to obtain ignition temperatures to about 30 on Z
  - Preheating reduces the implosion velocity needed to about 5-10 cm/μs
- Axial magnetic field strength required (about 5-10 T) feasible
  - Similar coil design parameters to coils for dynamic materials tests
- Simulations suggest 100 kJ yields on Z are possible
- Success of MagLIF hinges on <u>maintaining sufficient liner integrity</u>

* S. A. Slutz *et al.*, "Pulsed power driven cylindrical liner implosions with magnetized and preheated fuel", Phys. Plasmas **17** 056303 (2010).

Cylindrical geometry instabilities







g > 0: Implosion

g < 0: Stagnation

g = effective gravity in rest frame of interface

MRT (acceleration)

Sausage / m=0

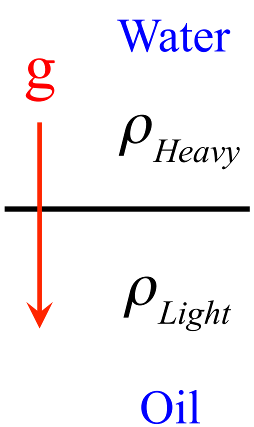
Kink / m=1

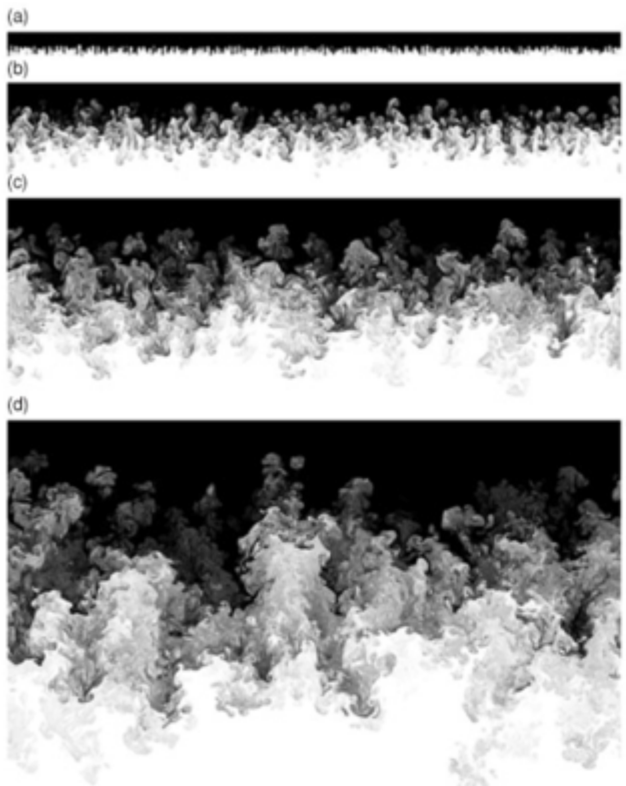
(present with no acceleration in a cylindrical current carrying plasma)

Rayleigh-Taylor Instability (RTI)

- Interchange instability from a light fluid pushing a heavy fluid
  - Water on top of oil in Earth's gravity
  - Deep water waters are the stable form of RTI (water supporting air)

Instability arises for: $\nabla p \cdot \nabla \rho < 0$





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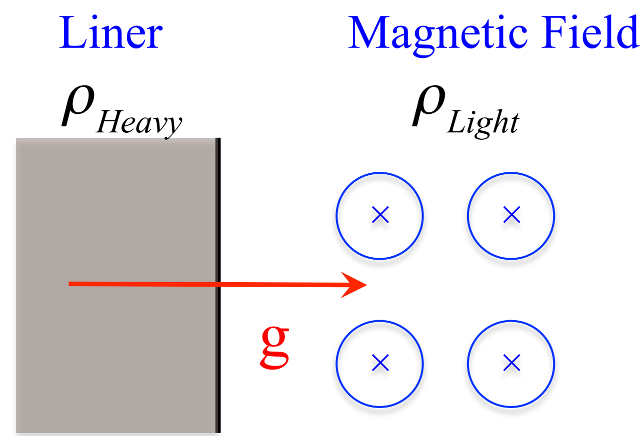
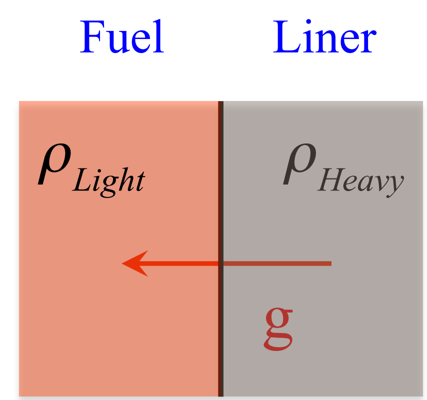




* LLNL ST & R Dec. 2004

Ideal MHD Equations

Mass Conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Momentum Conservation:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g}$$

Ampere's Law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Faraday/Ohm Law:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Perturbation of equilibrium

We assume that the time scale for perturbation growth is fast compared to liner dynamics, yielding an approx. instantaneous equilibrium:

$$\frac{dp}{dr}+\frac{1}{\mu_0}\left[B_z\frac{dB_z}{dr}+\frac{B_\theta}{r}\frac{d}{dr}\left(rB_\theta\right)\right]=\rho g$$

We perturb this equilibrium by a small displacement of the form:

$$\vec{\xi}(\vec{r},t)=\left\langle \xi_r(r),\xi_\theta(r),\xi_z(r)\right\rangle e^{\gamma t+ikz+im\theta}$$

We assume that the perturbed velocity is incompressible:

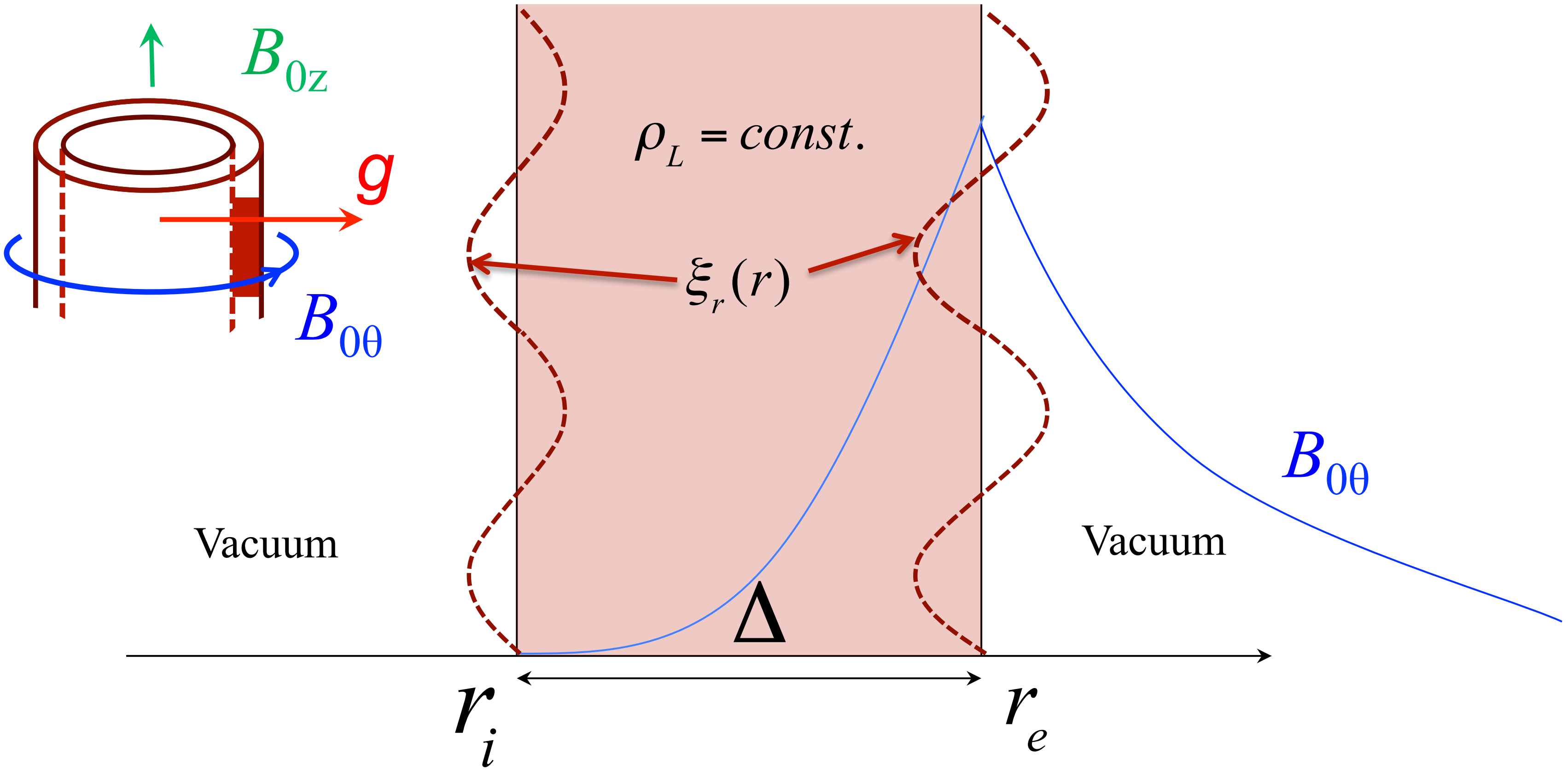
$$\nabla\cdot\frac{\partial\vec{\xi}}{\partial t}=\nabla\cdot\vec{\xi}=0$$

