Entropy Generation in Ultracold Neutral Plasmas

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Motivation and main questions

Ultracold neutral plasmas (UNPs) are plasmas that are:

- strongly coupled
- non-equilibrium
- relatively rarefied/cold

Can we use data from experiments of UNPs to:

- validate theories and computational models
- gain insight of processes occurring in UNPs via simulation

Ultracold neutral plasmas

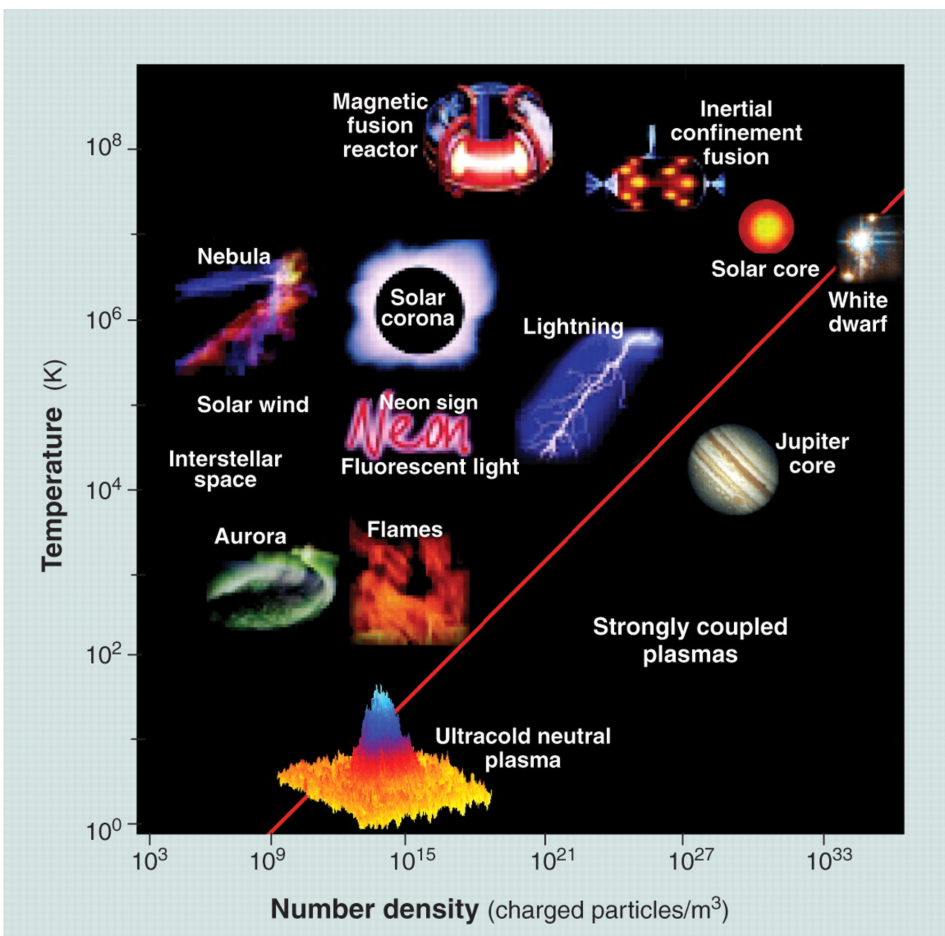

Figure 1: Parameter space for disparate types of plasmas. Ultracold neutral plasmas [1] are far less dense and hot than typical inertial confinement fusion plasmas.

