Development and Validation of a Quasi-one-dimensional Particle-in-cell Code for Magnetic Nozzle Simulation

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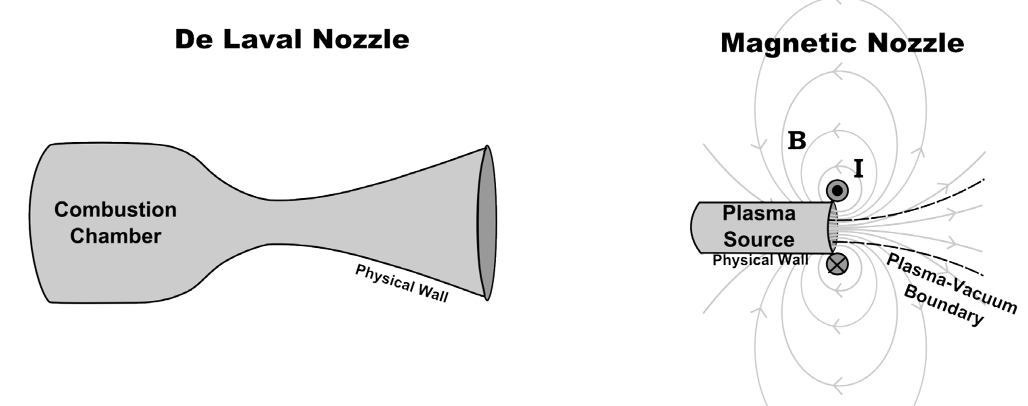
Abstract

The formulation and validation of a novel quasi-one-dimensional particle-in-cell code for the simulation of magnetic nozzles is presented.

Introduction

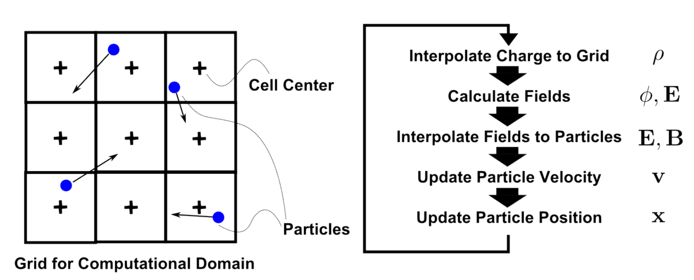
Strong guiding magnetic fields known as magnetic nozzles are key components in the design of electrodeless plasma thrusters. The operating regime of many magnetic nozzle devices (ex. Cubesat Ambipolar Thruster) lie near the edge of the continuum regime, making numerical simulation challenging.[1]



Comparison of magnetic nozzle to De Laval nozzle.[1]

Methodology

We developed a novel quasi-one-dimensional electrostatic particle-in-cell method which focuses on studying energy exchange and thermalization in the plasma.



Electrostatic Particle-In-Cell algorithm.

The centerline axis of the magnetic nozzle is modeled and three velocity dimensions are resolved.

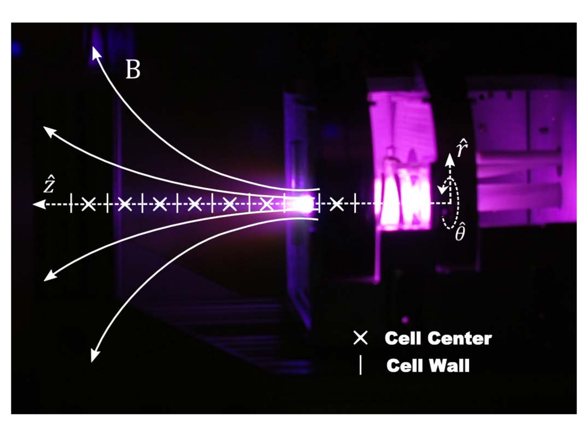


Illustration of the model domain overlaid on a firing of the Cubesat Ambipolar Thruster (CAT).