The growth rate, ω, is of the form:

$$\gamma^2\approx kg-\frac{\left(\mathbf{k}\cdot\mathbf{B}\right)^2}{\mu_0\rho}+C(m,\mathbf{k})$$

Where C includes the effects of azimuthal and current carrying modes

Sharp boundary model



Aspect ratio:

$$AR = \frac{r_e}{r_e - r_i} = \frac{r_e}{\Delta}$$

The *feedthrough* of instability from the outer to inner surface for a given mode, ω, is defined as:

$$\xi_r(r_i)/\xi_r(r_e) \equiv F(\omega)$$

We solve the linearized ideal MHD equations:

$$\rho \frac{\partial^2 \vec{\xi}}{\partial t^2} = -\nabla p_1 + \vec{J}_0 \times \vec{B}_1 + \vec{J}_1 \times \vec{B}_0 + \rho \vec{g} \cdot \vec{\xi}$$

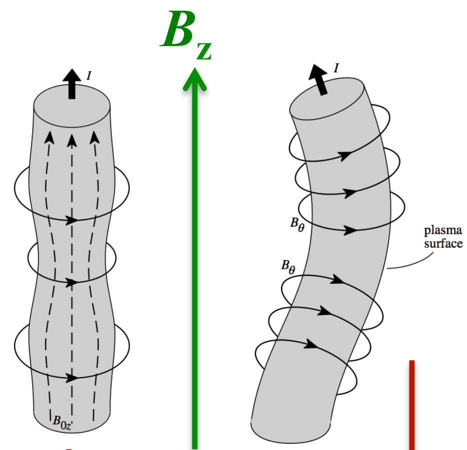
$$= -\nabla p_1 + (\nabla \times \vec{B}_0) \times [\nabla \times (\vec{\xi} \times \vec{B}_0)]/\mu_0 + \{\nabla \times [\nabla \times (\vec{\xi} \times \vec{B}_0)] \times \vec{B}_0\}/\mu_0 + \rho \vec{g} \cdot \vec{\xi}$$

- Subject to the boundary conditions of continuity of total pressure at each interface, which is an eigenvalue problem for the eigenfunction, ξ, and eigenvalue, ω
- The solution is analytically tractable for:
  - Constant density profiles (may be different in each region)
  - Constant Bz profiles (may be different in each region)
  - No magnetic diffusion of drive field
- Otherwise the problem is solved numerically using a shooting method

Sausage and kink modes are successfully recovered

- For **g = 0** and AR = 1 (solid plasma column undergoing no acceleration) give well known test problem

B_θ=1000 T drives instability but also stabilizes m > 0 modes to some extent (bent field lines)



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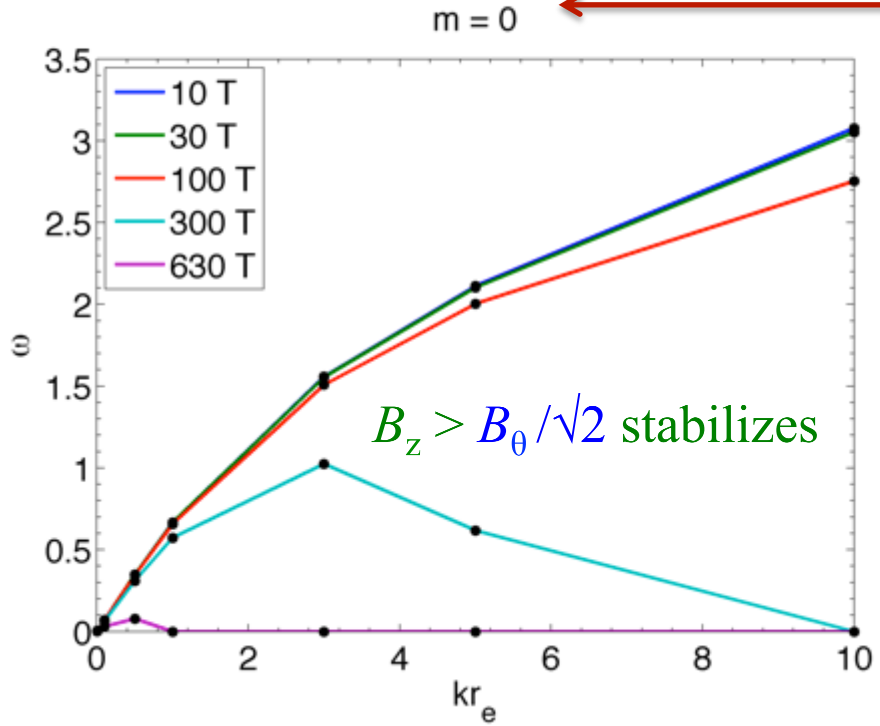
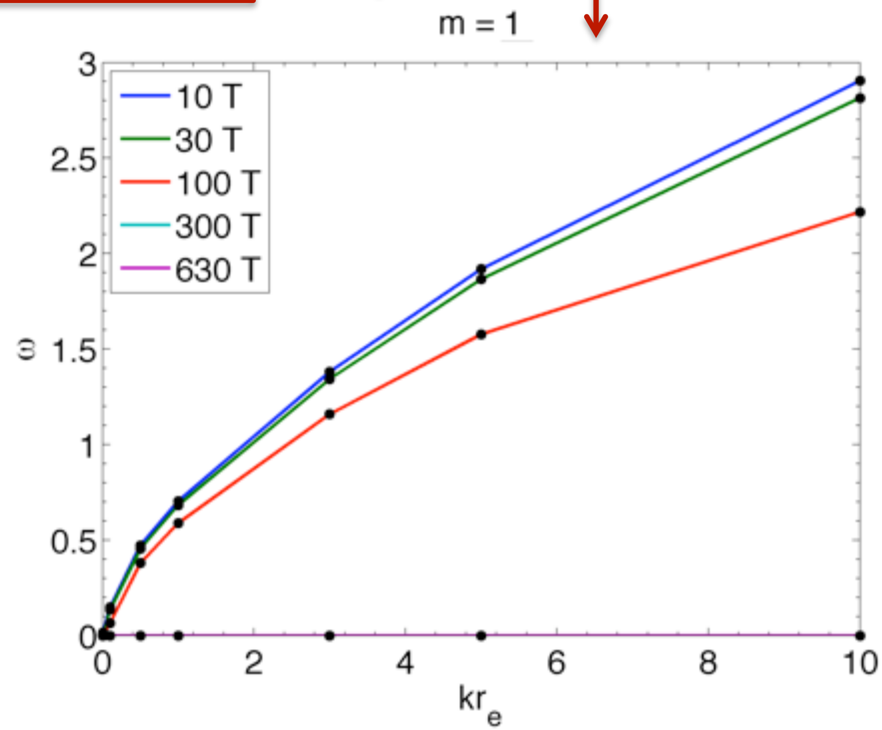


