The mobility of electrons in a Hall thruster simulation

Astrid L. Raisanen and Iain D. Boyd
University Of Michigan, Department of Aerospace Engineering, Ann Arbor, MI

Hall-effect thruster

Hall thrusters are electric propulsion devices for space vehicles, attractive for their highly efficient utilization of propellant mass.

Newton's 3rd Law: $T = m\frac{dV}{dt} = \dot{m}U_e = \dot{m}gI_{sp}$

Rocket equation: $\int_{V_i}^{V_f} \frac{-dV}{U_e} = \int_{m_i}^{m_f} \frac{dm}{m} \Rightarrow \frac{m_f}{m_i} = exp\left(\frac{-\Delta V}{U_e}\right)$

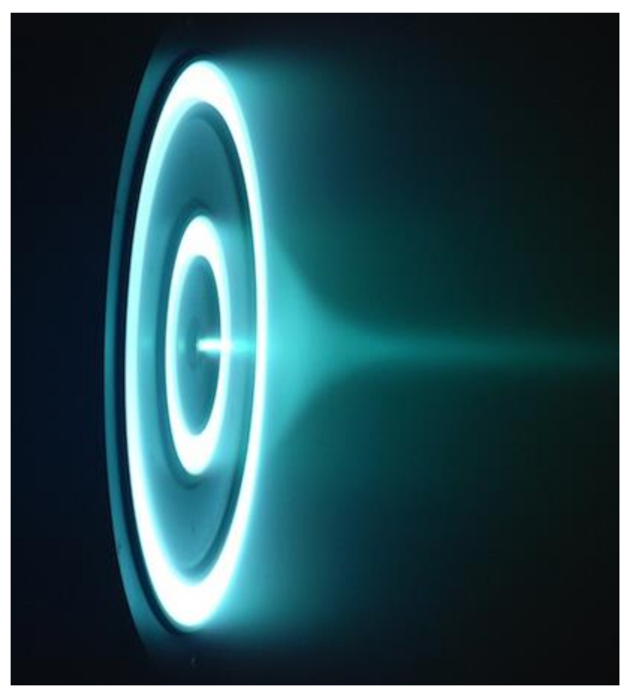

Fig. 1: Dual channel Hall thruster [1]

