# Formation and scaling of vortex rings ejected from shock-accelerated interfaces

MICHIGAN ENGINEERING
UNIVERSITY OF MICHIGAN

Michael Wadas, Loc Khieu, Griffin Cearley, and Eric Johnsen
Mechanical Engineering, University of Michigan, Ann Arbor, MI.

SCFP Laboratory

## Introduction

Vortex rings are a primary flow feature in many classical fluid applications, where their behavior is well understood, but are also important in flows unstable to the Richtmyer-Meshkov instability (RMI) including inertial confinement fusion (ICF) implosions and astrophysics. **Objective: generalize the classical vortex ring scaling theory to encompass rings ejected from shocked interfaces, enable advancements in ICF and astrophysics.**

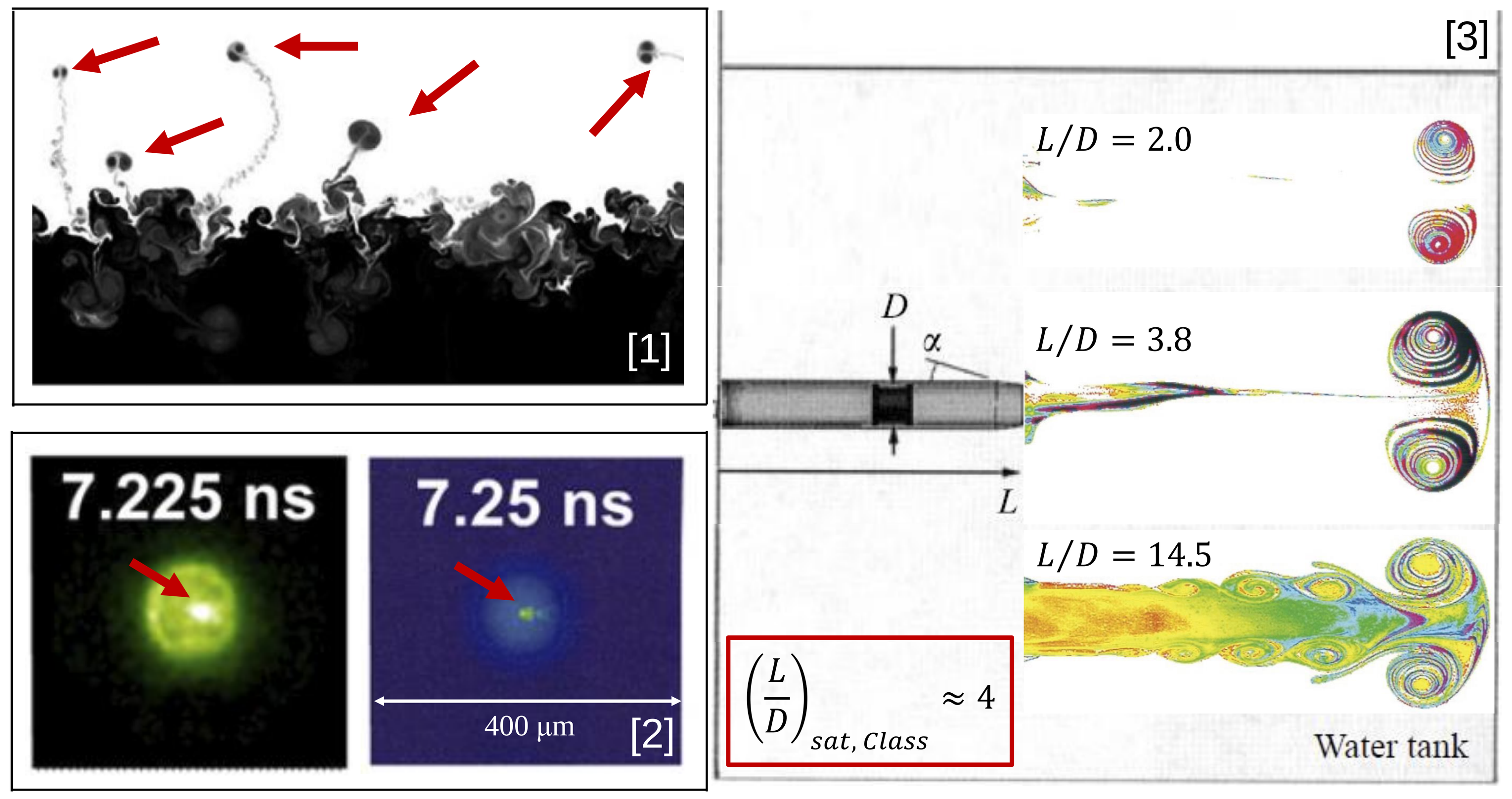


**Figure 1**: Vortex rings generated in the RMI (upper left), ICF implosions (lower left), and a water tank (right).

## Model

We extend the vortex ring scaling theory via an analogy between the classical piston-cylinder-generated vortex rings and those ejected from shock-accelerated interfaces.

