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Anisotropic Charge Transport and Current Crowding in Vertical Thin-Film Contacts with 2D Layered Materials

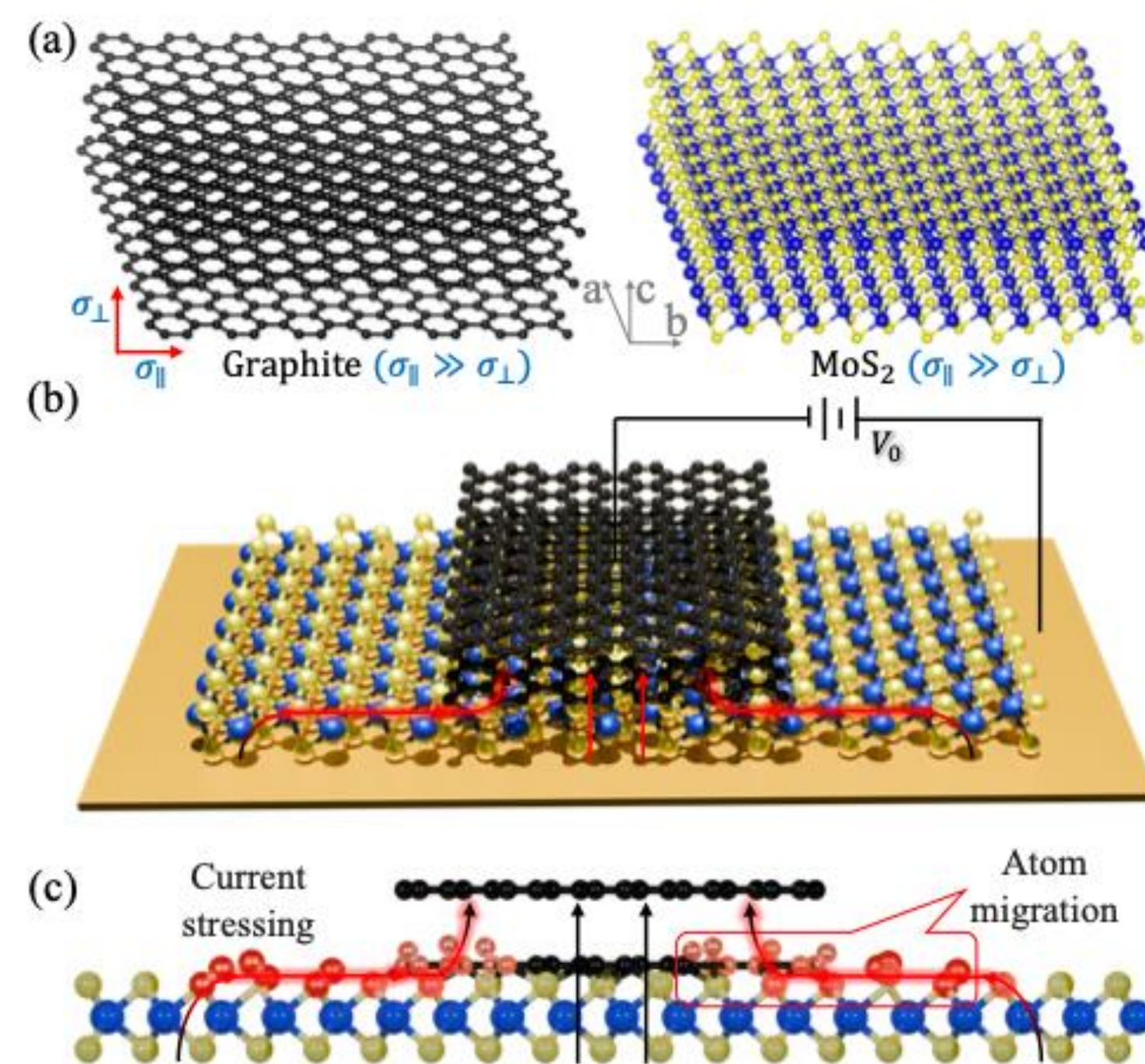
