# An Exact Hot-Tube Solution for Thin Tape Helix Traveling-Wave Tube*

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# Motivation

- Traveling-Wave Tubes (TWTs) are amplifiers used in satellite communications
- Gain is governed by Pierce dispersion relation for the beam-circuit interaction:

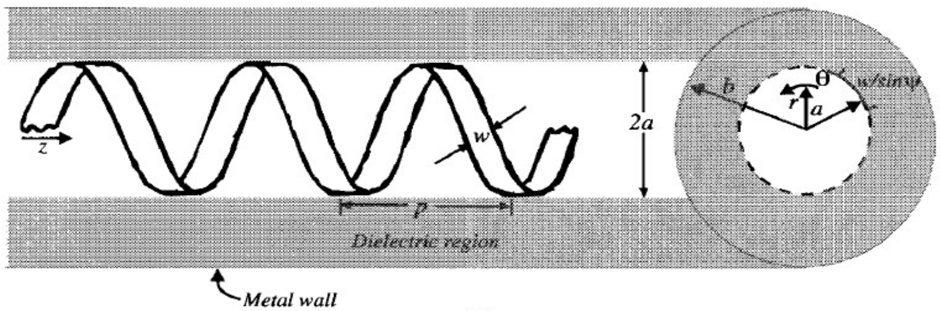
$$\underbrace{[(\beta-\beta_e)^2-4\beta_e^2 Q C^3]}_{\text{Beam Mode}}\underbrace{[\beta_{ph}-\beta]}_{\text{Circuit Mode}}=\underbrace{\beta_e^3 C^3}_{\text{Coupling}}$$

$\beta$ = propagation constant
$\beta_e = \omega/v_0$
$\beta_{ph} = \omega/v_{ph}$

$C$ = gain parameter
$Q$ = space-charge parameter, very difficult to determine

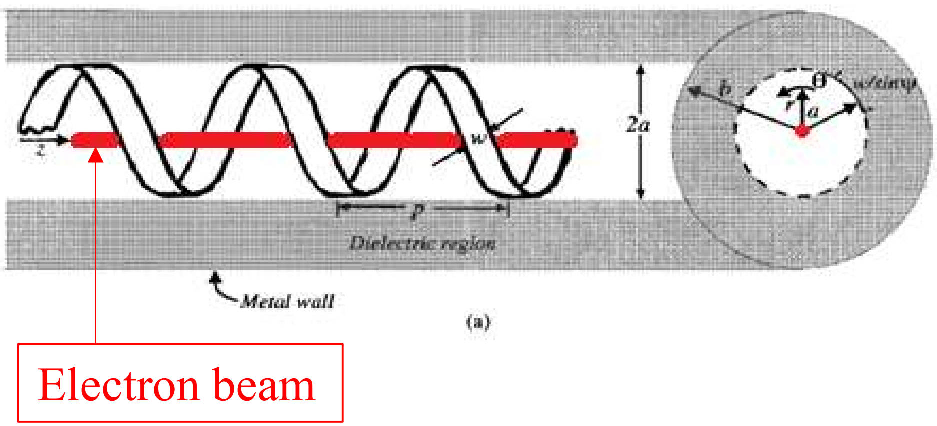
**This work:** Determine $Q$ reliably for first time from an exact theory.

# Exact Solution for Tape Helix TWT




(a)

- Cold-tube (no beam) dispersion relation derived exactly in [1]




- Add in a pencil beam. Obtain exact hot-tube dispersion relation


[1] D. Chernin, et al., *IEEE Trans. ED* **46**, 7 (1999).

# Previous Models of *Q* (or *QC*)

- Branch & Mihran [2]:
  - Replaces the helix with a perfectly-conducting tube (commonly used)
- Sheath Helix [3]:
  - Improvement on Branch & Mihran: current follows helical path
  - Used in the large-signal helix TWT codes

[2] G. M. Branch and T. G. Mihran, *IRE Trans. ED* **2**, 3 (1955).
[3] T. M. Antonsen, Jr. and B. Levush, NRL report NRL/FR/6840-97-9845 (1997).