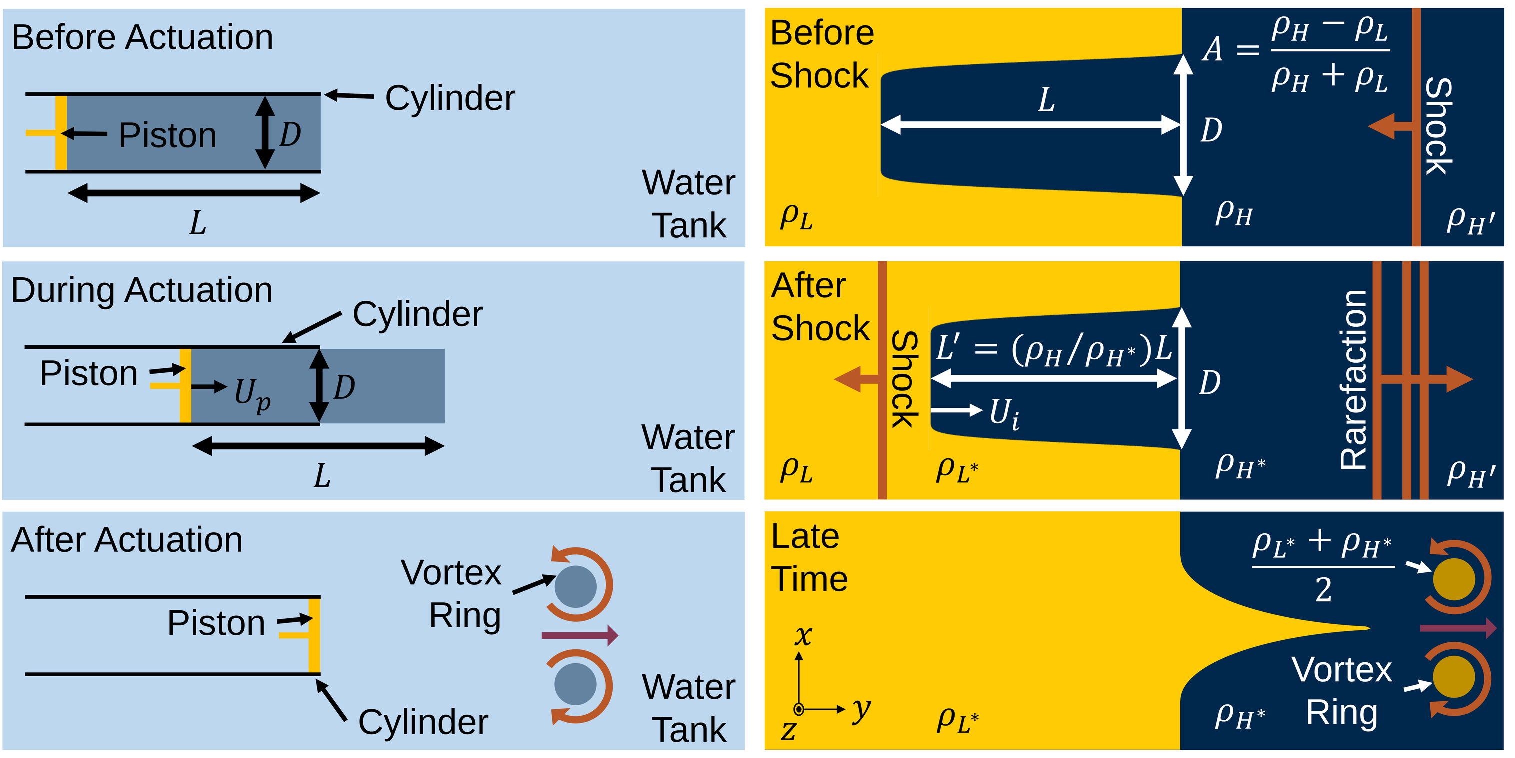


**Figure 2**: The analogy between classical rings (left) and those ejected form shock-accelerated interfaces (right).

## Model (continued)

Based on principles of energy, impulse, and circulation conservation, the formation number for vortex rings ejected from shock-accelerated interfaces is greater than their classical counterparts by a factor, $\sigma$, that is a function of the shock strength and interface Atwood number.

