

# Anisotropic Charge Transport and Current Crowding in Vertical Thin-Film Contacts with 2D Layered Materials

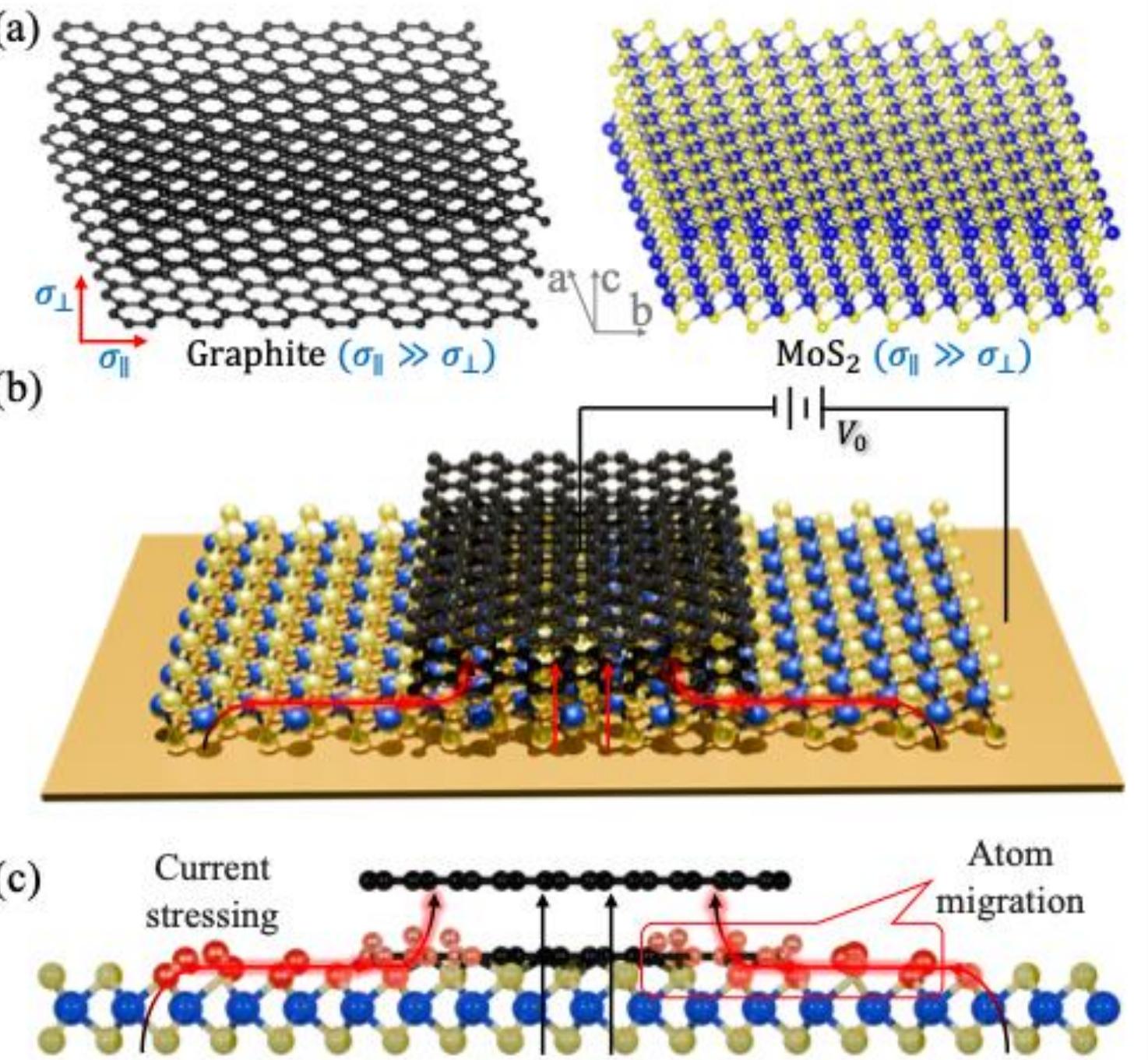
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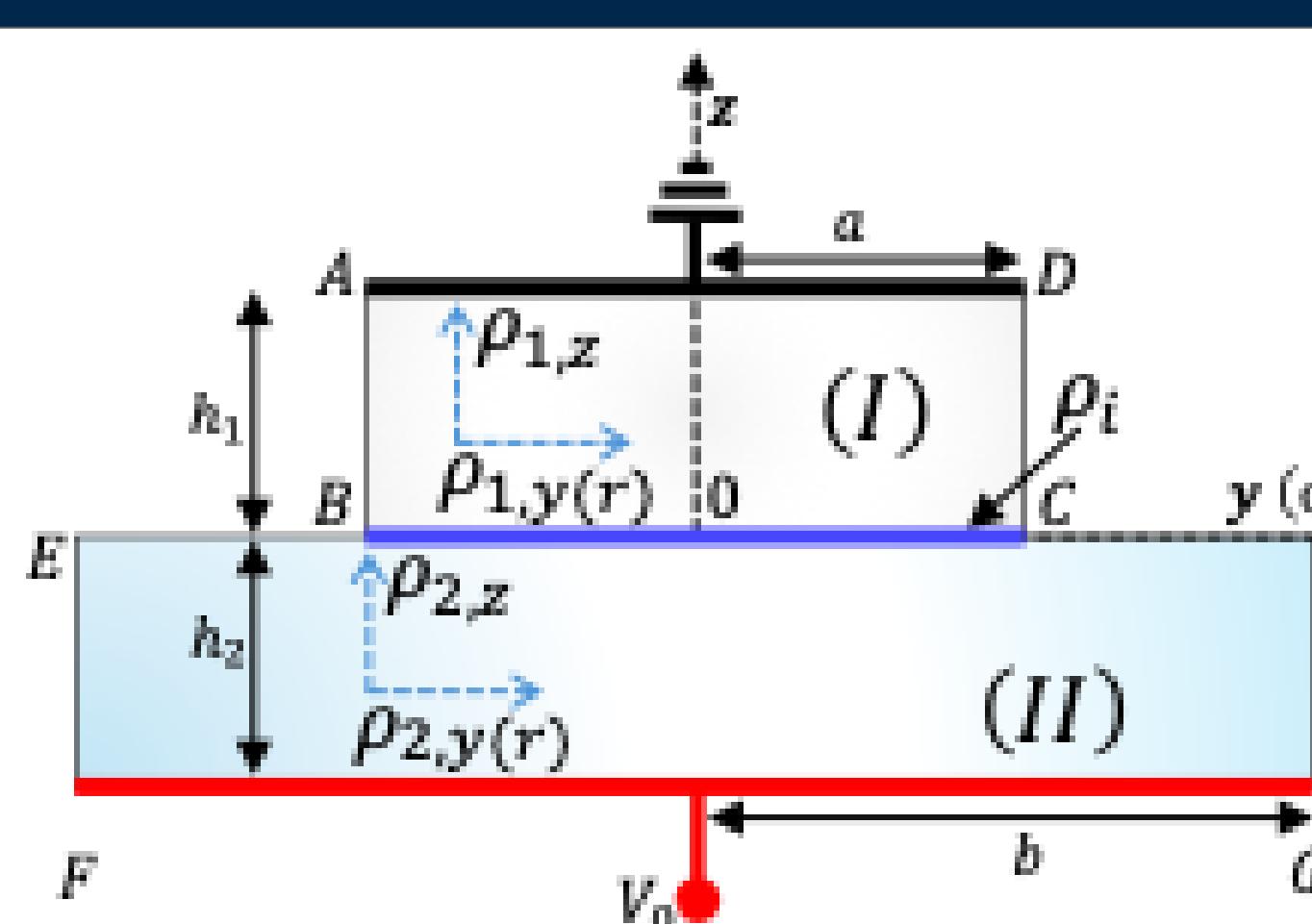
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## Introduction & Motivation

