Grating Optimization for Smith-Purcell Radiation: Direct Correlation between Spatial Growth Rate and Starting Current

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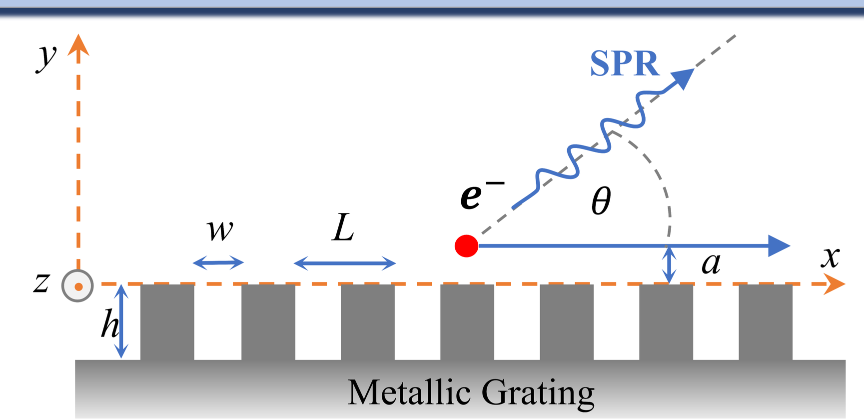
I. Motivation

- Back wave oscillation (BWO) based ***Smith-Purcell Radiation (SPR)*** has attracted strong interests for producing ***Terahertz (THz)*** radiation.
- High-power, efficient, and low-cost electromagnetic sources have significant uses in ***high-resolution imaging***, ***biomedical scanning***, ***material analysis***, ***security systems***, and ***high-rate data communications (6G)***, etc.

II. Smith-Purcell Radiation



When an electron beam emitted with the velocity v above a metallic periodic grating of period L, SPR can be generated with wavelength λ at an angle θ.[1]

$$\lambda = \frac{L}{n}\left(\frac{1}{\beta} - \cos\theta\right) \qquad (1)$$

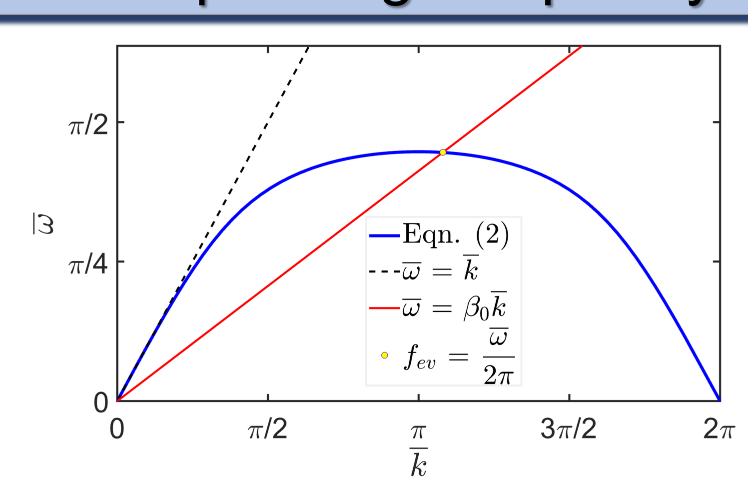
III. Cold-Tube Dispersion Relation

The ***cold-tube dispersion equation*** for the SPR model is [2,3],

$$\frac{\cot(\bar{\omega}\bar{H})}{\bar{\omega}\bar{H}} - \sum_{n=-\infty}^{\infty}\left(\frac{\sin\theta_n}{\theta_n}\right)^2 \frac{\bar{W}}{\gamma_n\bar{H}} = 0 \qquad (2)$$

where the normalized grating width $\bar{W} = w/L$, grating height $\bar{H} = h/L$, frequency $\bar{\omega} = \omega L/c$, wavenumber $\bar{k} = kL$, $\theta_n = P_n\bar{W}/2$, $\gamma_n = \sqrt{P_n{}^2 - \bar{\omega}^2}$, and $P_n = \bar{k} + 2n\pi$.

