A hydrodynamic mechanism for hot spot formation in SN1987A

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Introduction

SN1987A is a crucial system for supernova research due to its proximity to Earth in the Large Magellanic Cloud (LMC) and recency. About 20,000 years prior to the supernova blast, the progenitor star emitted a toroidal puff of heavy gas that may have encountered velocity gradients conducive to the formation of an expanding vortex dipole. Such a system is subject to the Crow instability [1].

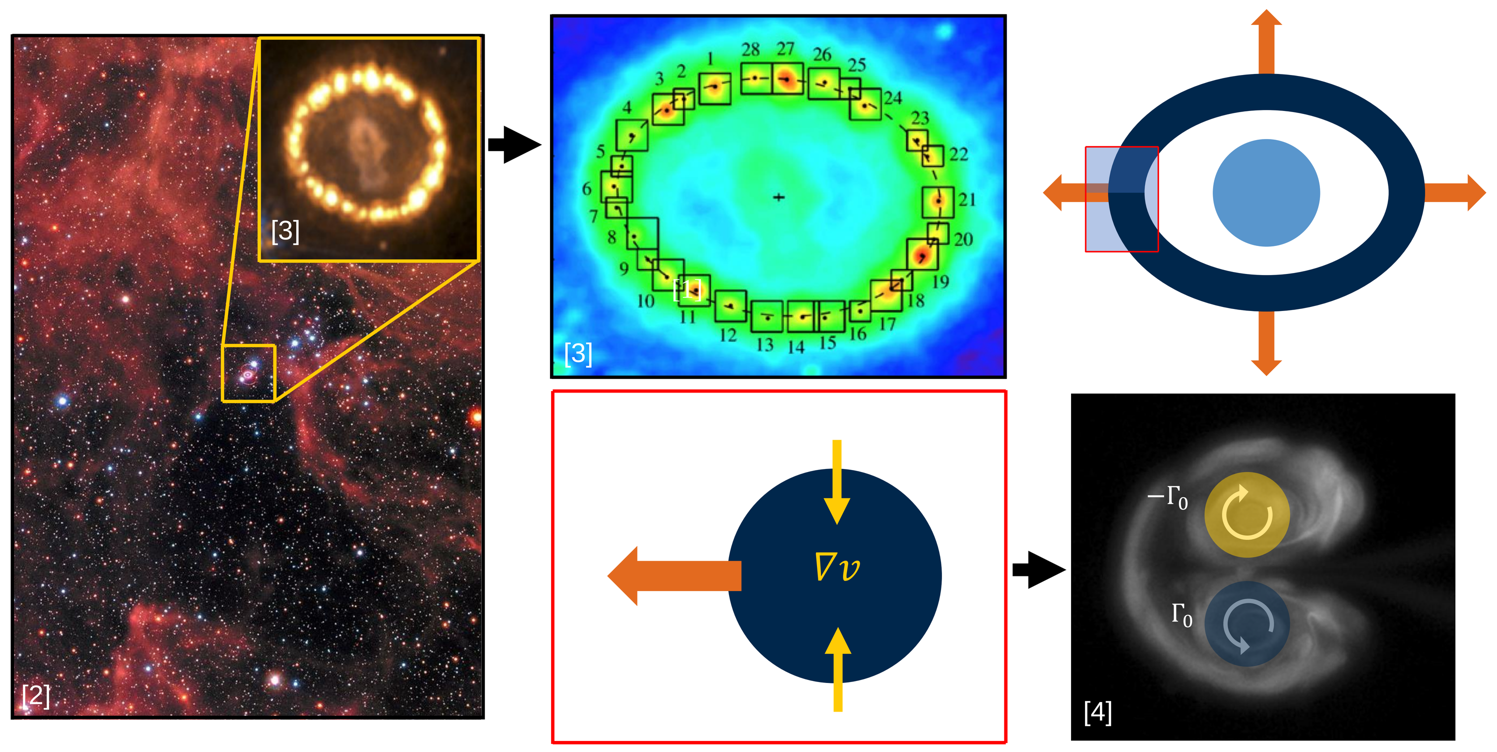


Figure 1: The LMC where SN1987A resides (left), an image showing the 28 hotspots (top center), a schematic showing the ejection of a gaseous torus (top right), a schematic showing a cross section of the ejected torus (bottom center), and the formation of a vortex dipole from an analogous fluid experiment (bottom right).

