Kinetic Vlasov-Poisson Simulations of Ion Acoustic Waves

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Introduction

1D1V Vlasov–Poisson equations for two species:

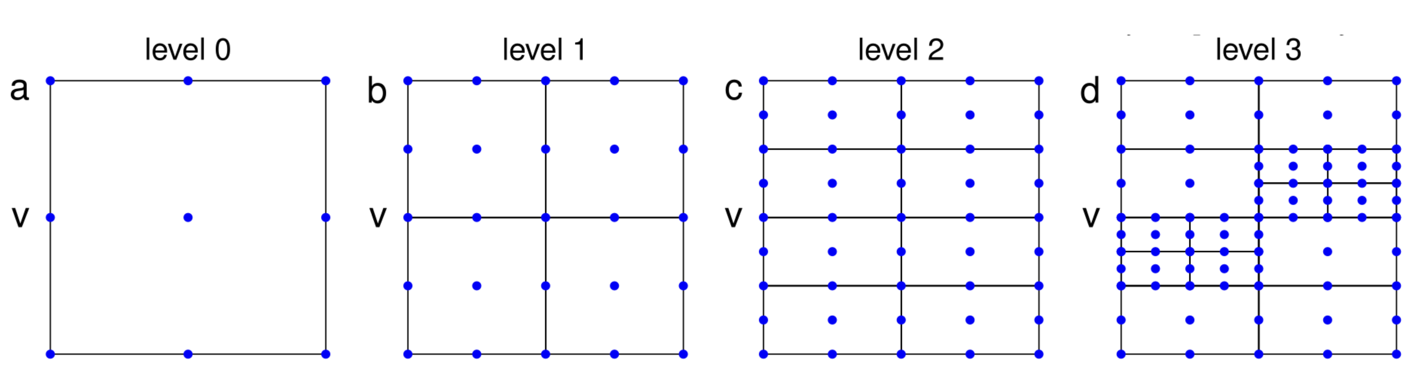
$$\partial_t f^e + v_e \partial_x f^e + \frac{q^e E}{m_e}\partial_v f^e = 0, \quad \partial_t f^i + v_i \partial_x f^i + \frac{q^i E}{m_i}\partial_v f^i = 0$$
$$E = -\partial_x \phi, \quad -\partial_x^2 \phi = \rho$$
$$\rho(x,t) = q^e \int_{-\infty}^{\infty} f^e(x,v,t)dv + q^i \int_{-\infty}^{\infty} f^i(x,v,t)dv$$

- $f^e(x,v,t)$: electron distribution function in $[0,L] \times [-v^e_{\max}, v^e_{\max}]$
- $f^i(x,v,t)$: ion distribution function in $[0,L] \times [-v^i_{\max}, v^i_{\max}]$
- $E(x,t)$: electric field
- $\phi(x,t)$: electric potential
- $\rho(x,t)$: charge density

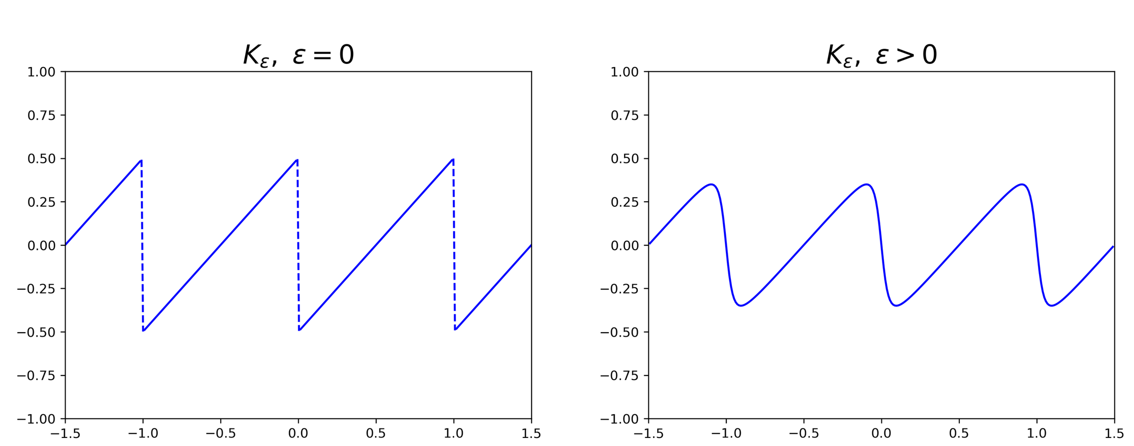
Charge neutrality (periodic BCs): $\int_0^L \rho(x,t)\,dx = 0$