IV. Operating Frequency



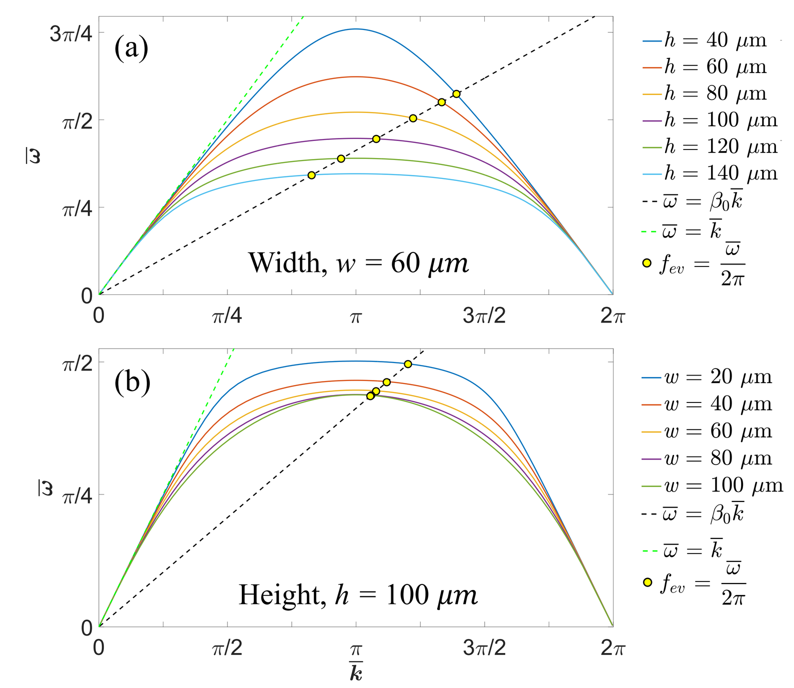
Intersection of ***beam line*** $(\bar{\omega} = (v_0/c)\bar{k} = \beta_0\bar{k})$ to the ***dispersion relation*** is the operating frequency (f_{ev}) for the grating parameters, where ***SPR occurs*** at the ***second harmonic*** $(2f_{ev})$ of this ***operating frequency***.

V. Main parameters for Analysis

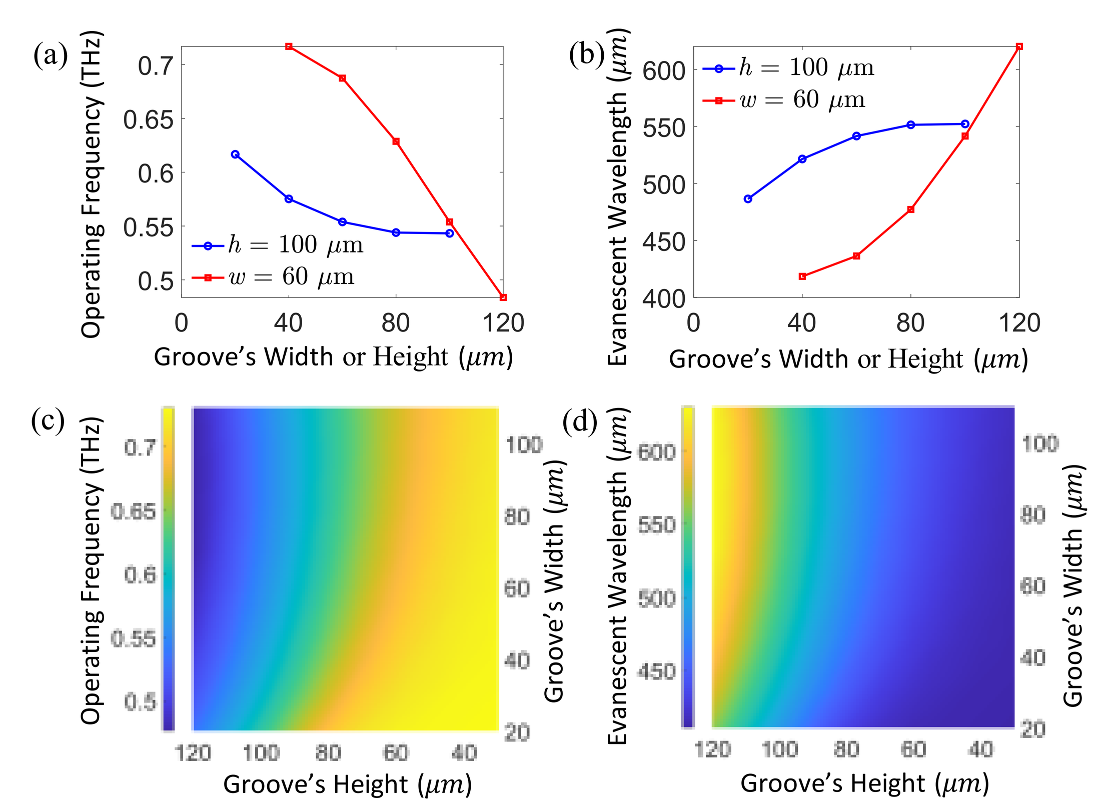
TABLE I. Main Parameters for the Calculation.