Our objective is to assess the viability of the Crow instability as the mechanism causing the formation of the observed 28 mass accumulations along the circumstellar torus. The Crow instability was initially explored in the context of the dissipation of wingtip vortices shed from large aircraft. Symmetric perturbations along the vortex cores grow until they touch, resulting in a series of vortex rings.

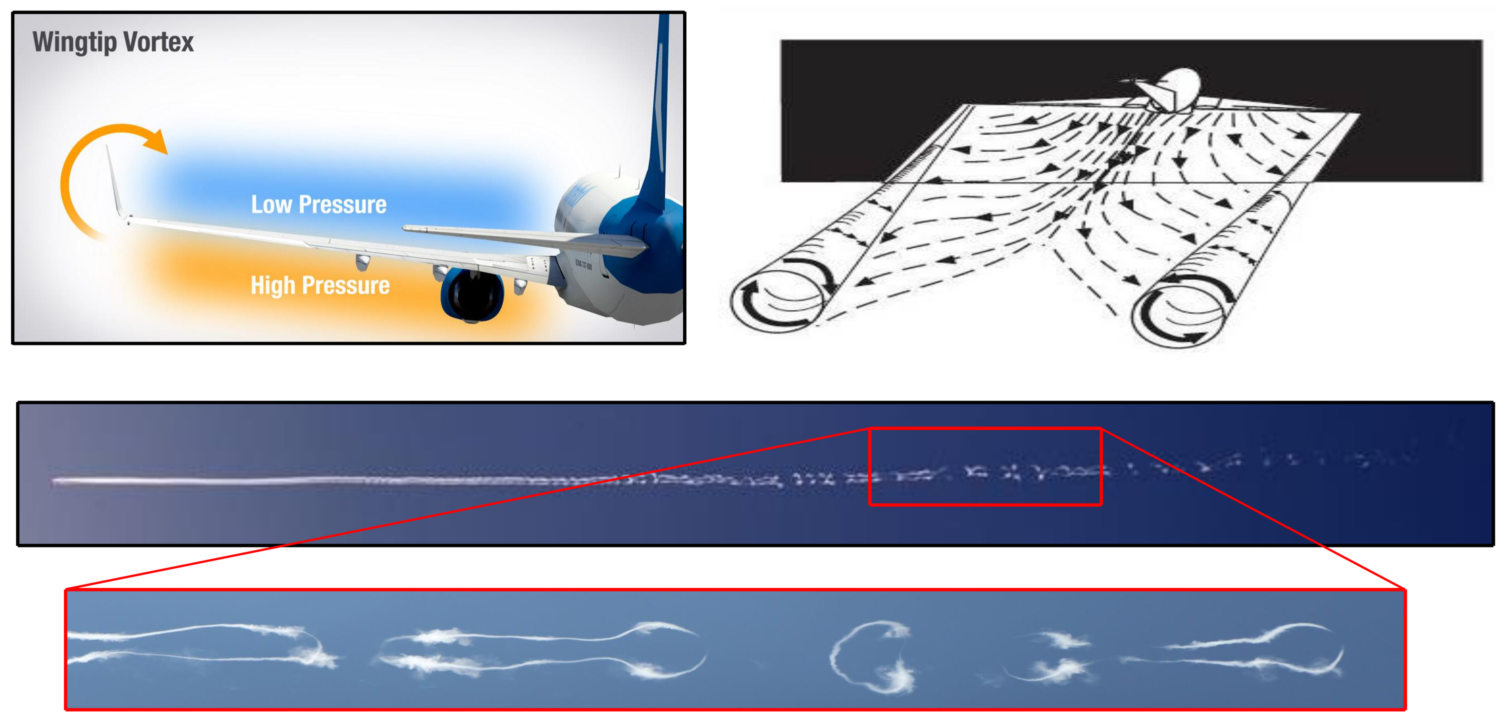
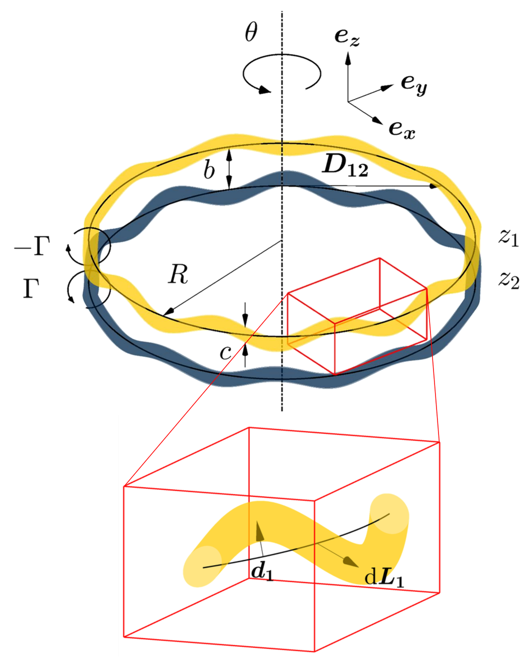


