Analyzing Spatial Growth Rate and Starting Current in Smith-Purcell Radiation Using Single- and Two-Layer Grating Structures

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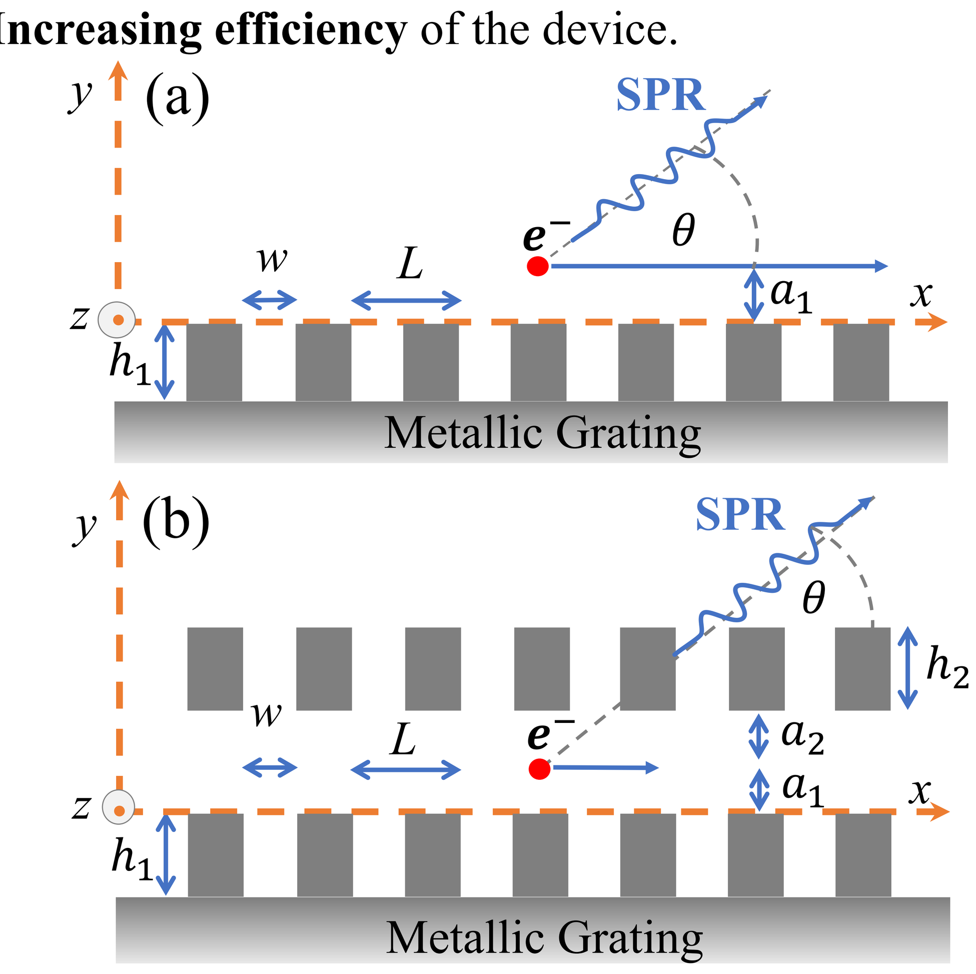
I. SPR from Two Layer Grating

Smith–Purcell radiation (SPR) [1] is generated when an electron beam passes near a periodic structure. Two-layer gratings are proposed to enhance SPR by improving electromagnetic coupling.

