Space-Charge and Circuit-Induced Distortion of Short-Pulse Beams in a Vacuum Diode

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Summary

- **Short-pulse electron beam** dynamics in vacuum diodes are strongly shaped by **space-charge** effects and the **external circuit**.
- **Multiple-sheet model** and **WarpX PIC simulations** self-consistently resolve the electric fields, charge transport, and circuit response.

Multiple-Sheet Model

A **grid-less solver** for one-dimensional (1D) diode with gap distance d and source voltage V_0, with a series resistor R, and with electron beam modeled as sheets inside the gap.

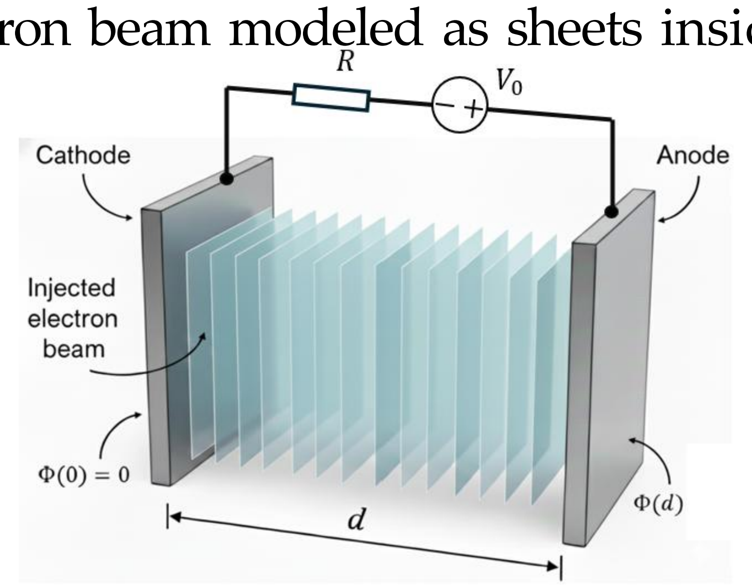


Figure 1: Sheet positions inside the diode gap

- The Electric field on sheet j of density ρ_j at position z_j with velocity v_j is

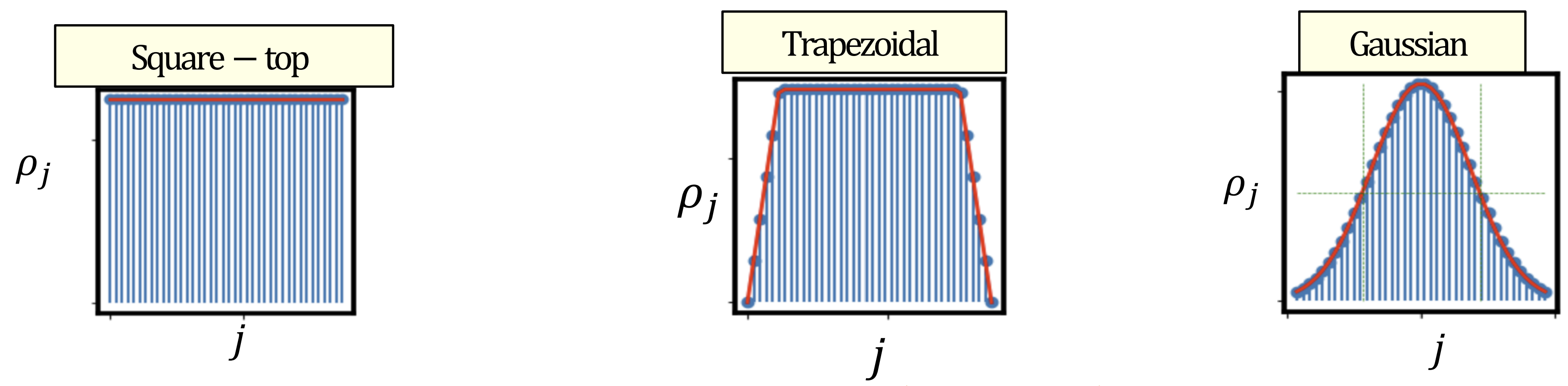
$$E_j = -\frac{V_g}{d} - \frac{1}{\varepsilon_0}\left(\sum_{i=1}^{M}\rho_i\frac{z_i}{d} - \sum_{i=1}^{j-1}\rho_i + \frac{1}{2}\rho_j\right) \quad (1)$$