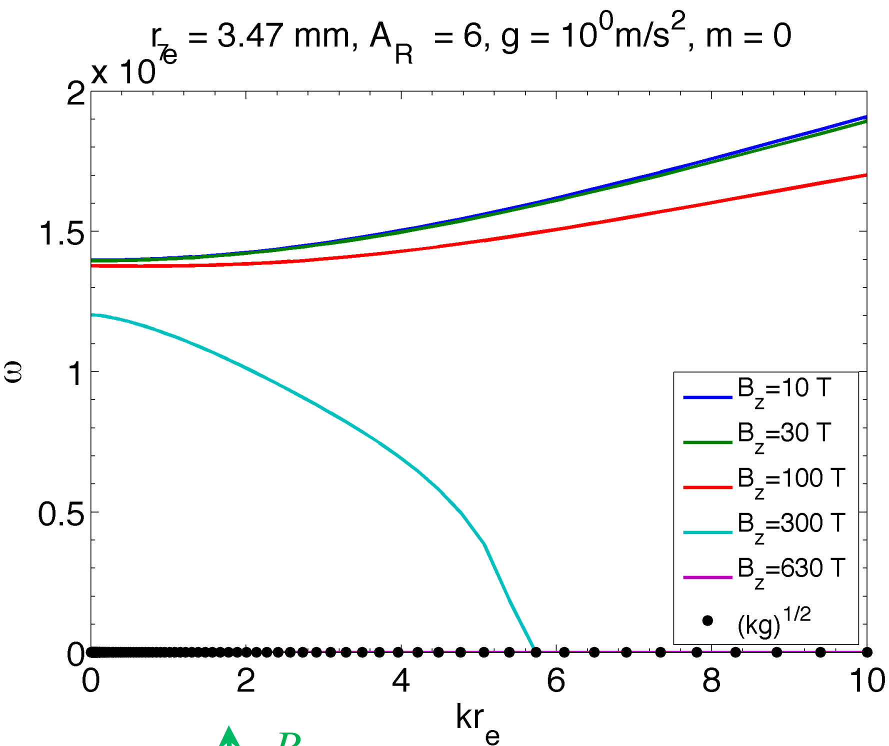


* Boyd and Sanderson. *The Physics of Plasmas*, Cambridge Press, 2003

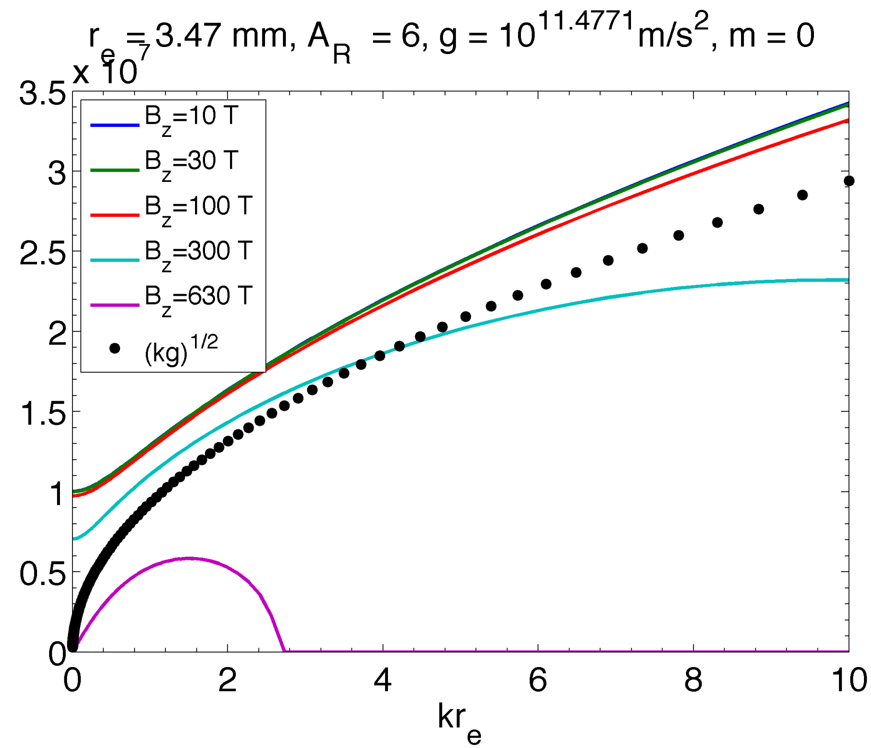
m = 0 modes will be stabilized by Bz only

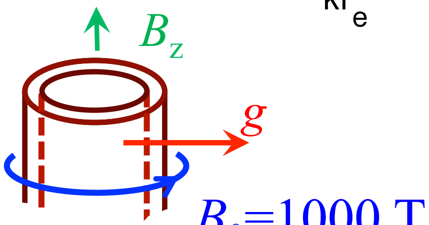
$$\omega^2 \approx kg - \frac{1}{\mu_0 \rho}\left[\left(\frac{m}{r}B_\theta + k_z B_z\right)\right]^2$$

No acceleration

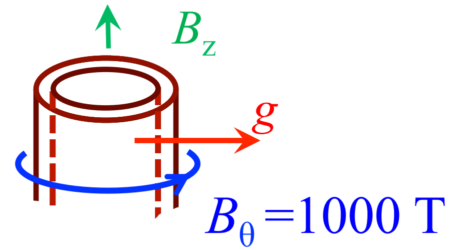


Implosion acceleration





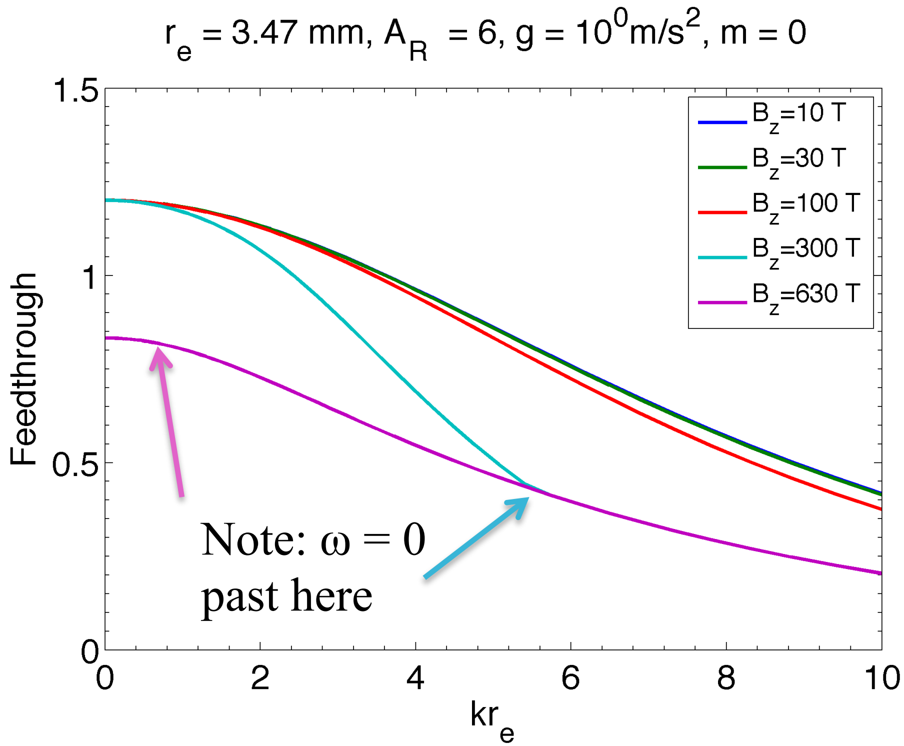
AR=6 liners show feedthrough reduction with Bz as expected