Numerical Method : FARSIGHT [1]

- Phase space distribution: quadrilateral panels with 3×3 particle grid



- $x_i'(t) = v_i(t), \quad v_i'(t) = qE(x_i,t) = -q^2 \sum_{j=1}^{N} K_\epsilon(x_i,x_j)\, f_{0j}\, \Delta x_{0j}\, \Delta v_{0j}\, \omega_j$
- 4th order Runge-Kutta
- Forward semi-Lagrangian - remesh: biquadratic interpolation on panels
- AMR criterion: $\max_{i=1:9} f_i - \min_{i=1:9} f_i > \epsilon$
- Barycentric Lagrange Treecode (BLTC): $O(N^2) \to O(N \log N)$ [2]
- GPU-accelerated: OpenACC version

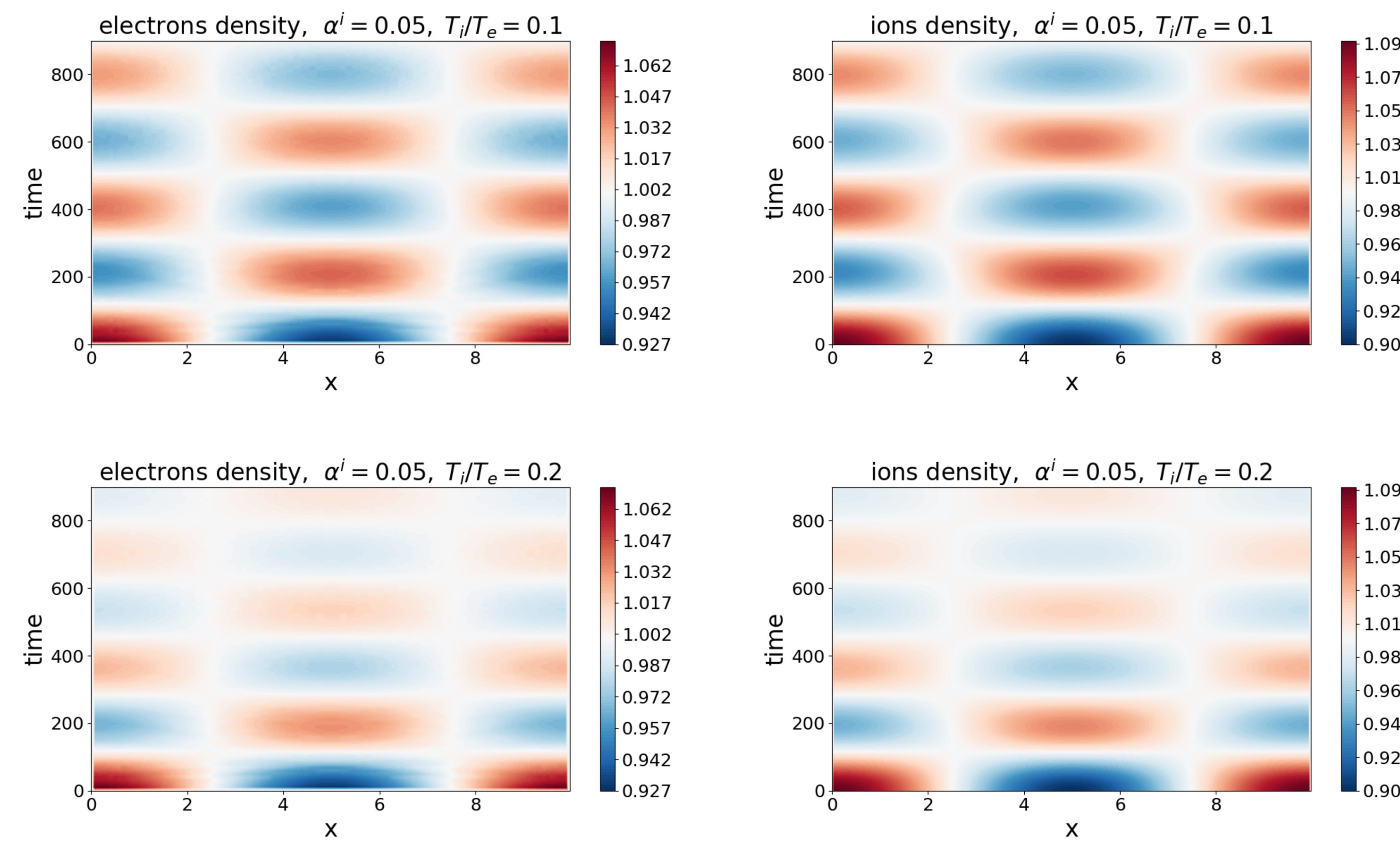


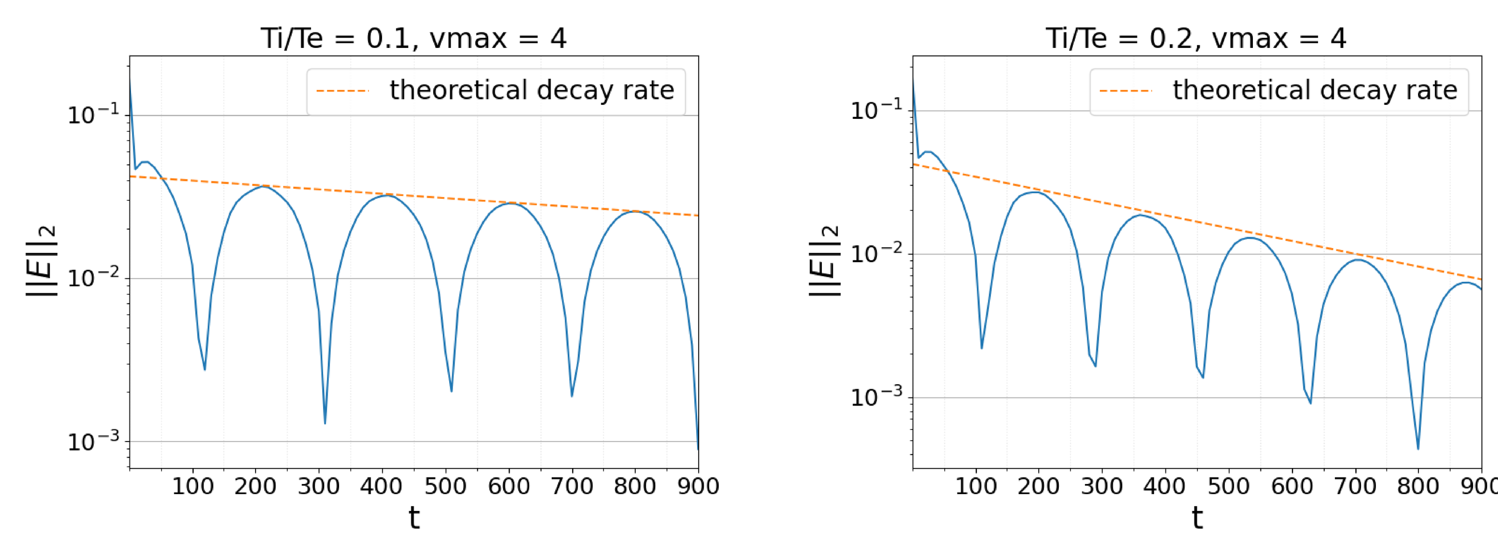
Maxwellian distributions are used for both electrons and ions in the following ion–acoustic wave simulations:

$$f^s = \frac{1}{\sqrt{2\pi} v_{th,s}} \exp\left(-\frac{(v-v_s)^2}{2v_{th,s}^2}\right)(1 + \alpha^s \cos(kx))$$

