Parametric Analysis of the Beam Kinetic Energy in Linear Beam Devices

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Motivation

- ❑ We provide a parametric analysis of beam-cavity interaction for large signals of linear beam device.
- ❑ We examine hollow (i.e. annular) beam effects for large signal analysis of electron beam interaction.
- ❑ We observe the effect of space charge on the electron trajectories and their kinetic energy which leads to lower conversion efficiency.

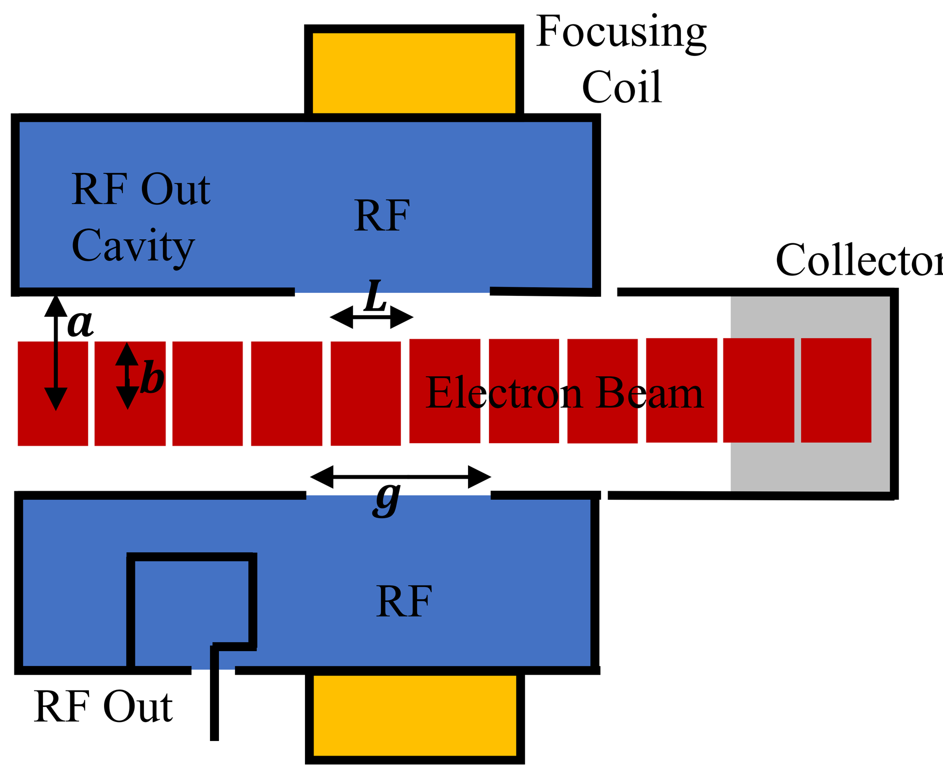

Fig 1: Schematic diagram of a Linear Beam Device

Models [1]

➢ Simple Model

- Input Power, $P_{in} = \frac{1}{8} I_{RF}^2 R_{in}\left(1 + \frac{R_{in}}{R_s}\right)$ (1)
- Wave number, $\beta = \frac{\omega}{u}$
- Gap Coupling Coefficient, $M(u) = \frac{2}{\beta b}\frac{I_1(\beta b)}{I_0(\beta a)}\frac{\sin(\frac{\beta g}{2})}{(\frac{\beta g}{2})}$
- Large Signal Gap Coupling Coefficient,

$$M\left(\frac{u_s + u_0}{2}\right) = \frac{2}{\beta b}\frac{I_1(\beta b)}{I_0(\beta a)}\frac{\sin(\frac{\beta g}{2})}{(\frac{\beta g}{2})} \quad (2)$$

- RF Output Power, $P_{out} = \frac{1}{2} M^2 I_{RF}^2 R_L$ (3)
- Electronic Efficiency, $\eta_e = \frac{P_{out}}{I_b V_a}$ (4)

TABLE I. Parameters for the Calculation

Anode Voltage, V_a	25kV
μ	150
External Impedance, R_L	26kΩ
Source Impedance, R_s	220Ω
Frequency, f	1.3Ghz
Number of Disks, N_D	50
η	$1.7e10^{11} Askg^{-1}$
Input Impedance, R_{in}	220Ω