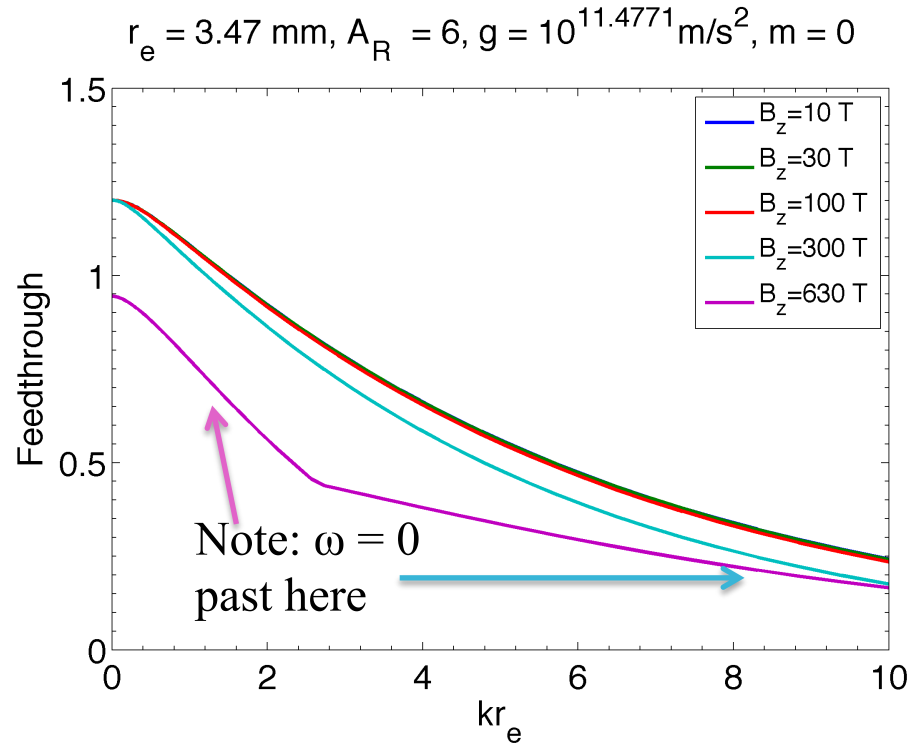
Reverse feedthrough also exists for small kr

- **This is not present in planar results!**
- **Increasing g reduces this effect**

No acceleration



Implosion acceleration



For significant feedthrough and MRT stabilization, require: $B_z \approx B_\theta$

- This is obtained by compressing the applied Bz seed field:

$$B_z(t) = B_{z0}\left(\frac{r_{i0}}{r_i(t)}\right)^2$$

$$= B_{z0} C_R^2$$

  - This assumes no loss of field from Nernst effect
- The outer surface MRT will never be stabilized but there is hope to slow growth on the inner surface
  - Minimize initial seeding from feedthrough
  - Stabilize growth via strong Bz
- The limits for: $kr << 1$ will need to be examined more closely due to the peculiar behavior seen
  - Sausage and kink mode may complicate this stabilization

Using realistic data as input into linearized model

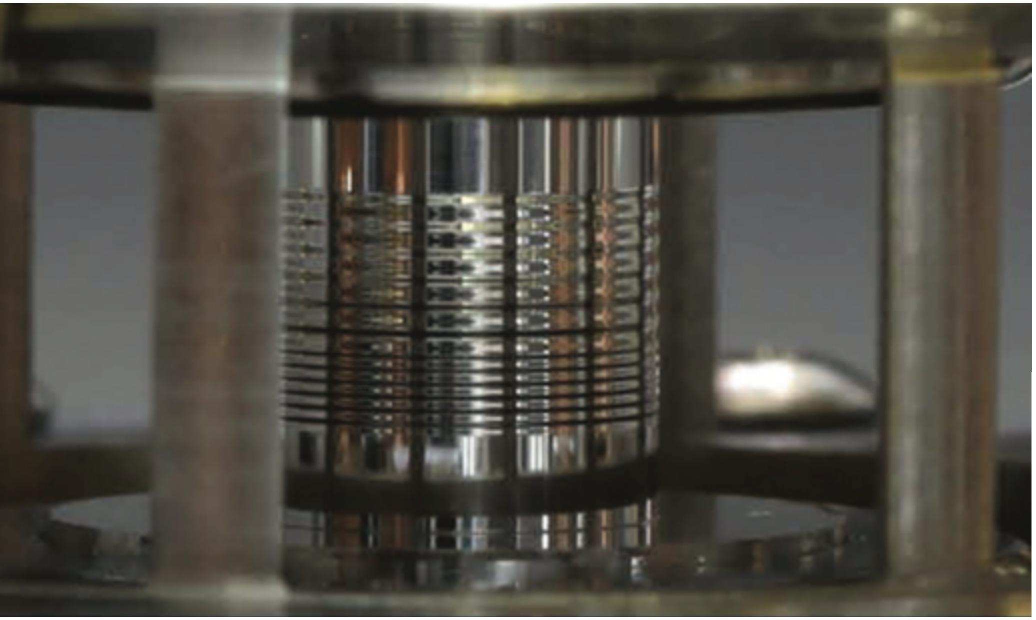
- Average physical quantities from 1D Hydra data in each 'region'
  - Running Lagrangian zones can be used to find liner/vacuum interfaces and, hence, the boundaries for averaging
- For a given wavelength we can calculate the instantaneous growth rate, ω(t) for each time step
  - The amplitude, η, of the instability is then determined by

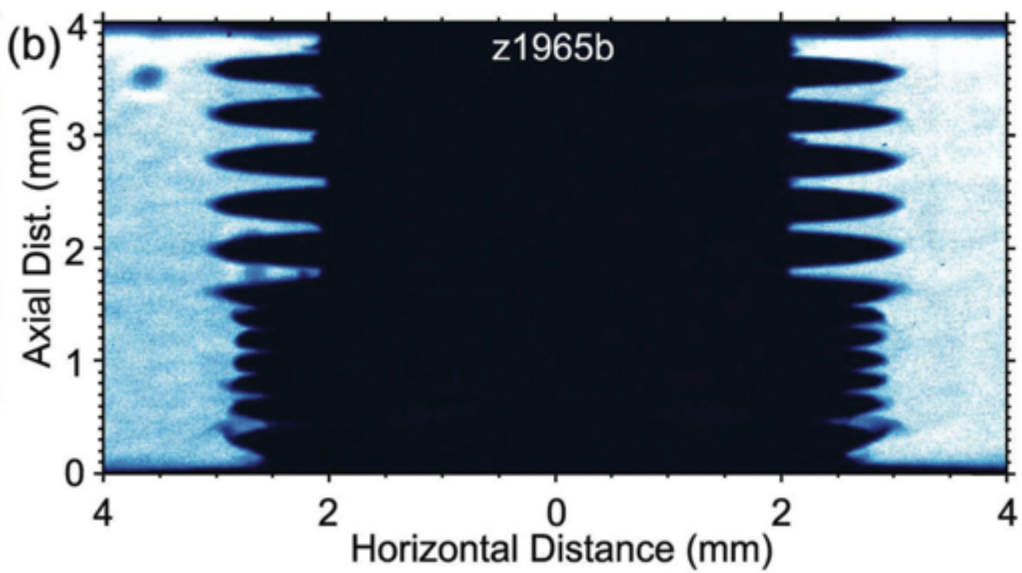
$$\frac{d^2}{dt^2}\eta(t) = \omega(t)^2\eta(t)$$

  - The feedthrough between interfaces is just the ratio of the eigenfunction at the inner and outer interface