- Layered 2D materials (graphene, MoS<sub>2</sub>, WSe<sub>2</sub>, etc.) exhibit dominant in-plane transport due to high anisotropy.
- High in-plane conductivity intensifies current crowding at contact edges.
- Consequences: higher spreading resistance[1], local heating, electromigration, interfacial degradation



## Models



Here,  $t = y$  for the Cartesian geometry and  $t = r$  for the cylindrical geometry and  $z$  = axis of rotation for cylindrical. Below  $R_1$  top,  $R_2$  bottom and  $R_i$  is the interface resistance. [3]

$$\begin{aligned} R_1 &= \rho_{1,z} h_1 / (2aW) \text{ (Cartesian)} \\ R_1 &= \rho_{1,z} h_1 / (\pi a^2) \text{ (Cylindrical)} \\ R_2 &= \rho_{2,z} h_2 / (2bW) \text{ (Cartesian)} \\ R_2 &= \rho_{2,z} h_2 / (\pi b^2) \text{ (Cylindrical)} \end{aligned}$$

$$\begin{aligned} R_i &= \rho_i / 2aW \text{ (Cartesian)} \\ R_i &= \rho_i / \pi a^2 \text{ (Cylindrical)} \\ W &\text{ width perpendicular to the paper.} \end{aligned}$$

