Origin of Second Harmonic Signals in Octave Bandwidth Traveling-Wave Tubes*

Patrick Y. Wong[1], Y. Y. Lau[1], David P. Chernin[2], Ronald M. Gilgenbach[1], and Brad W. Hoff[3]

[1]University of Michigan, Dept. of Nuclear Engineering and Radiological Sciences, Ann Arbor, MI

[2]Leidos Inc., Reston, VA

[3]Air Force Research Laboratory, Kirtland, NM

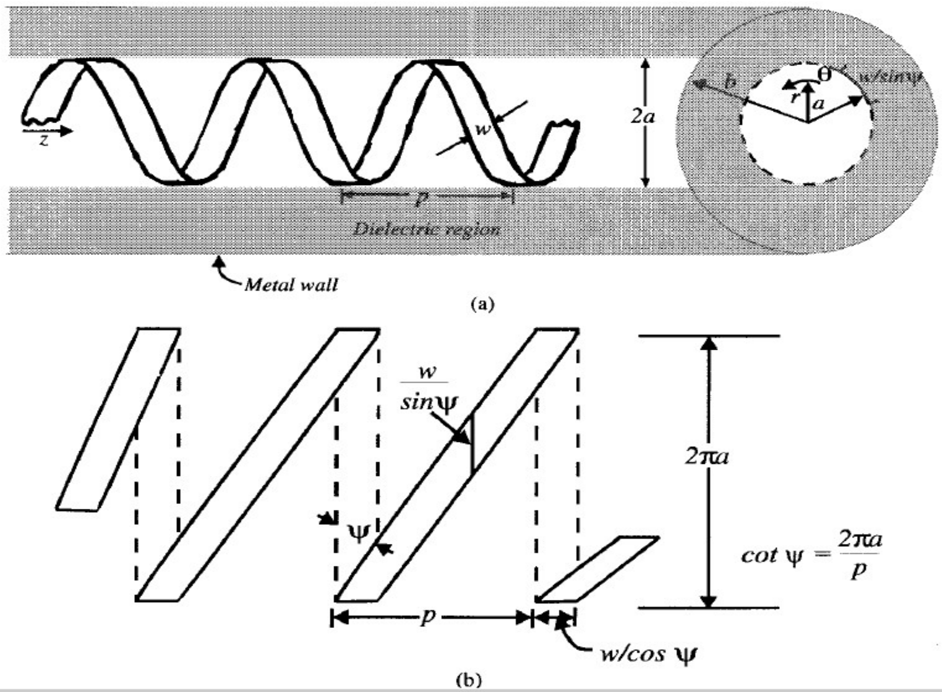
8th MIPSE Graduate Student Symposium

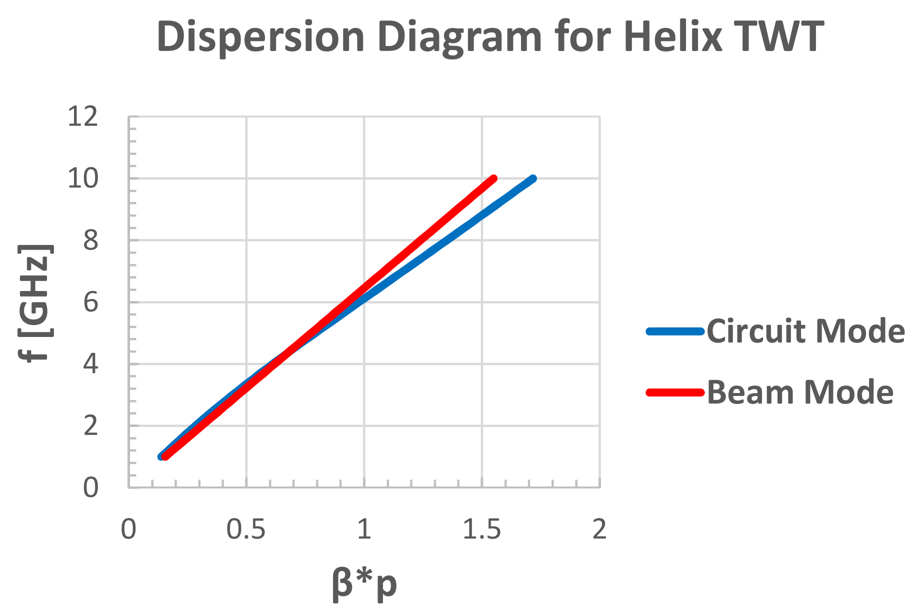
18 October 2017

*Work supported by AFOSR FA9550-15-1-0097, FA9550-14-1-0309, AFRL FA9451-14-1-0374, ONR N00014-16-1-2353, and L-3 Technologies.

Motivation

- In a helix Traveling-Wave Tube (TWT) that has a bandwidth exceeding one octave, the second harmonic of the input signal could be amplified
- General interest in harmonic generation in bunched beams





Question:
What is the harmonic content?

TWT Harmonic Content: Sources

[A] From linear orbits (charge overtaking, like in a klystron*)

[B] From non-linear orbits (e.g. '$v_1 \frac{\partial v_1}{\partial z}$' term in force law)

Turns out that [B] is much more important for harmonic generation

Reason: $v_1 \frac{\partial v_1}{\partial z} \propto e^{j(2\omega_0)t - j(2k_0)z}$ so that it drives resonantly in time *and* in space,

i.e., $v_{ph(2)} = \frac{2\omega_0}{2k_0} = \frac{\omega_0}{k_0} \approx v_0$, the condition for synchronism.

*C.F. Dong, P. Zhang, D. Chernin, Y.Y. Lau, B. W. Hoff, D.H. Simon, P. Wong, G. B. Greening, and R. M. Gilgenbach, "Harmonic Content in the Beam Current in a Traveling-Wave Tube," *IEEE T-ED, VOL. 62, p. 4285* (2015).