Ion Acoustic Waves

Charge density for different ion-electron temperature ratios:



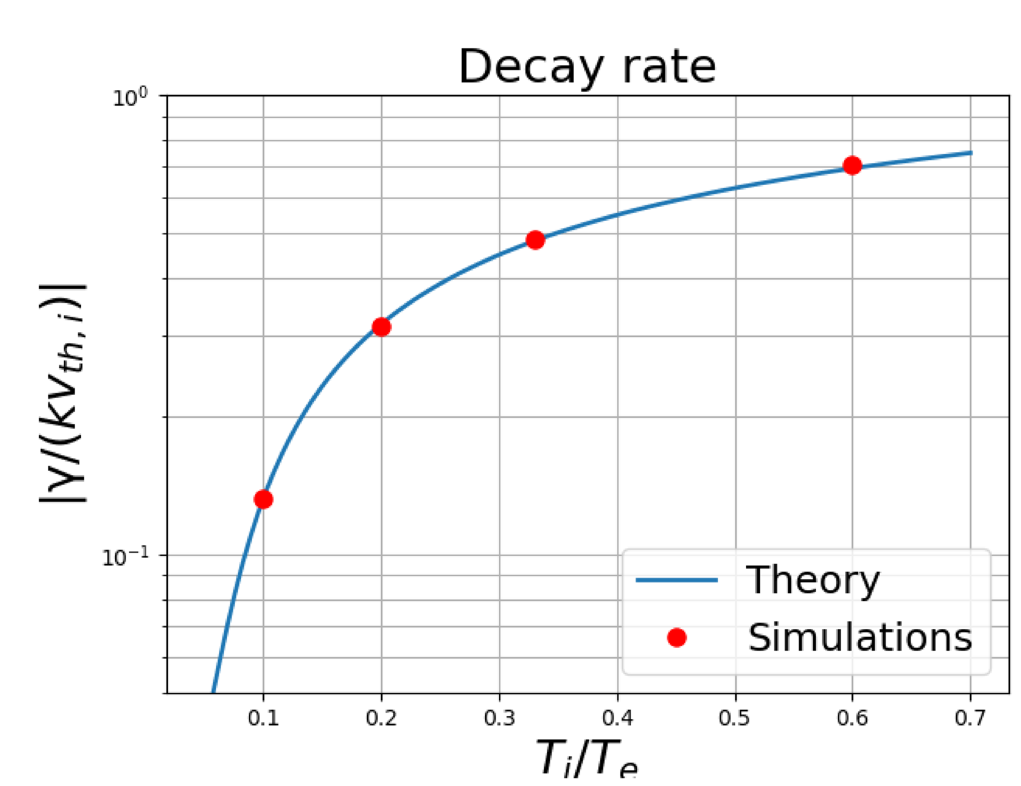


Dispersion Relation

The dispersion relation for ion-acoustic waves, accounting for kinetic effects of both electrons and ions [3], is