Beam energy	50 keV
Beam Height from Grating Surface, a	10 μm
Beam thickness, τ	10 μm
Beam Current, I	5000 A/m
Grating period, L	120 μm
Number of grating periods	35

VI. Effect of Grating Parameters



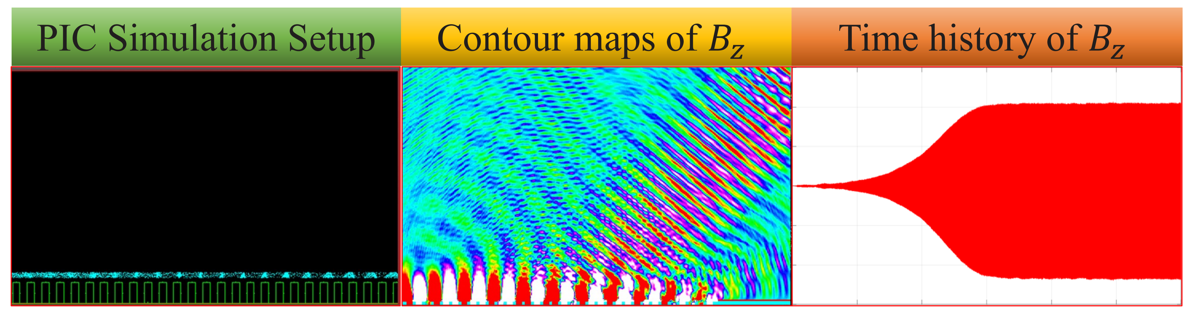
The cold-tube dispersion relation $\omega(k)$ (Eqn. (2)) for different groove's height h and width w. The yellow dots denote the ***operation points*** at the evanescent wave frequency f_{ev}. [4]



(a) The operating frequency (f_{ev}) and (b) the corresponding evanescent wavelength (λ_{ev}) as a function of grating groove's height [red line] and width [blue line] [4]. The surface plot of (c) f_{ev} and (d) λ_{ev} for a wide range of grating groove's heights and widths.

VII. Starting Current from PIC Simulation

PIC Simulation Setup	Contour maps of B_z	Time history of B_z



Increase Starting Current of Beam → Observed the Radiation Pattern → If B_z → Saturation, I → Starting Current. Else Increase I check B_z

Disadvantages

01 Computational cost is high
As each Particle-in-Cell (PIC) takes long time to compute the field, this method is time costly.

02 Small step for increasing start current
To get accurate starting current for Smith-Purcell Radiation, small step size for increasing current is needed.

