Molecular Dynamics Simulations of Ion-Electron Temperature Relaxation Rates for Strongly Magnetized Antimatter Plasmas

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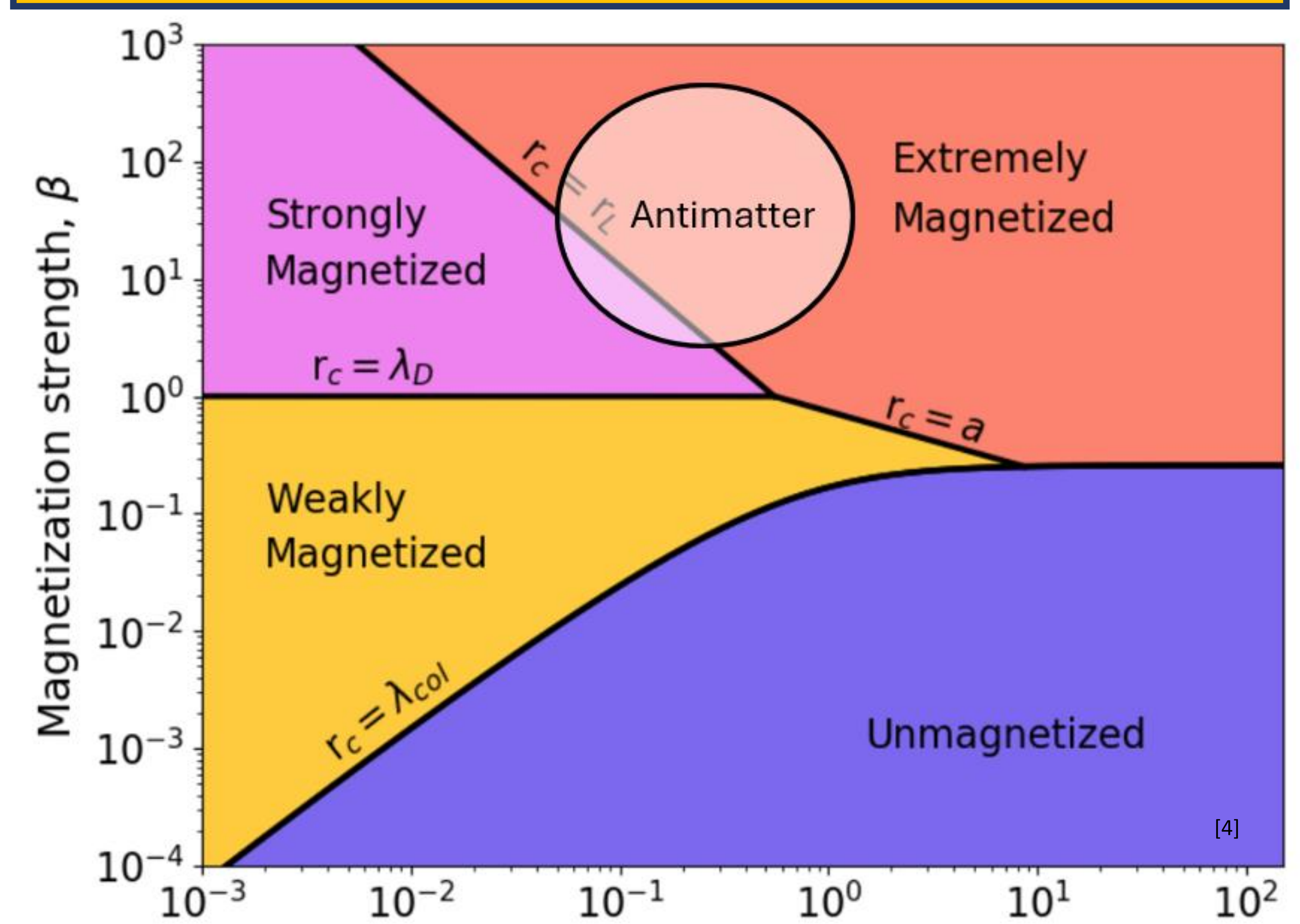
Antimatter Trap Experiments Provide the Opportunity to Test Novel Plasma Theory Predictions