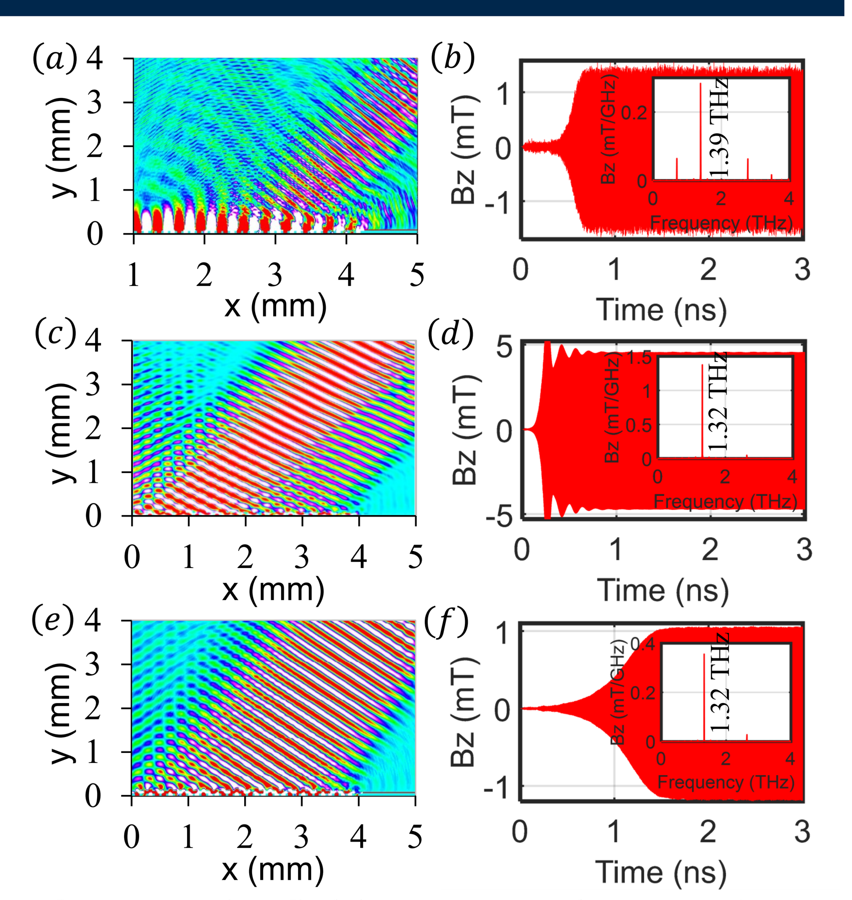
Motivations:

- **Reducing Cathode Current** without any special arrangement like two cathodes or side wall.
- **Increasing efficiency** of the device.



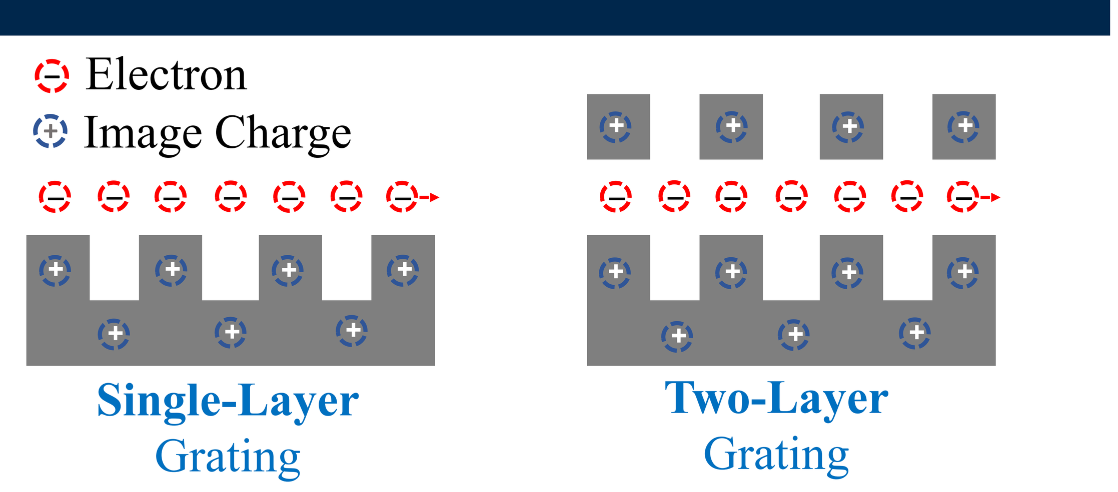
Schematic of Smith-Purcell (a) single-layer and (b) two-layer grating [2] and beam configuration.

