A Relativistic and Electromagnetic Correction to the Ramo-Shockley Theorem

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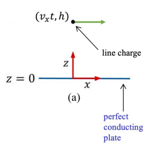
Background

- A moving charge induces currents in nearby conductors
- The induced current was formulated by Ramo (1939) and Shockley (1938) [RS]
- RS is widely used in radiation detection, discharge physics, accelerator theory, vacuum and solid state devices, and protein dynamics.
- Major assumptions in RS:
  - Charge motion is nonrelativistic.
  - Use only electrostatic fields.
- This paper: How do relativistic and electromagnetic effects modify RS?

Single plate – Parallel motion



Induced surface charge density

$$\sigma(x;t) = -\frac{\lambda z_c}{\pi\left[(x-x_c)^2+z_c^2\right]}$$

Classical RS induced surface current density

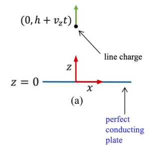
$$K_x = v_x\sigma = -\frac{\lambda h v_x}{\pi\left[(x-v_x t)^2+h^2\right]}$$

Electromagnetically and relativistically correct induced current:

$$K_x = -\frac{\gamma\,\lambda h v_x}{\pi\left[\gamma^2(x-v_x t)^2+h^2\right]}$$

$$\gamma = 1/(1-\beta^2)^{1/2},\ \beta = v_x/c$$

Single plate – Perpendicular motion



Electric current density

$$\boldsymbol{J} \equiv z J_z = z\begin{cases} 0, & t<0 \\ \lambda v_z\delta(x)\delta(z-h-v_z t), & t\geq 0\end{cases}$$

Classical RS induced current:

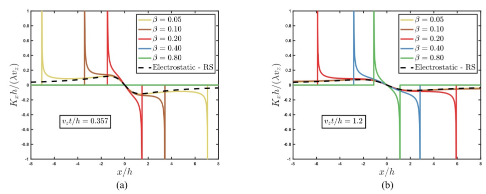
$$K_x = -\frac{\lambda v_z x}{\pi\left[x^2+(h+v_z t)^2\right]}$$

Electromagnetically and relativistically correct induced current:

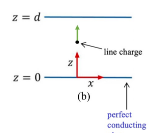
$$K_x = -\frac{\lambda\gamma^2 v_z x}{\pi\left[x^2+\gamma^2(h+v_z t)^2\right]}\times\frac{ct+hv_z/c}{\left[(ct)^2-x^2-h^2\right]^{1/2}}$$

$$\gamma = 1/(1-\beta^2)^{1/2},\ \beta = v_z/c$$

$$ct > \sqrt{x^2+h^2}$$



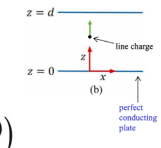
Parallel plates – Perpendicular motion



Classical RS induced current:

$$K_x = -\frac{\lambda v_z}{2d}\,\frac{\sinh\left(\frac{x\pi}{d}\right)}{\cosh\left(\frac{x\pi}{d}\right)-\cos\left(\frac{\pi}{d}(v_z t+h)\right)}$$

Electromagnetically and relativistically correct induced current



Method 1: Series expansion

$$\bar{K}_x(x;t) = -\frac{1}{2\pi i}\int_\Gamma d\bar{s}\,e^{\bar{s}\bar{t}}\left\{e^{-\bar{s}\beta\bar{x}}\frac{1}{2\bar{s}}\left(1-e^{-\bar{s}(1-\bar{h})}\right) + \sum_{n=1}^{\infty}e^{-\left(\bar{s}^2\beta^2+n^2\pi^2\right)^{1/2}\bar{x}}\frac{n\pi}{\bar{s}^2+n^2\pi^2} \times\left[-\sin\left(n\pi\bar{h}\right)+\frac{\bar{s}}{n\pi}\left(\cos\left(n\pi\bar{h}\right)-(-1)^n e^{-\bar{s}(1-\bar{h})}\right)\right]\right\}$$

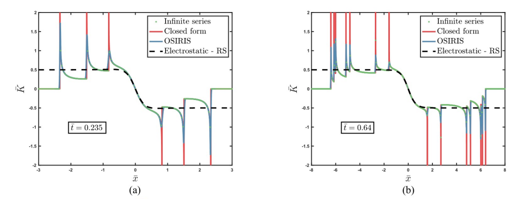
$\bar{K}_x(x;t) = K_x/(\lambda v_z/d)$, $\bar{s} = sd/v_z$, $\beta = v_z/c$, $\bar{h} = h/d$, $\bar{t} = v_z t/d$, $\bar{x} = x/d$

Method 2: Exact, closed form analytic solution.

$$\bar{K}_x(x;t) = -{\sum_n}'\left\{\frac{\bar{x}\left[\bar{t}/\beta+(\bar{h}-2n)\beta\right]}{\pi\left[(\bar{t}/\beta)^2-\bar{x}^2-(\bar{h}-2n)^2\right]^{1/2}\left[(\bar{t}+\bar{h}-2n)^2+\bar{x}^2/\gamma^2\right]}\right\} \equiv \boldsymbol{K}(\bar{x},\bar{t},\bar{h})$$

Method 3: OSIRIS Particle-in-cell Code

Next: Validation of the three (3) Methods

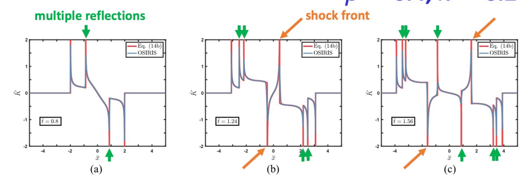


$\beta = 0.1,\ \bar{h} = 0.2$

Line charge strikes upper plate at t = T and is removed, EM shock is formed, with closed form solution in Eq. (14b)

$$\bar{K}_x(x;t) = \begin{cases}\boldsymbol{K}(\bar{x},\bar{t},\bar{h}), & 0<\bar{t}<\bar{T} \quad (14a)\\ \boldsymbol{K}(\bar{x},\bar{t},\bar{h})-\boldsymbol{K}(\bar{x},\bar{t}-\bar{T},1), & \bar{t}\geq\bar{T} \quad (14b)\end{cases}$$

$\beta = 0.4,\ \bar{h} = 0.2$



Conclusions

We illustrated, for the first time, how relativistic and electromagnetic effects modify the classical Ramo-Shockley theory in some simple, solvable examples. Discovered EM shock when electron strikes conductor.

RS ignored electromagnetic transients, wave reflections, and generation of electromagnetic shocks, otherwise works well for highly relativistic beams.

A mystery solved

Explains why RS works well for 80 years in vacuum electronics theory and simulations which assume: Sinusoidal steady state (ignores transients), eigenmode solutions (includes all reflections), ignore beam scraping (no EM shocks), RS adequate for relativistic beams.

A new question raised

Would the newly discovered EM shocks, caused by multipactor discharge, affect the signal quality in satellite communication?

References

1. S. Ramo, Proc. IRE 27, 584 (1939).
2. W. Shockley, J. Appl. Phys. 9, 635 (1938).
3. D. Li, D. Chernin, and Y. Y. Lau, *A Relativistic and Electromagnetic Correction to the Ramo-Shockley Theorem*, IEEE Trans. Plasma Sci., vol. 49, no. 9, pp. 2661-2669, Sept. 2021, doi: 10.1109/TPS.2021.3099512.