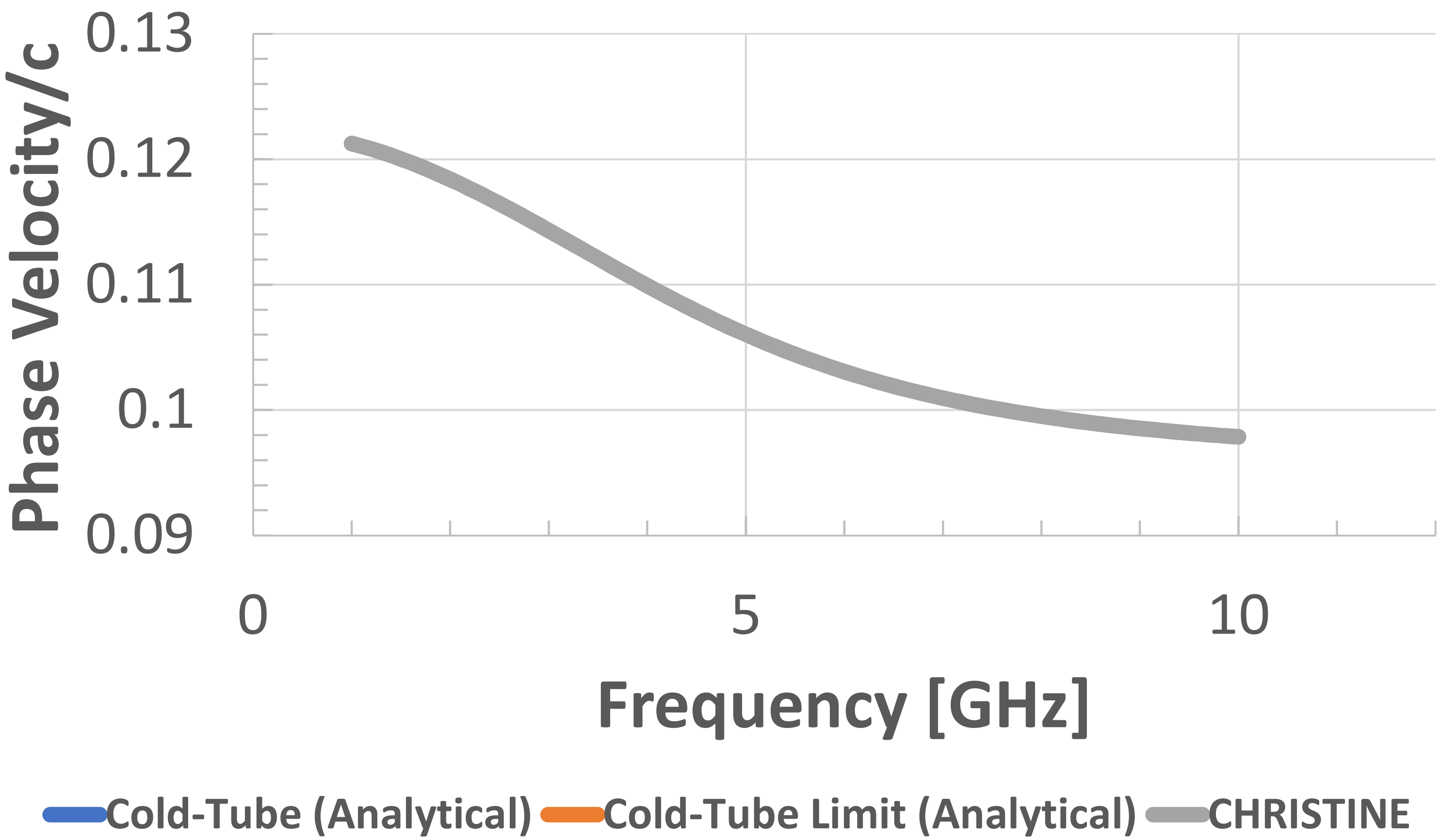
# The Hot-Tube Dispersion Relation

$$D(\beta;\omega) = \det(M) = 0,$$

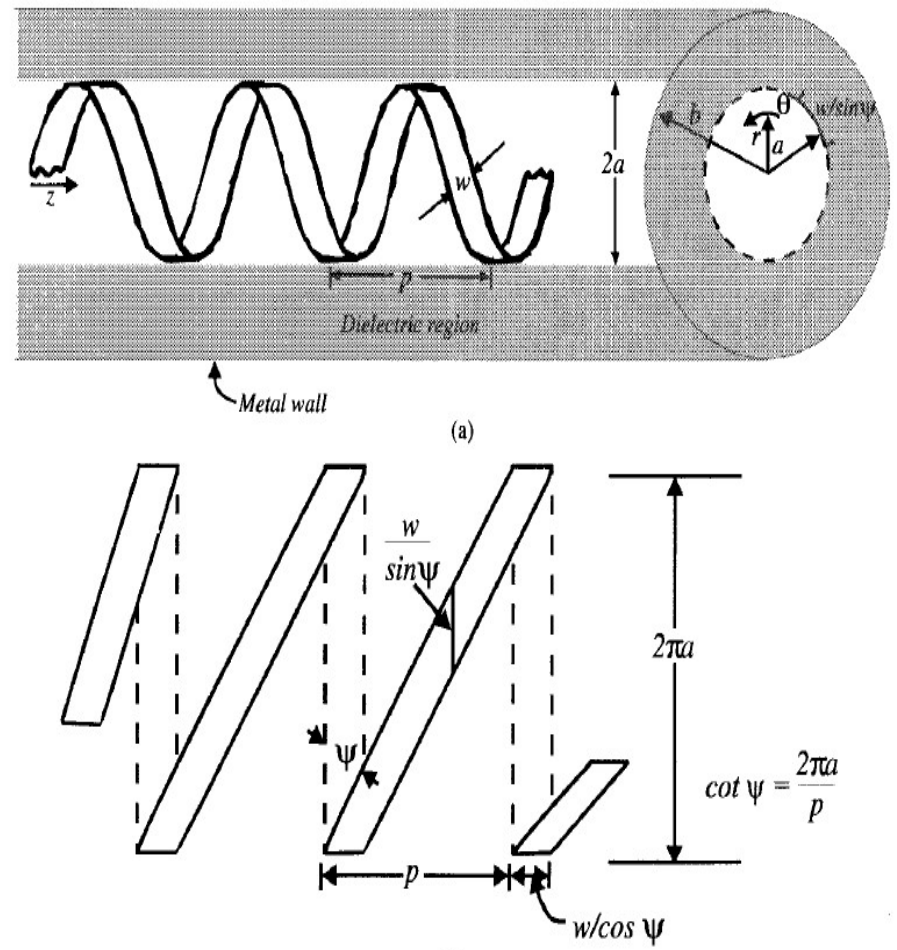
$$M_{ll'} = (-1)^l j^{l+l'} \sum_{n=-\infty}^{\infty} \begin{pmatrix} J_l(\alpha_n) & 0 \\ 0 & \frac{l+1}{\alpha_n} J_{l+1}(\alpha_n) \end{pmatrix} \widetilde{Z_n} \begin{pmatrix} J_{l'}(\alpha_n) & 0 \\ 0 & \frac{l'+1}{\alpha_n} J_{l'+1}(\alpha_n) \end{pmatrix}, l, l' = 0,1,2, \ldots$$

If beam current → 0, have verified analytically that the cold-tube dispersion relation is recovered

# Cold-tube limit
# (numerical)



# Test Case




(a)

(b)