TABLE II. List of Symbols

L	Length of a space charge disk
a	Tunnel/drift tube radius
b	Beam radius
g	Output gap length
ρ	Space charge density
Φ	Phase of gap field
ω	Angular frequency
c	Speed of Light
I_b	Beam Current
I_{RF}	RF current

➢ Disk Model

- Axial component of Electric Field,

$$E_z(z) = \frac{V_{gap}}{2\pi}\int_{-\infty}^{\infty}\frac{2}{\beta b}\frac{I_1(\beta b)}{I_0(\beta a)}\frac{\sin(\frac{\beta g}{2})}{(\frac{\beta g}{2})} e^{-j\beta z} d\beta \quad (5)$$

- Gap Voltage, $V_{gap} = I_{RF} M\left(\frac{u_0+u_s}{2}\right) R_L$ (6)
- Potential of Space charge,

$$V_{cyl} = \begin{cases} \frac{2ab\rho}{\varepsilon_0}\sum_{m=1}^{\infty}\frac{J_1(\mu_m b)J_0(\mu_m r)}{(\mu_m a)^3[J_1(\mu_m a)]^2} e^{-\mu_m z}\sinh\left(\frac{\mu_m L}{2}\right) & \text{for } |z| > \frac{L}{2} \\ \frac{2ab\rho}{\varepsilon_0}\sum_{m=1}^{\infty}\frac{J_1(\mu_m b)J_0(\mu_m r)}{(\mu_m a)^3[J_1(\mu_m a)]^2}[1 - e^{-\mu_m L/2}\cosh((\mu_m z)] & \text{for } |z| < \frac{L}{2} \end{cases} \quad (7)$$

- Differential Equation of motion,

$$\frac{d}{dt}(u_s) = \eta\sqrt[3]{1 - \frac{u_0^2}{c^2}} E_{total} \quad (8)$$

- Kinetic Energy,

$$KE = \frac{Q}{\eta}c^2\left(\frac{1}{\sqrt{1-\frac{u_s^2}{c^2}}} - 1\right) \quad (9)$$

Beam Filling Factor

- Increasing beam filling factor b/a increases the gap coupling factor M due to enhanced interaction and reduced space charge effects (see Fig 2).
- RF output power and efficiency are increased as M is increased (see Eq (3) & (4)).
- In the presence of space charge, the efficiency is lower due to distortion of electron trajectories.
- To prevent the interception of electrons on the drift tube, b is limited to 2a/3.