The Electronic Equation

$$\left(\frac{\partial}{\partial t} + v(z,t)\frac{\partial}{\partial z}\right) v(z,t) = -\frac{e}{m_e} E(z,t)$$

$$v(z,t) \equiv \frac{Ds}{Dt} = \left(\frac{\partial}{\partial t} + v(z,t)\frac{\partial}{\partial z}\right) s(z,t)$$

Expand:

$$s = s_0 + \varepsilon s_1 + \varepsilon^2 s_2 + \cdots,$$
$$v = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \cdots,$$
$$E = E_0 + \varepsilon E_1 + \varepsilon^2 E_2 + \cdots$$

ε is an expansion parameter in harmonics

The Electronic Equation

- ε^0: DC state
- ε^1: Linearized force law
  - $\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) v_1(z,t) = -\frac{e}{m_e} E_1(z,t)$
  - $v_1(z,t) = \left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) s_1(z,t)$
- ε^2: 2nd harmonic generation
  - $\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) v_2(z,t) = -\frac{e}{m_e} E_2(z,t) - \boldsymbol{v_1 \frac{\partial v_1}{\partial z}}$
  - $v_2(z,t) = \left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) s_2(z,t) + v_1 \frac{\partial s_1}{\partial z}$

  Note: Second harmonic is generated. It is phase-controlled by the input signal at the fundamental frequency.

The Electronic Equation (Continued)

- Write the nth harmonic in $E(z,t)$ as:
  - $E_n(z,t) = E_{Cn}(z,t) + E_{SCn}(z,t)$
    $$= \widetilde{E_{C(n)}}(z)e^{j(n\omega_0)t} + \frac{4(n\omega_0)^2 QC_{(n)}^3 \widetilde{s_{(n)}}(z)e^{j(n\omega_0)t}}{e/m_e}$$
    - E_{Cn} = Circuit electric field
    - E_{SCn} = Space-charge electric field

