Space Charge Effects on the Evolution of Short Pulse Beam Profiles

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Summary

- We investigate **space charge effects** on the dynamics of short pulse beam profile.
- We consider **short pulses** of different profiles for different charge densities and pulse widths.
- We analyze the electron sheet **phase-space trajectories** and pulse profile evolution during gap transit.

Multiple-Sheet Model

A one-dimensional (1D) planar diode with gap distance d and gap voltage V_g, with M sheets inside.

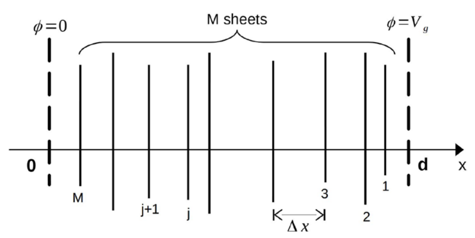

Figure 1: Sheet numbering inside the diode gap [1]

- Sheet j at position $\bar{x}_j = x_j/d$ has a normalized charge density $\bar{\rho}_j = \rho/\sigma_1$.
- The normalized electric field on sheet j is

$$\bar{E}_j = \frac{E_j}{E_0} = 1 + \left[\sum_{i=1}^{M}\bar{\rho}_i\bar{x}_i - \left(\sum_{i=1}^{j-1}\bar{\rho}_i + \frac{1}{2}\bar{\rho}_j\right)\right] \quad (1)$$

- The normalized Electric field at the cathode $(\bar{x} = 0)$

$$\bar{E}_K = 1 + \sum_{i=1}^{M}\bar{\rho}_i(\bar{x}_i - 1) \quad (2)$$

- The Space Charge Limited (SCL) charge density $\bar{\rho}_j^*$ is found for $\bar{E}_K = 0$

$$1 + \sum_{i=1}^{M}\bar{\rho}_i(\bar{x}_i - 1) = 0 \quad (3)$$

Model Parameters

Symbol	Meaning	Formula/Value
E_0	*Applied field*	$-V_g/d$
σ_1	*SCL density*	$\varepsilon_0 E_0$
τ_p	*Pulse length*	$[0.1, 1] \times T_0$
T_0	*Transit time*	$\sqrt{2d/(eE_0/m)}$
Δ	*Distortion*	$\delta\bar{x}_{final}/\delta\bar{x}_{init}$
J	*Current density*	$3\sum_{i=1}^{M}\bar{\rho}_i\bar{v}_i$

(a) Square-top Profile

- Sheets have **equal charge density** $\bar{\rho}$
- The charge density $\bar{\rho}^*$ is found from (3)

$$\bar{\rho}^* = \frac{1}{M}\left[\frac{1}{1 - \delta\bar{x}\left(\frac{M-1}{2}\right)}\right] \quad (4)$$

- We simulate 30 preloaded sheets inside the gap. The initial pulse intervals $\delta\bar{x} = \bar{x}_n - \bar{x}_{n+1}$ are assumed to be uniform

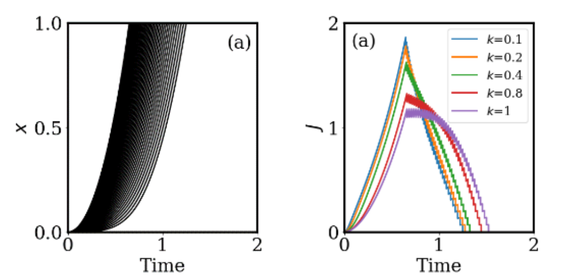

Figure 2: Trajectories & current density for T_0 and $\bar{\rho}^*$

(b) Trapezoidal Profile

- A **more general square-top model** that include a time of rise and a time of fall

$$\rho_j = \begin{cases} \rho_0 + (j-1)\dfrac{\rho_1 - \rho_0}{n_r}, & 1 \le j < n_r \\ \rho_1, & n_r \le j < (M - n_f) \\ \rho_0 + (M-j)\dfrac{\rho_1 - \rho_0}{n_f}, & (M - n_f) \le j \le M \end{cases}$$

- ρ_0 and ρ_1 are respectively the lowest and highest charge density.

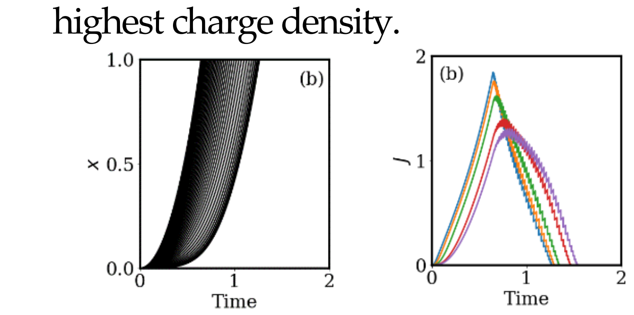

Figure 3: Trajectories & current density for T_0 and $\bar{\rho}^*$

(c) Gaussian Profile

- Sheet j has a **charge density**

$$\bar{\rho}_j = a\exp\left[-\frac{(j-\mu)^2}{b}\right] \quad (5)$$

where $\mu = (M+1)/2$, a and b are found by solving (3) with $\sum_{i=1}^{M}\bar{\rho}_i^* = 1$. (as the bulk of sheets tends to combine, looking like a single sheet).