## Solution of Laplace Equation ( $\nabla \cdot (\sigma \nabla \Phi) = 0$ )

Region I ( $0 < z < h_1$ ) [3]:  $\Phi_I(y, z) = A_0(z - h_1) + \sum_{n=1}^{\infty} A_n \frac{\sinh\left(\frac{n\pi}{a\sqrt{\alpha_1}}(z - h_1)\right)}{\cosh\left(\frac{n\pi}{a\sqrt{\alpha_1}}h_1\right)} \cos\left(\frac{n\pi}{a}y\right)$  [1a, Cartesian]

$$\Phi_I(r, z) = A_0(z - h_1) + \sum_{n=1}^{\infty} A_n \sinh\left(\frac{\beta_n}{\sqrt{\alpha_1}}(z - h_1)\right) J_0(\beta_n r) \quad [2a, \text{Cylindrical}]$$

Region II ( $-h_2 < z < 0$ ) [3]:  $\Phi_{II}(y, z) = V_0 + B_0(z + h_2) + \sum_{n=1}^{\infty} \frac{B_n \sinh\left(\frac{n\pi}{b\sqrt{\alpha_2}}(z + h_2)\right)}{\cosh\left(\frac{n\pi}{b\sqrt{\alpha_2}}h_2\right)} \cos\left(\frac{n\pi}{b}y\right)$  [1b, Cartesian]

$$\Phi_{II}(r, z) = V_0 + B_0(z + h_2) + \sum_{n=1}^{\infty} B_n \frac{\sinh\left(\frac{\lambda_n}{\sqrt{\alpha_2}}(z + h_2)\right)}{\cosh\left(\frac{\lambda_n}{\sqrt{\alpha_2}}h_2\right)} J_0(\lambda_n r) \quad [2b, \text{Cylindrical}]$$

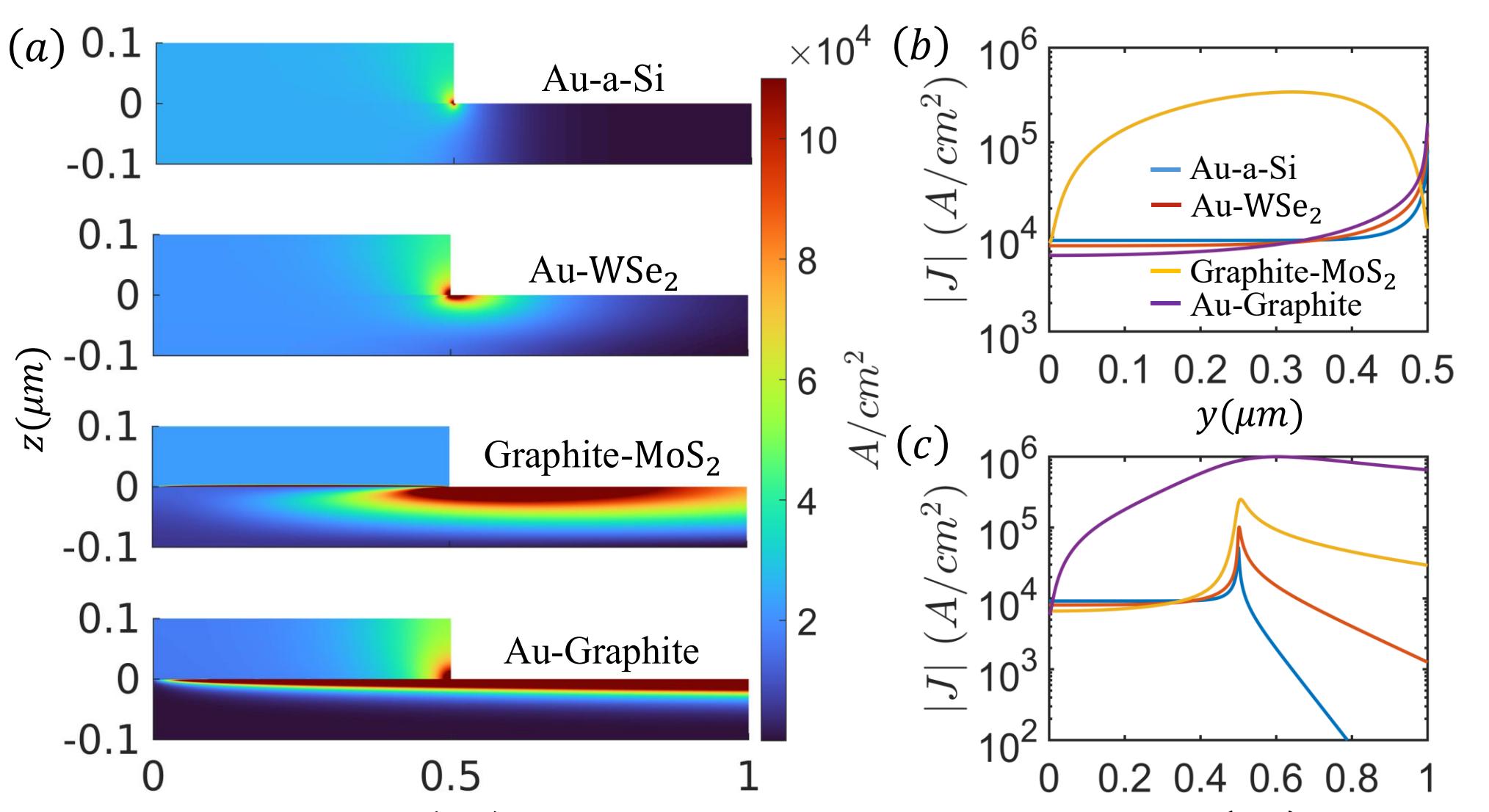
**Boundary conditions:**  $\rho_i J_z = \Phi_{II} - \Phi_I$  and  $J_z = -\frac{1}{\rho_{1,z}} \frac{\partial \Phi_I}{\partial z} = -\frac{1}{\rho_{2,z}} \frac{\partial \Phi_{II}}{\partial z}$ ,  $z = 0, |t| \in (0, a), \frac{\partial \Phi_{II}}{\partial z} = 0, z = 0, |t| \in (a, b)$