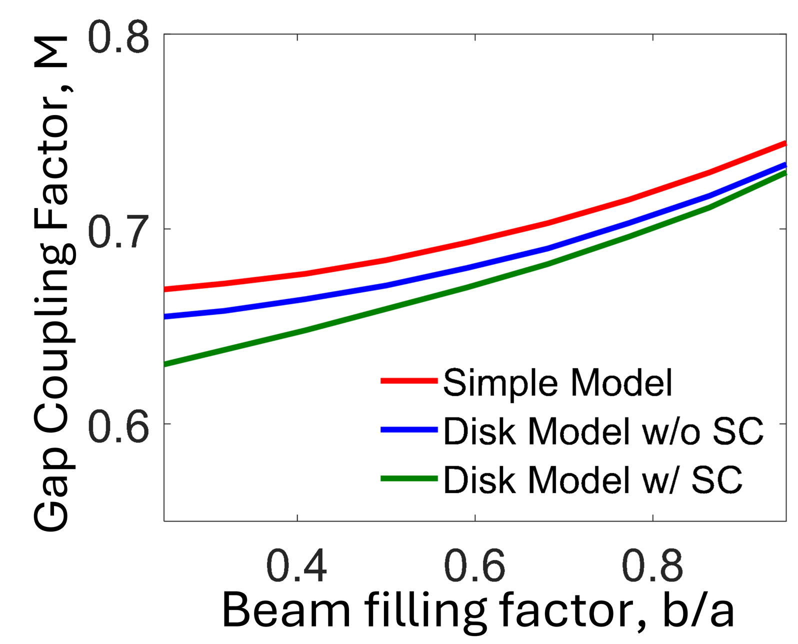

Fig 2: M for different beam filling factor

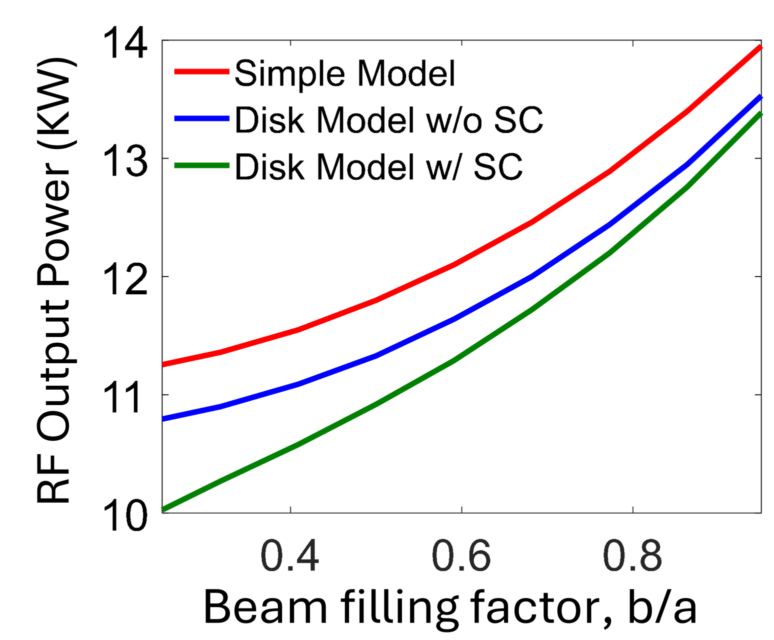

Fig 3: RF Output power for different beam filling factor

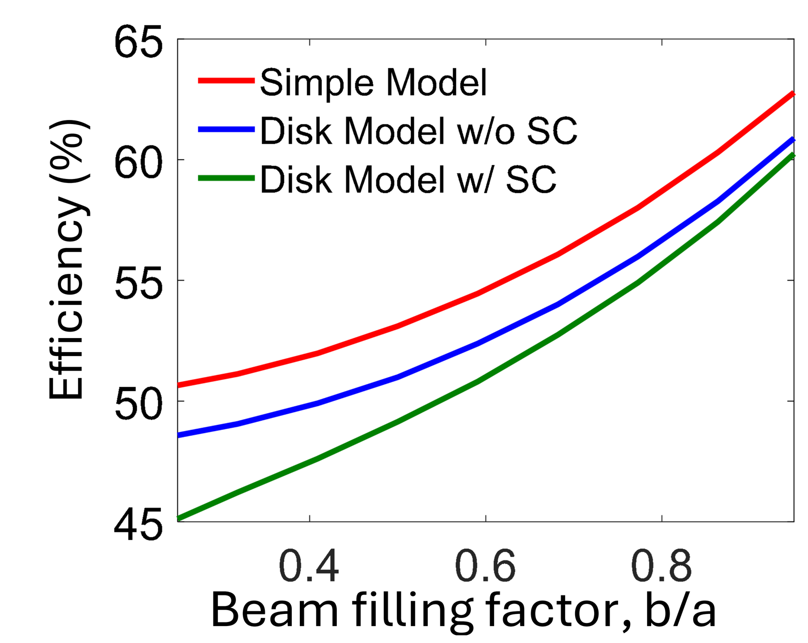

Fig 4: Efficiency for different beam filling factor

Kinetic Energy

- If we increase beam current I_b, the charge of the disk is increased as $Q = \frac{I_b}{f N_D}$ that increases the kinetic energy (see Eq (9)).
- Increasing beam filling factor lowers the KE that results in higher efficiency due to enhanced interaction and less repulsion (see Fig 7).