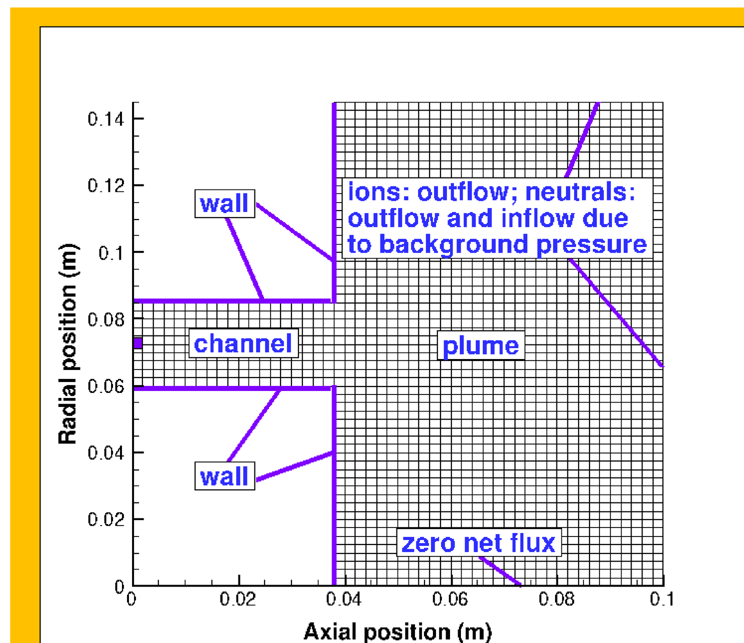

Figure 2: DK simulation domain

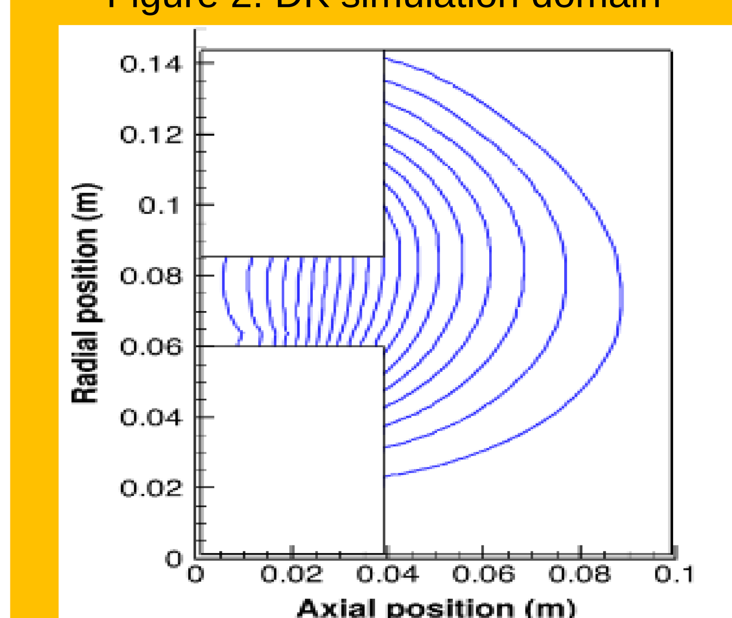

Figure 3: Electron simulation domain

Cross-field electron transport

For a steady, isothermal plasma:

$$\vec{v} = \frac{e}{mv}\vec{E} - \frac{k_b T}{mv}\frac{\nabla n}{n} = \mu\vec{E} - D\frac{\nabla n}{n}$$

In the presence of a magnetic field, classical cross-field transport is described by:

$$mn\frac{dv_\perp}{dt} = en\left(\vec{E} + v_\perp \times \vec{B}\right) - k_b T\nabla n - mnv\vec{v} = 0$$

$$\mu_\perp = \frac{\mu}{1+\omega_c^2\tau^2} \approx \frac{mv}{eB^2} \qquad D_\perp = \frac{D}{1+\omega_c^2\tau^2} \approx \frac{k_b T mv}{e^2 B^2}$$

Hall parameter: measure of electron confinement by a magnetic field

$$\Omega = \omega_c\tau \qquad \Omega \gg 1 \implies \mu, D \propto \frac{1}{B^2}$$

However, Bohm diffusion says that: $\mu, D \propto \frac{1}{B}$

and Hall thrusters are not dominated by classical transport

Modeling and theory: hybrid-direct kinetic (DK) method

Motivation: Particle-in-Cell (PIC) simulations contain statistical noise which may pollute modeled oscillatory behavior. The grid-based Vlasov, or direct kinetic (DK) method contains no statistical noise and is the subject of this work.

A 2D, finite volume DK solver (Figure 2) is coupled with a quasi-one dimensional electron solver (Figure 3).

Ion and neutral atom Boltzmann equations are described by:

$$\frac{\partial f_i}{\partial t} + v_z\frac{\partial f_i}{\partial z} + v_r\frac{\partial f_i}{\partial r} + \frac{eE_z}{m_i}\frac{\partial f_i}{\partial z} + \frac{eE_r}{m_i}\frac{\partial f_i}{\partial r} = S_i \qquad \frac{\partial f_n}{\partial t} + v_z\frac{\partial f_n}{\partial z} + v_r\frac{\partial f_n}{\partial r} = S_n$$

Thermalized potential for quasi-one-dimensional electron domain: $\phi^*(\lambda) = \phi(\lambda, r) - \frac{k_b T_e(\lambda)}{e} ln\left(\frac{n_e(\lambda, r)}{n_e^*}\right)$

It is necessary to consider coupling between boundary conditions for the potential and electron temperature, the anode plasma sheath, and the ***anomalous electron transport*** model.

Method: Two different electron mobility modeling approaches are used.

Result: Method 1 from the literature (below) yields insufficient results for the H6, a 6 kW Hall thruster. Method 2 (right) yields better results but indicates that the quasi-one-dimensional electron model does not yield the proper electron current density inside the channel, where diffusion should be high.

Method 1: 3-region Bohm mobility profile, similar to that in [2]