## Spreading (Constriction) Resistance, $R_s$

$$\begin{aligned} R_s &= \frac{\rho_{2,z}}{4\pi W} \bar{R}_s, \text{ and } \bar{R}_s = \bar{R}_s = 2\pi \left\{ \frac{\rho_{1,z}}{\rho_{2,z}} \sum_{n=1}^{\infty} B_n \left[ \tanh\left(\frac{n\pi}{\sqrt{\alpha_2}} b\right) + \frac{\rho_i}{\rho_{2,z} b} \frac{n\pi}{\sqrt{\alpha_2}} \right] \right. \\ &\quad \left. + \frac{\rho_i}{\rho_{2,z} b} \left( 1 - \frac{b}{a} \right) \right\} \text{ (1c, Cartesian) [3]} \\ R_s &= \frac{\rho_{2,z}}{4a} \bar{R}_s, \text{ and } \bar{R}_s = \frac{4}{\pi} \left\{ 2 \frac{\rho_{1,z}}{\rho_{2,z}} \sum_{n=1}^{\infty} B_n \left[ \tanh\left(\frac{\lambda_n b}{\sqrt{\alpha_2}} b\right) + \frac{\lambda_n b}{\sqrt{\alpha_2}} \frac{\rho_i}{\rho_{2,z} b} \right] \frac{J_1(\lambda_n a)}{\lambda_n a} + \frac{\rho_i}{\rho_{2,z} b} \left( \frac{a}{b} - \frac{b}{a} \right) \right\} \text{ (2c, Cylindrical) [3]} \end{aligned}$$

Where  $\alpha_1 = \frac{\rho_{1,t}}{\rho_{1,z}}$  and  $\alpha_2 = \frac{\rho_{2,t}}{\rho_{2,z}}$ ;  $A_n$  and  $B_n$  are calculated using boundary conditions, current density is  $|J| = \sqrt{(\sigma_t \partial_t \Phi)^2 + (\sigma_z \partial_z \Phi)^2}$ .

