Kinetic Theory of Strongly Magnetized Plasmas

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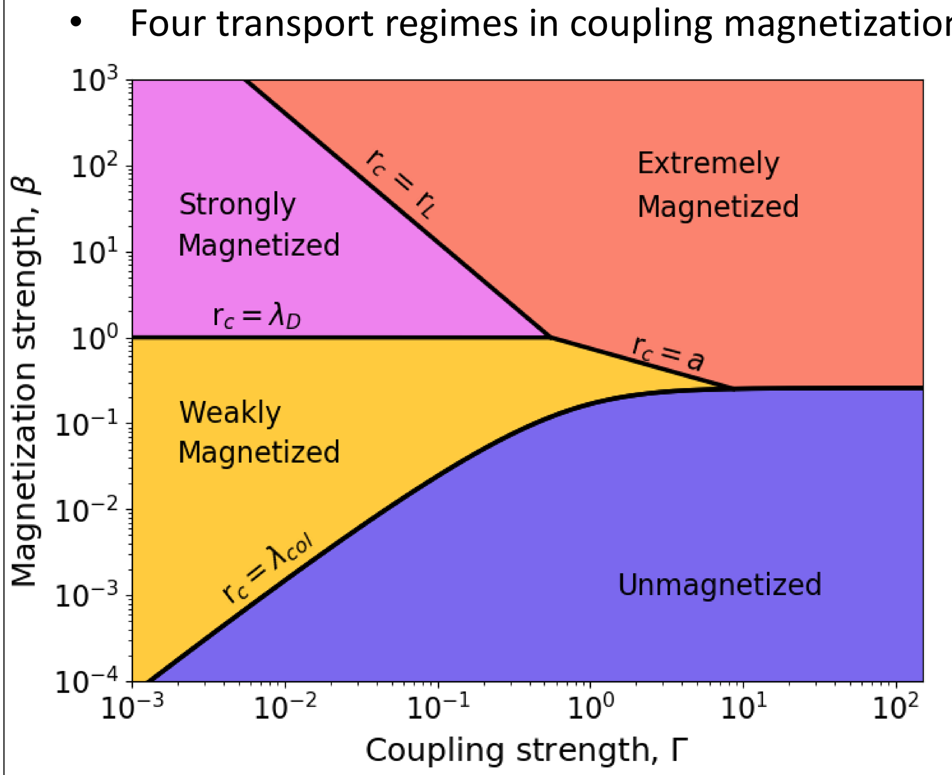
Research results

- Developed a generalized kinetic theory that can treat Coulomb collisions in strongly magnetized plasma
- Theory is used to compute friction force on a massive test charge moving through a strongly magnetized one-component plasma
- Friction force is found to have transverse and gyrofriction components in addition to the stopping power

There are four transport regimes in a plasma

- Four transport regimes in coupling magnetization phase space are [1]



λ_{col} = Coulomb collision mean free path
λ_D = Debye length
a = Average interparticle spacing
r_L = Landau length (i.e. distance of closest approach)
r_c = Gyroradius

$$\Gamma = \frac{e^2/a}{k_B T}$$

$$\beta = \frac{\lambda_D}{r_c} = \frac{\omega_c}{\omega_p}$$

- Unmagnetized plasma
  - $r_c > \lambda_{col}$
  - Magnetic field does not significantly influence the transport
  - Boltzmann, Fokker-Plank, Lennard-Balescu and Landau theories
- Weakly magnetized plasma
  - (max{λ_D , a} ≤ r_c ≤ λ_{col})
  - Magnetic field influences transport by distorting the distribution function
  - Boltzmann, Fokker-Plank, Lennard-Balescu and Landau theories
- Strongly magnetized plasma
  - $r_L < r_c < \lambda_D$
  - Gyromotion occurs on the same length scale as scattering
  - Extensions of Fokker-Plank and Rostoker's theory
- Extremely magnetized plasma
  - $r_c \leq \min\{r_L\, , a\, , \lambda_{col}\}$
  - Motion of the particles are effectively 1 D with 180° collisions
  - O'Neil's Boltzmann like collision operator

- Strongly coupled plasmas ($\Gamma > 1$) are described using effective potential theory such as mean force kinetic theory
- Strongly magnetized plasmas are found in experiments such as non-neutral plasmas, ultra-cold neutral plasmas and magnetized dusty plasmas