The Circuit Equation

Excitation of circuit wave:

$$\left(\frac{d}{dz} + \Gamma_{(n)}\right)\widetilde{E_{C(n)}}(z) = \frac{j}{v_0}\frac{m_e}{e}(n\omega_0)^3 C_{(n)}^3 \widetilde{s_{(n)}}(z),$$

$$\Gamma_{(n)} = j\beta_{p(n)} + \beta_{e(n)} C_{(n)} d_{(n)}$$

$\beta_{p(n)} = \dfrac{n\omega_0}{v_{ph(n)}}$ = cold circuit wavenumber

$\beta_{e(n)} = \dfrac{n\omega_0}{v_0}$ = electronic wavenumber

$C_{(n)}$ = Pierce's gain parameter

$d_{(n)}$ = Pierce's loss parameter

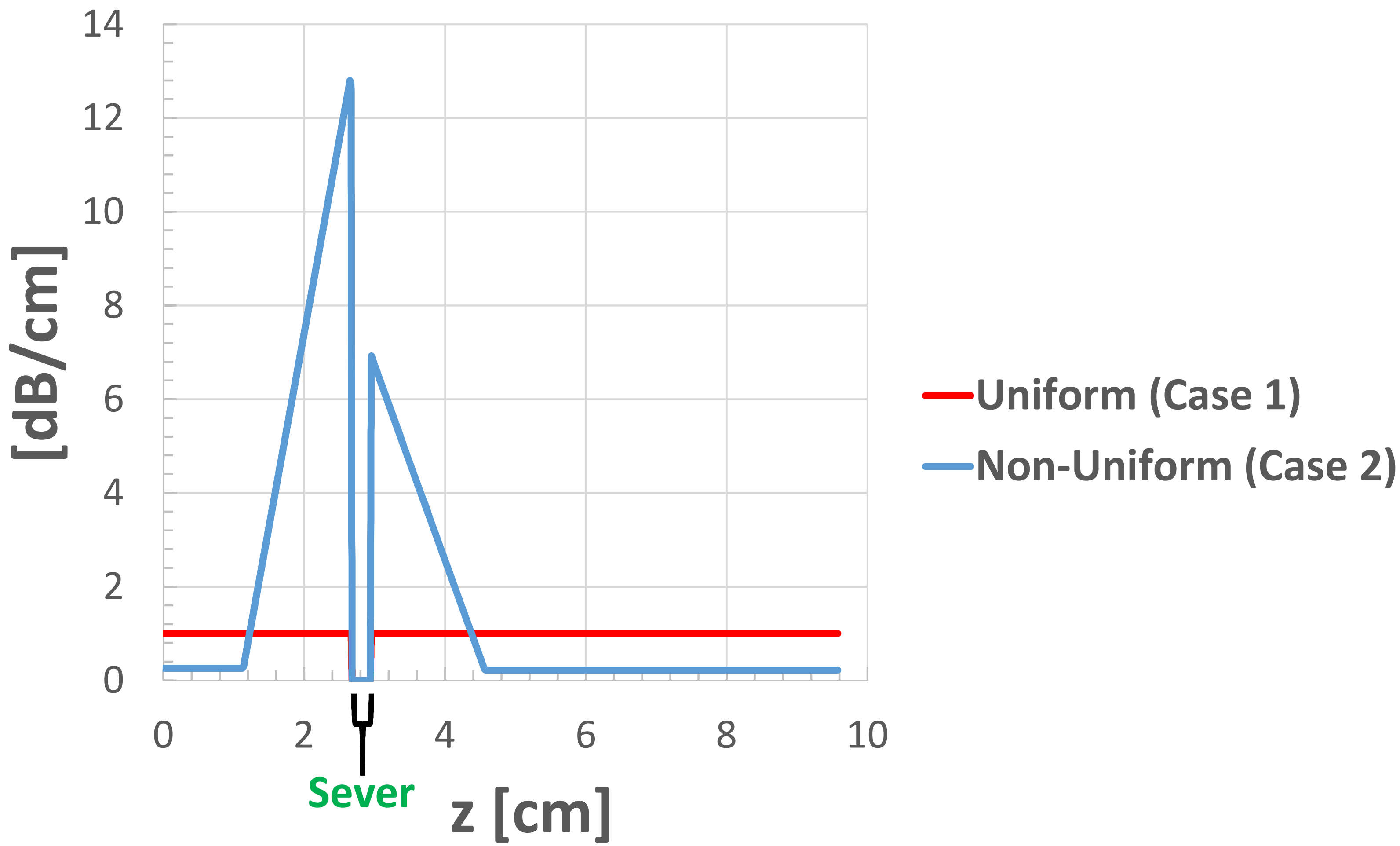
Test Cases

- Helix TWT with a mid-stream sever ($2.667 < z < 2.921$ cm)

Circuit Parameters					
n	$n^*\omega_0$	K	C	QC	b
1	2π(4.5 GHz)	111.27Ω	0.116	0.281	0.337
2	2π(9 GHz)	8.97Ω	0.050	1.053	2.961

Beam Parameters			
V_b	I_0	P_b	$I_0/V_b^{3/2}$
3.0 kV	0.17 A	$V_bI_0 = 510$ W	1.035 µP