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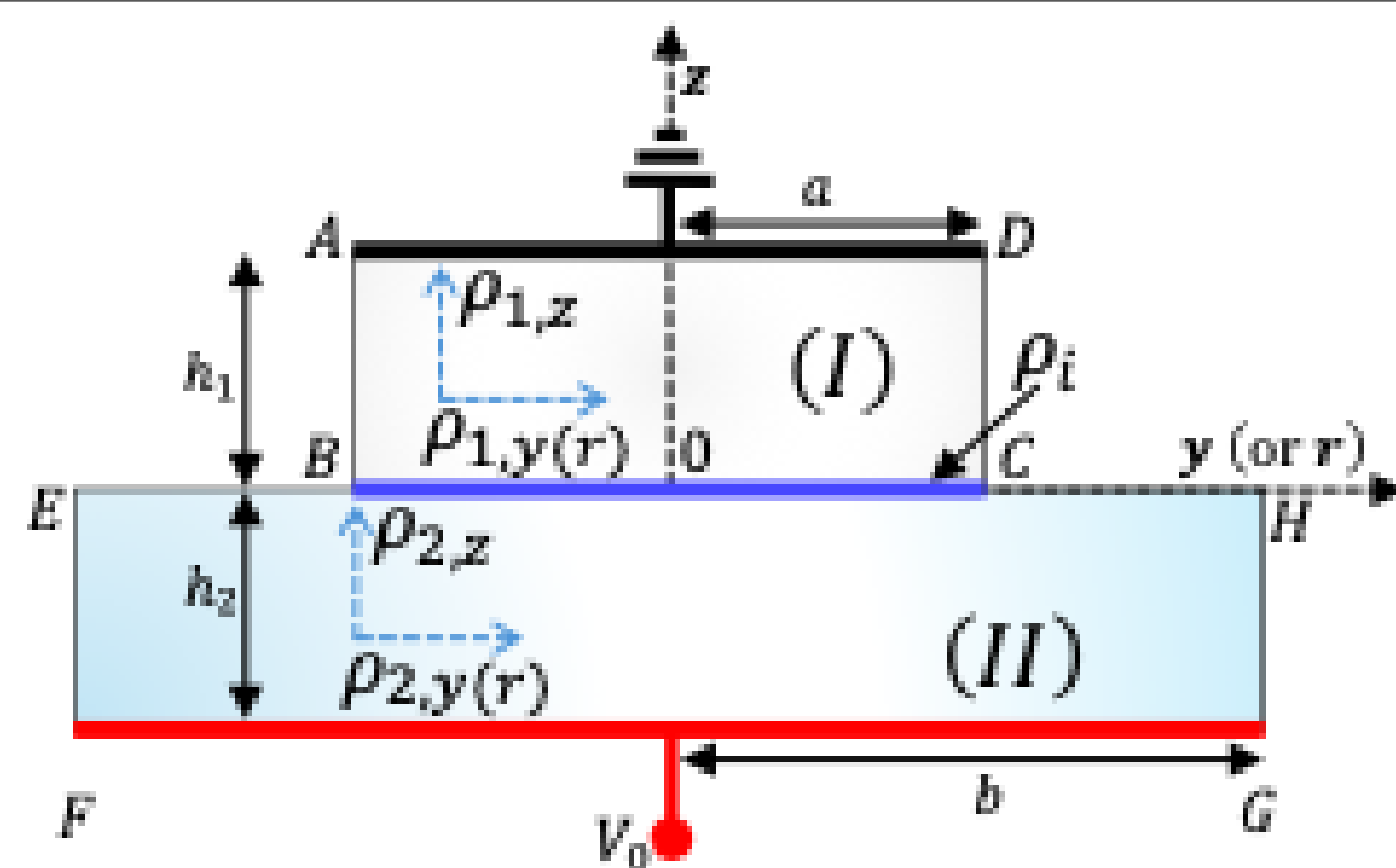
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Introduction & Motivation