We developed a generalized Boltzmann collision operator that can treat all the four transport regimes

- Form the first order BBGKY equation, following Grad's derivation we obtain the generalized collision operator

$$\mathcal{C} = \int d^3\mathbf{v}_2 \int_{S_-} ds\, |\mathbf{u}\cdot\hat{\mathbf{s}}|(f_1' f_2' - f_1 f_2)$$

- Cross section is replaced by an integral on the surface of collision volume
- Post collision velocities are obtained by solving equations of motion of colliding particles inside the collision volume

$$m_1\frac{d\mathbf{v}_1}{dt} = -\nabla_{\mathbf{r}_1}\phi(r) + e_1\left(\frac{\mathbf{v}_1}{c}\times\mathbf{B}\right)$$

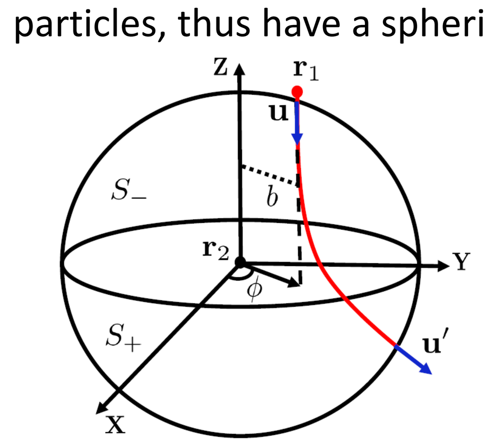
$$m_2\frac{d\mathbf{v}_2}{dt} = -\nabla_{\mathbf{r}_2}\phi(r) + e_2\left(\frac{\mathbf{v}_2}{c}\times\mathbf{B}\right)$$

No closed form solution
Solved numerically

- Particles interact inside the collision volume via potential of mean force
- For a weakly coupled plasma, the potential of mean force is the Debye–Hückel potential

Weakly magnetized limit: Boltzmann equation

- In this regime $\lambda_D < r_c$, the interaction depends only on the distance between the particles, thus have a spherical symmetry



The disk surface is perpendicular to the precollision relative velocity (u). Each point on the hemisphere (S_-) can be projected to a point on the disk as shown, making a one-to-one correspondence.

- On changing the integration surface from hemisphere to disk we get

$$\mathcal{C} = \int d^3\mathbf{v}_2 \int b\, db\, d\phi\, u(f_1' f_2' - f_1 f_2),$$

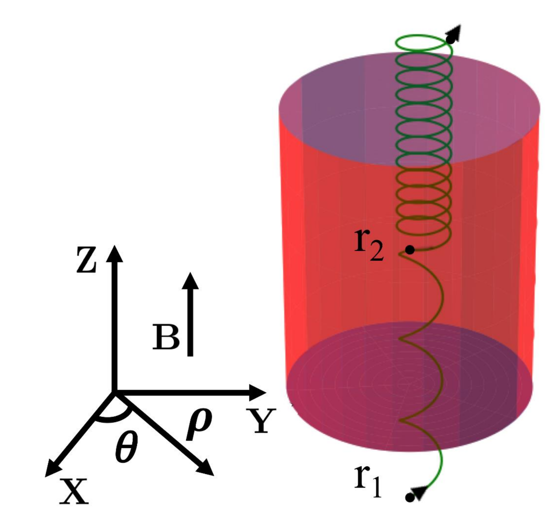
Using $b\, db\, d\phi \;=\; \sigma d\Omega$

$$\mathcal{C} = \int d^3\mathbf{v}_2 \int \sigma d\Omega\, u(f_1' f_2' - f_1 f_2)$$

- In the weakly magnetized limit closed form solution of differential scattering cross section (σ) exists

Extremely magnetized limit – O'Neil's equation

- A cylindrical collision volume aligned in the direction of magnetic field is considered, exploiting the cylindrical symmetry of the binary collisions.