Test Cases
Resistive Loss as a function of z

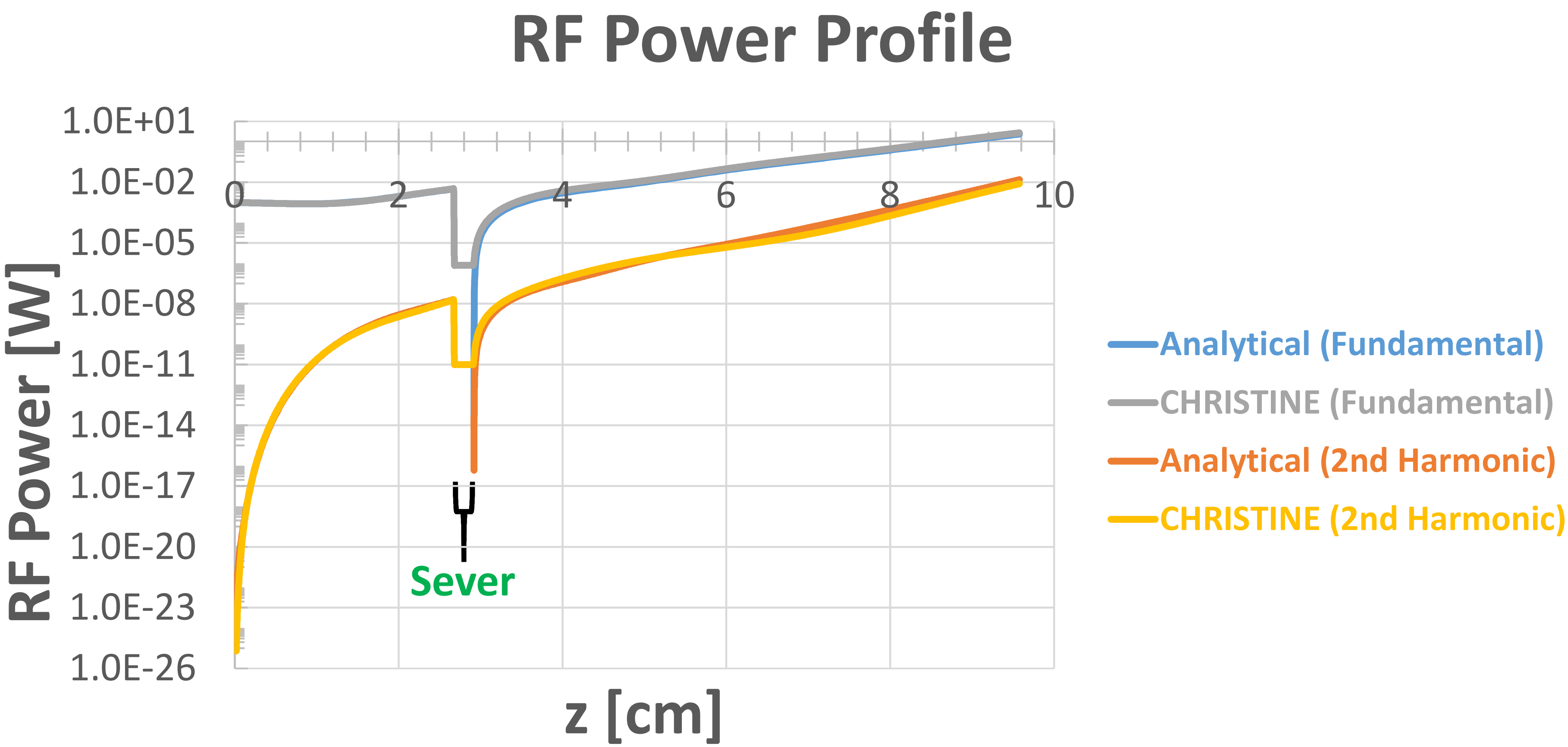


Initial/Boundary Conditions