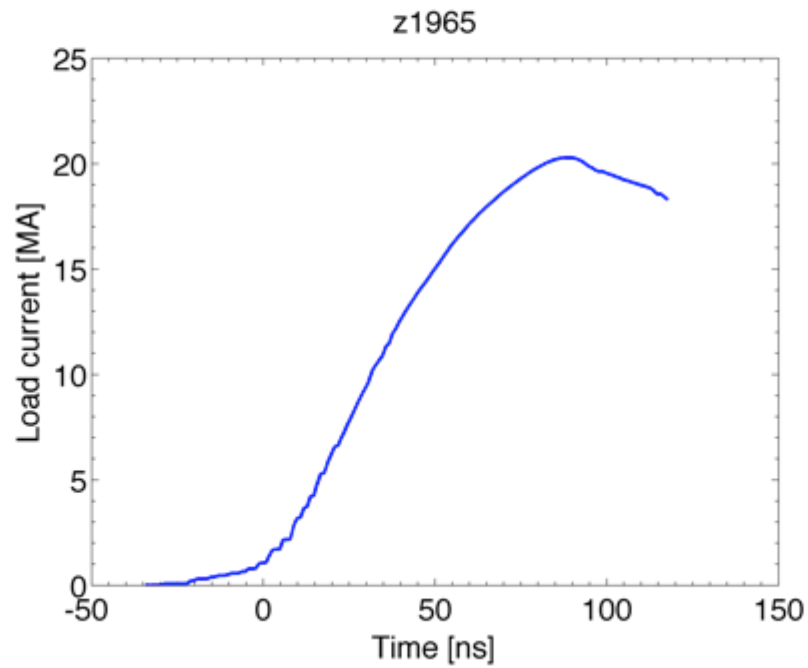
$$F(\gamma) = \xi(r_i)/\xi(r_e)$$

Aluminum liner experiments on Z with seeded MRT *





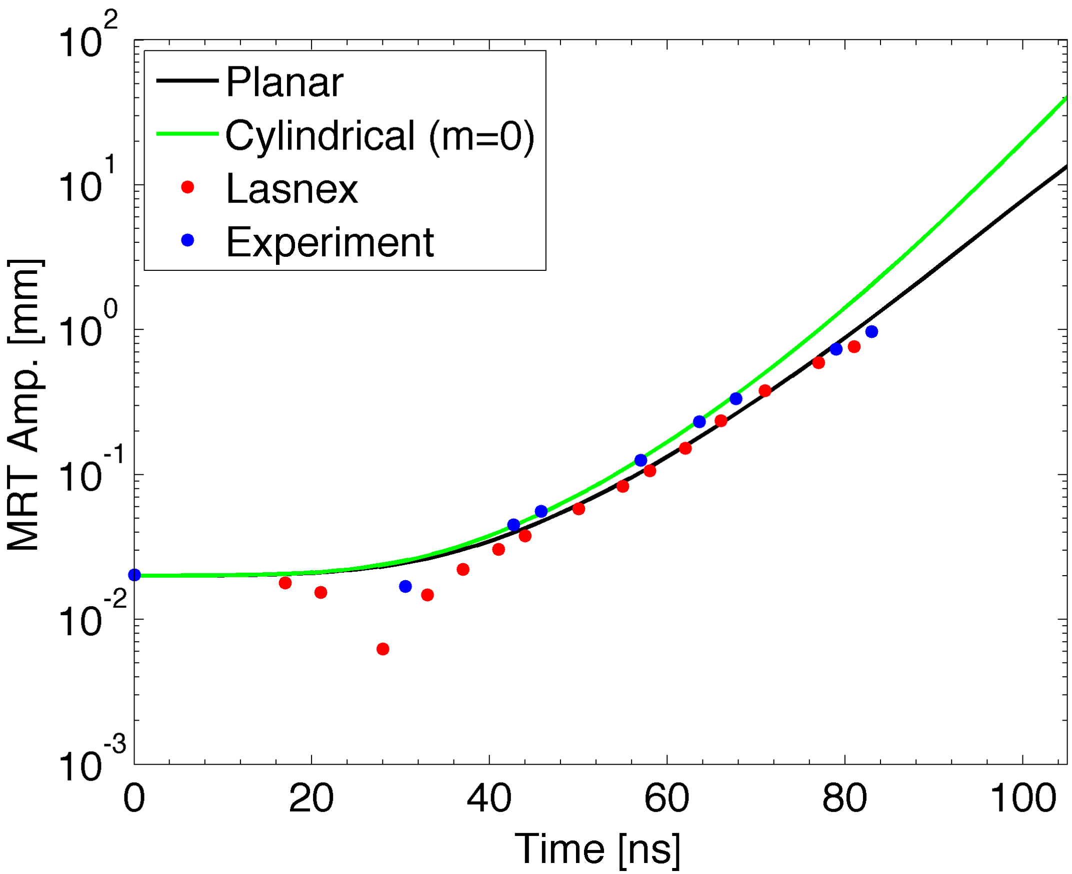
A 1D simulation with Hydra can be driven with the measured load current from which we can extract our averaged physical quantities

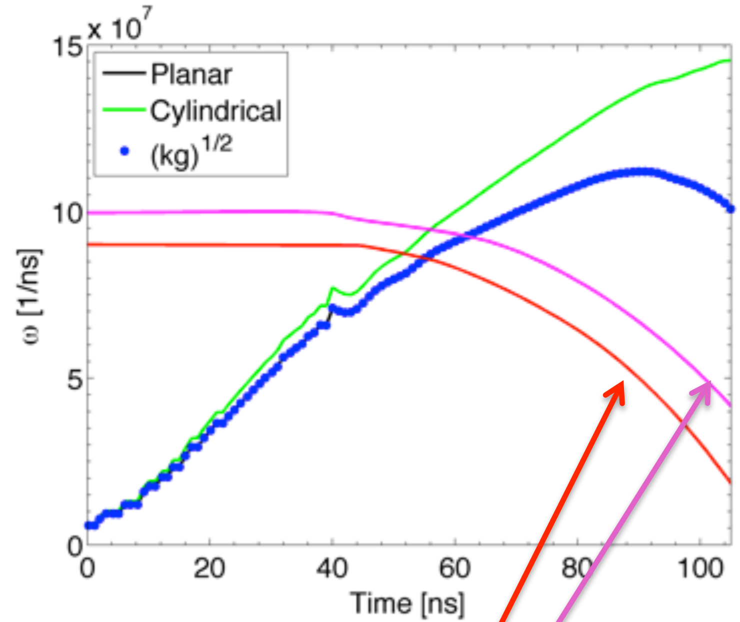


* Sinars et. al. Phys. Plasmas **18**, 056301 (2011)

Applying linearized model to Sinars et. al. * experiments shows good agreement while convergence is low