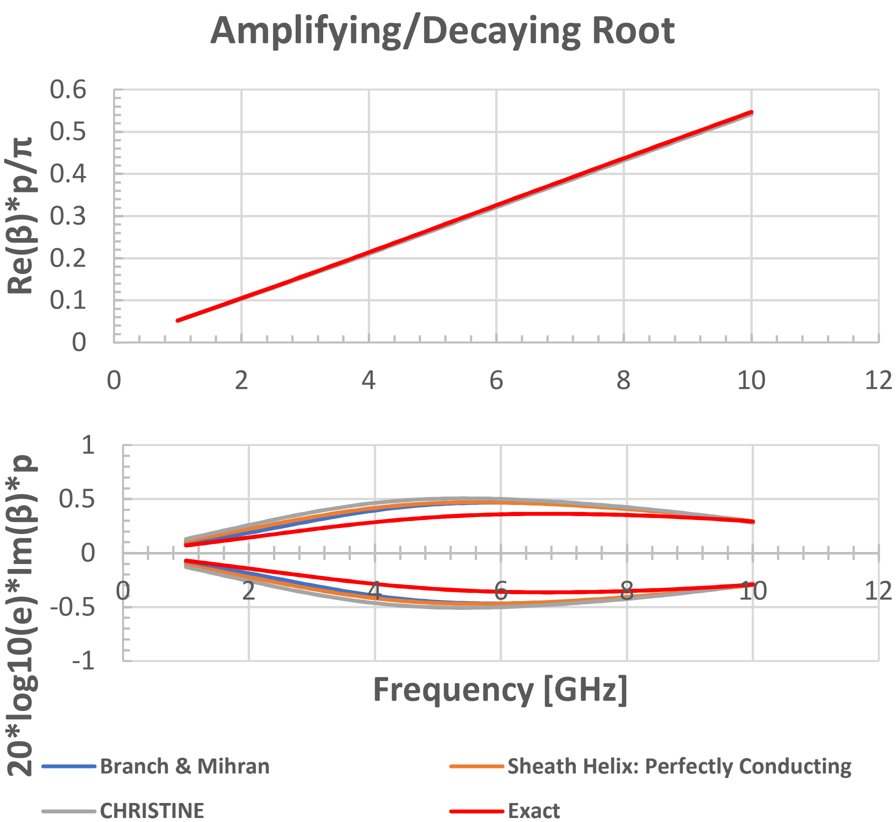
| Parameter | Value |
|---|---|
| **Tape radius (*a*)** | 0.1245 cm |
| **Pitch (*p*)** | 0.0801 cm |
| **Helix pitch angle ($\psi$)** | 5.85º |
| **Wall radius (*b*)** | 0.2794 cm |
| **Dielectric constant of supporting layer ($\varepsilon_r^{(2)}$)** | 1.25 |

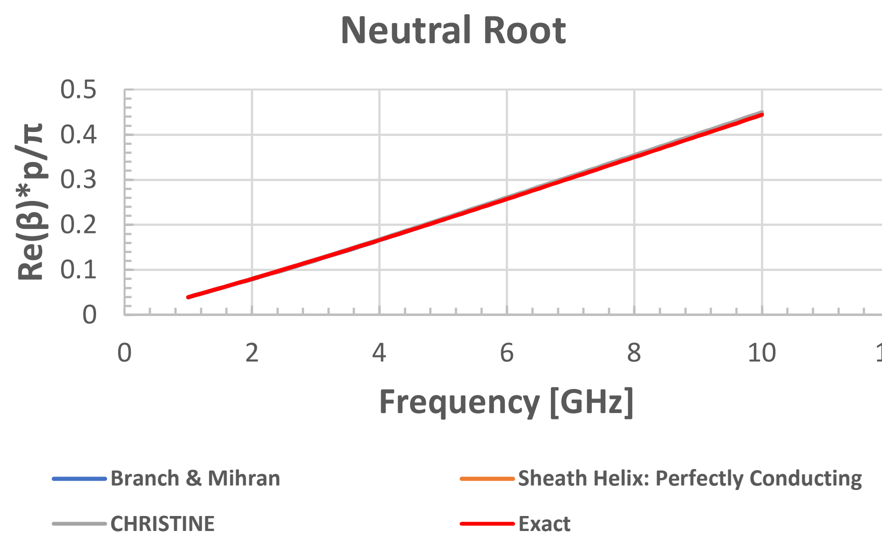
With $\frac{w}{p\cos(\psi)} = 0.2$

| Parameter | Value |
|---|---|
| Beam radius ($r_b$) | 0.05 cm |
| Beam voltage ($V_b$) | 3 kV |
| Beam current ($I_0$) | 0.17 A |