Magnetized particles are assumed to be displaced from the axis by their Larmor radii to incorporate axial magnetic forces and 2D effects.

$$B_r = -\frac{r_L}{2}\frac{\partial B_z}{\partial z}$$

The cross sectional area varies by assuming that particles approximately follow magnetic field lines.

$$A = \frac{B_{z,in}}{B_z}A_{in}$$

Quasi-one-dimensional Algorithms

Governing equations:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$$

Particle Movers:

- Standard Boris Algorithm for Axisymmetric Coordinates[2],[3]
- Modified Semi-Implicit Q1D Boris Algorithm

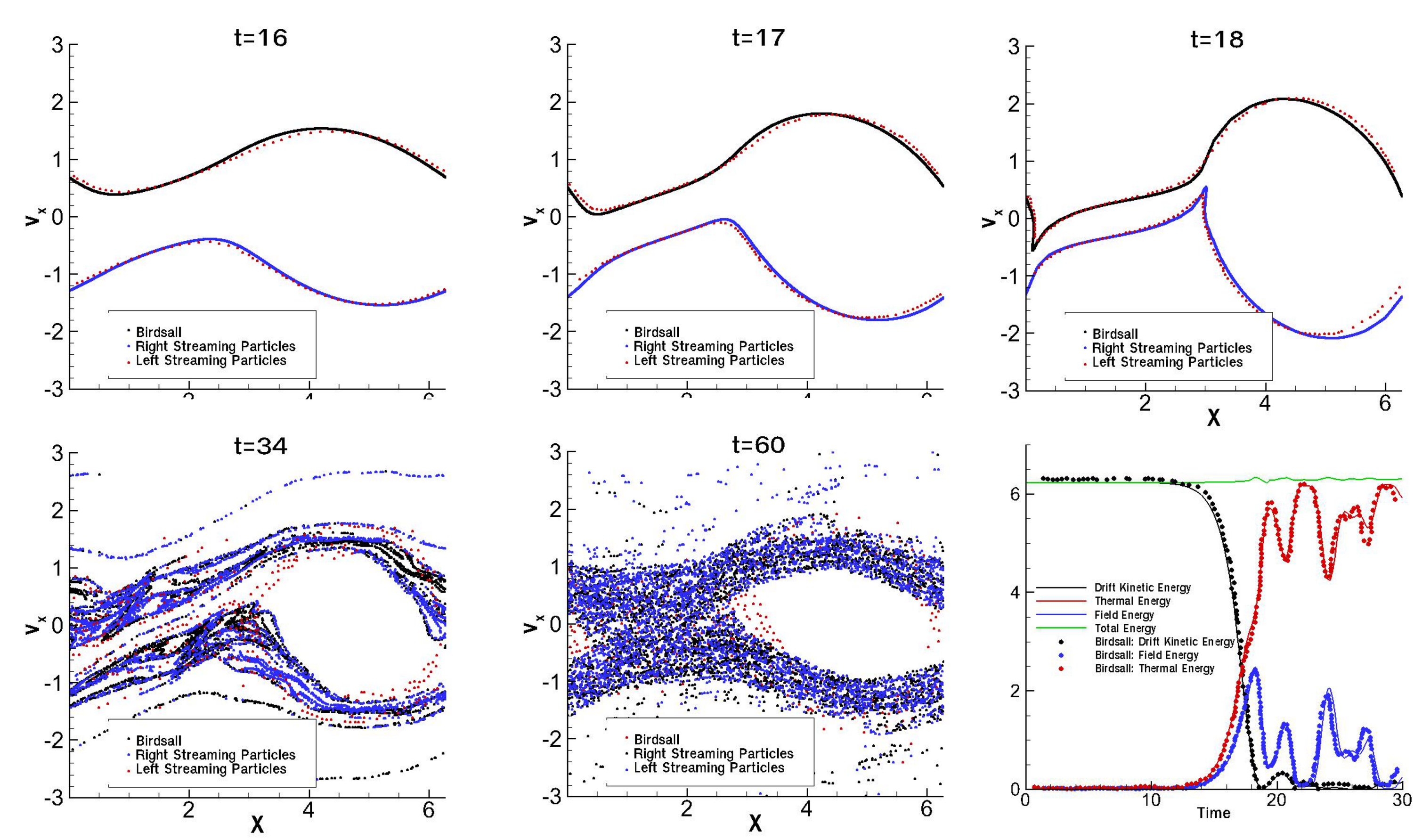
$$\frac{\boldsymbol{v}^{n+\frac{1}{2}} - \boldsymbol{v}^{n-\frac{1}{2}}}{\Delta t} = \frac{q}{m}\left[\boldsymbol{E} + \frac{\left(\boldsymbol{v}^{n+\frac{1}{2}} + \boldsymbol{v}^{n-\frac{1}{2}}\right)}{2}\times\boldsymbol{B}\right] + \frac{\left(\boldsymbol{a}_{cent}^{n+\frac{1}{2}} + \boldsymbol{a}_{cent}^{n-\frac{1}{2}}\right)}{2}$$