where $V_g = V_0 - RI(t)$ and $I(t)$ is the sum of the conduction and displacement current across the gap

$$I(t) = \frac{A}{d}\sum_{i=1}^{M}\rho_i v_i - \frac{\varepsilon_0 A}{d}R\frac{dI(t)}{dt} \quad (2)$$

- These equations are coupled to the Newton's equations of motion.

Short-Pulse Profiles



- The normalized charge density : $f = \rho/(-\varepsilon_0 V_0/d)$
- The normalized pulse duration : $k = \tau_p/\left(3d\sqrt{2eV_0/m_e}\right)$

with transit time : $T_{CL} = \tau_p/\left(3d\sqrt{2eV_0/m_e}\right)$

Electron trajectories using the Multiple-sheet solver

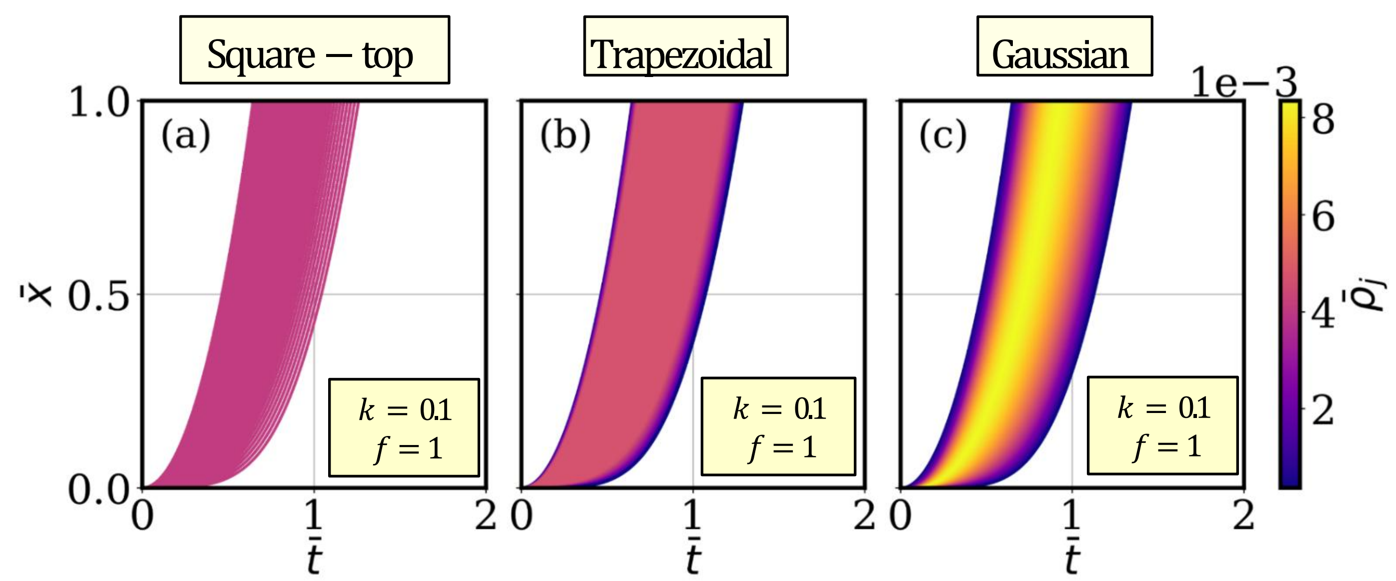


Figure 2: Electron sheet trajectories in vacuum with $R = 0\,\Omega$ [1]

WarpX Simulation with circuit coupling

We use the discretization of **Eq. (2),** presented in **Table 1**, to implement the circuit coupling in WarpX.