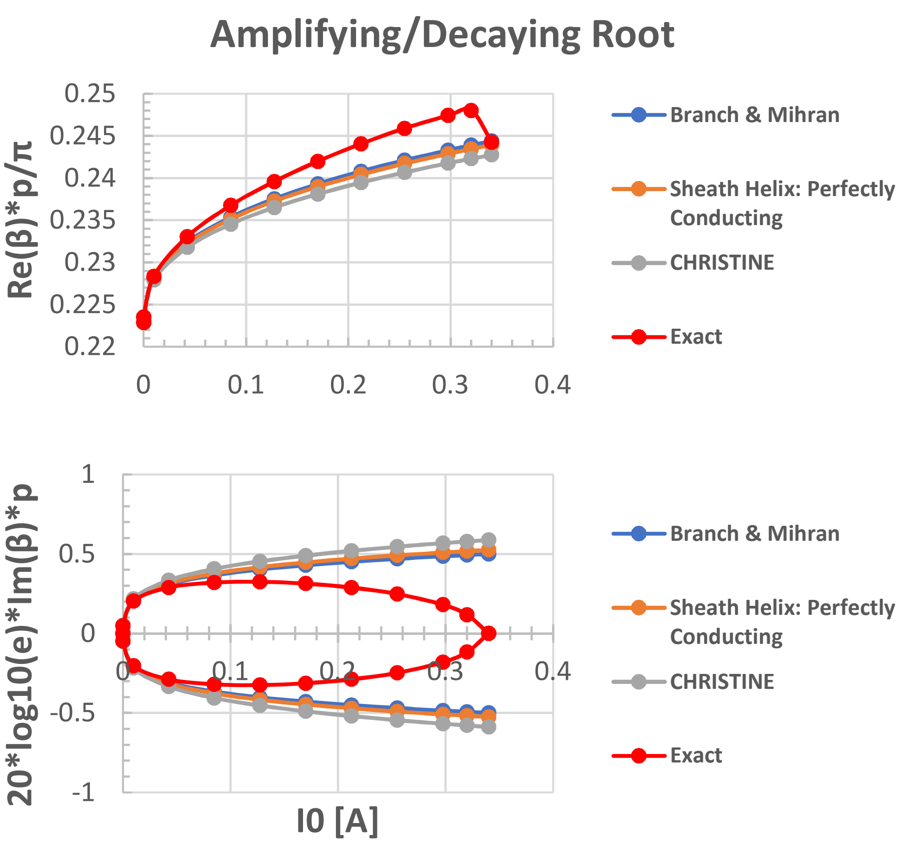
# Hot-Tube Results
# Fixed Beam Current $I_0$

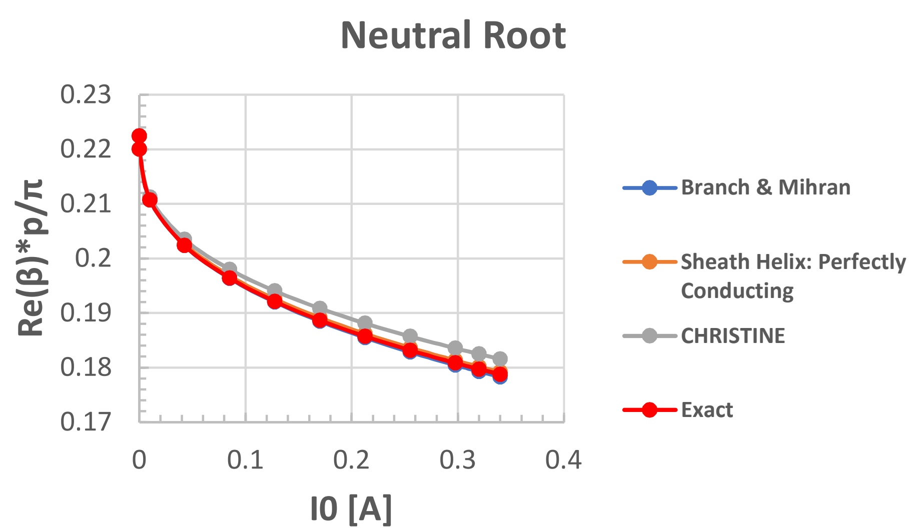
Always numerically found three roots, $\beta_1$, $\beta_2$, $\beta_3$, with two of the roots occurring in complex conjugates