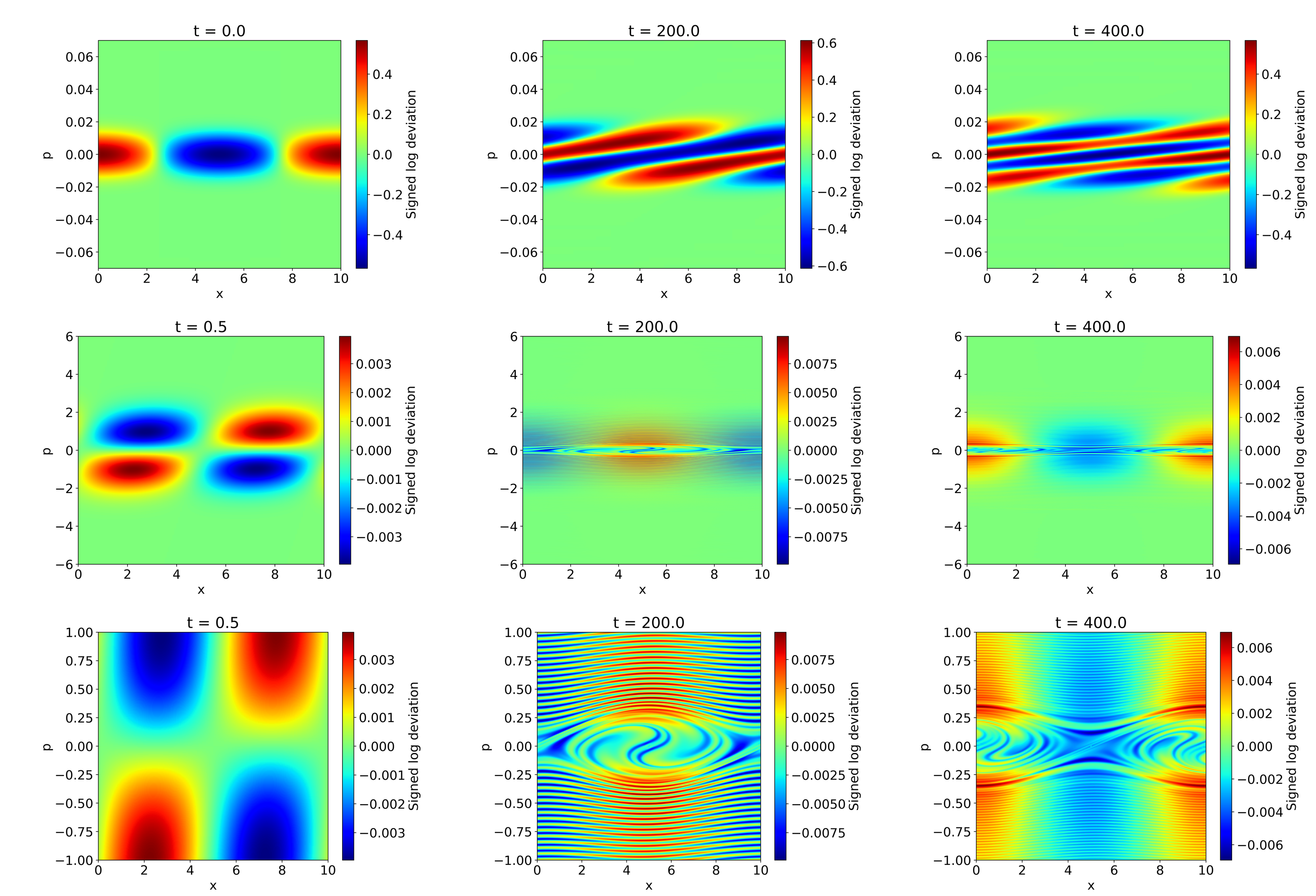
$$D(k,p) = 1 - \frac{1}{(k\lambda_{De})^2}\frac{1}{2}\left[Z'(\xi_e) + \frac{T_e}{T_i} Z'(\xi_i)\right] = 0, \quad \xi_s = \sqrt{\frac{m_s}{2\kappa T_s}}\frac{ip}{k}, \quad p = \gamma - i\omega.$$

Z is the plasma dispersion function.