- Layered 2D materials (graphene, MoS₂, WSe₂, etc.) exhibit dominant in-plane transport due to high anisotropy.
- High in-plane conductivity intensifies current crowding at contact edges.
- Consequences: higher spreading resistance[1], local heating, electromigration, interfacial degradation



Models



Here, $t = y$ for the Cartesian geometry and $t = r$ for the cylindrical geometry and $z =$ axis of rotation for cylindrical. Below R_1 top, R_2 bottom and R_i is the interface resistance. [3]

$$\begin{aligned} R_1 &= \rho_{1,z} h_1 / (2aW) \text{ (Cartesian)} \\ R_1 &= \rho_{1,z} h_1 / (\pi a^2) \text{ (Cylindrical)} \\ R_2 &= \rho_{2,z} h_2 / (2bW) \text{ (Cartesian)} \\ R_2 &= \rho_{2,z} h_2 / (\pi b^2) \text{ (Cylindrical)} \end{aligned}$$

$$\begin{aligned} R_i &= \rho_i / 2aW \text{ (Cartesian)} \\ R_i &= \rho_i / \pi a^2 \text{ (Cylindrical)} \end{aligned}$$

W width perpendicular to the paper.

Solution of Laplace Equation ($\nabla \cdot (\sigma \nabla \Phi) = 0$)

Region I ($0 < z < h_1$) [3]:

$$\Phi_I(y, z) = A_0(z - h_1) + \sum_{n=1}^{\infty} A_n \frac{\sinh\left(\frac{n\pi}{a\sqrt{\alpha_1}}(z - h_1)\right)}{\cosh\left(\frac{n\pi}{a\sqrt{\alpha_1}}h_1\right)} \cos\left(\frac{n\pi}{a}y\right) \quad [1a, \text{Cartesian}]$$

$$\Phi_I(r, z) = A_0(z - h_1) + \sum_{n=1}^{\infty} A_n \sinh\left(\frac{\beta_n}{\sqrt{\alpha_1}}(z - h_1)\right) J_0(\beta_n r) \quad [2a, \text{Cylindrical}]$$

Region II ($-h_2 < z < 0$) [3]:

$$\Phi_{II}(y, z) = V_0 + B_0(z + h_2) + \sum_{n=1}^{\infty} B_n \frac{\sinh\left(\frac{n\pi}{b\sqrt{\alpha_2}}(z + h_2)\right)}{\cosh\left(\frac{n\pi}{b\sqrt{\alpha_2}}h_2\right)} \cos\left(\frac{n\pi}{b}y\right) \quad [1b, \text{Cartesian}]$$

$$\Phi_{II}(r, z) = V_0 + B_0(z + h_2) + \sum_{n=1}^{\infty} B_n \frac{\sinh\left(\frac{\lambda_n}{\sqrt{\alpha_2}}(z + h_2)\right)}{\cosh\left(\frac{\lambda_n}{\sqrt{\alpha_2}}h_2\right)} J_0(\lambda_n r) \quad [2b, \text{Cylindrical}]$$

Boundary conditions: $\rho_i J_z = \Phi_{II} - \Phi_I$ and $J_z = -\frac{1}{\rho_{1,z}} \frac{\partial \Phi_I}{\partial z} = -\frac{1}{\rho_{2,z}} \frac{\partial \Phi_{II}}{\partial z}$, $z = 0$, $|t| \in (0, a)$, $\frac{\partial \Phi_{II}}{\partial z} = 0$, $z = 0$, $|t| \in (a, b)$