For a cylindrical collision volume

$$\mathcal{C} = \int d^3\mathbf{v}_2 \int \rho d\rho d\theta\, |\mathbf{u}\cdot\hat{\mathbf{z}}|(f_1' f_2' - f_1 f_2) + \int d^3\mathbf{v}_2 \int \rho d\theta dz\, |\mathbf{u}\cdot\hat{\boldsymbol{\rho}}|(f_1' f_2' - f_1 f_2)$$

- First term - Particle enters the collision volume through circular surface (blue)
- Second term - Particle enters the collision volume through cylindrical surface (red)
- In the extremely magnetized limit, particles are bound to their guiding center, scattering through the cylindrical surface is negligible. In this limit,

$$\mathcal{C} = \int d^3\mathbf{v}_2 \int \rho d\rho d\theta\, |\mathbf{u}\cdot\hat{\mathbf{z}}|(f_1' f_2' - f_1 f_2)$$

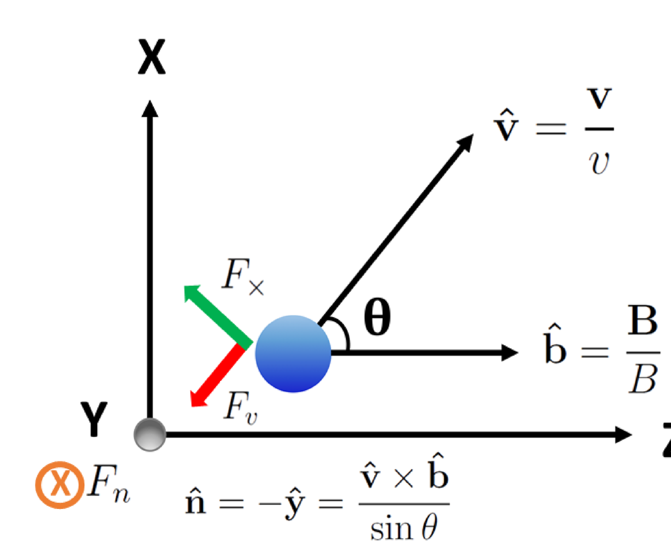
- This is O'Neil's collision operator [4] for extremely magnetized plasma.

Friction is not always anti-parallel to projectile's velocity in a strongly magnetized plasma

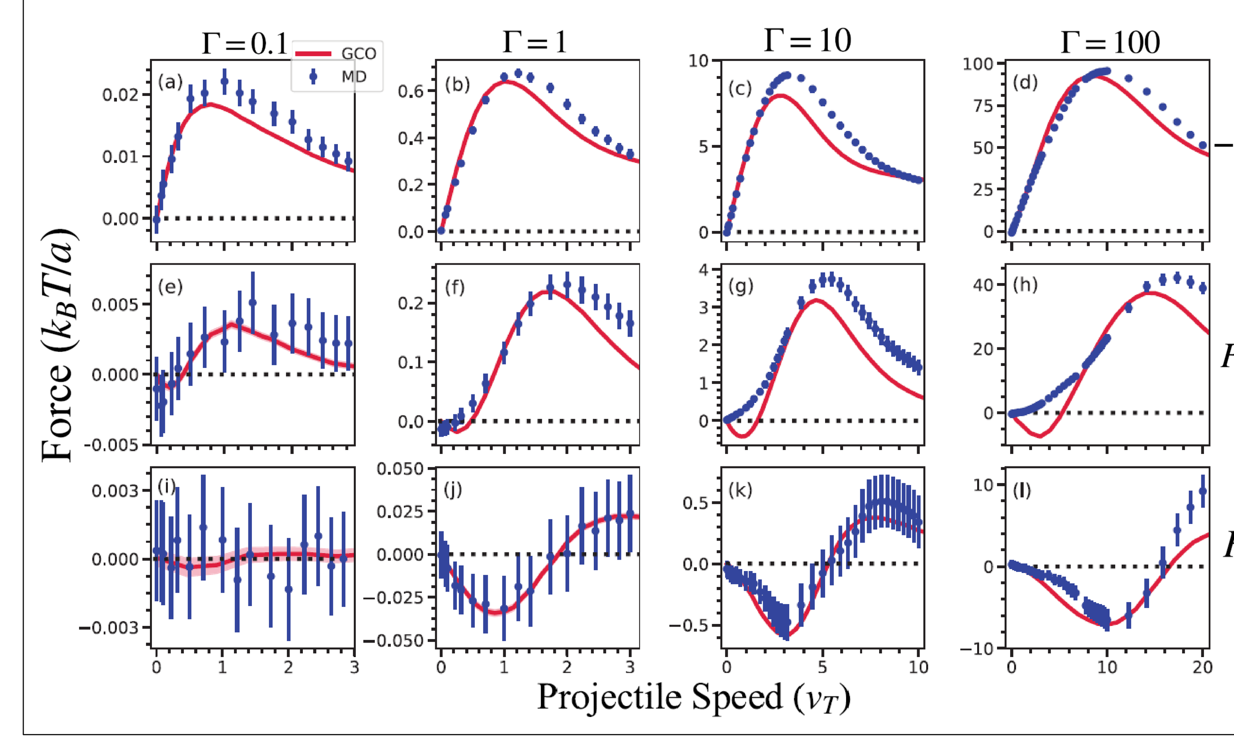
$$m_t\frac{d\mathbf{v}}{dt} = q_t\left(\frac{\mathbf{v}}{c}\times\mathbf{B}\right) - \boxed{m_t\, v\, \nu\, \hat{\mathbf{v}}} - \boxed{m_t\, v\, \nu_\times\, (\hat{\mathbf{v}}\times\hat{\mathbf{n}})} - \boxed{m_t\, v\, \nu_n\, \hat{\mathbf{n}}}$$