# Hot-Tube Results
# Fixed Signal Frequency (4.5 GHz)



# A New Space-Charge Parameter, $q$

- Impossible to construct the Pierce parameters $Q$ and $C$ self-consistently from the three exact roots, $\beta_1$, $\beta_2$, $\beta_3$:

$$(\beta - \beta_1)(\beta - \beta_2)(\beta - \beta_3) = 0$$

- Need to introduce an additional space-charge parameter, $q$

$$\underbrace{[(\beta - \beta_e)^2 - 4\beta_e^2 Q C^3]}_{\text{Beam Mode}}\underbrace{\left[(\beta_{ph} - \beta) + 4\beta_{ph} q C^3\right]}_{\text{Circuit Mode}} = \underbrace{\beta_e^3 C^3}_{\text{Coupling}}$$

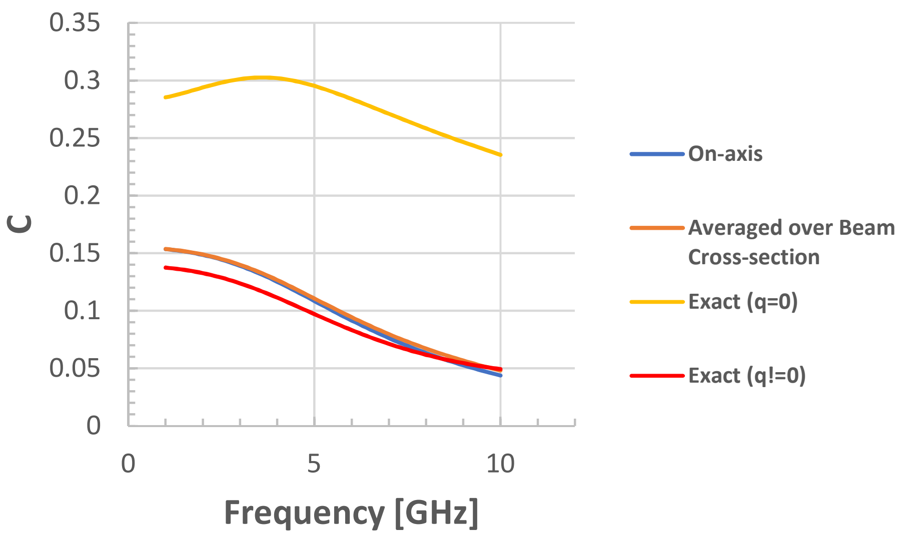
$Q$: Modification of beam mode by space-charge effects
$q$: Modification of circuit mode by space-charge effects

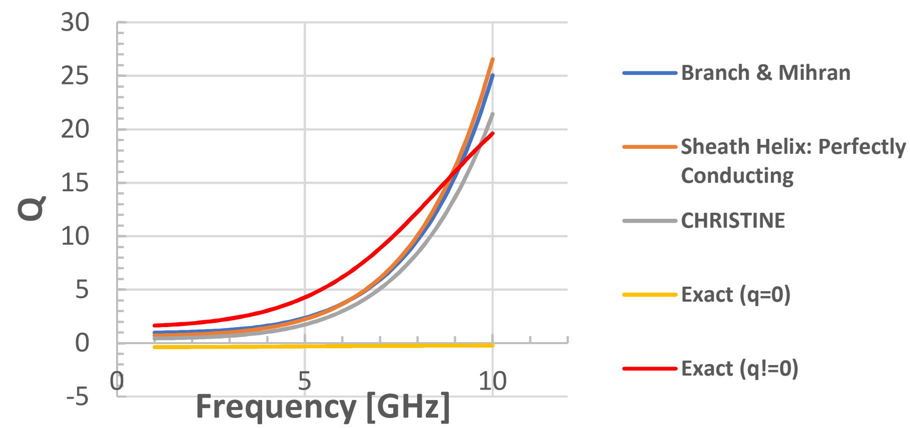
**Note:** $q$ is like a detune in the circuit, so it could be very important.