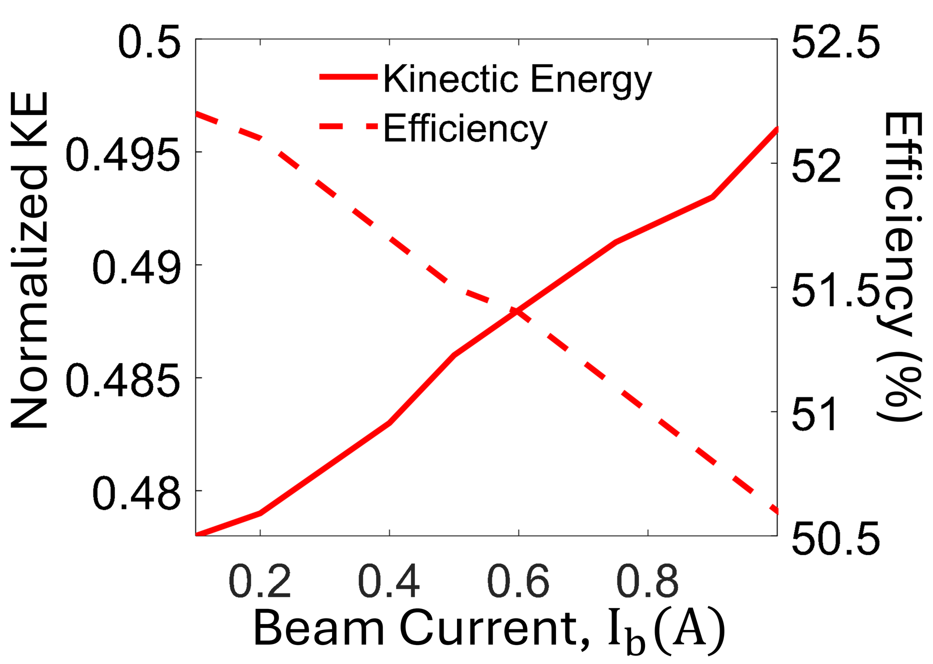

Fig 5: Normalized Kinetic Energy and Efficiency for different beam current, I_b(A)