$$\boldsymbol{a}_{cent}^{n+\frac{1}{2}} = \frac{\left(v_\theta^{n+\frac{1}{2}}\right)^2}{r_L}\hat{r} - \frac{v_r^{n+\frac{1}{2}}v_\theta^{n+\frac{1}{2}}}{r_L}\hat{\theta}$$

$$r_L = \frac{mv_\theta}{qB_z}$$

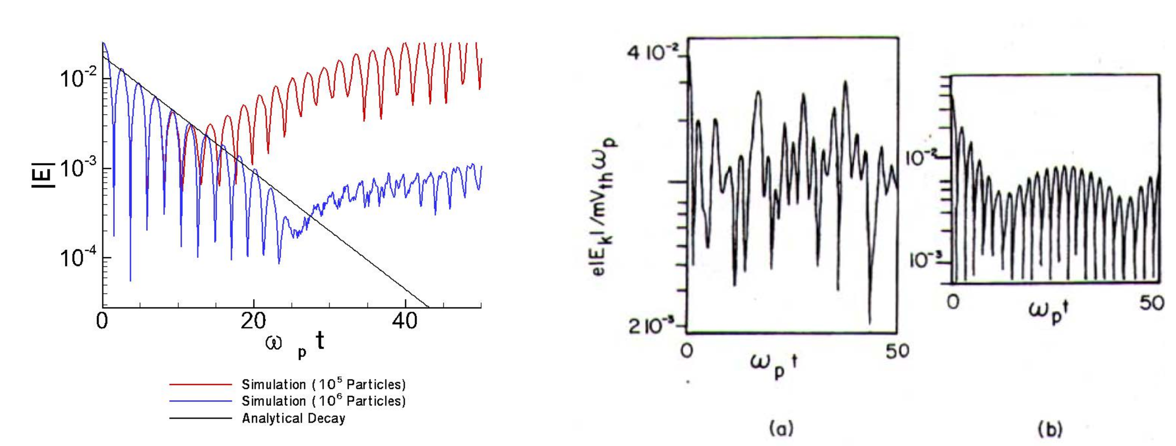
Validation Simulations

Electron-Electron Two Steam Instability



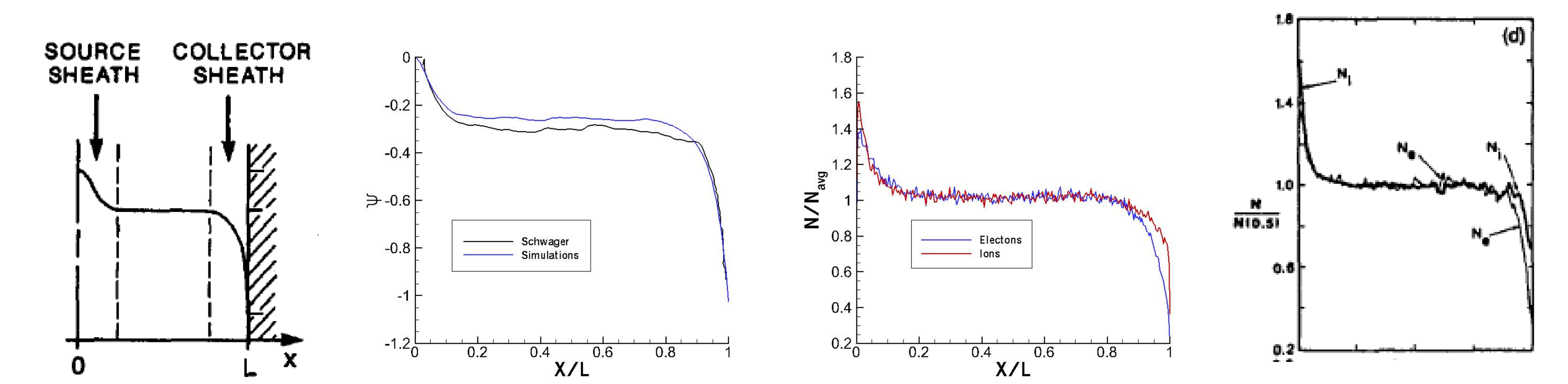
Velocity phase space and energy time history comparisons with results of Birdsall.[3]

Landau Damping



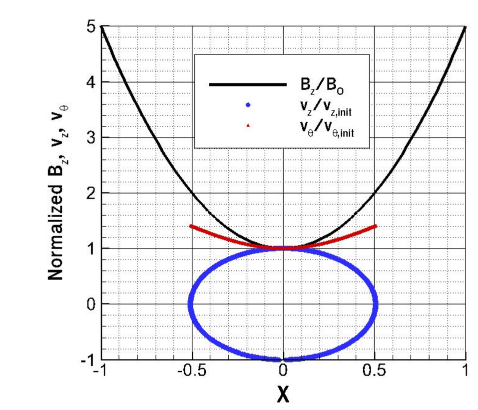
Electrostatic energy time history comparison with Denavit.[4]