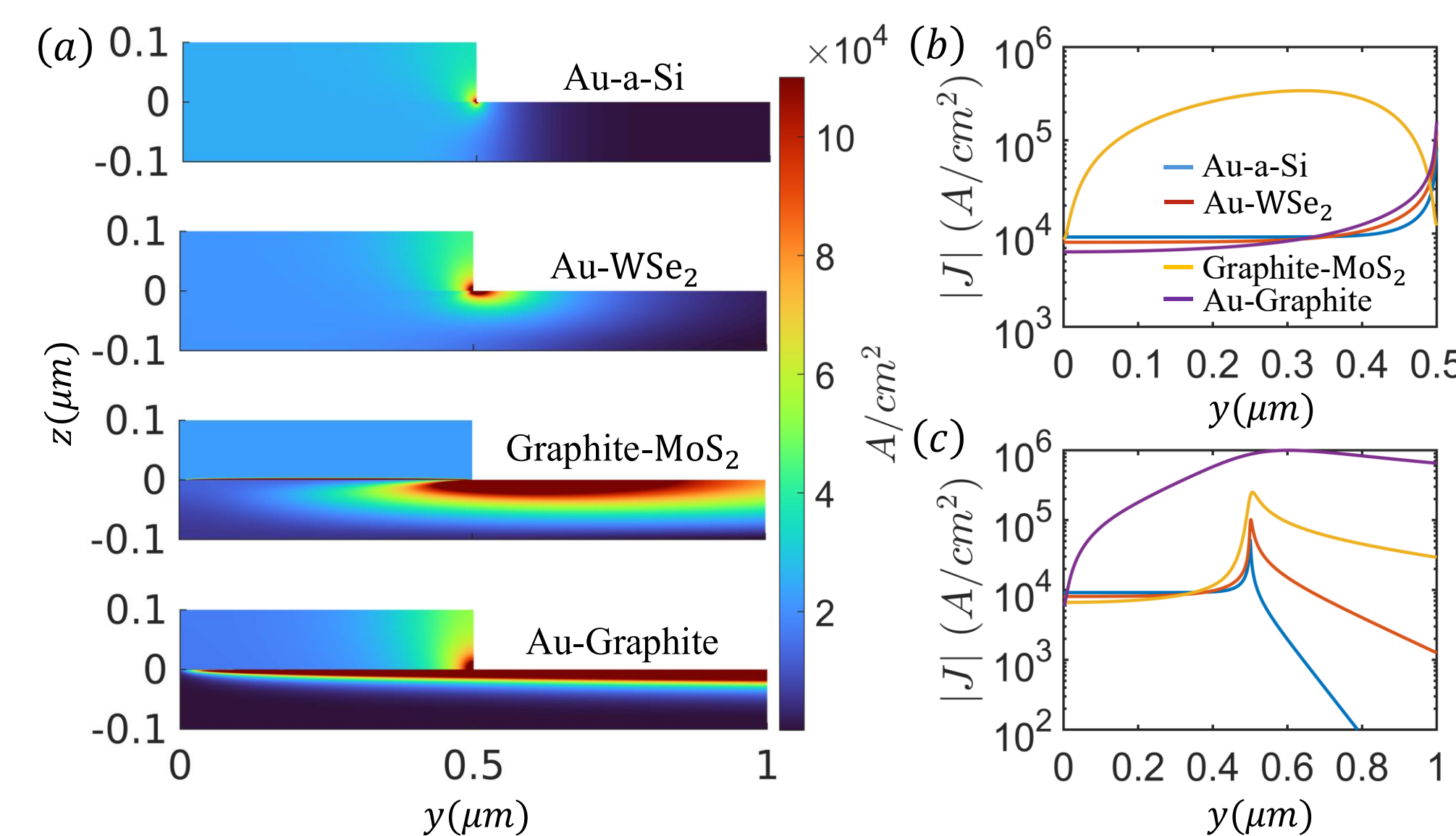
Spreading (Constriction) Resistance, R_S

$$R_S = \frac{\rho_{2,z}}{4\pi W} \bar{R}_S, \text{ and } \bar{R}_S = \bar{R}_S = 2\pi \left\{ \frac{\rho_{1,z}}{\rho_{2,z}} \sum_{n=1}^{\infty} B_n \left[\tanh\left(\frac{n\pi}{\sqrt{\alpha_2}} \frac{h_2}{b}\right) + \frac{\rho_i}{\rho_{2,z} b \sqrt{\alpha_2}} \frac{\sin(n\pi \frac{a}{b})}{n\pi \frac{a}{b}} + \frac{\rho_i}{\rho_{2,z} b} \left(1 - \frac{b}{a}\right) \right] \right\} \quad (1c, \text{Cartesian}) \quad [3]$$