| Slug Flow | Norbury Vortex Ring [4] |
|---|---|
| $E = \rho_H \frac{1}{8}\pi D^2 L U_i^2$ | $E = \frac{(\rho_{L^*} + \rho_{H^*})}{2}(\Omega\alpha l)^2 l^3 E_N$ |
| $\Gamma = \frac{\rho_H}{\rho_{H^*}}\frac{1}{2} L U_i$ | $\Gamma = (\Omega\alpha l) l \Gamma_N$ |
| $I = \rho_H \frac{1}{4}\pi D^2 L U_i$ | $\mathrm{I} = \frac{(\rho_{L^*} + \rho_{H^*})}{2}(\Omega\alpha l) l^3 \mathrm{I}_N$ |
| $U_{tr} = \frac{\partial E}{\partial I} = \frac{1}{2} U_i$ | $U_{tr} = (\Omega\alpha l) U_N$ [5] |

$$\frac{L}{D} = \sigma\sqrt{\frac{\pi}{2}}\, I_N^{\frac{1}{2}} \Gamma_N^{\frac{3}{2}} E_N^{-1} \qquad \frac{L}{D} = \sigma\sqrt{\frac{\pi}{2}}\, I_N^{-\frac{1}{2}} \Gamma_N^{\frac{3}{2}} U_N^{-1}$$

$$\boxed{\left(\frac{L}{D}\right)_{sat,\,RMI} = \sigma\left(\frac{L}{D}\right)_{sat,\,Class}} \longrightarrow \sigma = \sqrt{\frac{2\rho_{H^*}}{\rho_{L^*} + \rho_{H^*}}}\frac{\rho_{H^*}}{\rho_H} = f(\rho_{H'}/\rho_H, A)$$

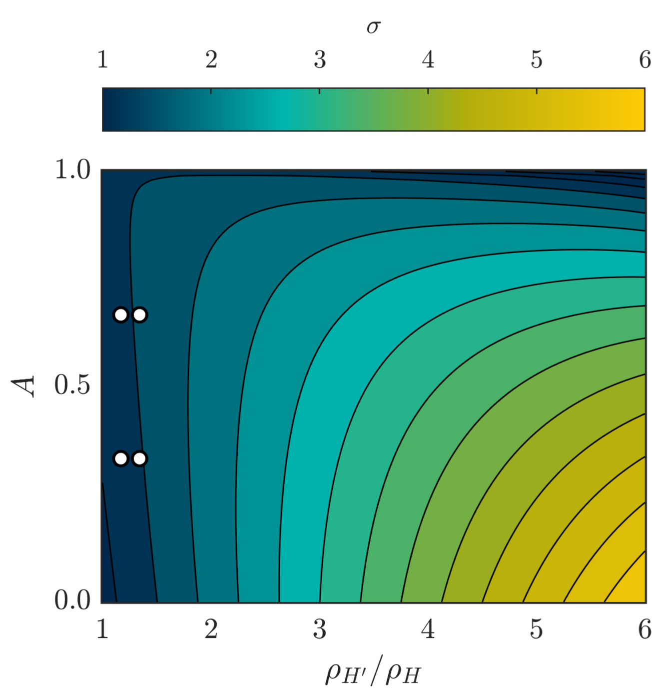


**Figure 3**: Theory (left) giving the formation number (right) of rings ejected from shock-accelerated interfaces expressed as a multiple of the classical formation number. Symbols represent verification simulation conditions.

## Results and Verification

Simulations performed with our in-house, high-order-accurate discontinuous-Galerkin code solving the 3D Euler equations appear to demonstrate that vortex rings ejected from shock-accelerated interfaces share many qualities with their classical counterparts.

