Effects of Annular Beam Properties on Gap Coupling in High-Frequency Microwave Devices

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Abstract

- This work studies the interaction of an annular electron beam with a cavity gap.
- We provide a parametric analysis of beam-cavity interaction for large signals of linear beam device.
- We compare the gap coupling factor of annular and solid beams for the same beam current.
- We observe the effect of space charge on the electron trajectories and their kinetic energy which leads to lower conversion efficiency.

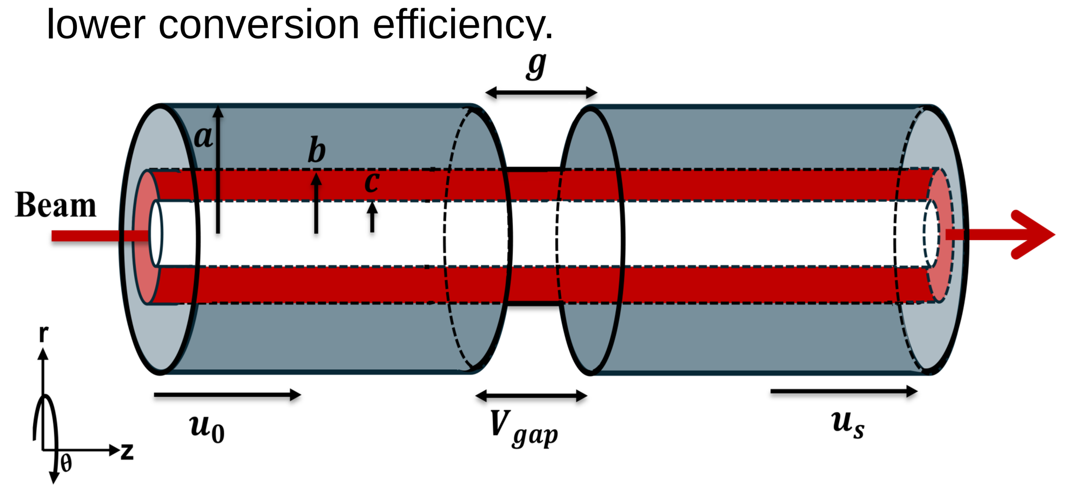

Fig. 1. Interaction gap of an annular linear beam device [1]

Methodology

TABLE I. Parameters for Calculation [2]

Anode Voltage, V_a	25kV
Load Impedance, R_L	26kΩ
Frequency, f	1.3Ghz
Number of Disks, N_D	50
Disk length, L	0.8mm
Output gap Length, g	11mm

TABLE II. List of Symbols

a	Tunnel radius
b	Outer Beam radius
c	Inner Beam radius
ρ_c	Space charge density
ω	Angular frequency
I_b	Beam current
I_{RF}	RF current
Q	Disk Charge
η_e	Charge/mass ratio

➢ Analytical Model

- Wave number, $\beta = \frac{\omega}{u}$
- Small Signal Gap Coupling Coefficient,

$$M = \frac{2[bI_1(\beta(u)b) - cI_1(\beta c)]}{\beta(u)(b^2 - c^2)I_0(\beta(u)a)} \frac{\sin(\frac{\beta(u)g}{2})}{(\frac{\beta(u)g}{2})}$$

- Large Signal Gap Coupling Coefficient,

$$M = \frac{2\left[bI_1\left(\beta\left(\frac{u_0+u_s}{2}\right)b\right) - cI_1\left(\beta\left(\frac{u_0+u_s}{2}\right)c\right)\right]}{\beta(\frac{u_0+u_s}{2})(b^2-c^2)I_0\left(\beta(\frac{u_0+u_s}{2})a\right)} \frac{\sin(\frac{\beta(\frac{u_0+u_s}{2})g}{2})}{\frac{\beta(\frac{u_0+u_s}{2})g}{2}}$$

- RF Output Power, $P_{RF} = \frac{1}{2}M^2 I_{RF}^2 R_L$ [3]
- Electronic Efficiency, $\eta = \frac{P_{out}}{I_b V_a}$

➢ Disk Model

- Axial component of Electric Field,

$$E_z(z) = \frac{V_{gap}}{2\pi}\int_{-\infty}^{\infty} \frac{2[bI_1(\beta b) - cI_1(\beta c)]}{\beta(b^2-c^2)I_0(\beta a)} \frac{\sin(\frac{\beta g}{2})}{(\frac{\beta g}{2})} e^{-j\beta z} d\beta$$