VIII. Hot-Tube Dispersion Relation

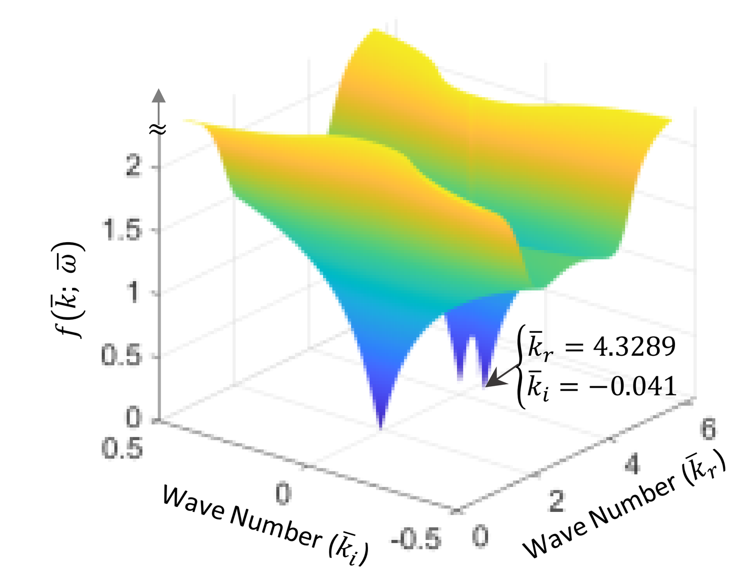
The ***hot-tube dispersion equation*** for the SPR model is [4],

$$f(\bar{k};\bar{\omega}) \equiv \frac{\cot(\bar{\omega}\bar{H})}{\bar{\omega}\bar{H}} - \sum_{n=-\infty}^{\infty} U_n\left(\frac{\sin\theta_n}{\theta_n}\right)^2 \frac{\bar{W}}{\gamma_n\bar{H}} = 0$$

$$\text{with } U_n = \frac{1 - \Psi\cosh(\gamma_n\bar{A})/e^{\gamma_n\bar{A}}}{1 - \Psi\sinh(\gamma_n\bar{A})/e^{\gamma_n\bar{A}}} \qquad (3)$$

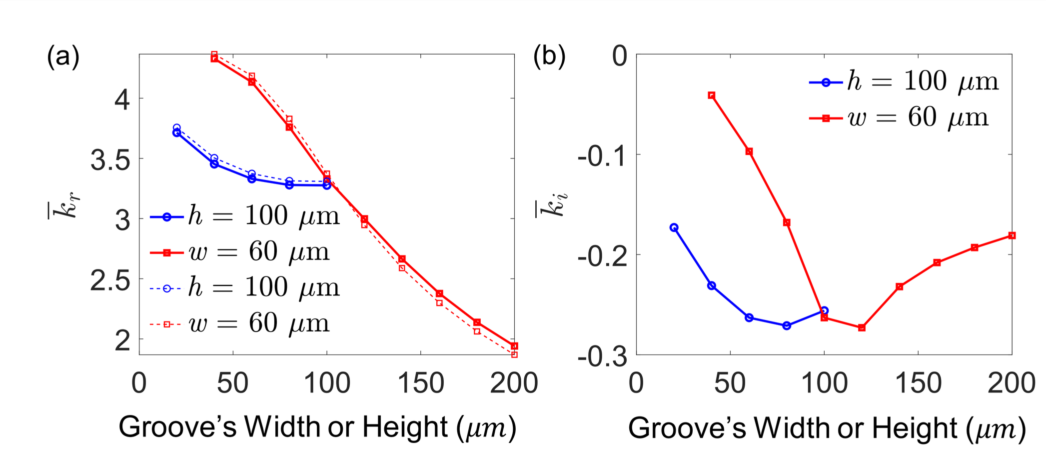
where the normalized beam distance from the grating $\bar{A} = a/L$, $\Psi = \bar{\omega}_a^2\gamma_n/(\bar{\omega} - p_n\beta_0)^2$, $\bar{\omega}_a = \omega_a\sqrt{L}/c$, $\omega_a^2 = (e^2 n_0/m\epsilon_0)\tau = \omega_p^2\tau$, ω_p is the plasma frequency, n_0 is the number of electrons per unit volume, τ is the thickness of beam.