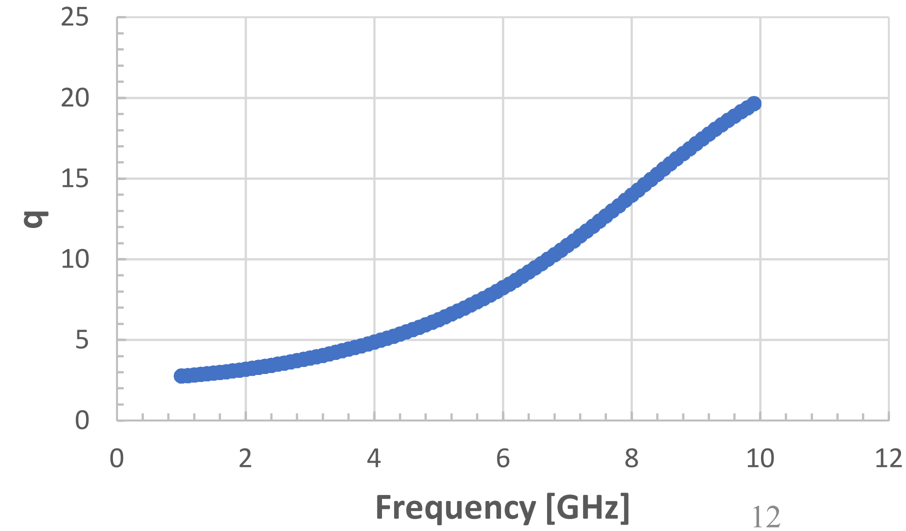
# Determination of $Q$, $q$, and $C$ from $\beta_1$, $\beta_2$, $\beta_3$

$$
\begin{cases}
qC^3 = \dfrac{1}{4}\left(\dfrac{\beta_1 + \beta_2 + \beta_3 - 2\beta_e}{\beta_{ph}} - 1\right) \\
QC^3 = \dfrac{1}{4}\left(1 - \dfrac{\beta_1\beta_2 + \beta_2\beta_3 + \beta_1\beta_3 - 2\beta_e\beta_{ph}(1 + 4qC^3)}{\beta_e^2}\right) \\
C^3 = \dfrac{\beta_e^2(1 - 4QC^3)\beta_{ph}(1 + 4qC^3) - \beta_1\beta_2\beta_3}{\beta_e^3}
\end{cases}
$$

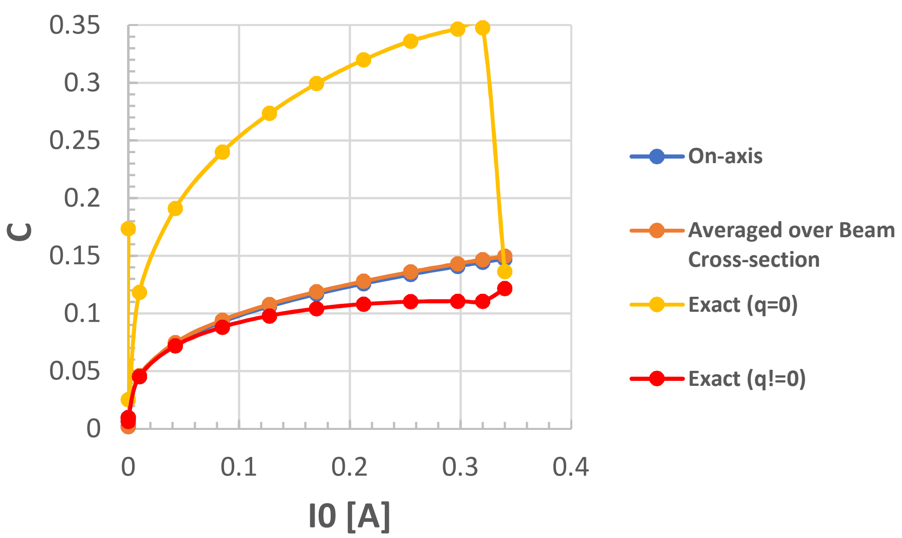
# The Pierce Parameters Fixed Beam Current $I_0$