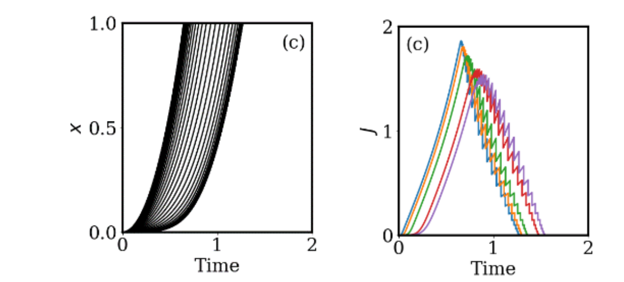

Figure 4: Trajectories & current density for T_0 and $\bar{\rho}^*$

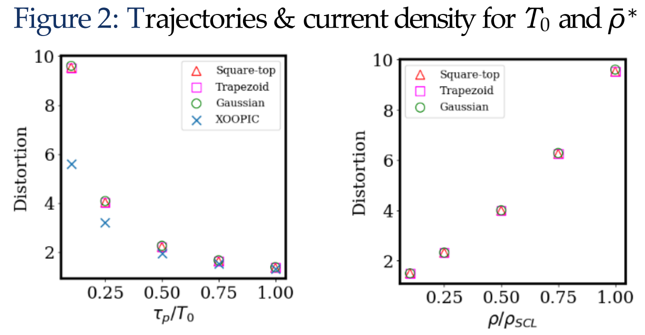

Figure 5(a): Profile distortion with initial pulse length and charge

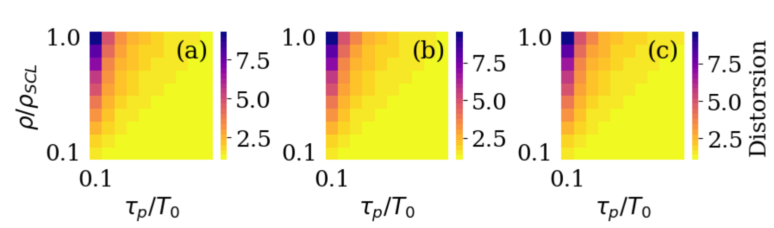

Figure 5(b): Profile distortion with initial pulse length and charge

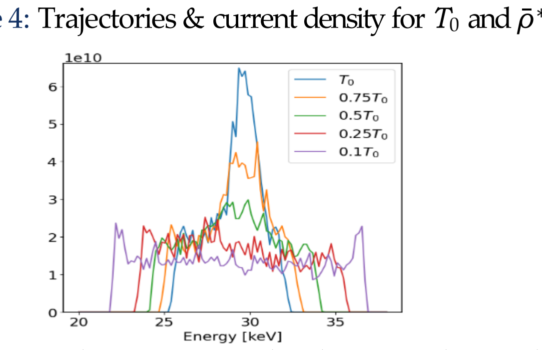

Figure 6: Electron energy distribution at the anode

Comparison of pulse profiles

- We compare pulses of different charges and shapes.

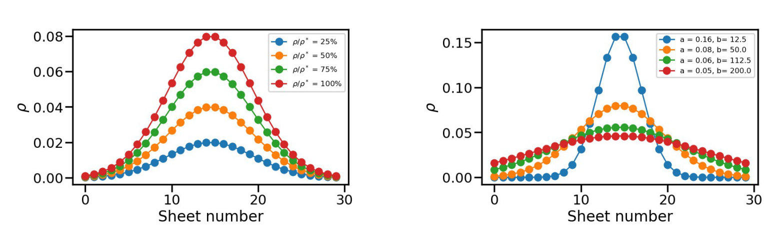

Figure 7: Comparison Case 1 and Case 2

- **Case 1**: We vary the total charge but keep the pulse shape unchanged.
- **Case 2**: We maintain the total charge but vary the shape

Child-Langmuir limit

- We simulate Gaussian pulses with $M = 30$ preloaded sheets. We fixed $\delta\bar{x} = 1/M^2 = 1/900$.

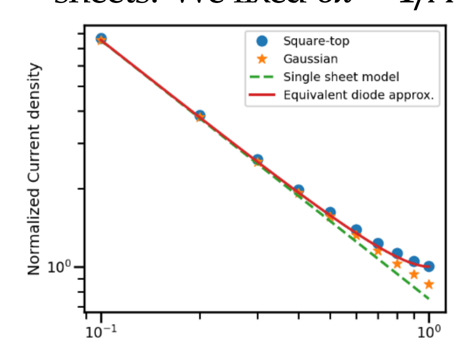

Figure 8: Normalized critical current density, J/J_{CL} as a function of the normalized pulse length.