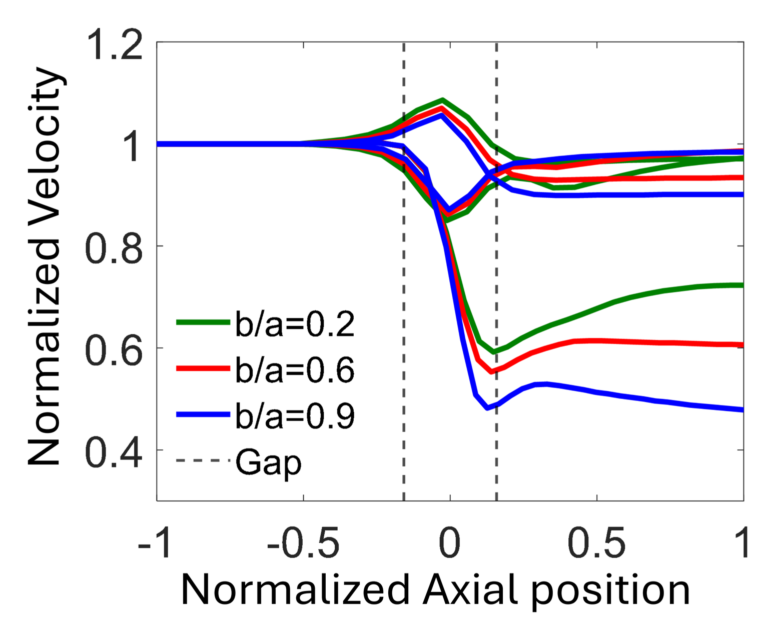

Fig 6: Normalized velocity with axial position for different beam filling factor

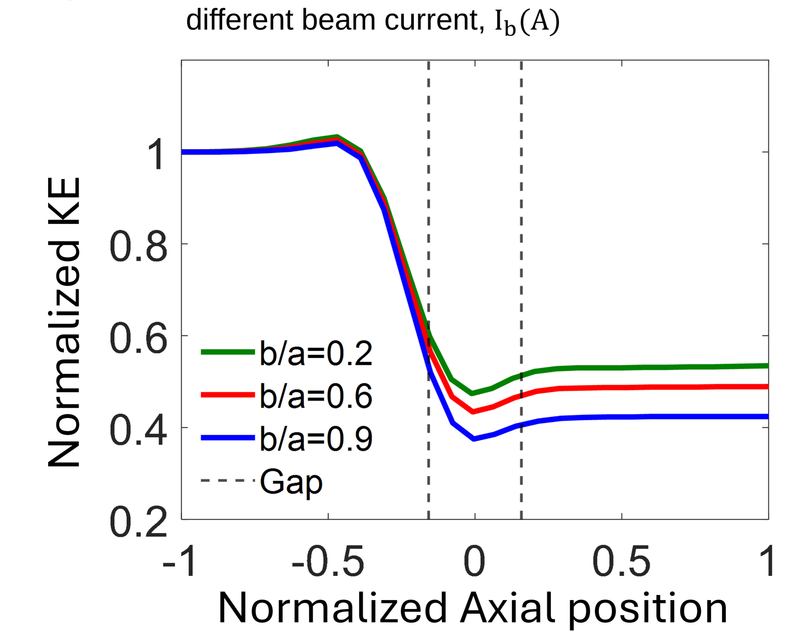

Fig 7: Normalized Kinetic Energy with axial position for different beam filling factor