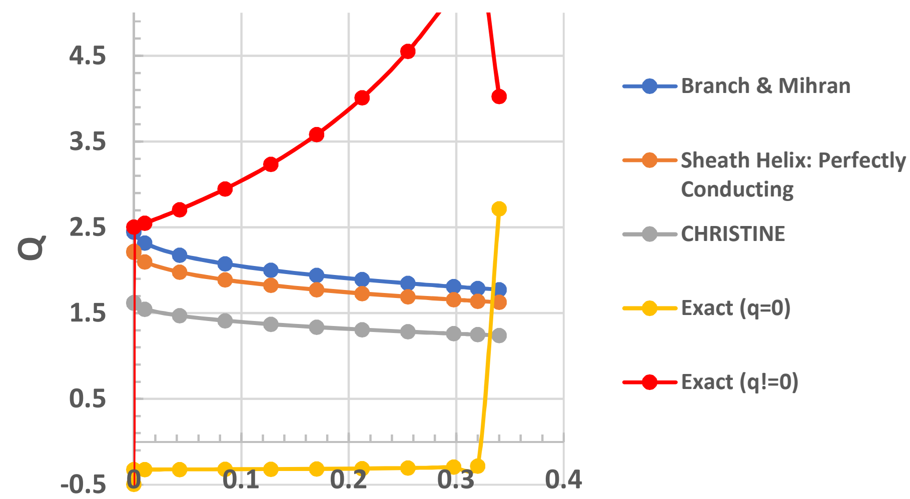


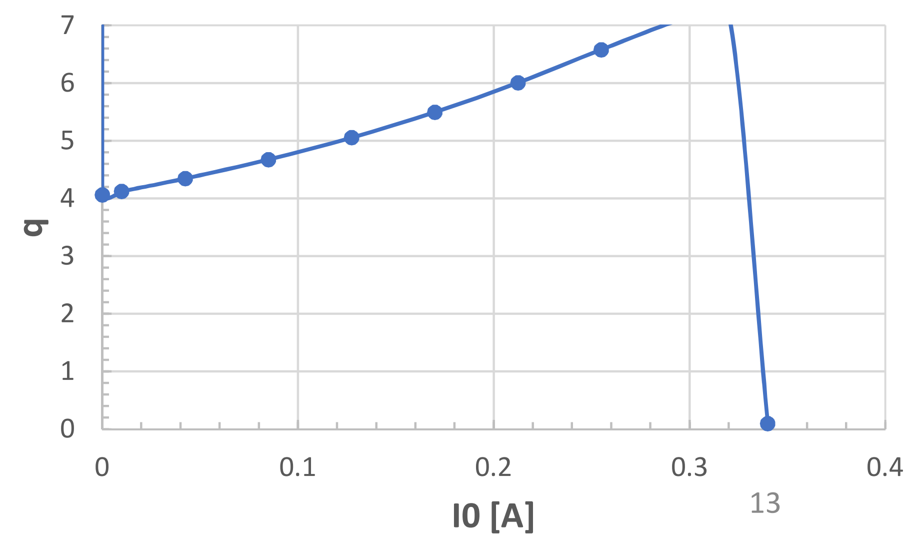



# The Pierce Parameters
# Fixed Signal Frequency (4.5 GHz)



# Conclusions

- An *exact* hot-tube dispersion relation for a realistic tape helix TWT was derived, for the first time.
- Discovered a new parameter $q$ that accounts for space-charge effects on circuit mode, possibly as important as the well-known $Q$ that accounts for space-charge effects on beam mode.
- $Q$, $q$, and $C$ are extracted from the exact dispersion relation, necessary for non-linear code development.