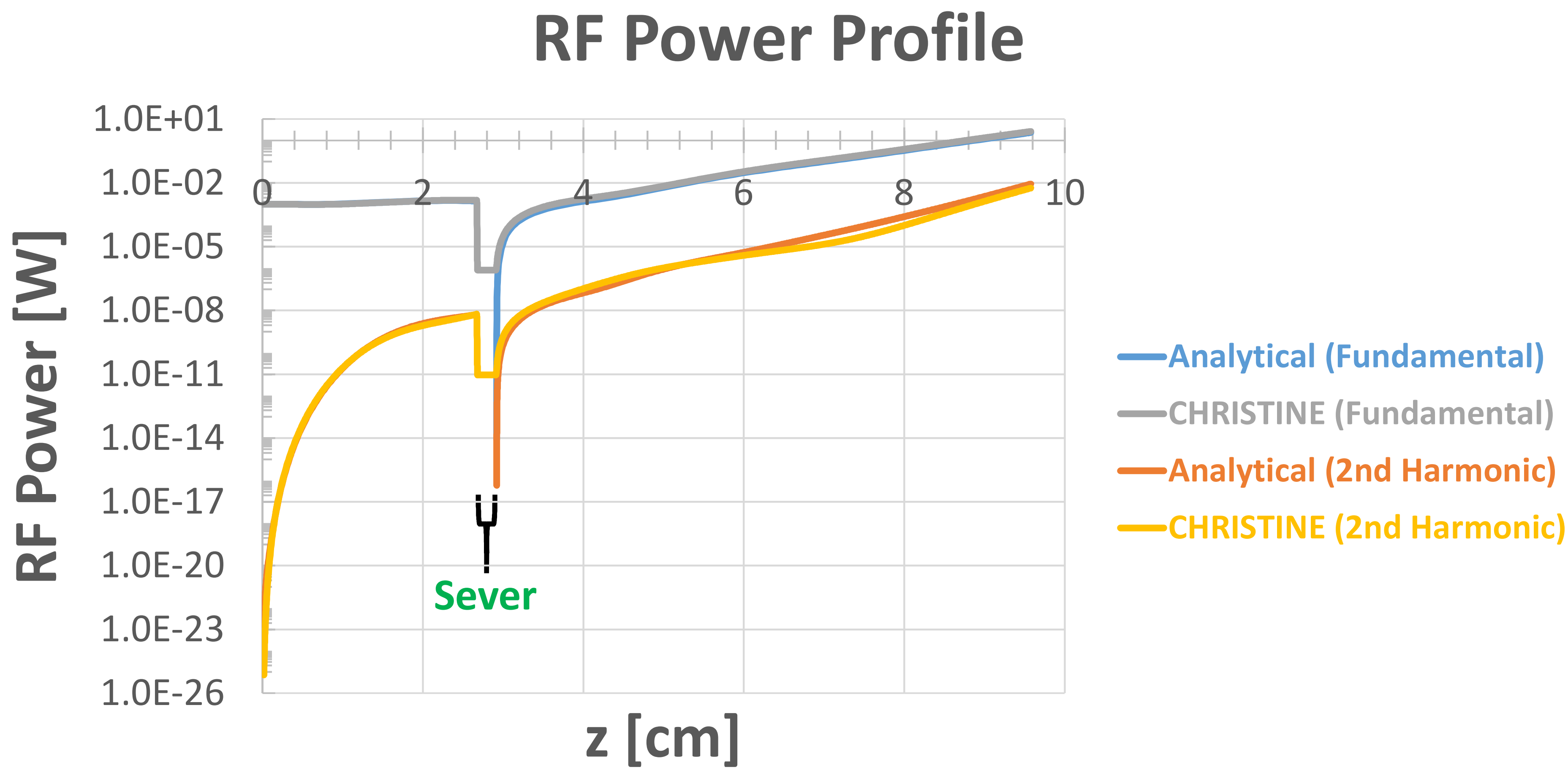
- The governing equations in each region are solved subject to the appropriate initial/boundary conditions:

Pre-sever	
Fundamental (n=1)	2nd Harmonic (n=2)
$\widetilde{s_{(1)}}(z = 0) = 0$	$\widetilde{s_{(2)}}(z = 0) = 0$
$\widetilde{v_{(1)}}(z = 0) = 0$	$\widetilde{v_{(2)}}(z = 0) = 0$
$\widetilde{E_{(1)}}(z = 0) = E_{10}$	$\widetilde{E_{(2)}}(z = 0) = 0$
Sever	
Fundamental (n=1)	2nd Harmonic (n=2)
$\widetilde{s_{(1)}}(z^- = 2.667\text{ cm}) = \widetilde{s_{(1)}}(z^+ = 2.667\text{ cm})$	$\widetilde{s_{(2)}}(z^- = 2.667\text{ cm}) = \widetilde{s_{(2)}}(z^+ = 2.667\text{ cm})$
$\widetilde{v_{(1)}}(z^- = 2.667\text{ cm}) = \widetilde{v_{(1)}}(z^+ = 2.667\text{ cm})$	$\widetilde{v_{(2)}}(z^- = 2.667\text{ cm}) = \widetilde{v_{(2)}}(z^+ = 2.667\text{ cm})$
$\tilde{E}(z) \equiv 0\ \forall z$	
Post-sever	
Fundamental (n=1)	2nd Harmonic (n=2)
$\widetilde{s_{(1)}}(z^- = 2.921\text{ cm}) = \widetilde{s_{(1)}}(z^+ = 2.921\text{ cm})$	$\widetilde{s_{(2)}}(z^- = 2.921\text{ cm}) = \widetilde{s_{(2)}}(z^+ = 2.921\text{ cm})$
$\widetilde{v_{(1)}}(z^- = 2.921\text{ cm}) = \widetilde{v_{(1)}}(z^+ = 2.921\text{ cm})$	$\widetilde{v_{(2)}}(z^- = 2.921\text{ cm}) = \widetilde{v_{(2)}}(z^+ = 2.921\text{ cm})$
$\widetilde{E_{(1)}}(z = 2.921\text{ cm}) = 0$	$\widetilde{E_{(2)}}(z = 2.921\text{ cm}) = 0$