Stopping Power (F_v) Transverse force ($F_\times$) Gyrofriction (F_n)

- Friction force arises due to Coulomb collisions of the test charge with the background particles
- In the weakly magnetized limit friction has only stopping power component
- In the strongly magnetized limit, transverse force was predicted using linear response theory [3] and confirmed using molecular dynamics simulations [4]
- Gyrofriction is observed in molecular dynamics simulation of strongly coupled strongly magnetized plasmas [5]



Friction force computed using the generalized kinetic theory is in good agreement with the molecular dynamics simulations

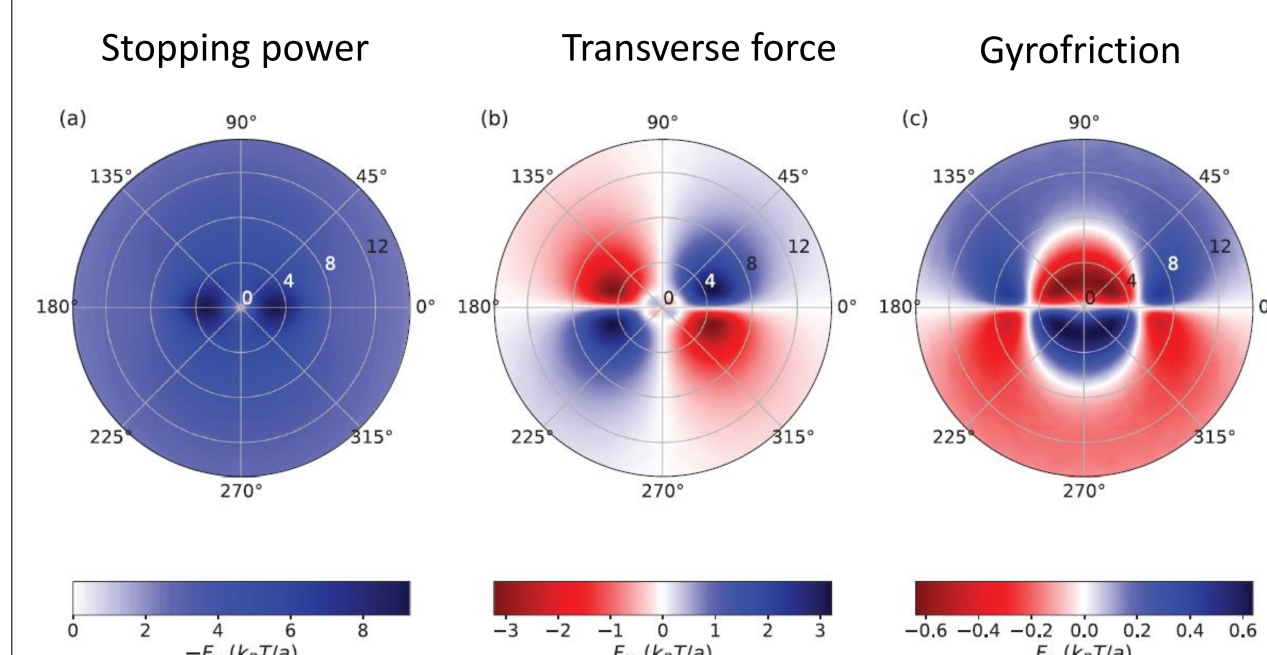


Gyrofriction and transverse force is observed in GCO calculation

GCO captures all the qualitative features observed in MD

When coupling strength increases
- Curves broaden
- Bragg peak shifts to higher speeds