- Gap Voltage, $V_{gap} = I_{RF} M \left(\frac{u_0+u_s}{2}\right) R_L$
- Kinetic Energy,

$$KE_k = \frac{Q}{\eta_e} c^2 \sum_{j=1}^{N_D} (1/\sqrt{1 - \frac{u_{s_{k,j}}^2}{c^2}} - 1)$$

Methodology

➢ Disk Model

- Space charge Field, $E_{SC} =$

$$\begin{cases} \frac{4\rho_c}{\varepsilon_0}\sum_{m=1}^{\infty}\frac{1}{\mu_m}\frac{[bJ_1(\mu_m b)-cJ_1(\mu_m c)]^2\, e^{-\mu_m|z-z_i|}}{(b^2-c^2)(\mu_m a)^3[J_1(\mu_m a)]^2}\sinh\left(\frac{\mu_m L}{2}\right) & \text{for } |z| > \frac{L}{2} \\ \frac{4\rho_c}{\varepsilon_0}\sum_{m=1}^{\infty}\frac{1}{\mu_m}\frac{[bJ_1(\mu_m b)-cJ_1(\mu_m c)]^2\, e^{-\frac{\mu_m L}{2}}}{(b^2-c^2)(\mu_m a)^3[J_1(\mu_m a)]^2}\sinh \mu_m(z - z_i) & \text{for } |z| < \frac{L}{2} \end{cases}$$

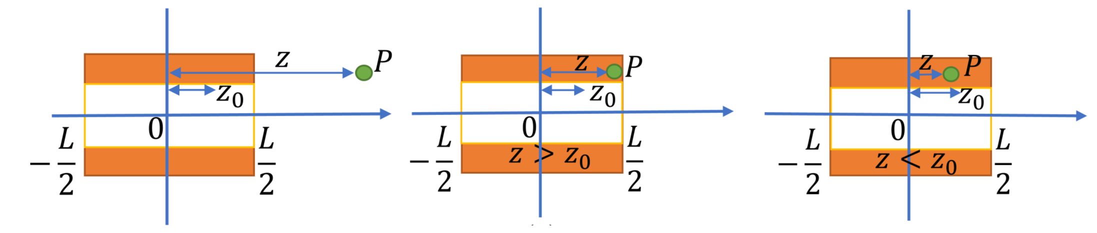

Fig. 2. Space charge potential for outside ($z > z_0$) and inside ($z > z_0$, $z < z_0$)

- Equation of motion,

$$\frac{du_{s_i}}{dt} = \eta_e\left(1 - \frac{u_0^2}{c^2}\right)^{3/2}\left[E_z(z_i)\cos(\omega t) + \sum_{j=1}^{N_D}\frac{Q}{e}E_{sc}(z_i - z_j)\right]$$

- DC beam power of the disk model,

$$P_{DC} = KE_0 \times f$$

- RF power transferred to the gap from the change in KE

$$P_{RF} = (KE_0 - KE_{max}) \times f$$

- Efficiency of the linear beam device,

$$\eta = \frac{P_{RF}}{P_{DC}} = \frac{(KE_0 - KE_{max}) \times f}{KE_0 \times f}$$

Results

A. Constant Current Density and Beam Current

- Default outer and inner beam radius : b=8.2mm & c=5mm.
- c/b=0.6 has same current density as solid beam b=6.5mm [2].
- Gap coupling factor, M is higher for annular beam.
- At I_{RF} = 1.4A, M calculated from annular disk model is 0.7, which is 4.64% higher than the solid beam [2].