Figure 2: A schematic showing wingtip vortex formation (top), and wingtip vortex breakup (bottom).

Model

The linear stability analysis considers perturbations along neighboring vortex cores governed by the Biot-Savart law and the condition that perturbations move at the local flow velocity. Both symmetric and anti-symmetric perturbations experience growth.



$$\boldsymbol{U_n} = \sum_{m=1}^{2} \frac{\Gamma_m}{4\pi} \int \frac{\boldsymbol{D_{mn}} \times d\boldsymbol{L_m}}{|\boldsymbol{D_{mn}}|^3} = \boldsymbol{e_x} u_n + \boldsymbol{e_y} v_n + \boldsymbol{e_z} w_n$$

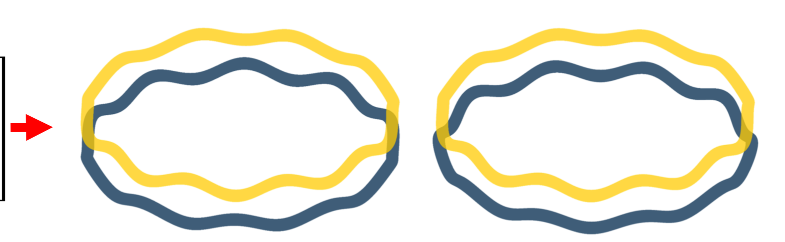
$$\boldsymbol{D_{mn}} = \boldsymbol{e_x} R(\cos\theta_{m'} - \cos\theta_n) + \boldsymbol{e_y} R(\sin\theta_{m'} - \sin\theta_n) + \boldsymbol{e_z}(z_m - z_n) + (\boldsymbol{d'_m} - \boldsymbol{d_n})$$

$$\boldsymbol{d_n} = \boldsymbol{e_x} h_n(\theta_n, t)\cos\theta_n + \boldsymbol{e_y} h_n(\theta_n, t)\sin\theta_n + \boldsymbol{e_z} s_n(\theta_n, t) = \widetilde{\boldsymbol{d_n}} e^{at+ik\theta_n}$$

$$d\boldsymbol{L_n} = \left(-\boldsymbol{e_x} R\sin\theta_n + \boldsymbol{e_y} R\cos\theta_n + \partial\boldsymbol{d_n}/\partial\theta_n\right) d\theta_n$$

$$\partial\boldsymbol{d_n}/\partial t + u_n(\partial\boldsymbol{d_n}/\partial x_n) + v_n(\partial\boldsymbol{d_n}/\partial y_n) = \boldsymbol{e_x} u_n + \boldsymbol{e_y} v_n + \boldsymbol{e_z} w_n$$

$$a\begin{bmatrix}\hat{h}_1\\ \hat{s}_1\\ \hat{h}_2\\ \hat{s}_2\end{bmatrix} = M\begin{bmatrix}\hat{h}_1\\ \hat{s}_1\\ \hat{h}_2\\ \hat{s}_2\end{bmatrix}$$