Temperature relaxation data from UNPs experiments

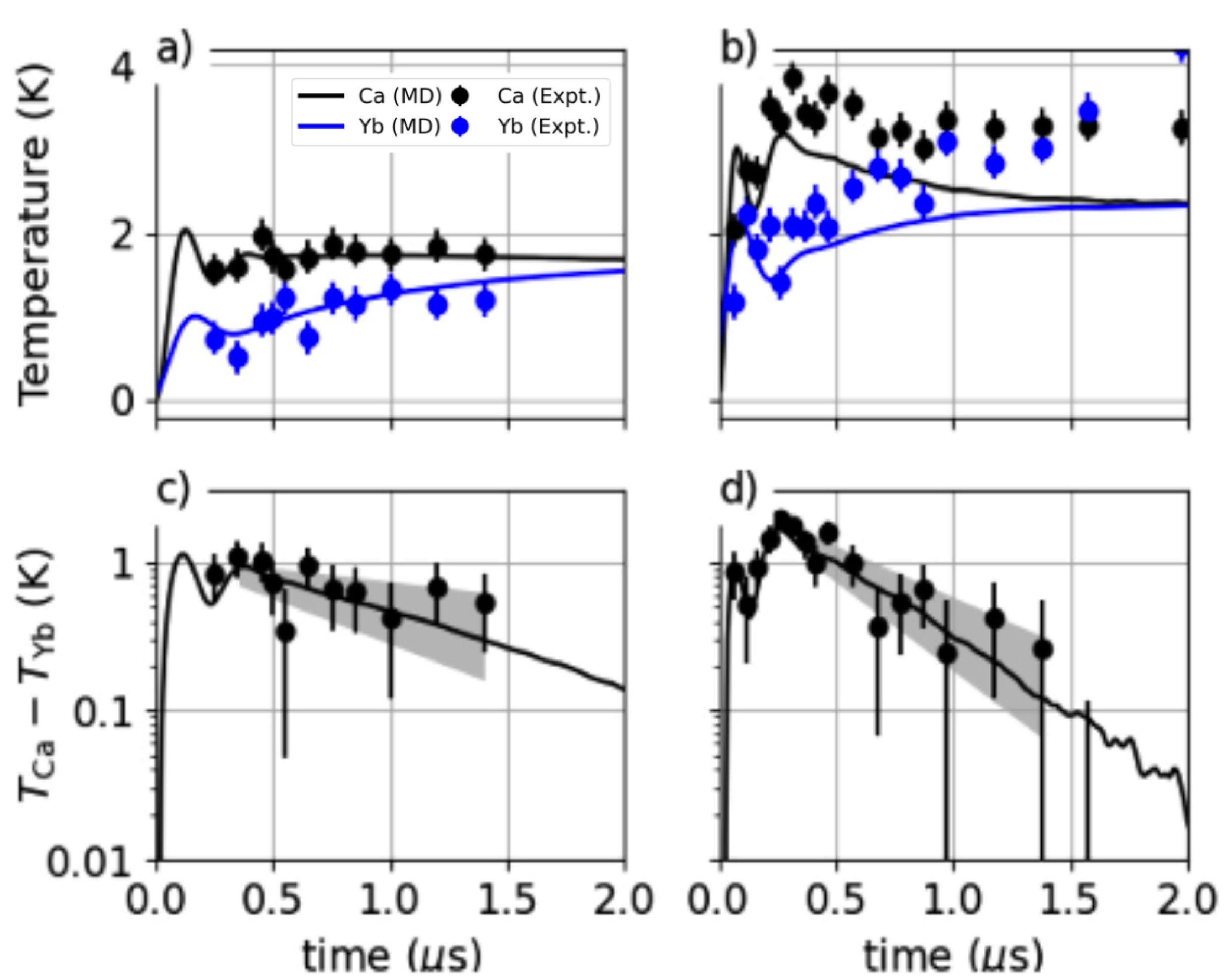

Figure 2: Temperature versus time for two UNP configurations [2] (see Fig. 6) The circles represent laboratory data and the lines represent molecular dynamics data.

Kinetic Equations and Their Approximations

Kinetic equations evolve a phase-space distribution $f(\mathbf{x}, \mathbf{v}, t)$ in time. Some examples are given in Fig. 3.

general 1D-1V kinetic equation

$$\frac{\partial f}{\partial t} + v\frac{\partial}{\partial x}f + \frac{qE}{m}\frac{\partial}{\partial v}f = Q[f]$$

"collisionless" → Vlasov equation

$$\frac{\partial f}{\partial t} + v\frac{\partial}{\partial x}f + \frac{qE}{m}\frac{\partial}{\partial v}f = 0$$

"BGK" collision operator → Vlasov-Bhatnagar-Gross-Krook (V-BGK) equation

$$\frac{\partial f}{\partial t} + v\frac{\partial}{\partial x}f + \frac{qE}{m}\frac{\partial}{\partial v}f = \nu\left(\mathcal{M}(x,v,t) - f\right)$$

Figure 3: Diagram depicting different treatments of the collision operator $Q[f]$ in kinetic equations. By neglecting particle collisions, we obtain the Vlasov equation. By assuming the system is close to equilibrium and particle collisions are not negligible, we obtain the V-BGK equation.