- Novel plasma physics predications could be tested[1],[2]
- Antimatter experiments at ALPHA at CERN often consist of ion-electron plasmas in the strongly magnetized regime[3]

Research Results: Relaxation of $T_{e\perp}$ is Heavily Suppressed in the Strongly Magnetized Regime

- In unmagnetized theory, 1 rate $\left(\nu^{ie}\right)$ describes the relaxation
- In strongly magnetized regime, 2 rates $\left(\nu_{00}^{ie}, \nu^{ee}\right)$ describe the relaxation
- First: $T_{e\parallel}$ relaxes with T_i
- Second: $T_{e\perp}$ relaxes with T_i & $T_{e\parallel}$
- Molecular Dynamics (MD) Simulations show excellent agreement with theory

Experimental Conditions Exist in Novel Plasma Transport Regimes



$$\Gamma_e = \frac{e^2/a}{k_B T} \qquad \beta_e = \frac{\lambda_{De}}{r_{ce}} = \frac{\omega_{ce}}{\omega_{pe}}$$

Unmagnetized Theory Requires only 1 Rate to Describe the Temperature Evolution

$$\frac{dT_i}{dt} = -\nu^{ie}(T_i - T_e)$$
$$\frac{dT_e}{dt} = -\nu^{ie}(T_e - T_i)$$

Accounting for Anisotropy Formation, 4 Distinct Temperatures can Exist in an Ion-Electron Plasma of Arbitrary Magnetization Strength (11 rates)

$$\frac{dT_{i\parallel}}{dt} = -\nu_{\parallel 00}^{ie}(T_i - T_e) + \nu_{\parallel 10}^{ie}\Delta T_i + \nu_{\parallel 01}^{ie}\left(1 + \chi_{\parallel 01}^{ie}\right)\Delta T_e - 2\nu^{ii}\Delta T_i$$
$$\frac{dT_{i\perp}}{dt} = -\nu_{\perp 00}^{ie}(T_i - T_e) + \nu_{\perp 10}^{ie}\Delta T_i + \nu_{\perp 01}^{ie}\left(1 + \chi_{\perp 01}^{ie}\right)\Delta T_e + \nu^{ii}\Delta T_i$$
$$\frac{dT_{e\parallel}}{dt} = -\nu_{\parallel 00}^{ei}(T_e - T_i) - \nu_{\parallel 01}^{ei}\Delta T_i - \nu_{\parallel 10}^{ei}\left(1 + \chi_{\parallel 10}^{ei}\right)\Delta T_e - 2\nu^{ee}\Delta T_e$$
$$\frac{dT_{e\perp}}{dt} = -\nu_{\perp 00}^{ei}(T_e - T_i) - \nu_{\perp 01}^{ei}\Delta T_i - \nu_{\perp 10}^{ei}\left(1 + \chi_{\perp 10}^{ei}\right)\Delta T_e + \nu^{ee}\Delta T_e$$

Reduced Models for 3 Timescales at $\Gamma = 1, \beta = 34$

- Breaking up the evolution into timescales allows for comparison of rate values between theory & MD
- Only valid from: $25 \lesssim \beta \lesssim 60$
- Ion anisotropy relaxation (short timescale)
- Parallel electron relaxation (intermediate timescale)
- Perpendicular electron relaxation (slowest timescale)