$$R_S = \frac{\rho_{2,z}}{4a} \bar{R}_S, \text{ and } \bar{R}_S = \frac{4}{\pi} \left\{ 2 \frac{\rho_{1,z}}{\rho_{2,z}} \sum_{n=1}^{\infty} B_n \left[\tanh\left(\frac{\lambda_n b}{\sqrt{\alpha_2}} \frac{h_2}{b}\right) + \frac{\lambda_n b}{\sqrt{\alpha_2}} \frac{\rho_i}{\rho_{2,z} b} \right] \frac{J_1(\lambda_n a)}{\lambda_n a} + \frac{\rho_i}{\rho_{2,z} b} \left(\frac{a}{b} - \frac{b}{a}\right) \right\} \quad (2c, \text{Cylindrical}) \quad [3]$$

Where $\alpha_1 = \frac{\rho_{1,t}}{\rho_{1,z}}$ and $\alpha_2 = \frac{\rho_{2,t}}{\rho_{2,z}}$; A_n and B_n are calculated using boundary conditions, current density is $|J| = \sqrt{(\sigma_t \partial_t \Phi)^2 + (\sigma_z \partial_z \Phi)^2}$.

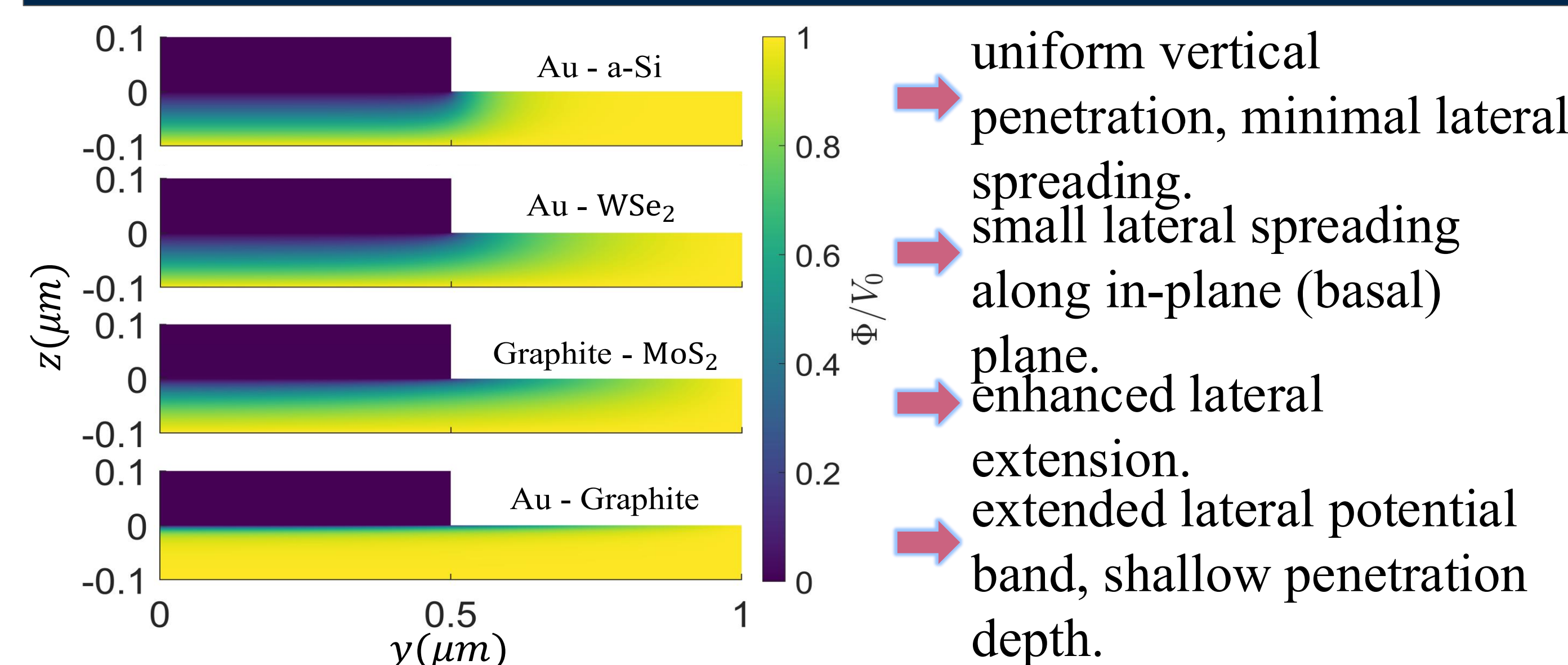
Current Density Distributions