II. Enhanced Coherent SPR



All contour maps of magnetic field, B_z snapshots at 3 ns. Here magnetic field $B_z(t)$ and its FFT for single- and two-layer grating with a continuous DC beam were detected at $X = 4.9\ mm$, $Y = 3\ mm$ using XOOPIC [3] PIC simulation. All the parameters are mentioned in the Table I.

III. Radiation Mechanism of SPR



Two-layer grating experiences more dipoles oscillation and induces stronger velocity modulations than single-layer structure, hence PIC simulation shows that two-layer gratings exhibit greater radiation intensity than single layer gratings.

IV. Main parameters for Analysis

TABLE I. Main Parameters for the Calculation.

Beam energy	50 keV
Beam Height from Grating Surface, a_1	10 μm
Beam thickness, τ	10 μm
Beam Current, I	5000 A/m
Grating period, L	120 μm
Number of grating periods	35

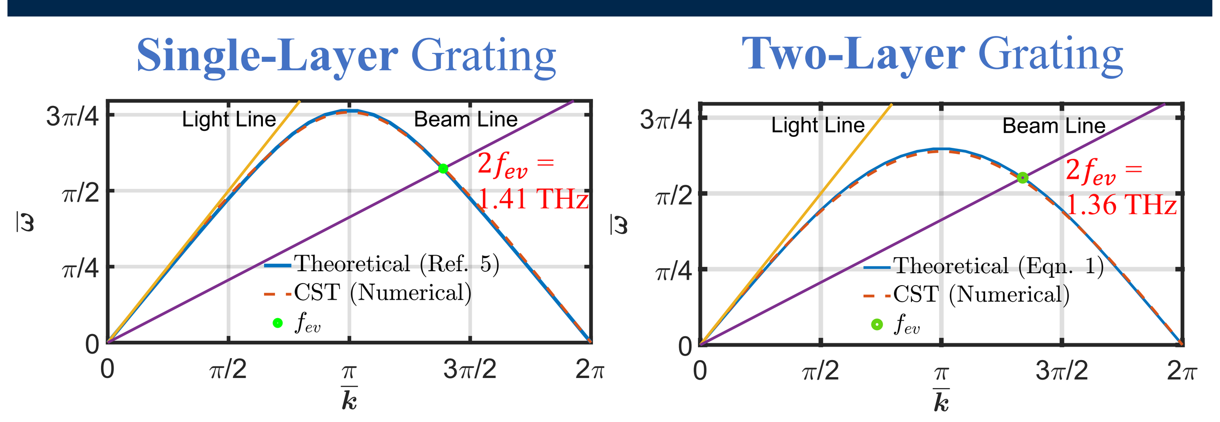
V. Cold-Tube Dispersion Relation

The cold-tube dispersion equation for the SPR model with two-layer grating is,

$$1-Y_n = \sum_{n=-\infty}^{\infty}\left((A-BE)e^{-\gamma_n(\overline{A_1}+\overline{A_2})}+(A-DE)e^{\gamma_n(\overline{A_1}+\overline{A_2})}\right)\left(\frac{e^{ip_n\overline{W}}-1}{ip_n\overline{W}}\right) \quad (1)$$

where A, B, D, E, and Y_n are functions of grating parameters. Other parameters are grating width $\overline{W} = w/L$, grating height $\overline{H_1} = h_1/L$ and $\overline{H_2} = h_2/L$, beam height $\overline{A} = a/L$, angular frequency $\overline{\omega} = \omega L/c$, wavenumber $\overline{k} = kL$, $\gamma_n = \left(P_n{}^2 - \overline{\omega}^2\right)^{1/2}$, and $p_n = \overline{k} + 2n\pi$.

VI. Operating Frequency



The dispersion relation $\omega(k)$ (a) for *single-layer* grating [5] and (b) *two-layer* grating (Eqn. 1).

Intersection of ***beam line*** $\left(\overline{\omega} = (v_0/c)\overline{k} = \beta_0\overline{k}\right)$ to the ***dispersion relation*** is the operating frequency (f_{ev}) for the grating parameters, where ***SPR*** occurs at the ***second harmonic*** $(2f_{ev})$ of this ***operating frequency***.