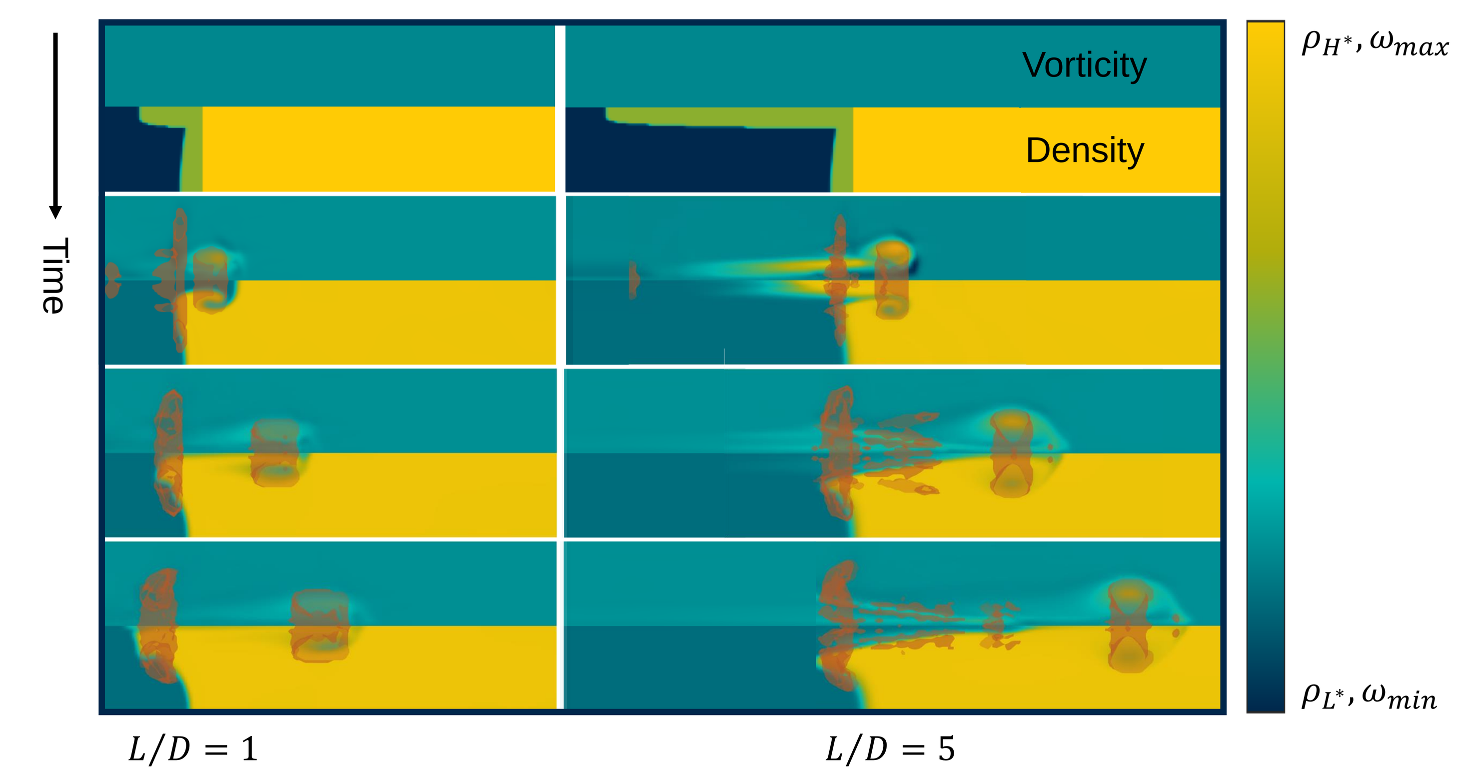


**Figure 4**: The evolution of a heavy-fluid hole with aspect ratios $L/D = 1$ (left) and $L/D = 5$ (right) along an $A = 0.33$ interface following the passage of a shock of strength $\rho_{H'}/\rho_H = 1.34$ showing the ejection of a vortex ring.

## Results and Verification (continued)

Examining the vortex ring circulation as a function of the initial hole aspect ratio indicates that the formation number has been reached. Furthermore, the formation number model prediction agrees very well with that inferred from simulations.