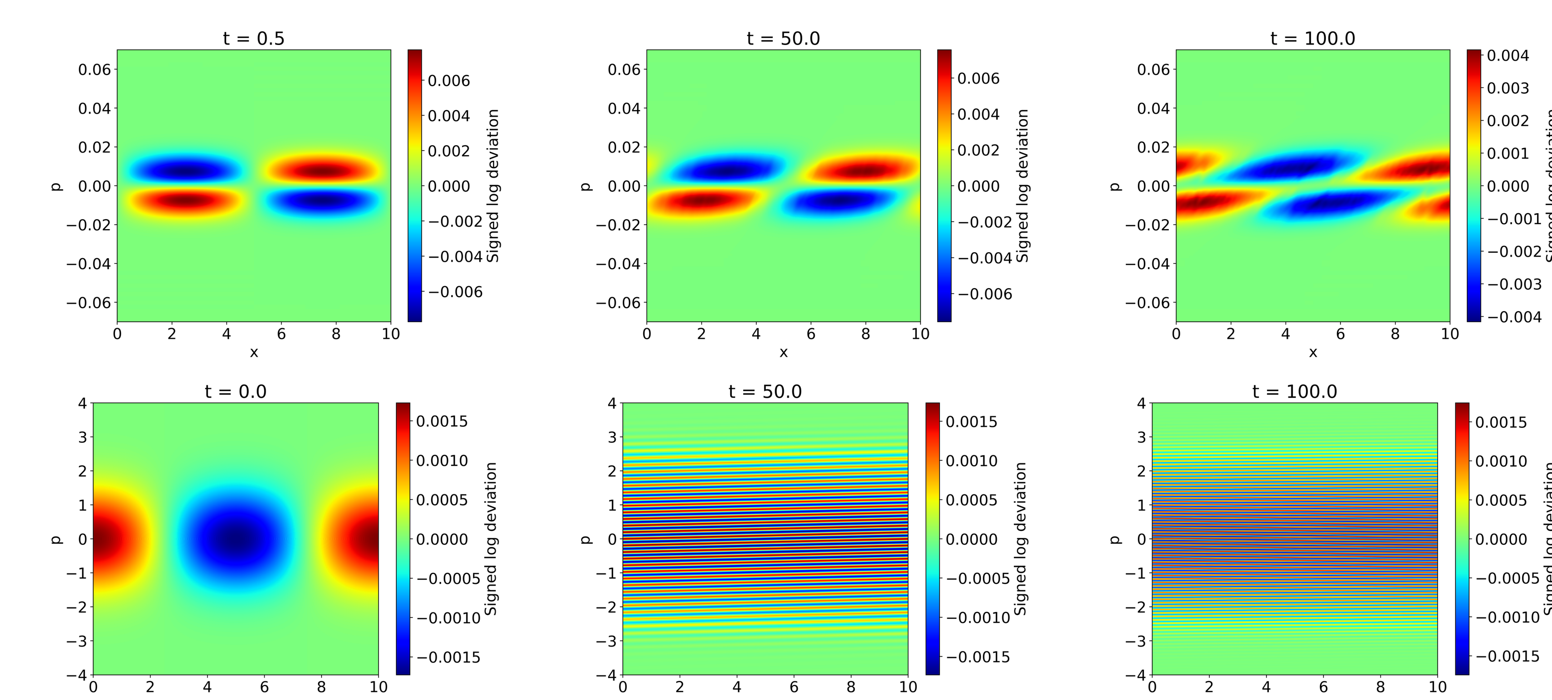
Phase Space with ion perturbation

Plotting $\log(|f(x,v,t) - f_M(v; v_{\text{th}})|)$ in phase space for ions(1st row) and electrons(2nd row):



Phase Space with electron perturbation

ions(1st row) and electrons(2nd row):



Future Work

- 1D2V Vlasov–Maxwell
- 2D resistive MHD code

References

[1] R. Krasny R. T. Sandberg and A. G. Thomas. The FARSIGHT vlasov-poisson code. *Journal of Computational Physics*, 523:113664, 2025.

[2] R. Krasny L. Wang and S. Tlupova. A kernel-independent treecode based on barycentric lagrange interpolation. *Communications in Computational Physics*, 28(4):1415–1436, 2020.

[3] Donald A. Gurnett and Amitava Bhattacharjee. *Introduction to Plasma Physics: With Space, Laboratory and Astrophysical Applications*. Cambridge University Press, 2 edition, 2017.

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