$\hat{s}_s = \hat{s}_2 - \hat{s}_1 \quad \hat{h}_s = \hat{h}_2 + \hat{h}_1 \qquad \hat{s}_a = \hat{s}_2 + \hat{s}_1 \quad \hat{h}_a = \hat{h}_2 - \hat{h}_1$

Figure 3: A schematic showing the relevant parameters in the stability analysis (left), the governing equations (top right), and a schematic showing symmetric and anti-symmetric perturbations (bottom right).

Results

Our analysis predicts three separate wavenumber bands that experience perturbation growth. The fastest growth rates are those associated with the high-frequency symmetric band and the anti-symmetric band. However, these bands constantly migrate to larger wavenumbers while the low-frequency symmetric band consistently excites lower wavenumbers. The dominant wavenumber is 28, which is consistent with the number of observed hotspots in SN1987A.

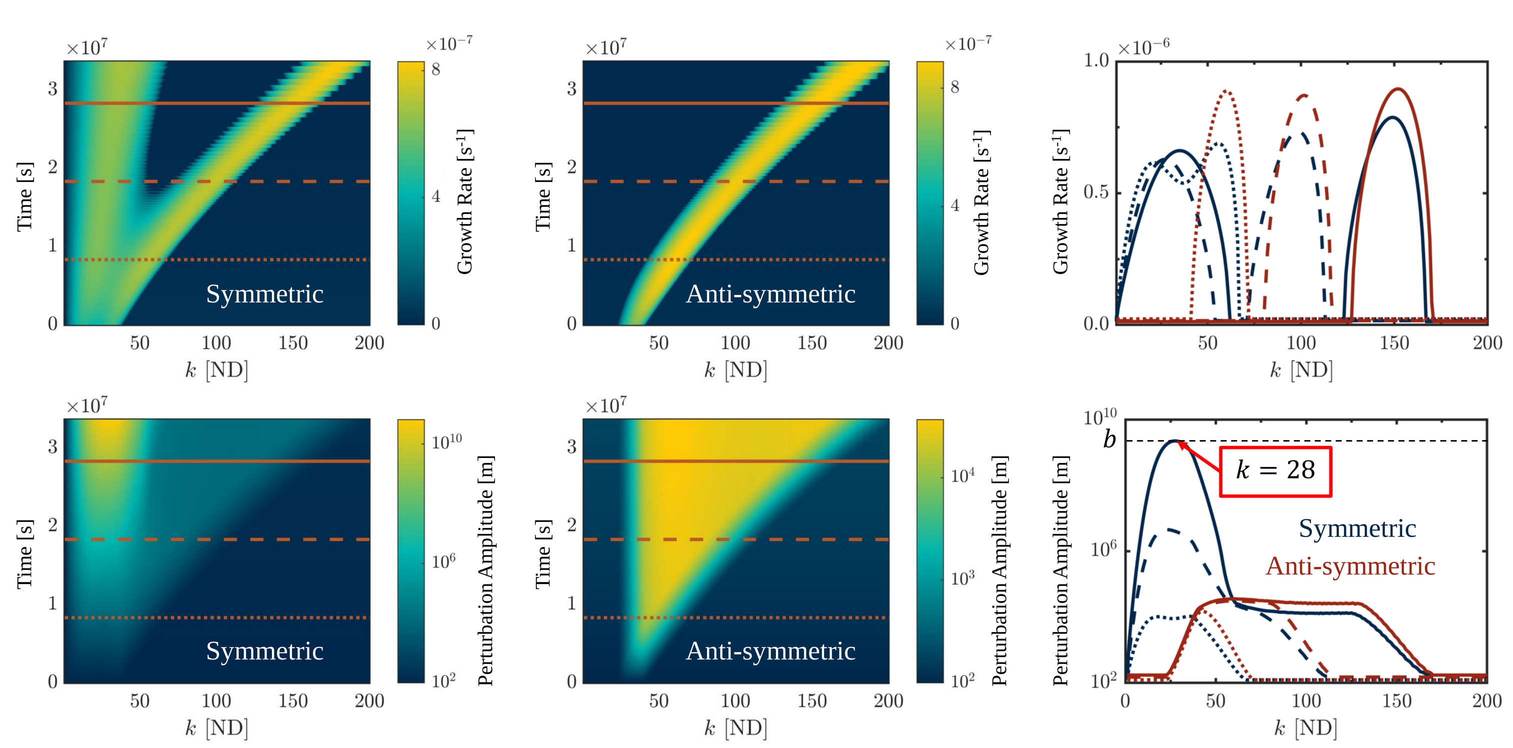


Figure 4: The growth rates (top) and perturbation amplitudes (bottom) as a function of wavenumber and time for symmetric (left) and antisymmetric (center) perturbations at three separate times with lineouts (right).

Sensitivity

The initial perturbation spectrum is assumed to be uniform with an amplitude given by the mean free path of the flow, which is difficult to estimate. However, the dominant wavenumber only varies by roughly 10 when varying the initial amplitude over six orders of magnitude .