Short Timescale Reduced Model $\left(\nu^{ii}\right)$

- Ions are still weakly magnetized ($\beta_i \approx 1$)

$$\frac{d\Delta T_i}{dt} = -3\nu^{ii}\Delta T_i$$

- Simple analytic solution

$$\Delta T_i(t) = \Delta T_i(0)e^{-3\nu^{ii}t}$$

Intermediate Timescale Reduced Model $\left(\nu_{00}^{ie}\right)$

- In the intermediate timescale, $\Delta T_i \approx 0$
- $T_{e\perp}$ is roughly constant ($T_{e\perp} = T_{e\perp 0}$)

$$\frac{dT_i}{dt} = -\nu_{00}^{ie}(T_i - T_{e\parallel})$$
$$\frac{dT_{e\parallel}}{dt} = -3\nu_{00}^{ie}(T_{e\parallel} - T_i)$$

- Simple analytic solutions

$$T_{i(t)} = \tilde{T} - \frac{1}{4}(T_{e0} - T_{i0})e^{-4\nu_{00}^{ie}t}$$
$$T_{e\parallel}(t) = \tilde{T} + \frac{3}{4}(T_{e0} - T_{i0})e^{-4\nu_{00}^{ie}t}$$

- Where $\tilde{T} = \frac{1}{4}T_{e0} + \frac{3}{4}T_{i0}$

Slowest Timescale Reduced Model $\left(\nu^{ee}\right)$

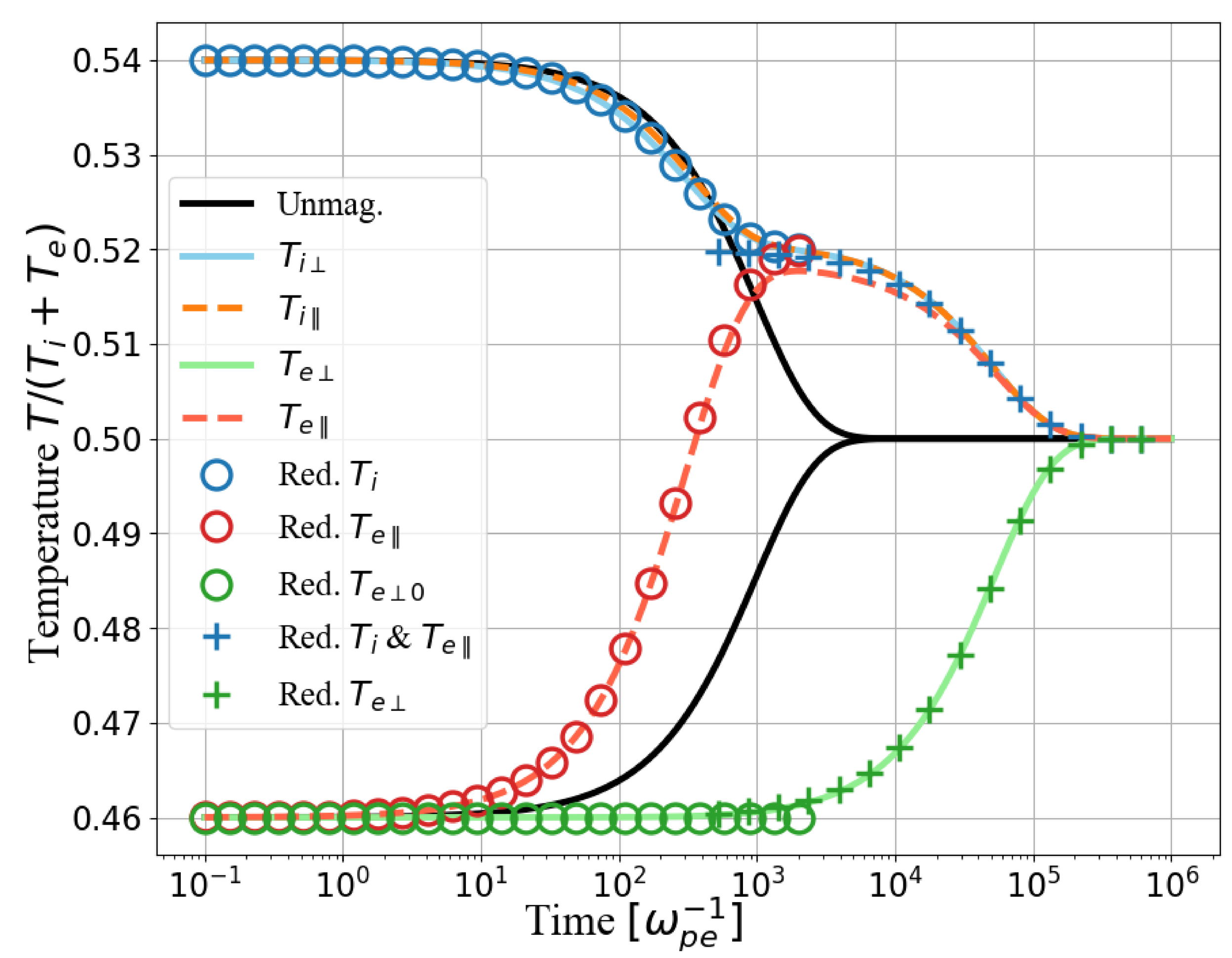
- On the long timescale, $T_i \approx T_{e\parallel}$ and $\Delta T_i \approx 0$
- $\nu_{\perp 00}^{ei}$ and $\nu_{\perp 10}^{ei}$ terms are small compared to ν^{ee} ($\beta \lesssim 60$)