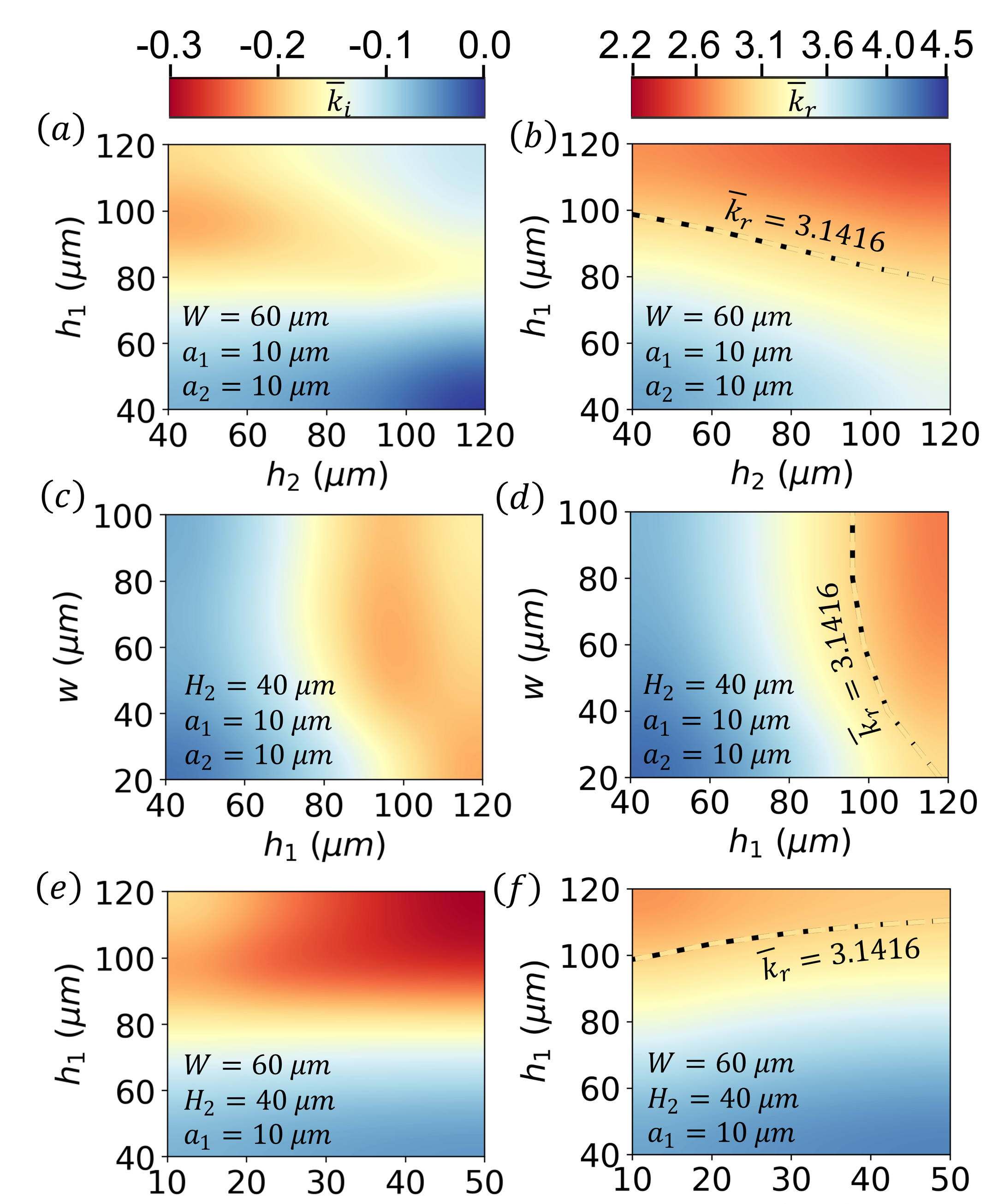
VII. Hot-Tube Dispersion Relation

The ***hot-tube dispersion equation*** for the SPR model with two-layer grating is,

$$1-Y_n = \sum_{p=-\infty}^{\infty}\Big((1+M)(A-BE)e^{-\gamma_n(\overline{A_1}+\overline{A_2})} - M(A-DE)e^{\gamma_n(\overline{A_1}-\overline{A_2})} + M(A-BE)e^{-\gamma_n(\overline{A_1}-\overline{A_2})} + (1-M)(A-DE)e^{\gamma_n(\overline{A_1}+\overline{A_2})}\Big)\left(\frac{e^{ip_n\overline{W}}-1}{ip_n\overline{W}}\right) \quad (2)$$