Friction force has a strong dependence on the orientation of projectile's velocity

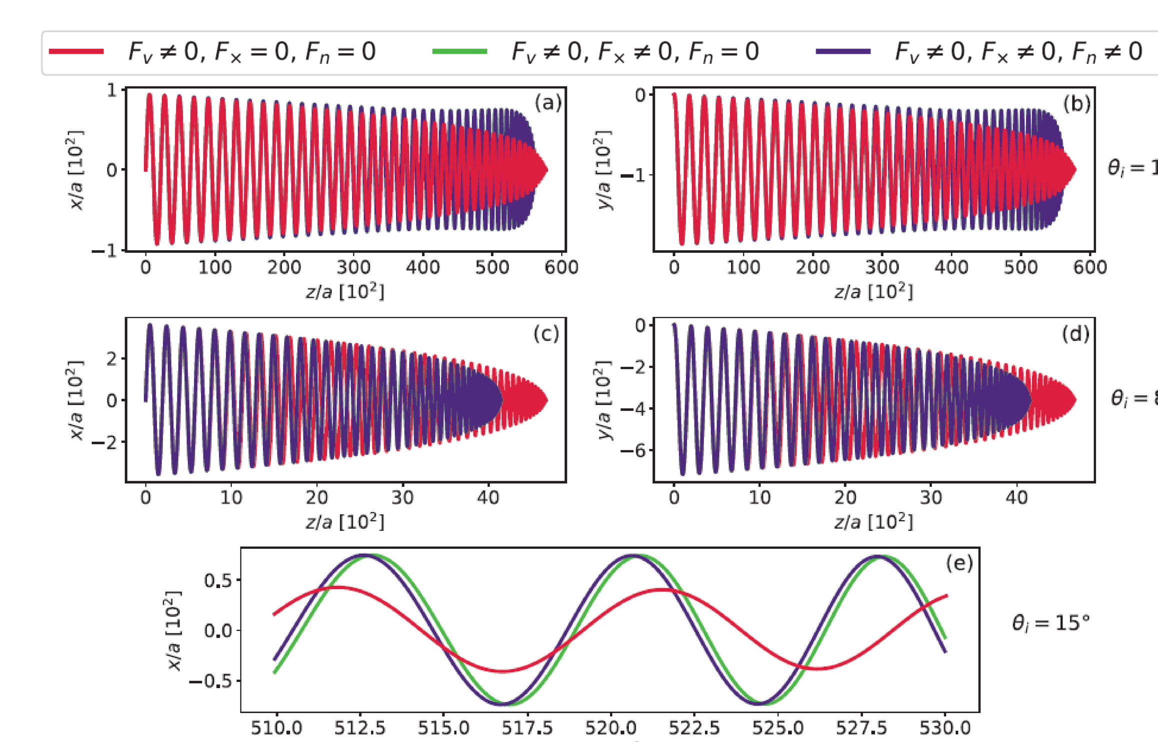


Transverse force is zero when θ=0° or 90°

Gyrofriction is zero when θ=0° and maximum when θ=90°

Stopping power is maximum when test particle moves parallel to the magnetic field

Friction force strongly influences the particle trajectories



Gyroradius is significantly altered by the transverse force

Stopping distance of the projectile is decreased

Gyrofriction slightly changes the gyrofrequency of the test charge

References

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[2] T. M. O'Neil, *Phys. Plasmas 26, 2128 (1983).*
[3] T. Lafleur and S. D. Baalrud, *Plasma Physics and Controlled Fusion* 61, 125004 (2019).
[4] D. J. Bernstein, T. Lafleur, J. Daligault, and S. D. Baalrud, *Phys. Rev. E* 102, 041201 (2020).
[5] D. J. Bernstein, and S. D. Baalrud, *Phys. Plasmas* 28, 062101 (2021).

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