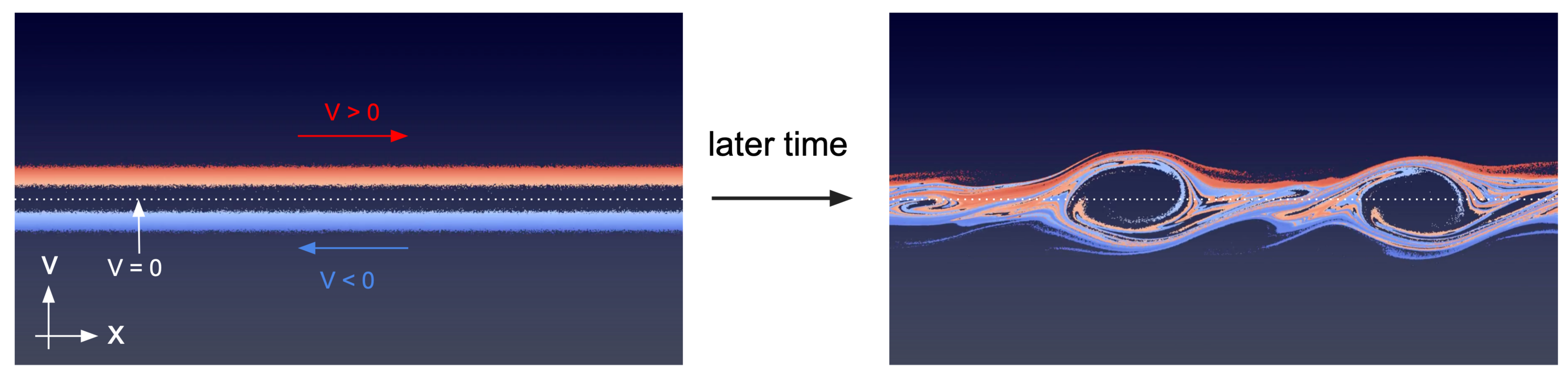

Figure 4: Two time snapshots of a distribution function $f(x, v, t)$ for the "two-stream instability." A particle-in-cell simulation was carried out to numerically solve the Vlasov equation.

Entropy density equations: How does entropy evolve in time?

First, why doe we care about entropy? Here are two reasons:

1. second law of thermodynamics states: entropy must be increasing for an isolated system
2. when change in entropy is zero, system is in equilibrium

$$\frac{\partial \rho s}{\partial t} = -k_B \int \frac{\partial f}{\partial t} \ln f \, dv$$

$\frac{\partial f}{\partial t}$ from Vlasov equation →

$$\frac{\partial \rho s}{\partial t} = -\frac{\partial}{\partial x}\left(\rho s u + J_s\right)$$

entropy density advection equation

$\frac{\partial f}{\partial t}$ from V-BGK equation →

$$\frac{\partial \rho s}{\partial t} = -\frac{\partial}{\partial x}\left(\rho s u + J_s\right) \quad + k_B \nu \int (f - \mathcal{M}) \ln\left(\frac{f}{\mathcal{M}}\right) dv$$

entropy density advection equation plus a source term

Figure 5: Diagram of different entropy density equations based on different choices of kinetic equation. J_s denotes the entropy flux relative to a reference velocity. The Vlasov equation does not generate entropy while the V-BGK equation does.

Multi-component BGK model

The single-species V-BGK model can be extended to handle multiple species via [3]

$$\frac{\partial f_i}{\partial t} + \mathbf{v}\cdot\nabla_x f_i + \mathbf{a}_i \cdot \nabla_v f_i = \nu_{ij}(M_{ij} - f_i), \qquad \text{for } i, j = 1, 2, \cdots, N_s \text{ and } j \neq i,$$

where

$$M_{ij}(\mathbf{v}) = n_i \left(\frac{m_i}{2\pi T_{ij}}\right)^{3/2} \exp\left(-\frac{m_i(\mathbf{v} - \mathbf{u}_{ij})^2}{2T_{ij}}\right).$$