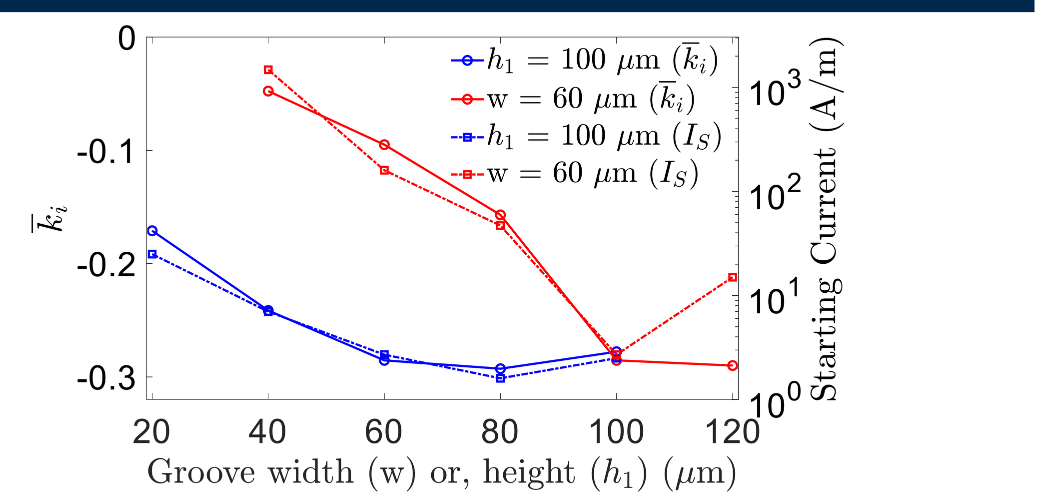
where $M = \overline{\omega_a^2}\gamma_n/2(\overline{\omega} - p_n\beta_0)^2$, $\overline{\omega_a} = \omega_a\sqrt{L}/c$, $\omega_a^2 = (e^2 n_0/m\epsilon_0)\tau = \omega_p^2\tau$, ω_p is the plasma frequency, n_0 is the number of electrons per unit volume, τ is the thickness of beam. Other parameters are mentioned in Equation 1.

VIII. Grating Optimization



From the Figures 2 (a), (c), and (e), it is observed that the spatial growth rate $(\overline{k}_i)$ is higher when the real part of the wave number, $\overline{k}_r$ (Figs. 2 (b), (d), and (f)) approaches $\pi(= 3.1416)$ (upper band edge in the cold-tube dispersion relation) [4].

IX. Growth Rate and Starting Current



This figure shows a good correlation between Growth Rate (calculated from hot-tube dispersion relation, Eqn. 2) and electron beam's Starting Current (numerically calculated using PIC simulation) [4]. Here, second layer height, $h_2 = 40\mu m$, and beam distance from second layer, $a_2 = 50\mu m$ and all the other parameters are mentioned in the Table I.