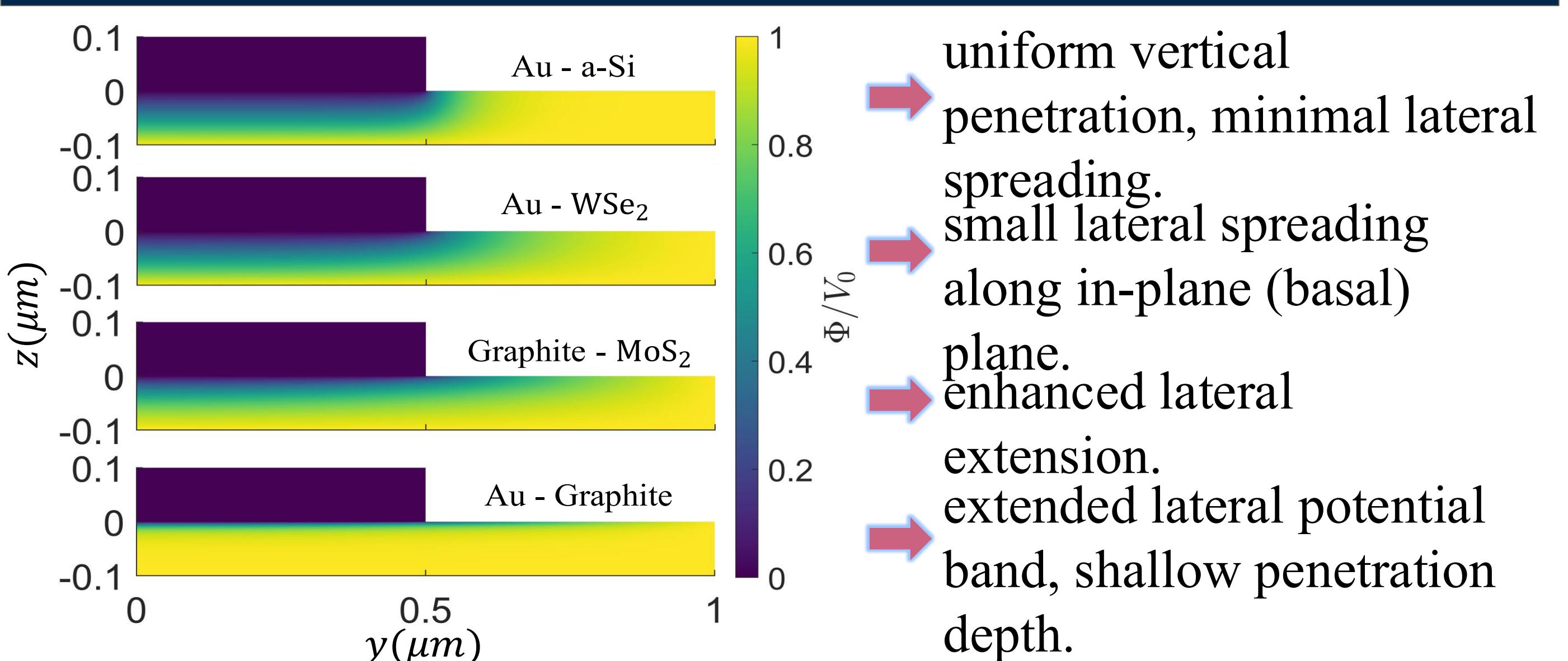
## Current Density Distributions



(a) Contour maps of current density  $|J|$ , (b) Current profiles 5 nm above the interface and (c) Current profiles 5 nm below the interface [3].