The hot-tube dispersion relation (Eqn. (3)) is solved numerically for obtaining the complex wavenumber $\bar{k} = \bar{k}_r + j\bar{k}_i$ for a given operating frequency $\bar{\omega}$. With $\sim e^{-j\omega t + jkx}$ dependence for the fields when deriving Eqn. (3), a negative ***imaginary part*** of the wavenumber $k = k_r + jk_i$ indicates a ***growing wave*** as a function of position as $e^{-k_i x}$, with $-k_i$ being the ***growth rate***.



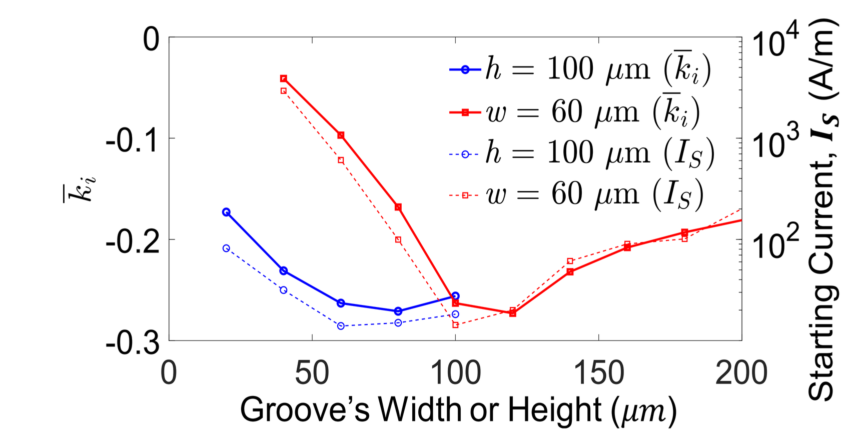
This figure shows the roots of the Eqn. (3) where the grating parameters are $w = 60\ \mu m$ and $h = 40\ \mu m$ and the corresponding operating frequency $\bar{\omega} = 1.80444$ (Calculated from Eqn. (2)).

IX. Growth Rate



The above figure shows the (a) real and (b) imaginary part of the wavenumber calculated from ***hot-tube dispersion relation*** (Eqn. (3)) at the operating frequency $\bar{\omega} = 2\pi f_{ev}L/c$ calculated from Eqn. (2), as a function of the groove height h (width w fixed at 60 μm) as well as groove width w (height h fixed at 100 μm), where $e^{-k_i x}$ (Fig. (b)) represents the ***growth rate***.

X. Minimizing the Starting Current



Here, we compare the growth rate with the starting currents calculated from ***particle-in-cell (PIC)*** simulations in Ref. [5] for the same sets of grating parameters. It can be seen that the pattern of the imaginary part of the wavenumber follows very closely that of the ***starting current***. Note the linear and log scale used to plot the $-\bar{k}_i$ and the starting current respectively, which precisely confirms that the exponential wave growth $e^{-k_i x}$ follow the scaling of the starting current as a function of the grating parameters w and h.

Advantages

01 We can avoid PIC simulation
For finding the starting current scaling we can use above procedure and avoid costly Particle-in-Cell simulation.

02 Computational cost is very low
As we need to solve only cold-tube and hot-tube dispersion relation, Computational cost is very low.

XI. Conclusion

The operating frequencies of SPR have changed with different grating parameters using cold-tube dispersion relations where the grating period has been fixed. The ***growth rate*** calculation using ***hot-tube dispersion*** relation can be used to predict the optimal grating parameters to minimize the starting current of SPR. This approach has a significantly reduced computation cost compared to direct PIC simulations. As both the ***operation frequency*** of SPR and its ***growth rate*** depend strongly on the grating parameters, both the ***cold-tube*** and ***hot-tube dispersion relations*** can be used in combination to ***minimize*** the ***starting current*** at a desired radiation frequency. While we apply our analysis to the effects of grating parameters on SPR, we expect that our dispersion relation treatment to grating optimization is applicable to study linear free-electron beam-based vacuum devices in general and in various geometries (e.g., cylindrical geometry).

References

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