(a) Contour maps of current density $|J|$, (b) Current profiles 5 nm above the interface and (c) Current profiles 5 nm below the interface [3].

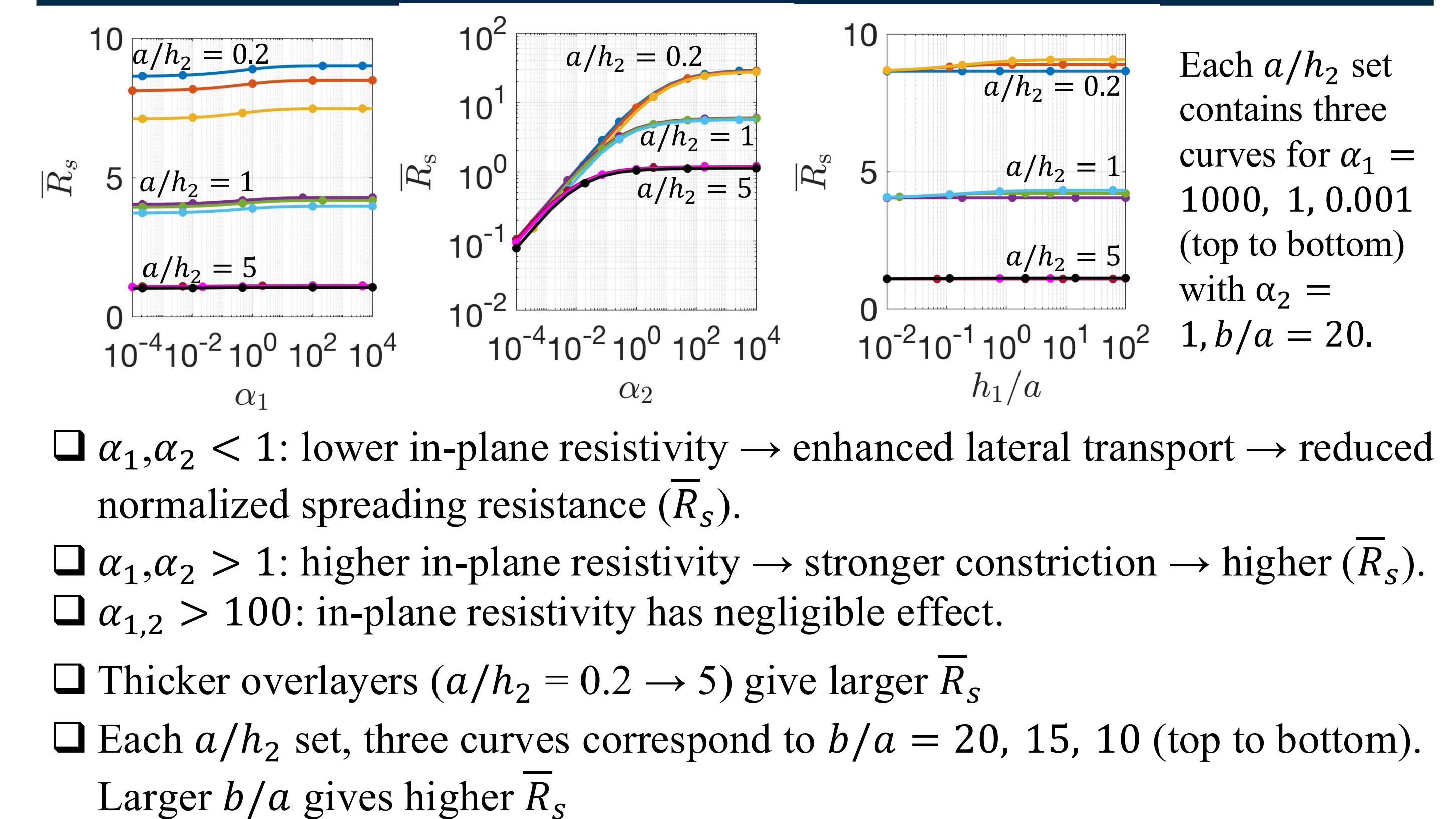
- isotropic** (Au - a-Si; $\alpha_1 = \alpha_2 = 1$): nearly uniform spreading.
- moderately anisotropic bottom layer** (Au - WSe₂; $\alpha_1 = 1$; $\alpha_2 = 0.1333$): small lateral spreading before injection
- both anisotropic layers** (graphite - MoS₂; $\alpha_1 = 2.7 \times 10^{-4}$, $\alpha_2 = 0.009$): amplified lateral flow in both layers
- strongly anisotropic bottom** (Au - graphite; $\alpha_1 = 1$, $\alpha_2 = 2.7 \times 10^{-4}$): extreme in-plane conductivity \rightarrow delayed vertical injection \rightarrow extended high- $|J|$ band