Source and Collector Plasma Sheath

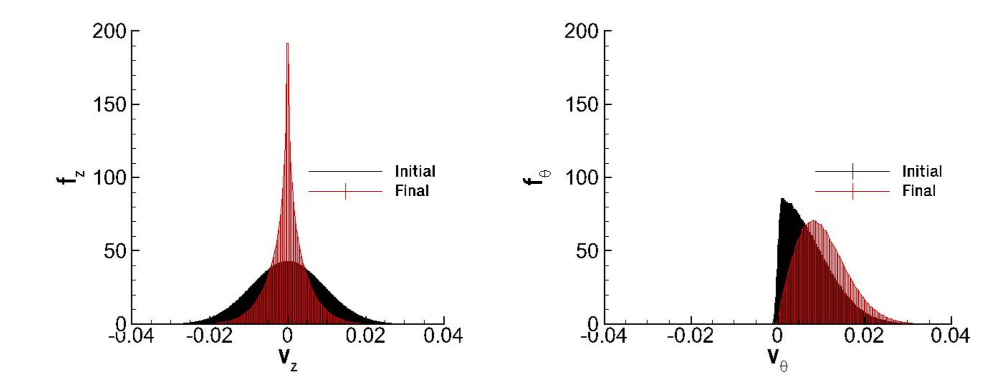


Comparison between our simulations and those of Schwager[6] for a source and collector sheath.

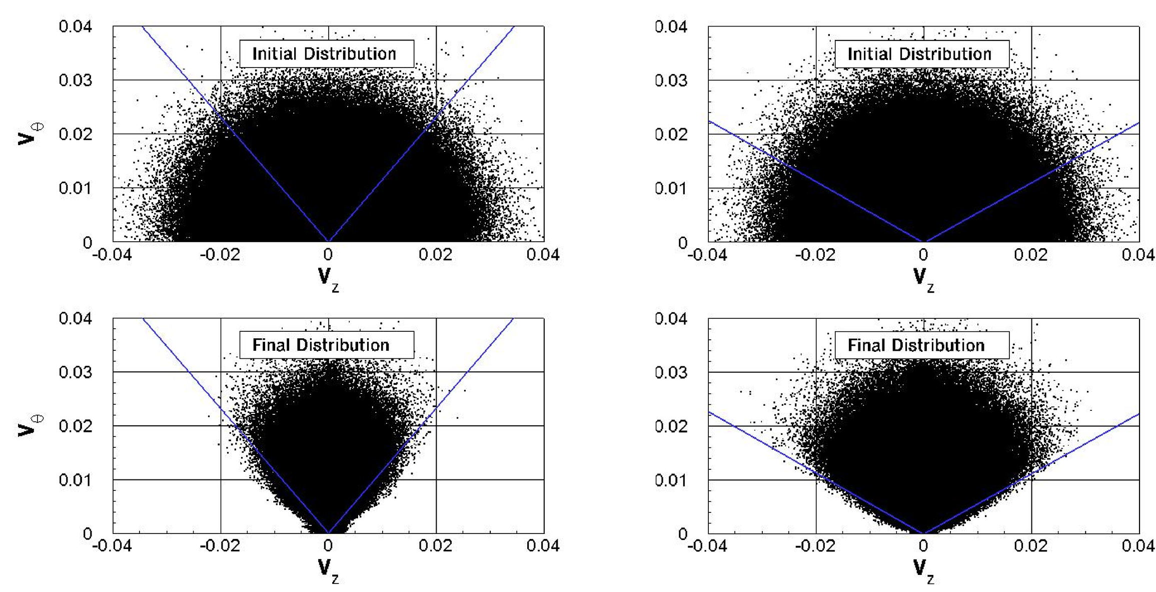
Magnetic Mirror



Particle motion in a magnetic bottle.



Initial and final velocity distribution function in magnetic mirror.



Initial and final velocity space for magnetic mirror simulation with analytical loss cone (blue).

Left: $B_{max} = 1.75$ T
Right: $B_{max} = 4.0$ T

Conclusions

The code is validated with standard one-dimensional test cases. A quasi-one-dimensional method for magnetic nozzle simulation is developed and its implementation shows promising results for magnetic mirror test cases. Further evaluation of Q1D algorithms and implementation will be done in the future.

Acknowledgements

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