- Aluminum liner seeded with 400 um surface perturbation





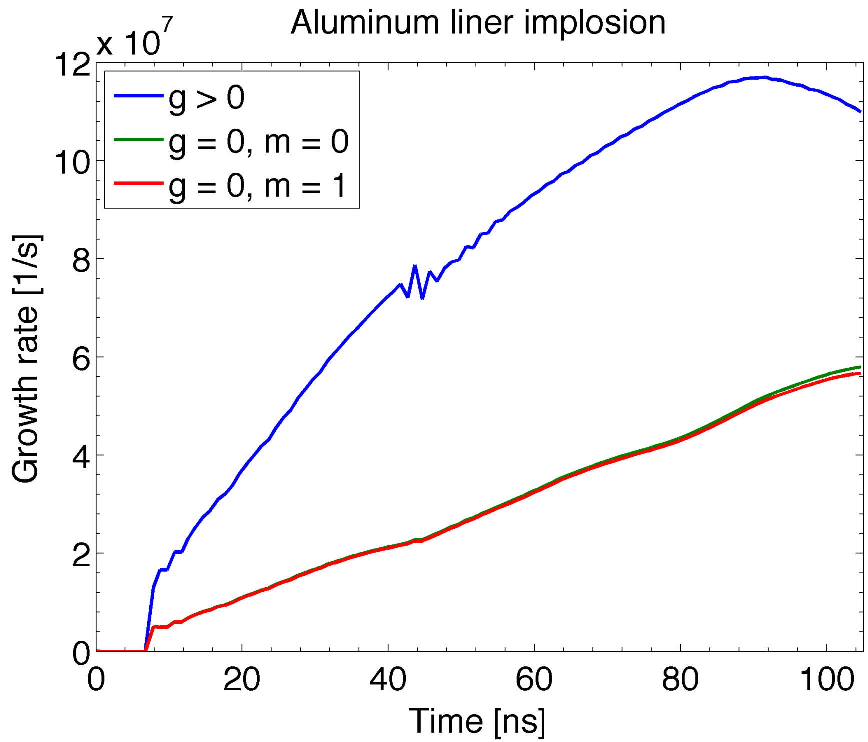
Inner/outer radii

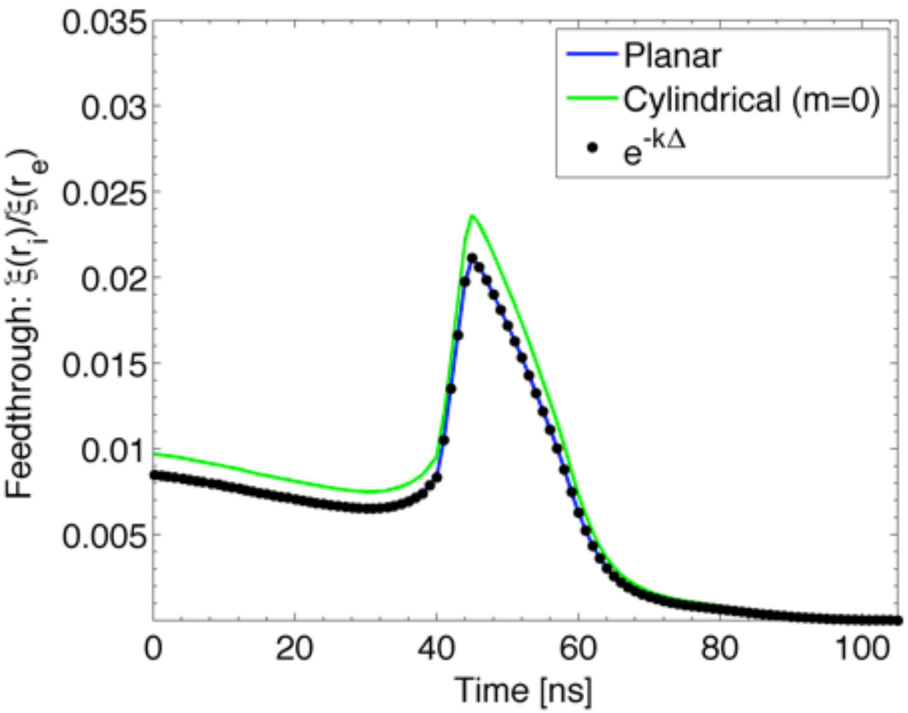
As convergence increases, growth rate becomes more complicated

* Sinars et. al. Phys. Plasmas **18**, 056301 (2011)

While g is large and convergence is small, growth is dominated by classical Rayleigh-Taylor growth rate:

$$\omega^2 \approx kg >> -\frac{(\mathbf{k}\cdot\mathbf{B})^2}{\mu_0\rho} + C(m,\mathbf{k})$$



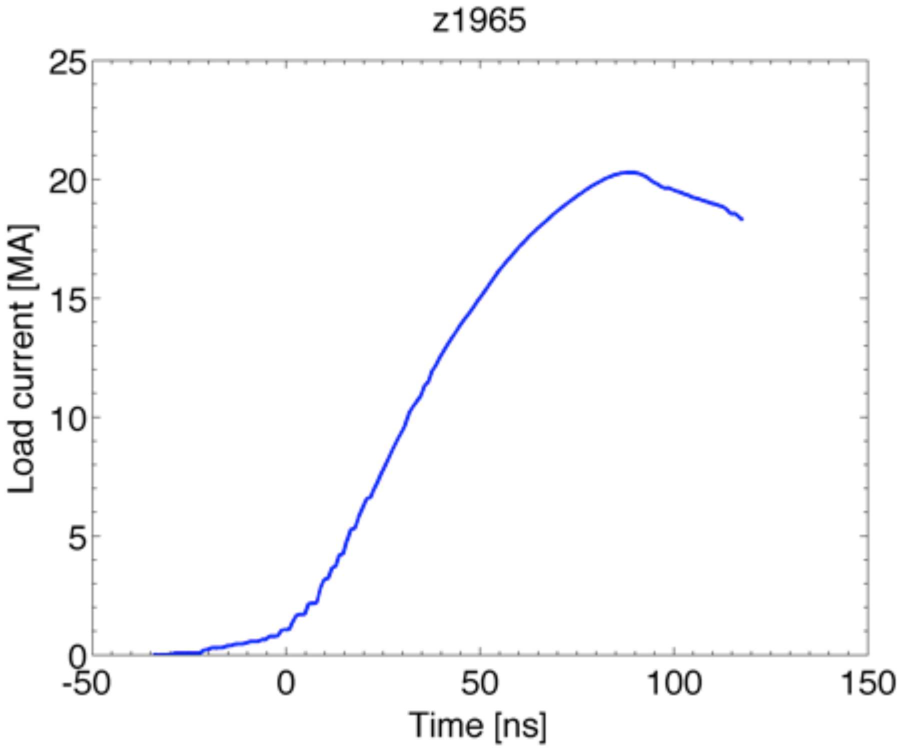


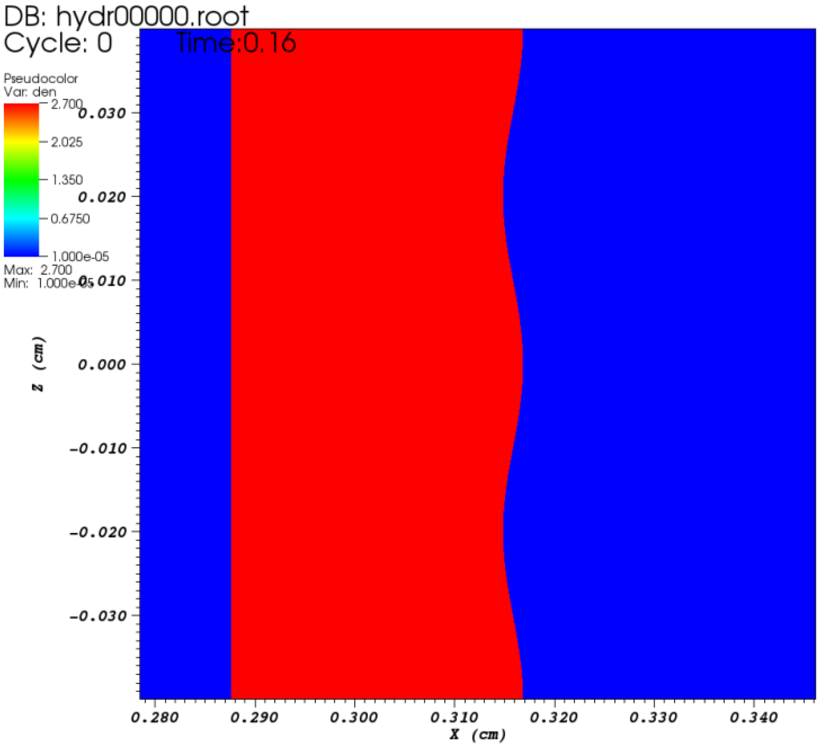
If we remove g for the same problem, we see the remaining physics gives much lower growth