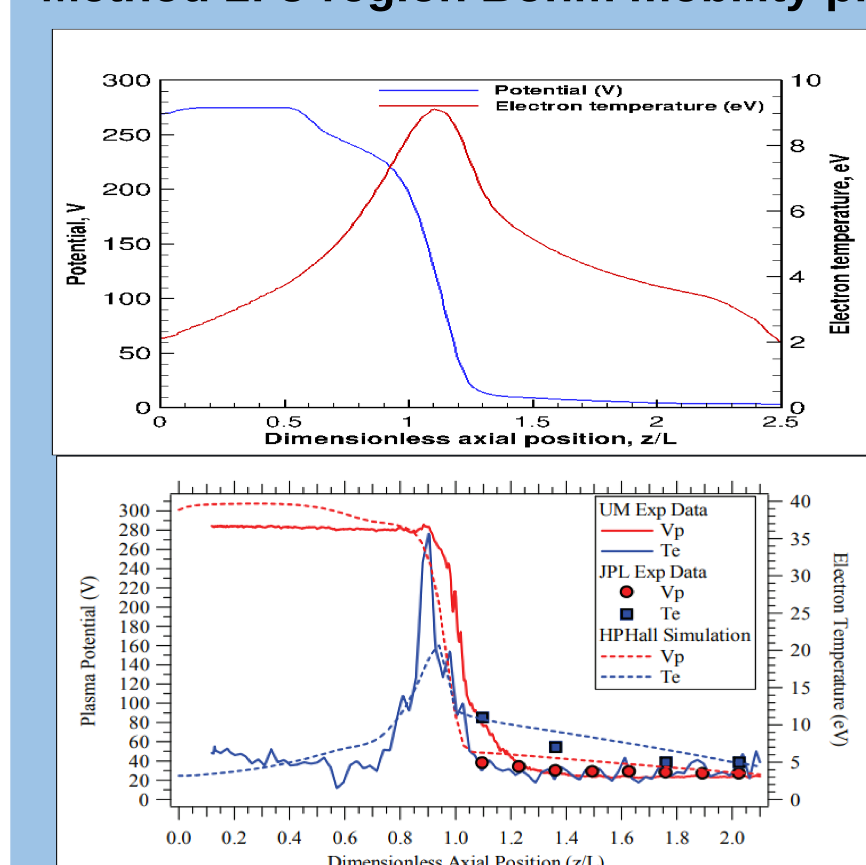

Figure 4: Average plasma properties along thruster channel centerline (top: method 1 and bottom: published [2])

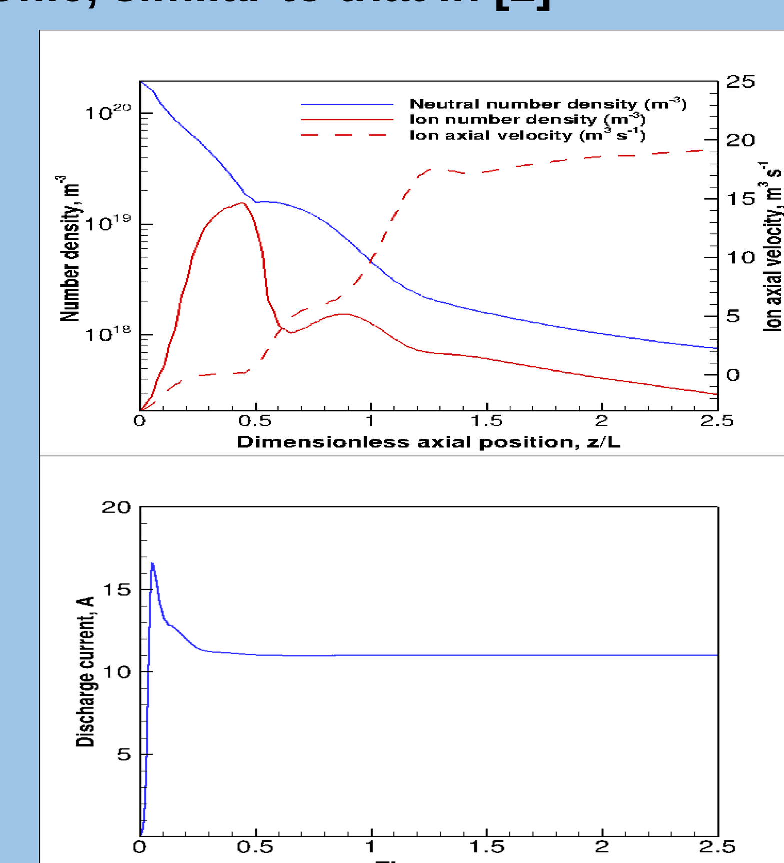

Figure 5: Average plasma properties along thruster channel centerline (top) and discharge current vs time (bottom)

Observations:

- Electron properties are qualitatively similar to published results.
- Ionization occurs rapidly, and the system reaches a steady state configuration quickly.
- This behavior is not representative of a typical Hall thruster.

Method 2: self-correcting mobility profile

- Determine the mobility near the channel exit based on electron-neutral and electron-wall collisions.
- Calculate the mobility elsewhere to satisfy the required electron current density.