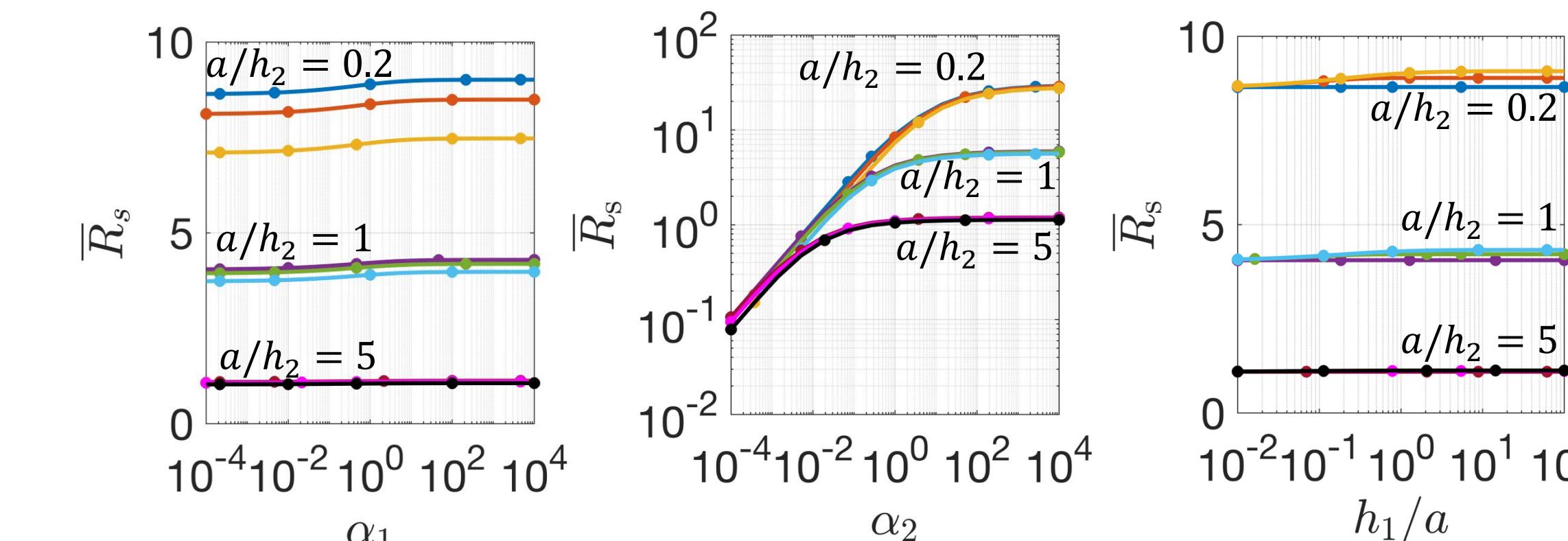
- isotropic** (Au - a-Si;  $\alpha_1 = \alpha_2 = 1$ ): nearly uniform spreading.
- moderately anisotropic bottom layer** (Au - WSe<sub>2</sub>;  $\alpha_1 = 1; \alpha_2 = 0.1333$ ): small lateral spreading before injection
- both anisotropic layers** (graphite - MoS<sub>2</sub>;  $\alpha_1 = 2.7 \times 10^{-4}, \alpha_2 = 0.009$ ): amplified lateral flow in both layers
- strongly anisotropic bottom** (Au - graphite;  $\alpha_1 = 1, \alpha_2 = 2.7 \times 10^{-4}$ ): extreme in-plane conductivity  $\rightarrow$  delayed vertical injection  $\rightarrow$  extended high- $|J|$  band

## Potential Profiles



uniform vertical penetration, minimal lateral spreading.  
small lateral spreading along in-plane (basal) plane.  
enhanced lateral extension.  
extended lateral potential band, shallow penetration depth.