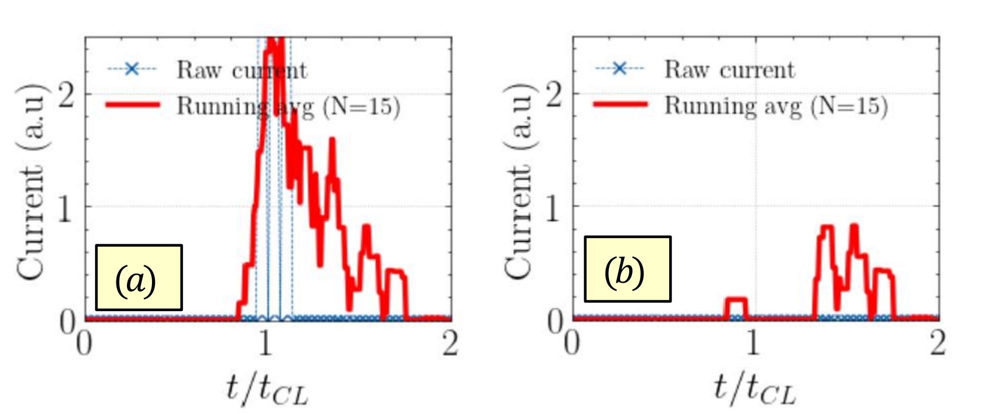


Figure 3: Diode current for (a) $R = 0\,\Omega$, and (b) $R = 10^3\,\Omega$ for a Square-top pulse, with $f = 0.95$, $k = 0.1, d = 2.8\ mm$ and $V_0 = 50$ V

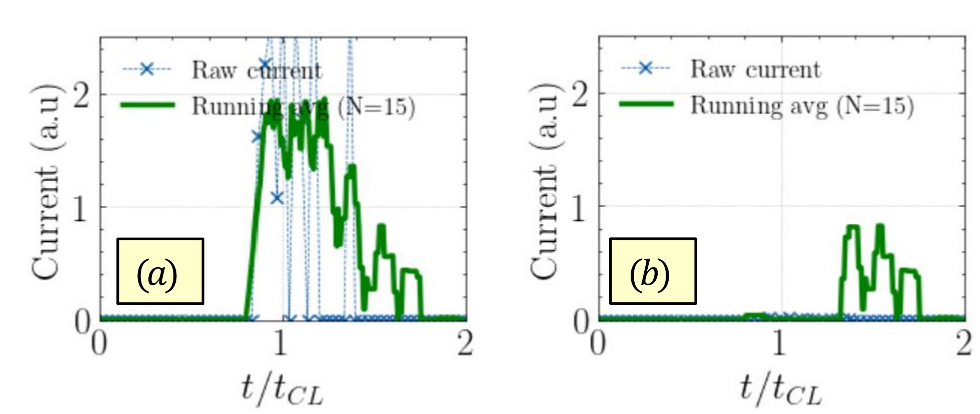


Figure 4: Diode current for (a) $R = 0\,\Omega$, and (b) $R = 10^3\,\Omega$ for a Gaussian pulse, with $f = 0.95$, $k = 0.1, d = 2.8\ mm$ and $V_0 = 50$ V

Table 1: Discretization with a backward-Euler

$I^{n+1} = I^{n+1}_{\text{cond}} - \beta\frac{I^{n+1} - I^{n}}{\Delta t}$	(A)
$I^{n+1}\left(1 + \frac{\beta}{\Delta t}\right) = I^{n+1}_{\text{cond}} + \frac{\beta}{\Delta t}I^{n}$	(B)
$I^{n+1} = \frac{I^{n+1}_{\text{cond}} + \beta/\Delta t\, I^{n}}{(1 + \beta/\Delta t)}$	(C)

Future Work

1. Systematic parametric studies for vacuum diodes with series resistor.
2. Extend the model to plasma-filled gaps by incorporating ionization, charge exchange, etc.
3. Benchmark the plasma-gap formulation experiments data, and relevant scaling laws, on current, voltage with respect to gas pressure and diode parameters.

References & Acknowledgement

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