- **Single sheet model**

$$J_{crit} = \frac{3J_{CL}}{4X_{CL}}$$

- **Equivalent diode approximation**

$$J_{crit} = 2\,\frac{1 - \sqrt{1 - \frac{3}{4}X_{CL}^2}}{X_{CL}^3}\,J_{CL}$$

Evolution of the Gaussian Pulse Profiles Inside the Gap

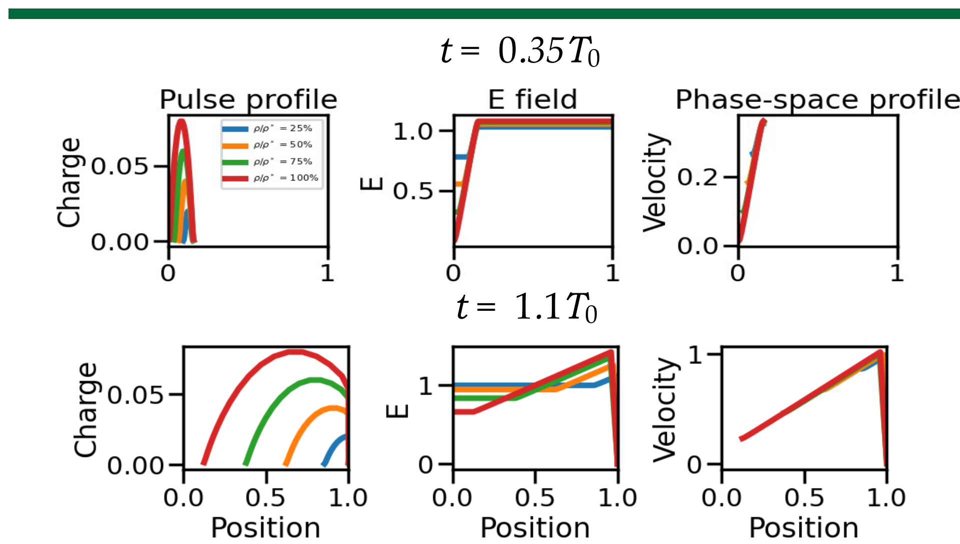

Figure 9: Evolution of pulse profile, electric field, and velocity for Case 1.

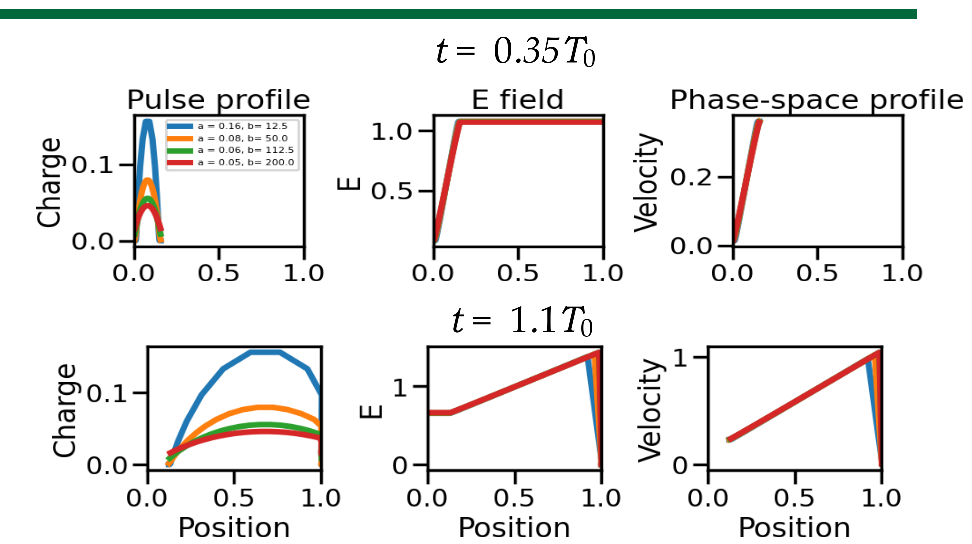

Figure 10: Evolution of pulse profile, electric field, and velocity for Case 2.

Algorithm 1 Calculation of distortion

Input: $M, \delta\bar{x}$

$\delta\bar{x}_{init} \leftarrow (M-1)\delta\bar{x}$

$t \leftarrow 0$

while $\bar{x}_1(t) < 1$ **do**

  $t \leftarrow t + 1$

end while

$\delta\bar{x}_{final} \leftarrow \bar{x}_1(t) - \bar{x}_M(t)$

$\Delta \leftarrow \delta\bar{x}_{final}/\delta\bar{x}_{final}$

return Δ

Conclusion & Future Work

1. For the same total charge, square-top and Gaussian pulses undergo similar distortion (fig.5).
2. The shorter the pulse length, the more significant the distortion becomes.
3. The smaller the charge, the faster the tail of the pulse travels through the gap (fig. 8).
4. The Child-Langmuir limit increases as the pulse length decreases.

References & Acknowledgement

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