Potential Profiles



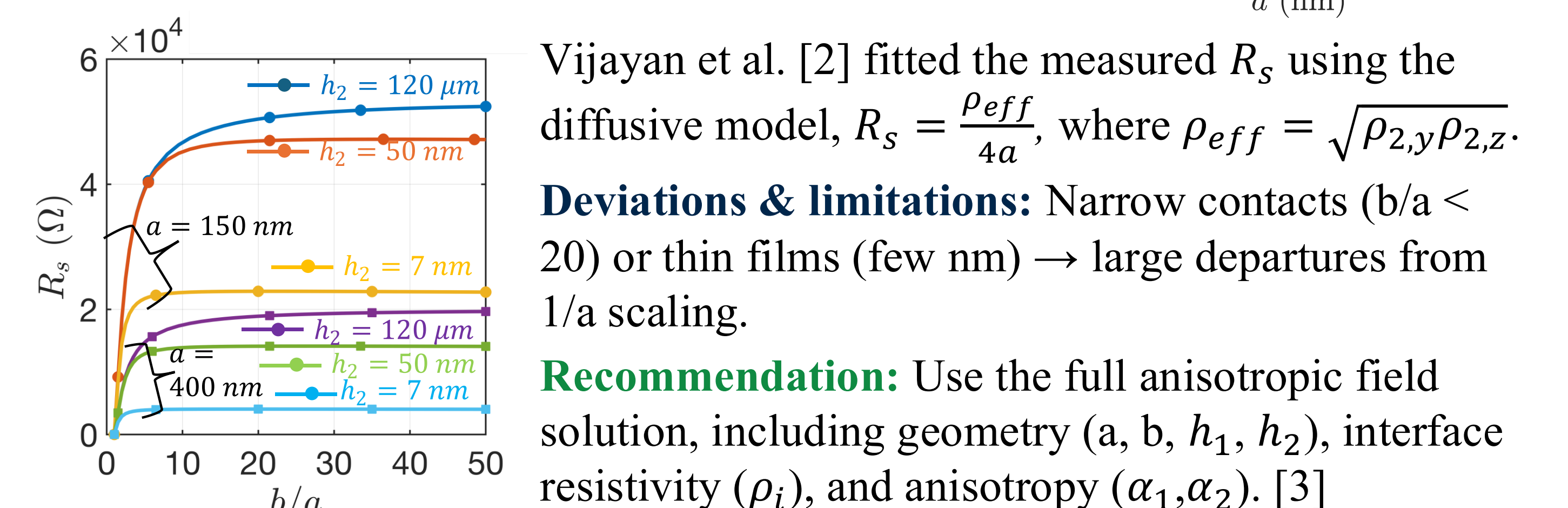
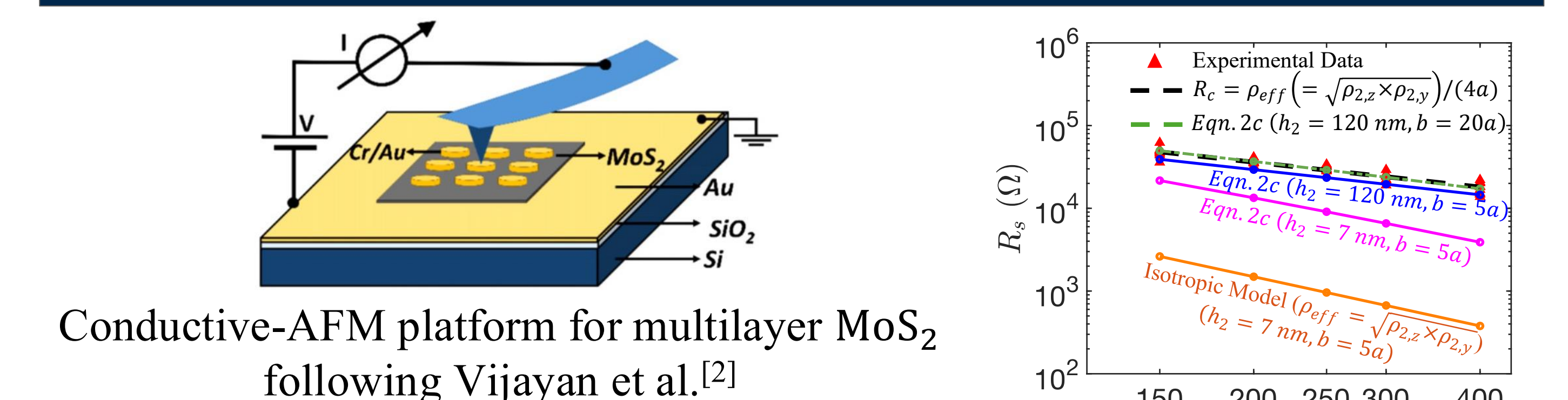
uniform vertical penetration, minimal lateral spreading.
small lateral spreading along in-plane (basal) plane.
enhanced lateral extension.
extended lateral potential band, shallow penetration depth.