Multi-component BGK (McBGK) Simulations

Two experimental UNP configurations

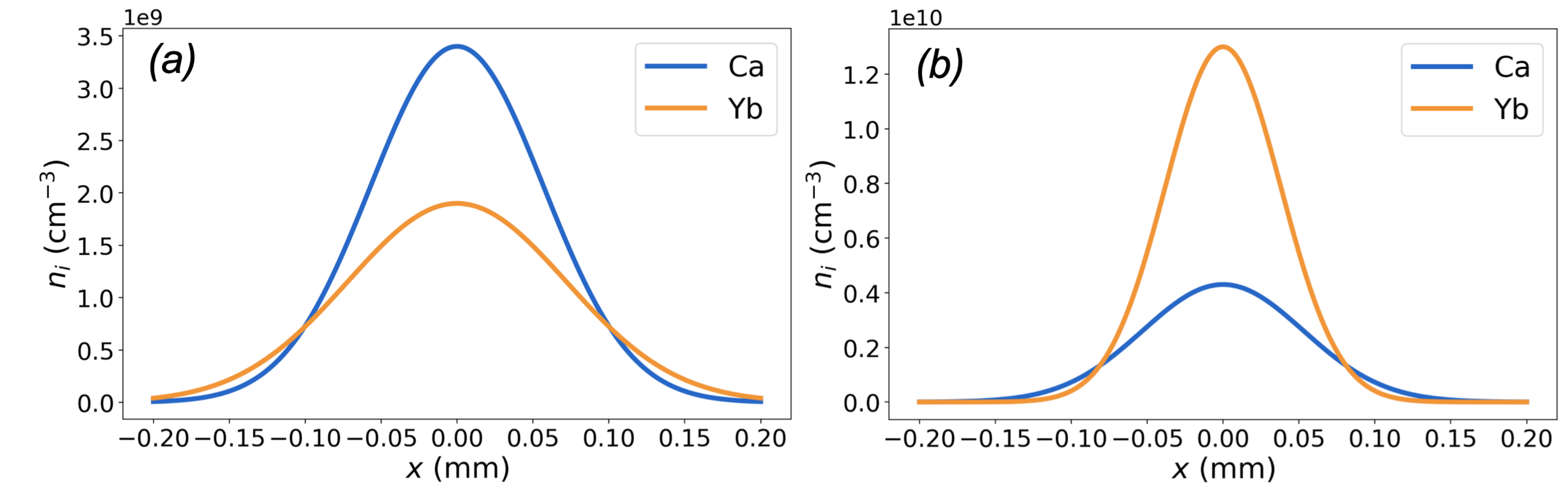

Figure 6: Two UNP configurations for a mixture of the elements Ca and Yb. In case (*a*), the Ca species was more dense that the Yb species. Case (*b*) shows the opposite scenario. In both cases, $T_e \approx 96$K.