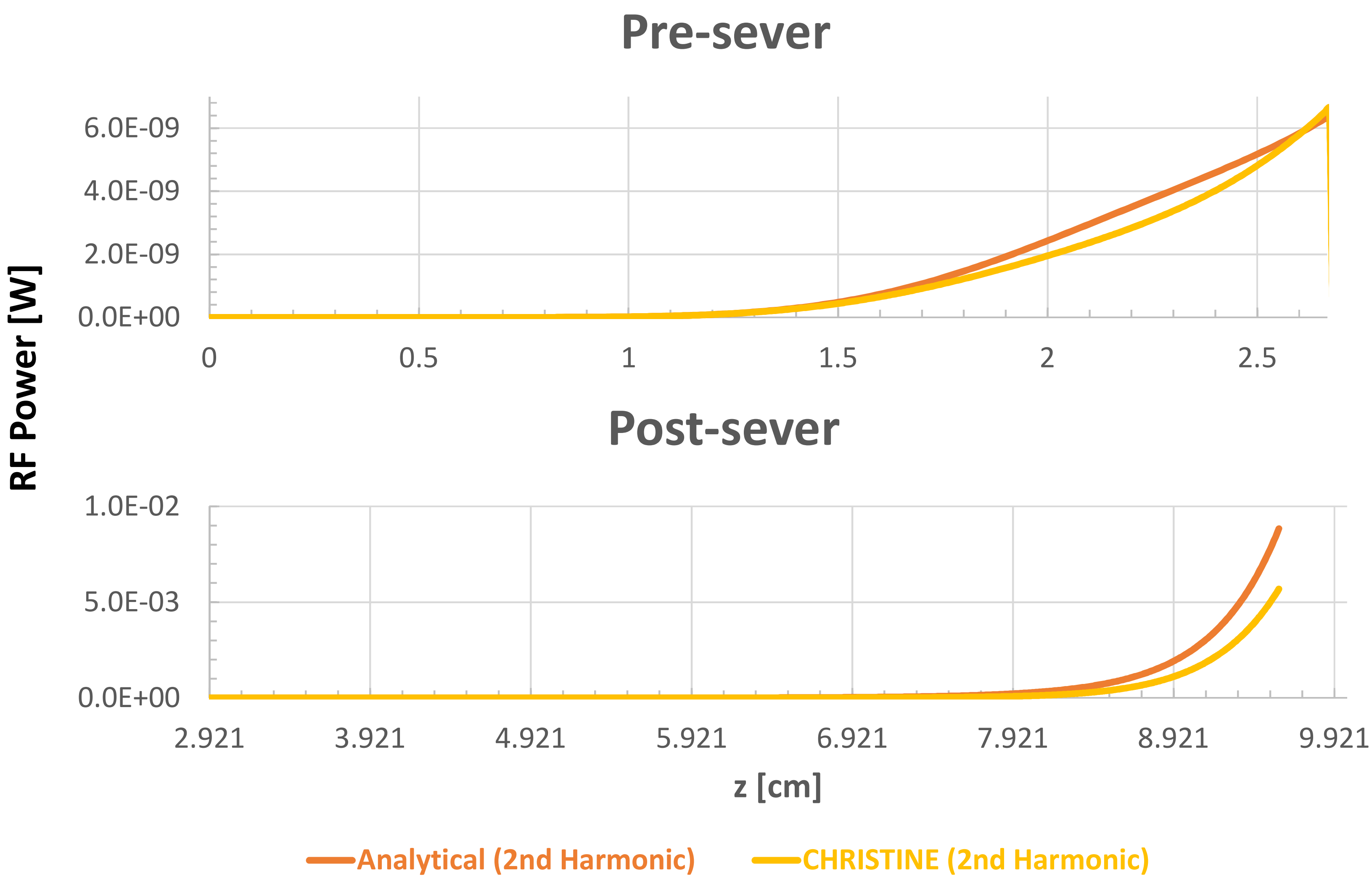
Case 1: Uniform Cold-Tube Loss (1 dB/cm attenuation)



Case 2: Non-Uniform Cold-Tube Loss



Results: In Linear Scale RF Power Profile



Conclusions

- A wideband TWT may have the 2nd harmonic within its amplification band and hence gain at this $2\omega_0$ frequency with input only at the fundamental ω_0
- The non-linear orbital terms, leading to $e^{j(2\omega_0)t-j(2k_0)z}$, resonantly interact with the beam mode at second harmonic, $(2\omega_0)/(2k_0) = v_0$
- The analytical formulation of the equations governing the evolution of this 2nd harmonic was found, including effects of spatial taper
- Reasonable agreement between the analytic theory and simulations using the CHRISTINE code was observed