Effect of Anisotropy on R_S



- $\alpha_1, \alpha_2 < 1$: lower in-plane resistivity \rightarrow enhanced lateral transport \rightarrow reduced normalized spreading resistance (\bar{R}_S).
- $\alpha_1, \alpha_2 > 1$: higher in-plane resistivity \rightarrow stronger constriction \rightarrow higher (\bar{R}_S).
- $\alpha_{1,2} > 100$: in-plane resistivity has negligible effect.
- Thicker overlayers ($a/h_2 = 0.2 \rightarrow 5$) give larger \bar{R}_S
- Each a/h_2 set, three curves correspond to $b/a = 20, 15, 10$ (top to bottom). Larger b/a gives higher \bar{R}_S

Experimental Benchmarking of R_S



Vijayan et al. [2] fitted the measured R_S using the diffusive model, $R_S = \frac{\rho_{eff}}{4a}$, where $\rho_{eff} = \sqrt{\rho_{2,z} \rho_{2,y}}$.

Deviations & limitations: Narrow contacts ($b/a < 20$) or thin films (few nm) \rightarrow large departures from $1/a$ scaling.

Recommendation: Use the full anisotropic field solution, including geometry (a, b, h_1, h_2), interface resistivity (ρ_i), and anisotropy (α_1, α_2). [3]

Acknowledgements

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References

- [1] Zhang, P.; Lau, Y. Y. IEEE J. Electron Devices Soc. 1, 83–90 (2013). DOI: 10.1109/JEDS.2013.2261435
- [2] Vijayan, G.; Uzhansky, M.; Koren, E. Appl. Phys. Lett. 124, 133101 (2024)
- [3] Faisal, M. A.; Zhang, P. Impact of Anisotropic Conductivity on Current Crowding and Spreading Resistance in Vertical Contacts to 2D Thin Films. ACS Applied Electronic Materials 2026, 8, 854-864. (updated this reference after the symposium)