Feedthrough is similarly dominated by the classical expression

Hydra has been used to model Al liner implosions with seeded MRT

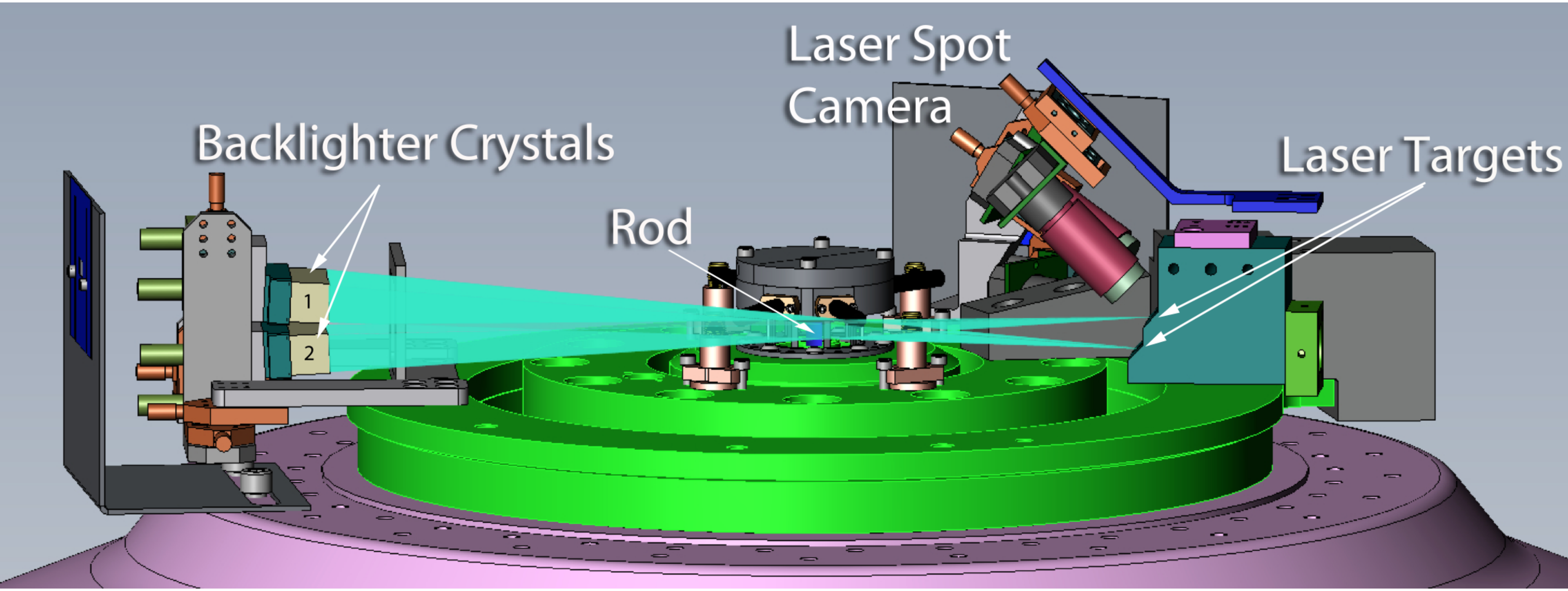
- A sinusoidal perturbation of λ=400 um was applied to the surface of an Al liner and an implosion was driven using the load current on shot z1965 in attempt to replicate the MRT growth rates shown earlier *



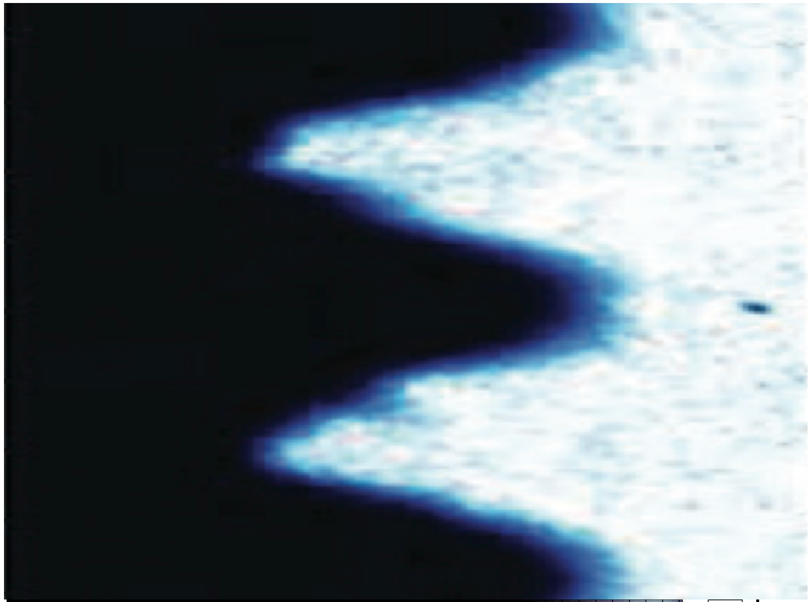


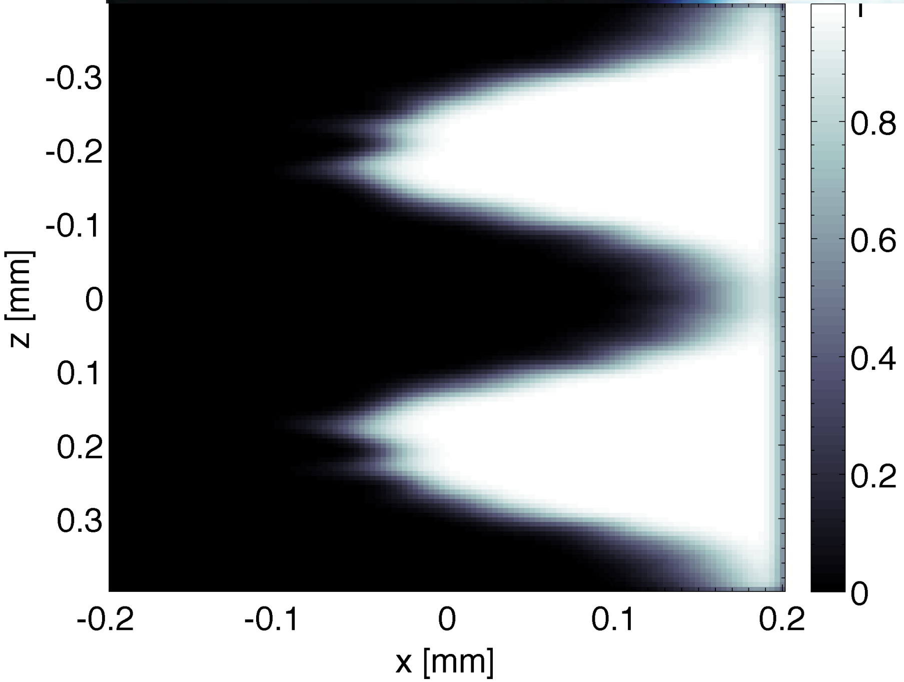
* Sinars et. al. Phys. Plasmas **18**, 056301 (2011)

ZBL is used to create the one or two frame 6.151 keV radiograph images



- **Radiograph lines of sight are ± 3° from horizontal when using two frame radiograph**
  - **This can introduce shadowing of short wavelength modes**
- **Straight on (0°) radiographs can alleviate this but only can take one frame**





Simulated radiographs (from SPECT3D) are generated from X-ray transmission through plasma onto a submicron resolution detector and a 15 um blur is added (ZBL resolution)

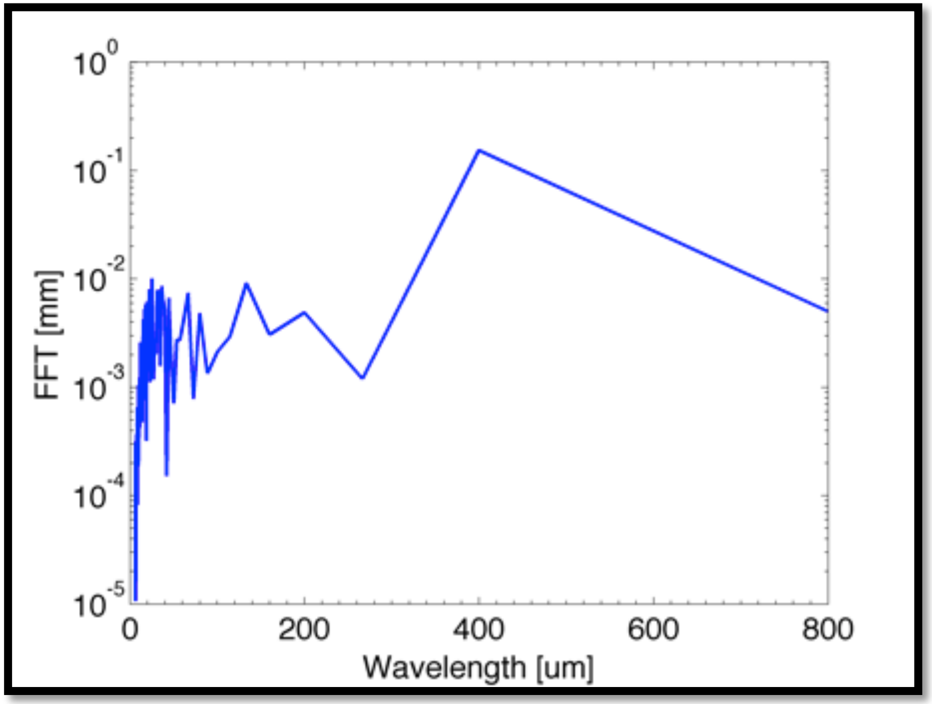
Comparing to radiographs from Sinars et. al. (2011) at t = 63.6 ns show excellent agreement both in amplitude and gross features even at 0°