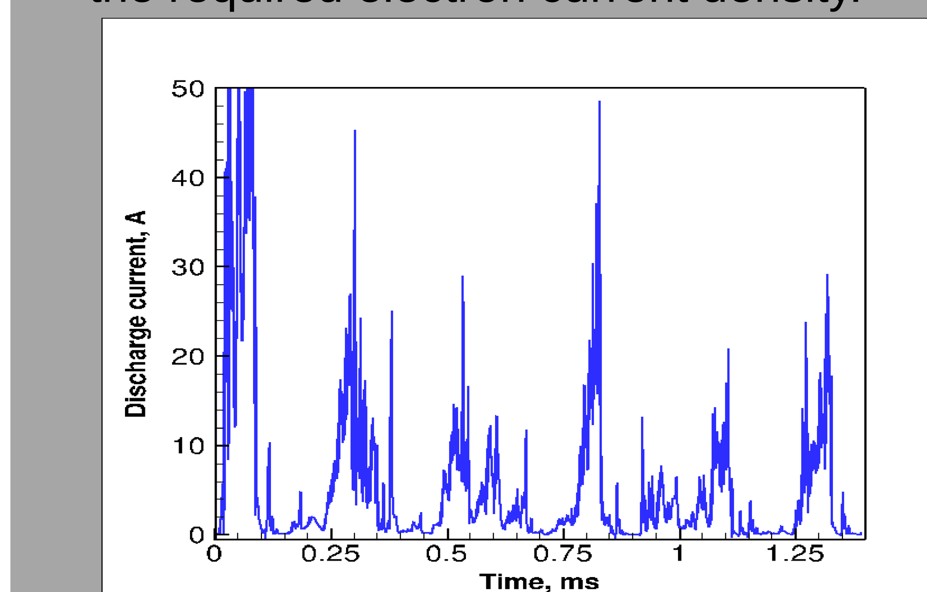

Figure 6: Discharge current vs time

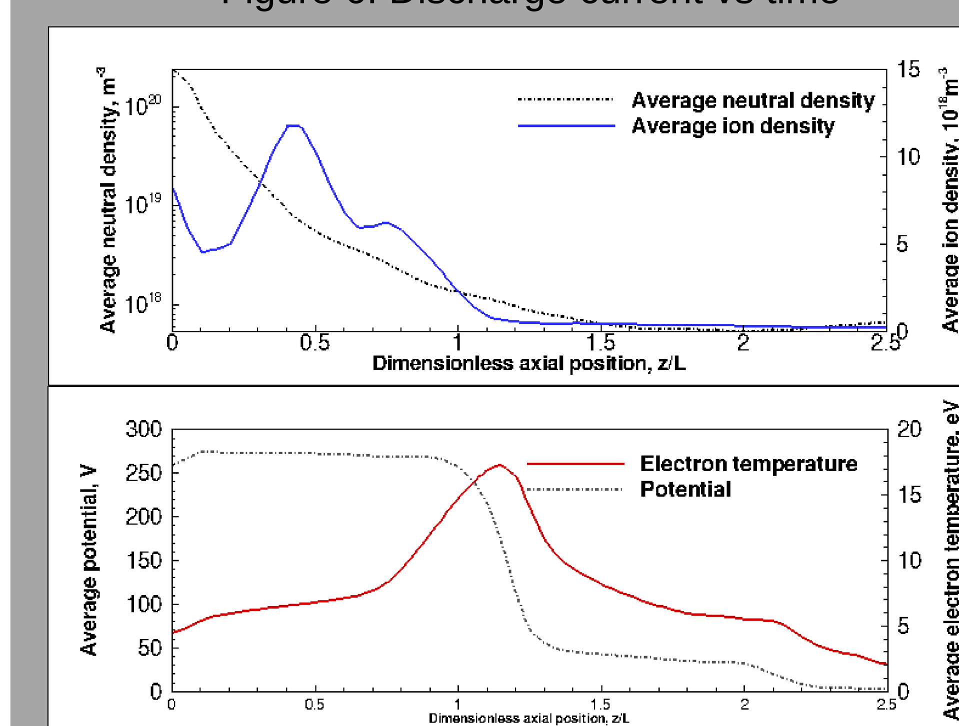

Figure 7: Average plasma properties along thruster channel centerline

Observations:

- Gradients in plasma properties are similar to those seen in plasma measurements.
- The current profile is oscillatory, but the corresponding mobility profile varies significantly over time.

Summary

A two-dimensional, hybrid-Direct Kinetic (DK) simulation is used to model the evolution of plasma in a Hall effect thruster channel and its near-field plume. Although a stable simulation configuration has been achieved in previous work for another thruster [3], the quasi-one-dimensional electron model in its current configuration is inadequate to model the physics in the channel of the H6 hall thruster.

Acknowledgments and References

The authors gratefully acknowledge financial support provided by Air Force Office of Scientific Research Grant No. FA95550-17-1-0035.

[1] Liang, R., *The Combination of Two Concentric Discharge Channels in a Nested Hall-Effect Thruster*, Ph. D. Thesis, University of Michigan, 2013.
[2] Hofer, R., *et. al,* "Efficacy of electron mobility models in hybrid-PIC Hall Thruster Simulations," AIAA-2008-4924, July 2008.
[3] Raisanen, A.L., Hara, K., and Boyd, I.D., "Two-dimensional Hybrid-Direct Kinetic Simulation of a Hall Thruster," AIAA-2018-4809, July 2018.