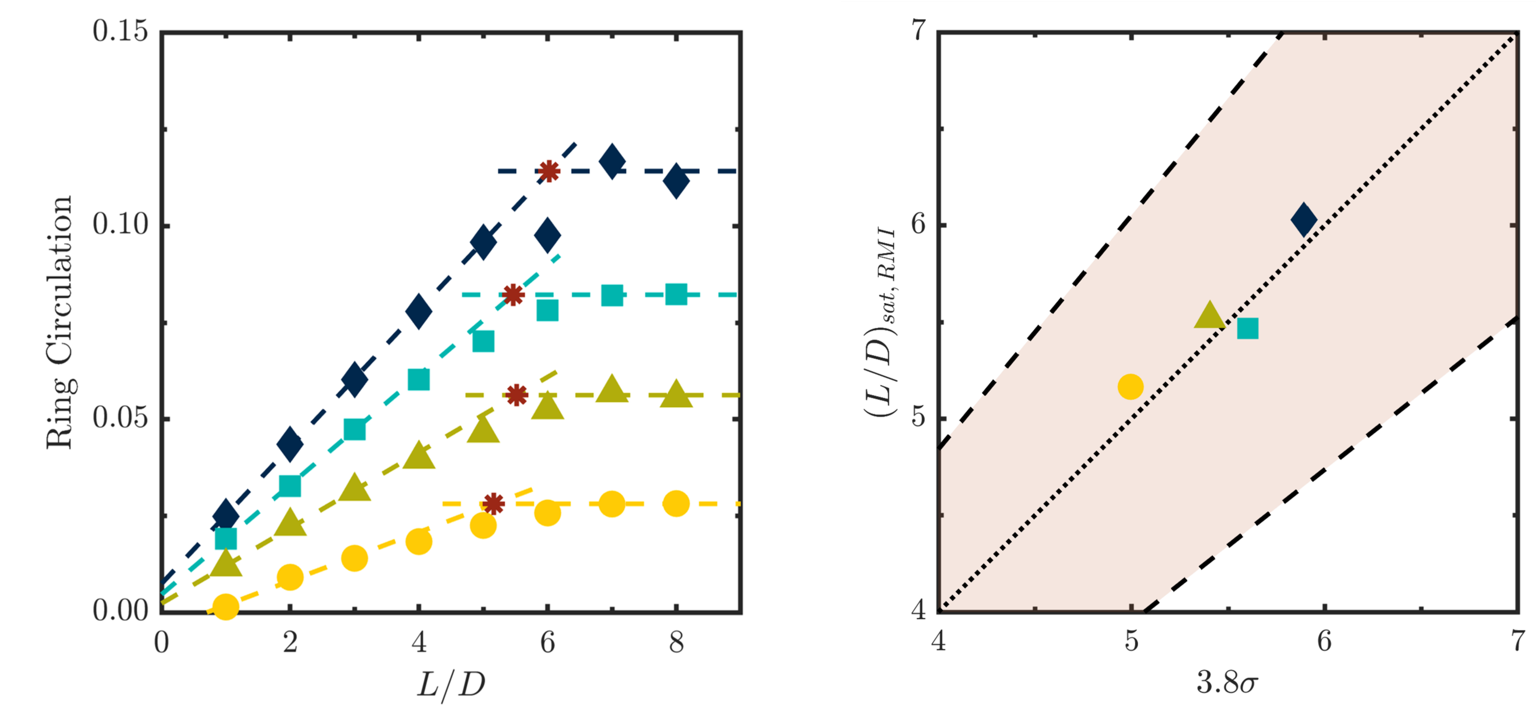


**Figure 5**: The vortex ring circulation as a function of the initial hole aspect ratio (left) and the formation number inferred from simulation as a function of the prediction of the model (right).

| Case | $\rho_{H'}/\rho_H$ | $A$ | $3.0\sigma$ | $4.6\sigma$ | $(L/D)_{sat,\,RMI}$ |
|---|---|---|---|---|---|
| 1 | 1.17 | 0.33 | 3.94 | 6.05 | 5.16 |
| 2 | 1.34 | 0.33 | 4.27 | 6.54 | 5.52 |
| 3 | 1.17 | 0.67 | 4.42 | 6.78 | 5.47 |
| 4 | 1.34 | 0.67 | 4.65 | 7.13 | 6.03 |

**Table 1**: The conditions for the four sets of verification simulations, the range of predicted formation numbers given as multiples of the classical range, and the formation numbers inferred from simulation.

## Conclusions and Acknowledgement

The classical vortex ring scaling has been generalized to include those ejected from shock-accelerated interfaces. Our model is able to accurately predict the formation number of such rings, with critical implications to ICF implosions and astrophysical mixing, as well as the development of more general RMI flows.

[1] Thornber and Zhou, Phys. Plasmas, 22 (2015) [2] Baker et al., Phys. Plasmas, 27 (2020) [3] Gharib et al., J. Fluid Mech., 360 (1998) [4] Norbury, J. Fluid Mech., 57 (1973) [5] Mohseni and Gharib, Phys. Fluids, 10 (1998)

This work is supported by the Lawrence Livermore National Laboratory under Subcontract No. B632749 and the U. S. Department of Energy as part of the Stewardship Science Graduate Fellowship Program under Grant No. De-NA0003960. Computational resources were provided by the Extreme Science and Engineering Discovery Environment Comet system, USA under grant TG-CTS130005 and the Oak Ridge Leadership Computing Facility, a DOE Office of Science User Facility supported under Contract DE-AC05-00OR22725.