Comparing McBGK temperature relaxation to experiments

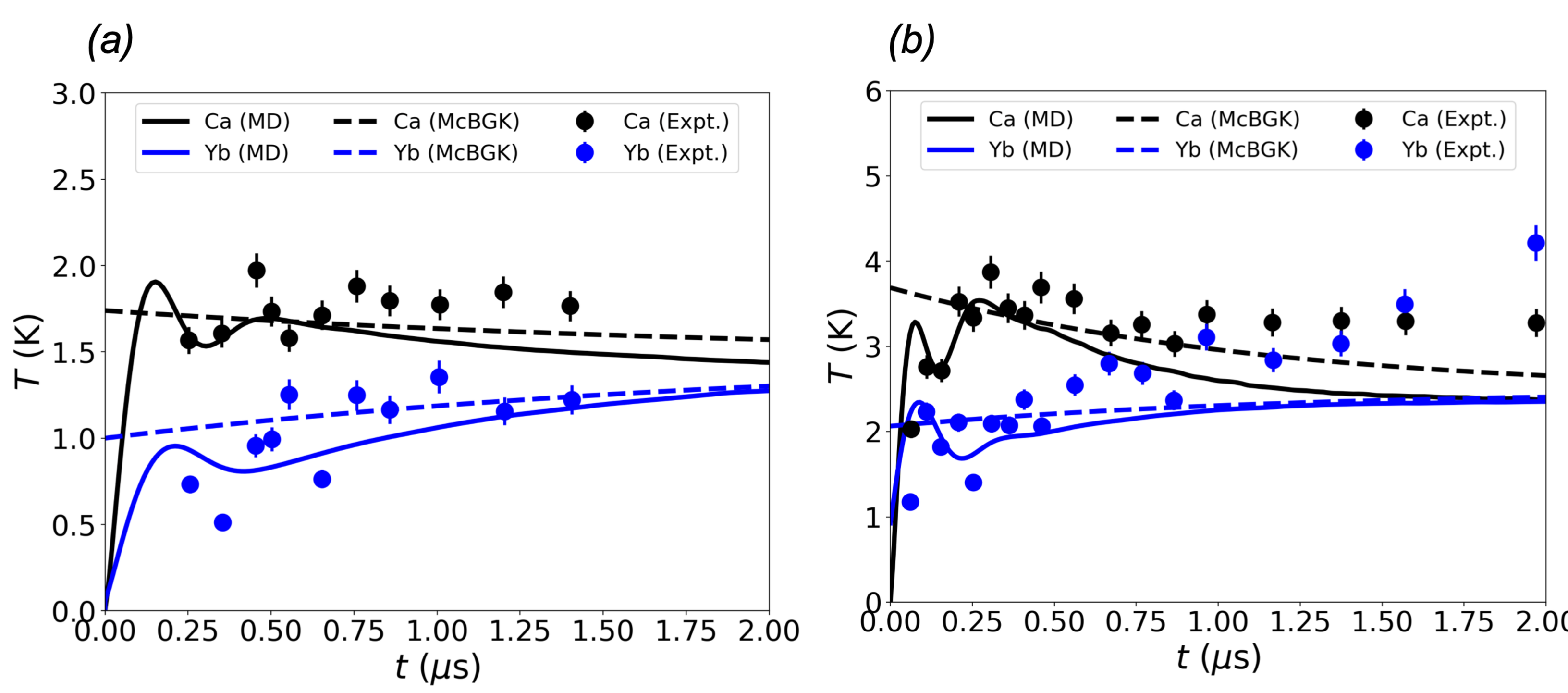

Figure 7: Temperature relaxation of cases (*a*) and (*b*) using the McBGK model. the McBGK results agree well with both molecular dynamics and experimental data.

Entropy generation for both UNP configurations

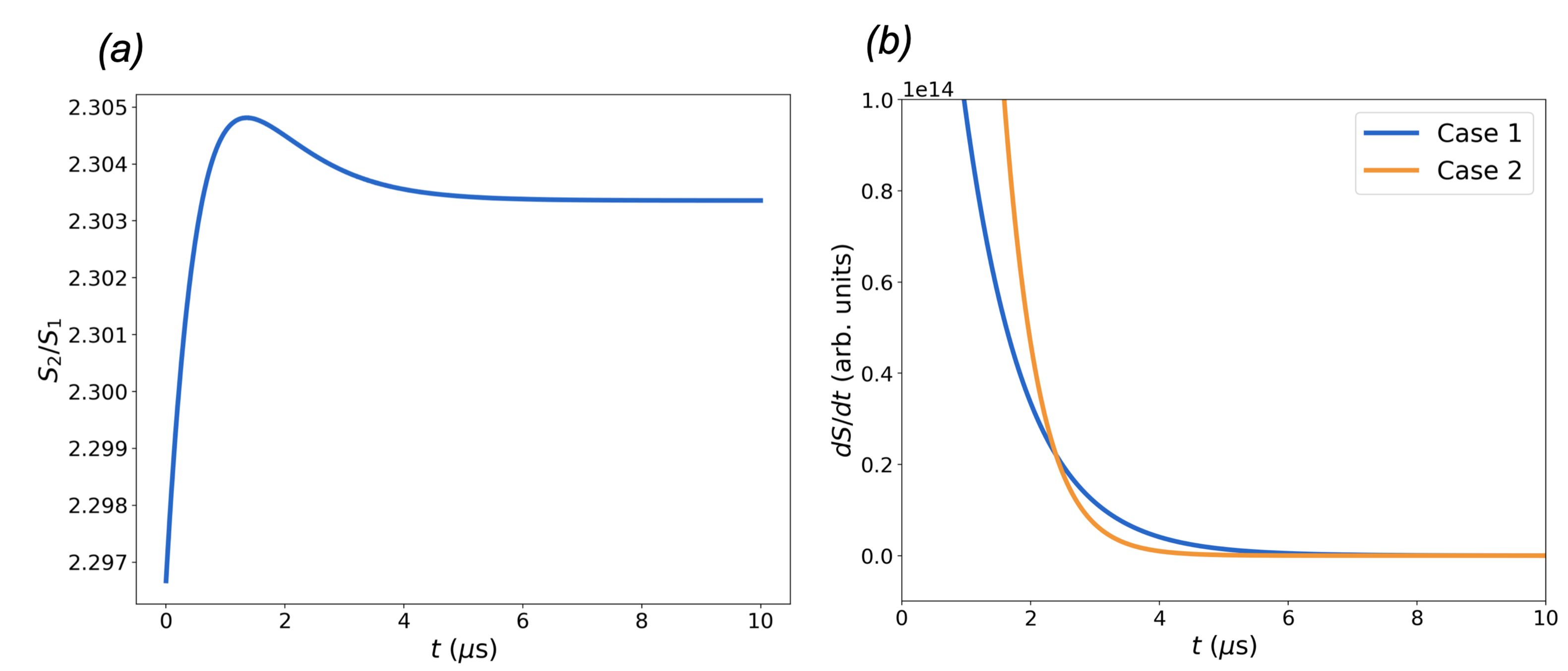

Figure 8: Entropy versus time for two UNP configurations (see Fig. 6). Panel (*a*) shows the ratio of entropy generation for case 2 versus case 1. Case 2 generates twice as much entropy than case 1. Panel (*b*) shows the derivatives of each case showing that case 2 equilibrates more quickly than case 1.

Bibliography

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