We can also estimate the growth by FFT or direct calculation

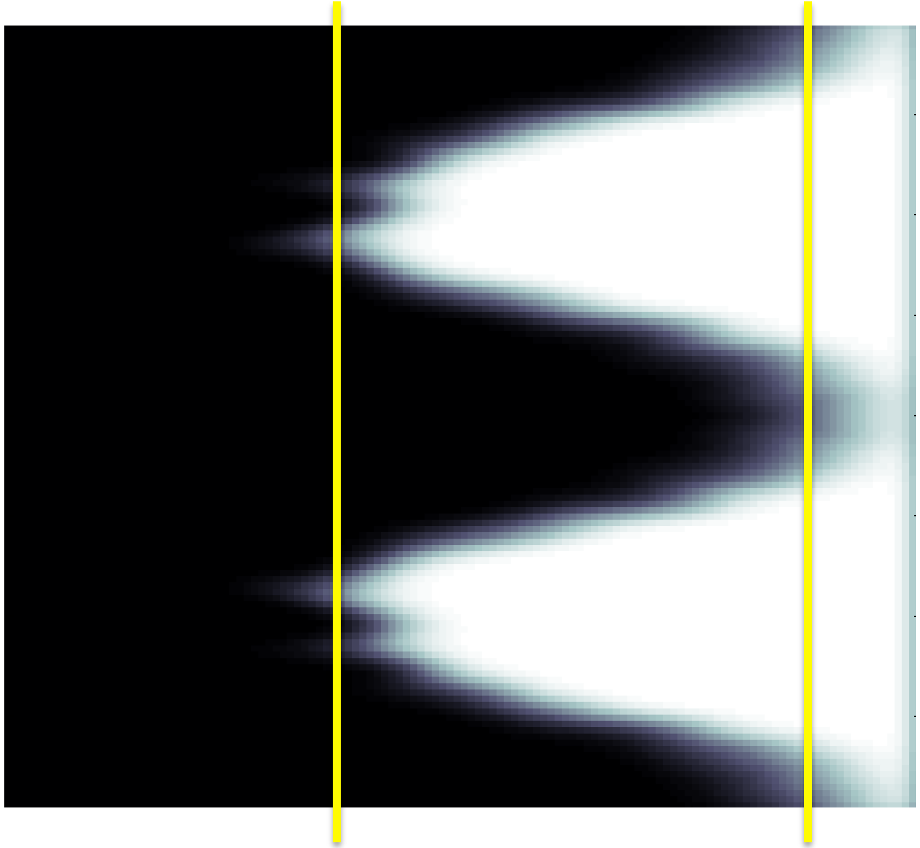
Axial FFT of result

$$\int \rho(r,z)r\,dr = m_L(z)$$

$$m_L(k) \approx \int m_L(z)e^{-ikz}\,dz$$

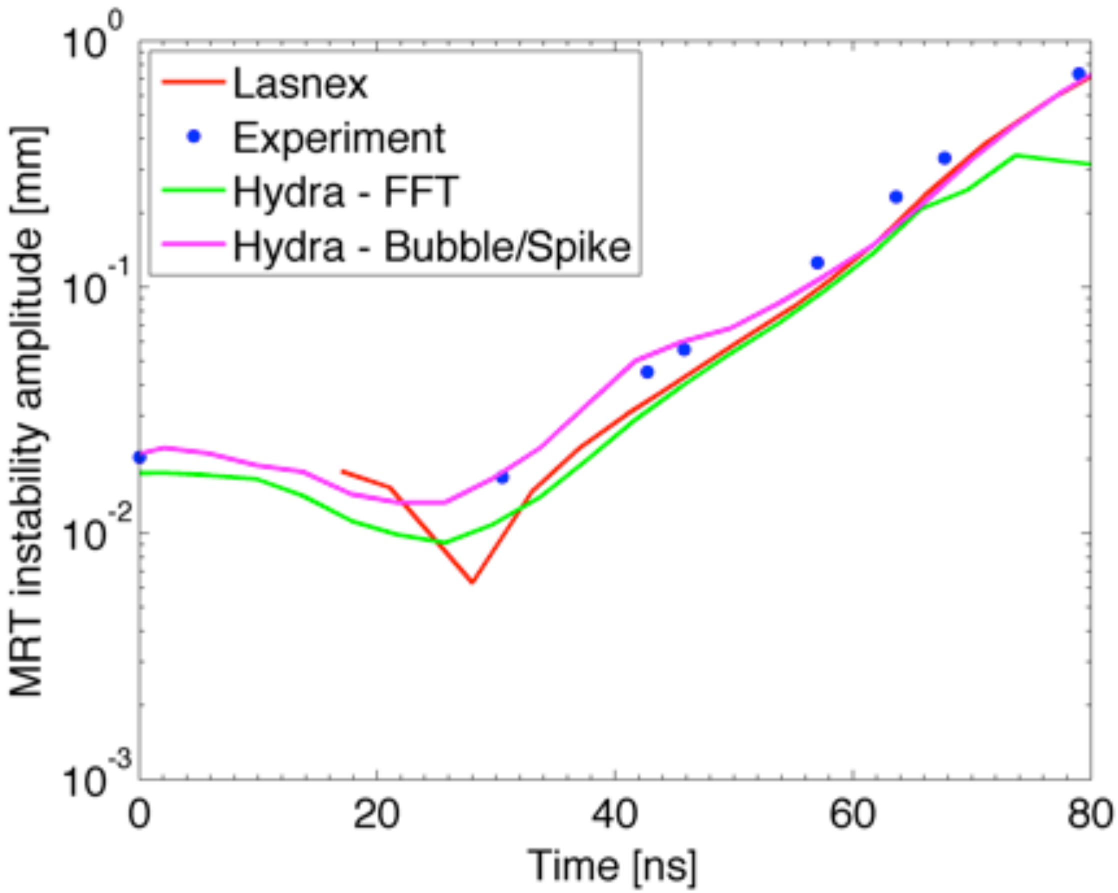


For example: choose 50% transmission contour



Bubble radius **Spike radius**

Hydra shows excellent agreement



- FFT method chronically underestimates growth
  - Possibly due to resolution issues
  - Later times show 400 um peak is broadening to couple with nearby modes
- Though the FFT growth calculation slows, bubble/spike shows continued growth as expected

Summary of results

- Analytic calculations show:
  - Early time feedthrough is minimal and classical RT dominates
  - At high convergence, Bz can do some good, cylindrical modes could cause problems
- Hydra seems to do a good job of getting MRT correct
  - Amplitude growth as a function of time matches data well
  - Simulated radiographs match data well for most times
    - Tilted views tend to smooth over stranger structure and give better agreement
- As non-linear MRT starts to dominate agreement with radiographs begins to degrade which could be due to any number of issues
  - Insufficient resolution
  - Meshing issues
  - Missing physics (3D, Hall, etc.)

Future work

- Analyze MagLIF implosions at high convergence with analytic calculations
  - Analyze inner surface behavior for seeding of MRT at early times
  - Effect of shock propagating through liner
- Use Hydra output to characterize feedthrough and compare to analytic theory
  - Inner interface is invisible to radiography for aluminum
  - Analyze inner surface stability (ET, MRT)
  - Feedthrough should be most important at high convergence which is difficult to image anyway
- Further stress Hydra's predictive capabilities with the latest experiments on the Z-machine