$$\frac{dT_{e\parallel}}{dt} \approx -\frac{1}{2}\nu^{ee}\Delta T_e$$
$$\frac{dT_{e\perp}}{dt} \approx -\nu^{ee}\Delta T_e$$

- Simple analytic solutions

$$T_{e\parallel} = \frac{1}{2}(T_{i0} + T_{e0}) + \frac{1}{4}(T_{i0} - T_{e0})e^{-\frac{3}{2}\nu^{ee}t}$$
$$T_{e\perp} = \frac{1}{2}(T_{i0} + T_{e0}) - \frac{1}{2}(T_{i0} - T_{e0})e^{-\frac{3}{2}\nu^{ee}t}$$

Reduced Models Agree with Full Theory

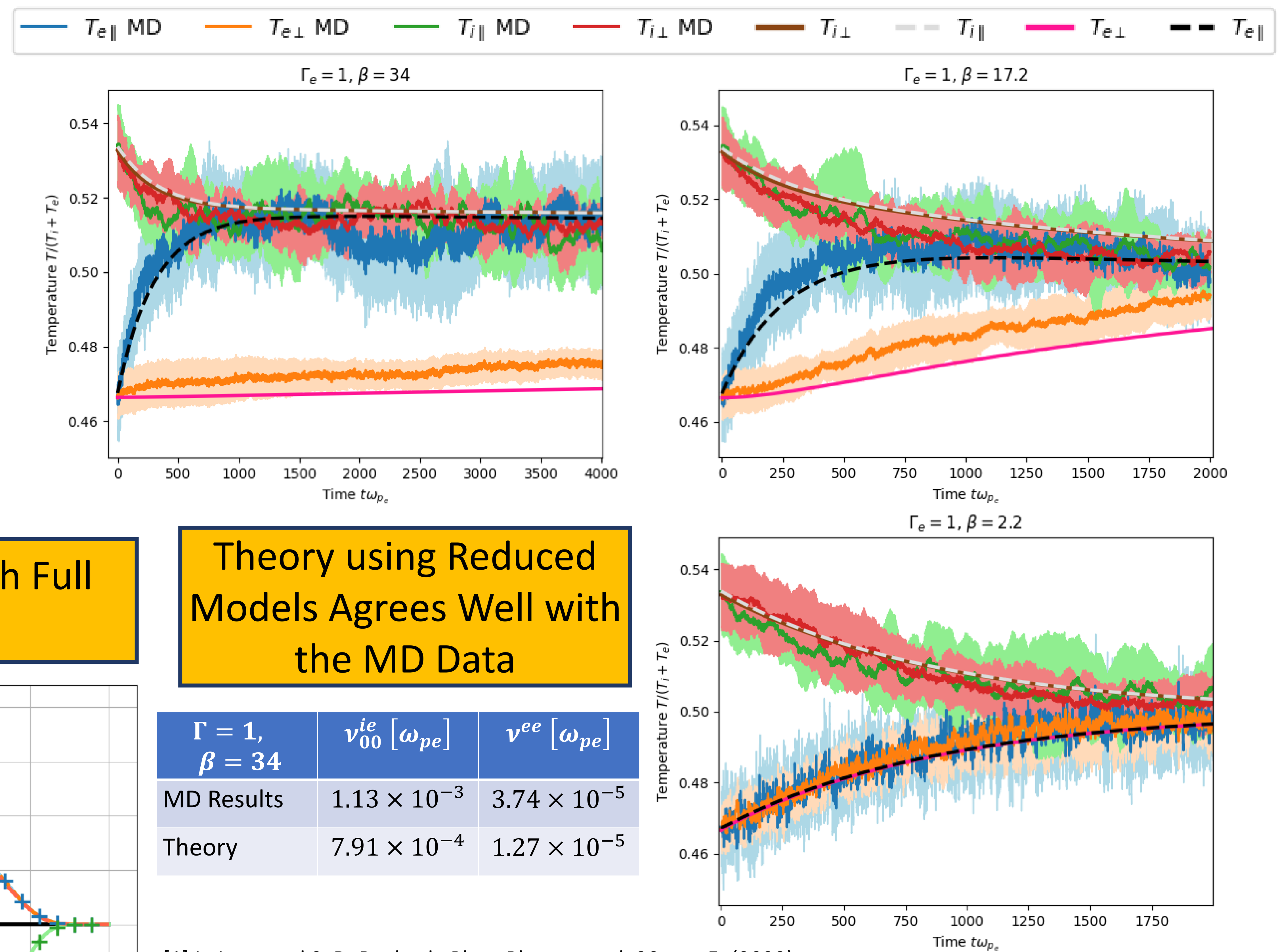


Molecular Dynamics (MD) Simulations are a First-Principles Approach to Benchmark Theory

- MD simulations performed with LAMMPS [5]
- Thermostat brings system to equilibrium configuration
- Scale velocities of each species such that a 10% temperature difference is created
- Extra thermostat is used to bring each species to its equilibrium spatial configuration at its new temperature
- Turn on magnetic field
- Start NVE simulation and record temperatures

MD Shows Good Agreement with Full Theory