X. Single and Two-layer Grating Comparison

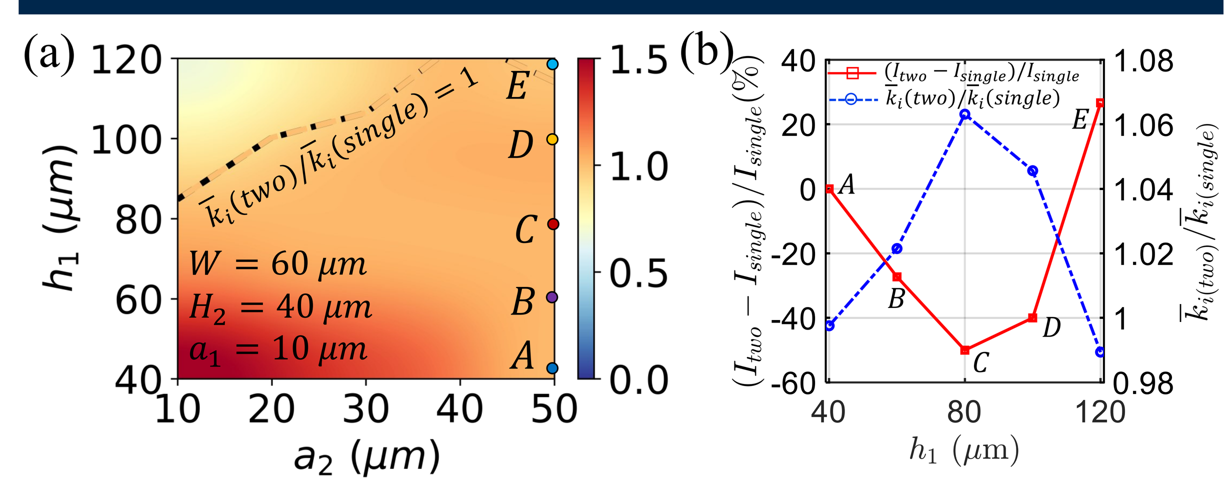
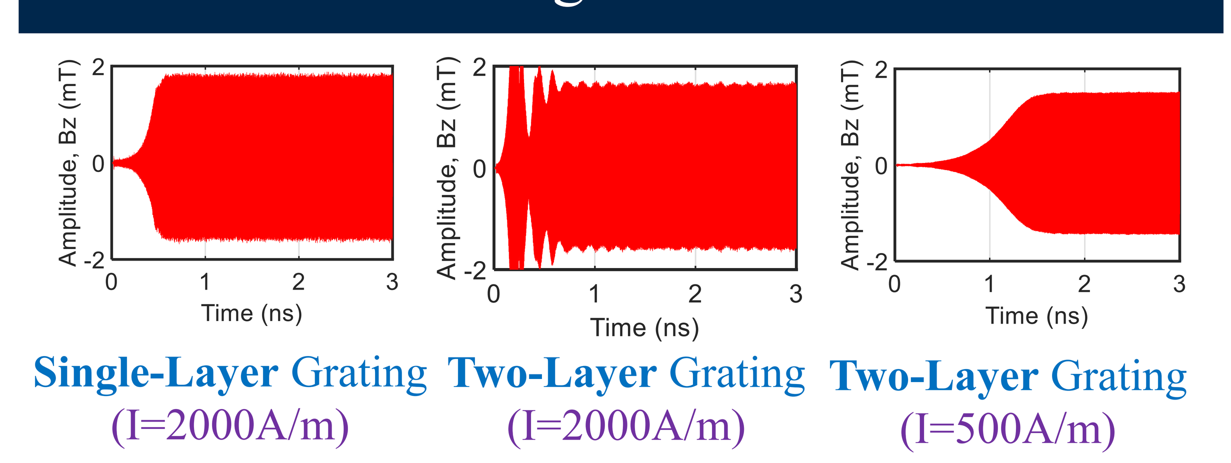


Figure (a) illustrates the spatial growth rate ratio between a two-layer grating structure and a single-layer grating structure [4]. In Figure (b), when the ratio exceeds 1, the two-layer grating structure exhibits a lower starting current [6] compared to the single-layer grating structure.

XI. Heating on the Device



The tangential magnetic field $B_z(t)$ on the grating structure shows that we may get enhanced electromagnetic field from two-layer grating without increasing heating on the grating.

XII. Conclusion

Theory and PIC simulations reveal that the two-layer gratings exhibit larger spatial growth rate and significantly higher radiation intensity. Some grating parameters are demonstrated to generate coherent SPR with a reduced beam starting current [4]. This helps prolonging cathode lifetime, and mitigating the heating of beam-carrying components to generate higher power.

References

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Acknowledgment

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