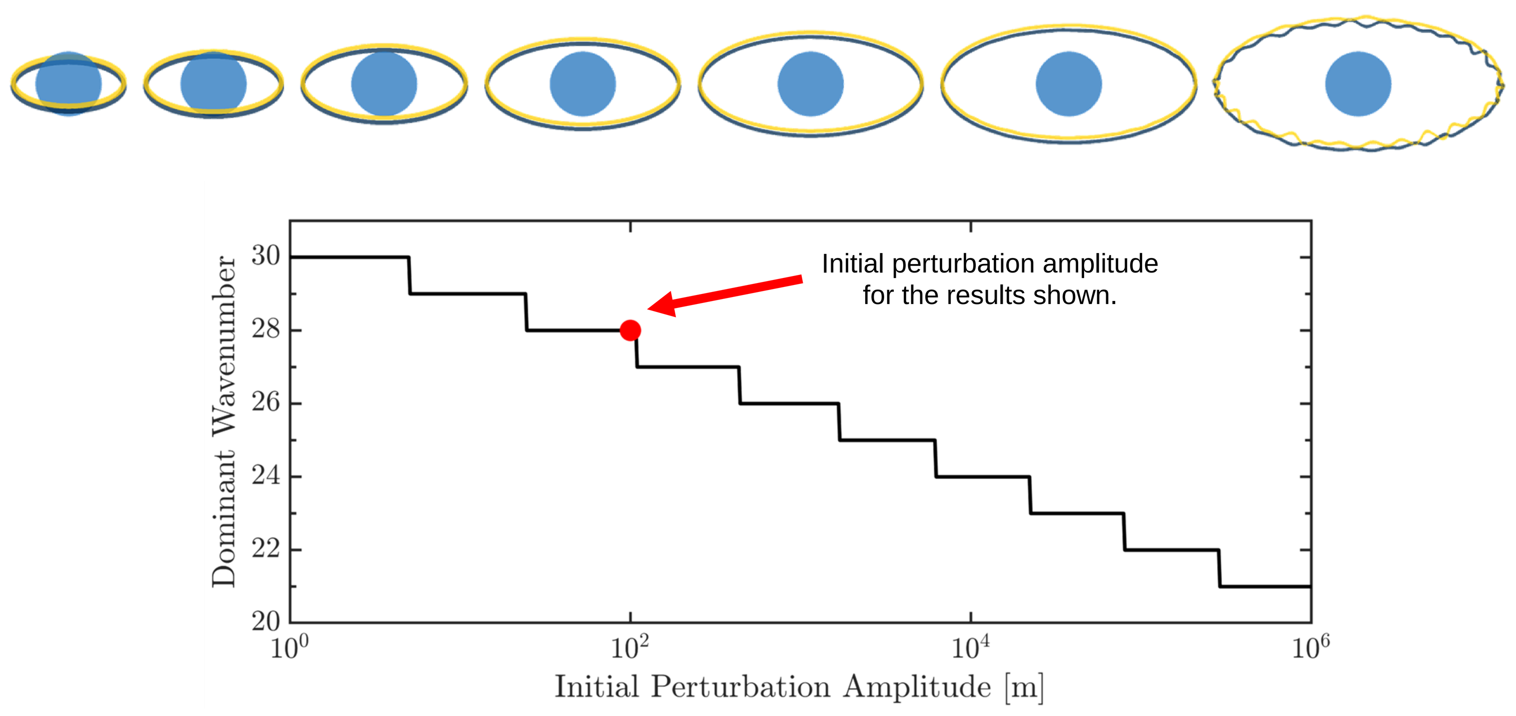


Figure 5: A time series showing the evolution of the expanding vortex dipole (top), and the sensitivity of the dominant wave number to the amplitude of the initial perturbation spectrum (bottom).

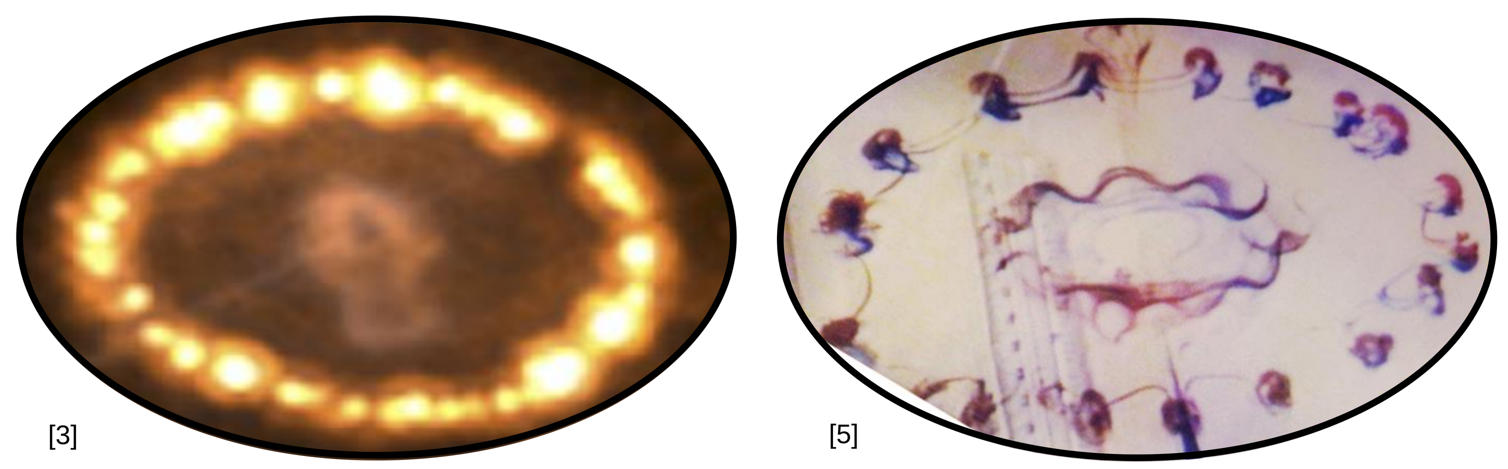


Figure 6: Hotspots along the circumstellar torus of SN1987A [3] (left) and secondary vortex structures formed along an expanding vortex dipole resulting from the direct collision of two vortex rings [5] (right).

Conclusions and Acknowledgement

The dominant mode predicted by our stability analysis is consistent with the number of observable hot spots along the circumstellar torus of SN1987A. The Crow instability may therefore be a viable mechanism facilitating the accumulation of mass along radial tori ejected by some star systems.

[1] Crow, AIAA J., 8 (1970) [2] NASA, ESA, Krishner, Mutchler, and Avila [3] Fransson et al., Astrophys. J., 806 (2015) [4] Olmstead et al., Exp. Fluids, 58 (2017) [5] Lim and Nickels, Nat. Lett., 357 (1992)
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