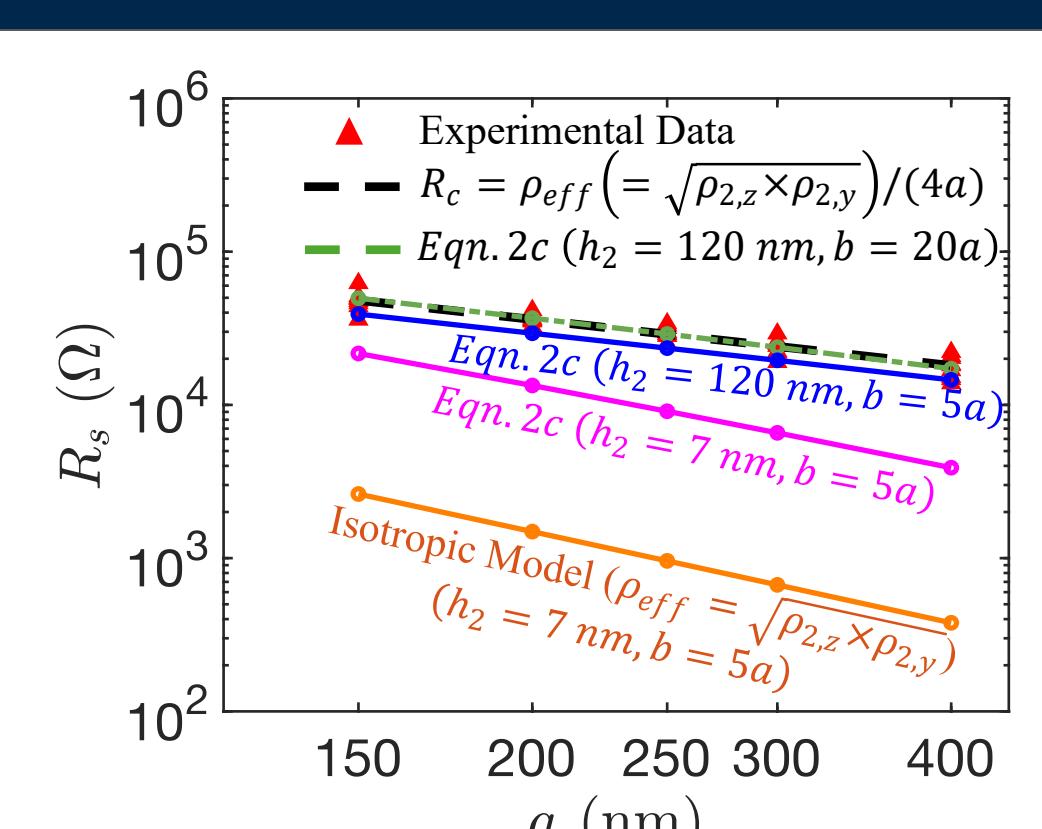
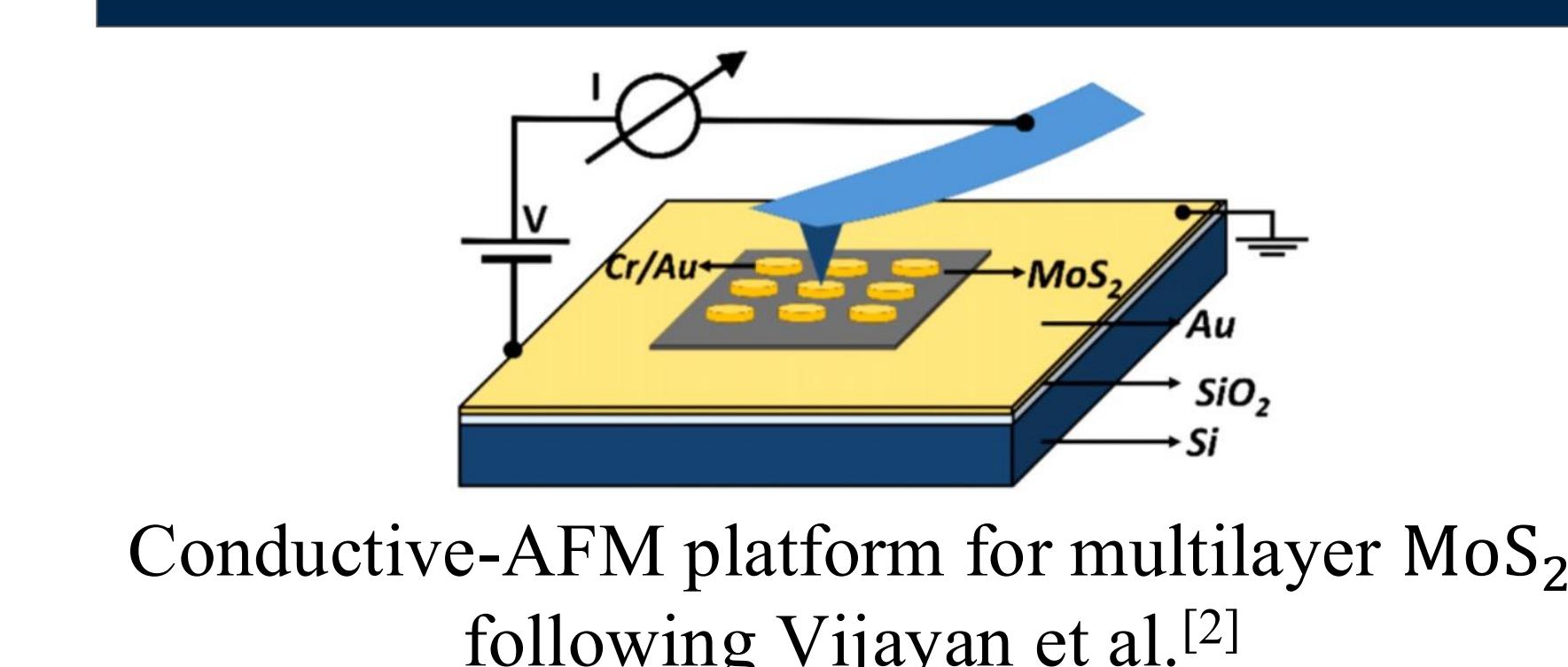
## Effect of Anisotropy on $R_s$



Each  $a/h_2$  set contains three curves for  $\alpha_1 = 1000, 1, 0.001$  (top to bottom) with  $\alpha_2 = 1, b/a = 20$ .