Hollow (annular) Beam Effects

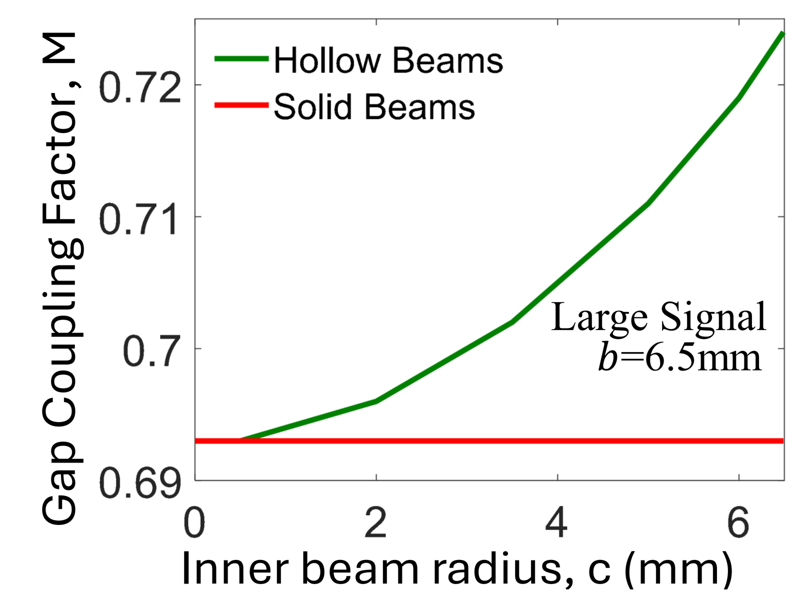

Fig 8: Gap coupling factor for different inner beam radius

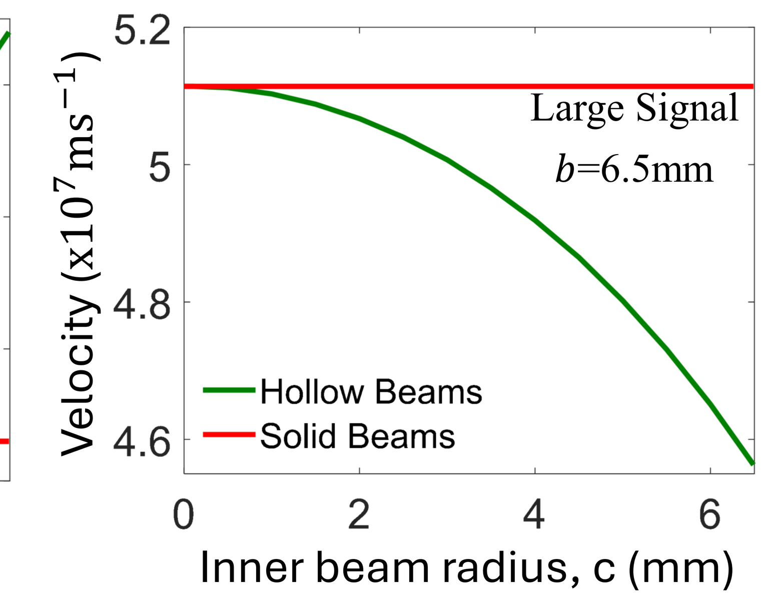

Fig 9: Electron velocity for different inner beam radius

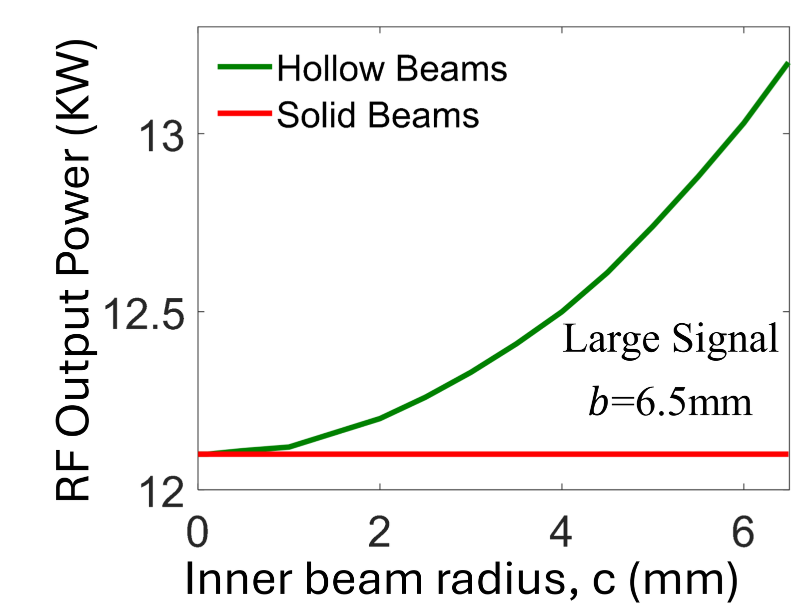

Fig 10: RF output power for different inner beam radius

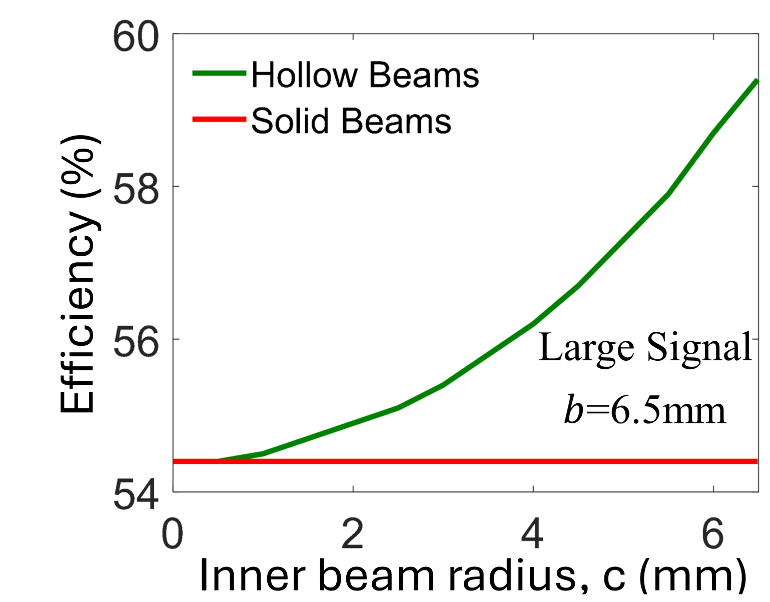

Fig 11: Efficiency for different inner beam radius

- Increasing inner beam radius increases gap coupling factor and decreases the final velocity of electrons.
- Decreasing electron velocity lowers the kinetic energy that increases the RF output power & efficiency.
- When the inner radius is close to zero, it act as a solid beam, that is shown as red line in the figures.

Conclusion & Future Work

➢ Conclusion

- For solid beam, higher conversion efficiency can be achieved by increasing beam filling factor, b/a.
- For hollow beam, higher conversion efficiency can be achieved by increasing c/b.
- Space charge increases the kinetic energy which leads to lower conversion efficiency.

➢ Future Work

- We will identify the appropriate inner beam radius from the disk model.
- We will compare our theoretical model of linear beam device with CST simulation.

References

[1] Carter, R.G., 2010. Simple model of an inductive output tube. IEEE transactions on electron devices, 57(3), pp.720-725
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