- The regime was chosen because it's relevant to conditions at ALPHA ($B = 3T, n_e = 7.6 \times 10^{16} m^{-3}, T = 11.4K$) [3]
- Evaluate the full theory at, ($\Gamma_e = 1, \beta_e = 34$), ($\Gamma_e = 1, \beta_e = 17.2$), ($\Gamma_e = 1, \beta_e = 2.2$)
- 10 MD simulations were run and averaged



Theory using Reduced Models Agrees Well with the MD Data

$\Gamma = 1, \beta = 34$	$\nu_{00}^{ie}\ [\omega_{pe}]$	$\nu^{ee}\ [\omega_{pe}]$
MD Results	1.13×10^{-3}	3.74×10^{-5}
Theory	7.91×10^{-4}	1.27×10^{-5}

[1] L. Jose, and S. D. Baalrud. Phys. Plasmas, vol. 30, no. 5, (2023).
[2] L. Jose and S. D. Baalrud, Phys. Plasmas 27, 112101 (2020)
[3] G. B. Andresen, et al. Phys. Rev. Lett., vol. 100, p. 203401, (2008).
[4] S. D. Baalrud and J. Daligault, Phys. Rev. E 96, 043202 (2017).
[5] A. P. Thompson et al, Comput. Phys. Commun. 271 108171 (2022)

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