- $\alpha_1, \alpha_2 < 1$ : lower in-plane resistivity  $\rightarrow$  enhanced lateral transport  $\rightarrow$  reduced normalized spreading resistance ( $\bar{R}_s$ ).
- $\alpha_1, \alpha_2 > 1$ : higher in-plane resistivity  $\rightarrow$  stronger constriction  $\rightarrow$  higher ( $\bar{R}_s$ ).
- $\alpha_{1,2} > 100$ : in-plane resistivity has negligible effect.
- Thicker overlayers ( $a/h_2 = 0.2 \rightarrow 5$ ) give larger  $\bar{R}_s$
- Each  $a/h_2$  set, three curves correspond to  $b/a = 20, 15, 10$  (top to bottom). Larger  $b/a$  gives higher  $\bar{R}_s$

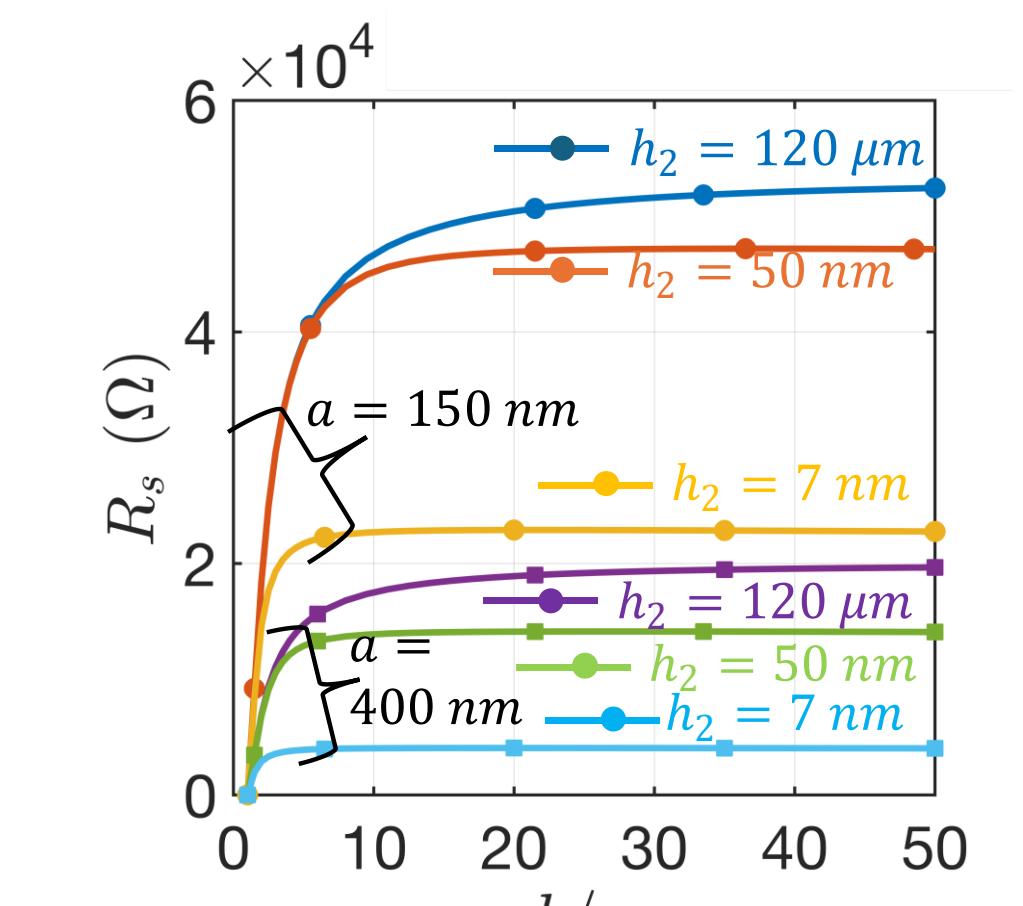
## Experimental Benchmarking of $R_s$



Vijayan et al. [2] fitted the measured  $R_s$  using the diffusive model,  $R_s = \frac{\rho_{eff}}{4a}$ , where  $\rho_{eff} = \sqrt{\rho_{2,y} \rho_{2,z}}$ .

**Deviations & limitations:** Narrow contacts ( $b/a < 20$ ) or thin films (few nm)  $\rightarrow$  large departures from  $1/a$  scaling.

**Recommendation:** Use the full anisotropic field solution, including geometry ( $a, b, h_1, h_2$ ), interface resistivity ( $\rho_i$ ), and anisotropy ( $\alpha_1, \alpha_2$ ). [3]



## Acknowledgements

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## References

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- Faisal, M. A.; Zhang, P. Impact of Anisotropic Conductivity on Current Crowding and Spreading Resistance in Vertical Contacts to 2D Thin Films. ACS Applied Electronic Materials 2026, 8, 854-864. (updated this reference after the symposium)