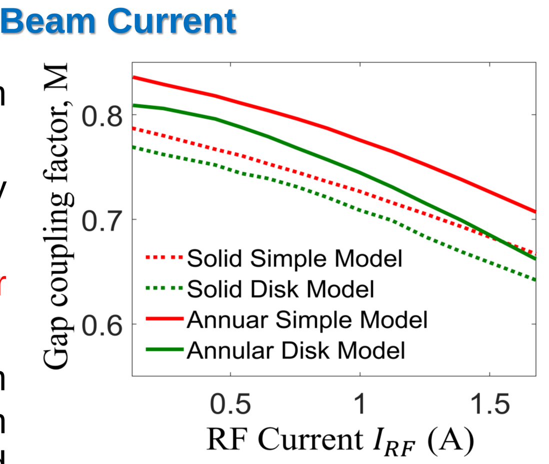

Fig. 3. Comparison of M of simple and disk model as a function of different RF current for annular and solid beam

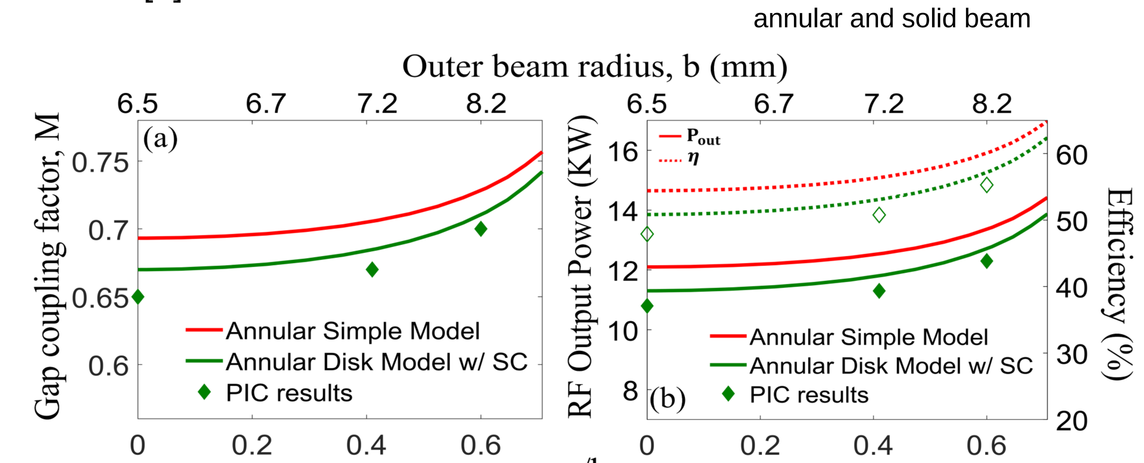

Fig. 4. Comparison of (a) M and (b) P_{RF} (solid lines) and η (dotted lines) from simple and disk model with space charge for different *c/b and compared with* PIC results using XOOPIC.

- For a larger c/b, simple and disk models show a higher M due to closer interaction with the gap and reduced space charge effects than in a solid beam (c/b = 0).
- Reduction of M due to space charge effects is smaller in an annular beam.
- P_{RF} and η increase with a larger c/b,
- Enhanced electromagnetic coupling & reduced space charge effects lead to more effective energy transfer from beam to cavity fields.

Results

➢ Normalized Gap Coupling Factor

- Length scale $L = 1/\beta\left(\frac{u_0+u_s}{2}\right)$, normalized tunnel, outer & inner beam radius, and gap length are $\bar{a} = \frac{a}{L}, \bar{b} = \frac{b}{L}, \bar{c} = \frac{c}{L}, \ \bar{g} = \frac{g}{L}$. Normalized M:

$$M = \frac{2}{(\bar{b}^2 - \bar{c}^2)} \frac{[\bar{b}I_1(\bar{b}) - \bar{c}I_1(\bar{c})]}{I_0(\bar{a})} \frac{\sin(\frac{\bar{g}}{2})}{(\frac{\bar{g}}{2})}$$

- Increase of normalized $\bar{a}$ and $\bar{g}$ lowers M for the annular beam, which is similar to solid beam [2]
- Increase of normalized $\bar{b}$ and $\bar{c}$ increases M.

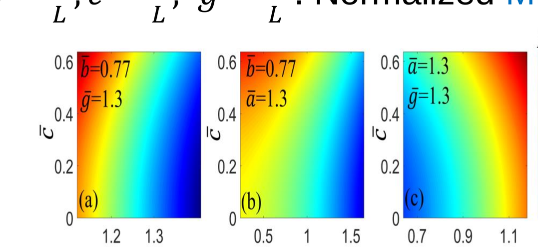

Fig. 5. Comparison of M as a function of (a) $\bar{c}$ and $\bar{a}$, (b) $\bar{c}$ and $\bar{g}$, (c) $\bar{c}$ and $\bar{b}$

B. Constant Outer Beam Radius and Beam Current

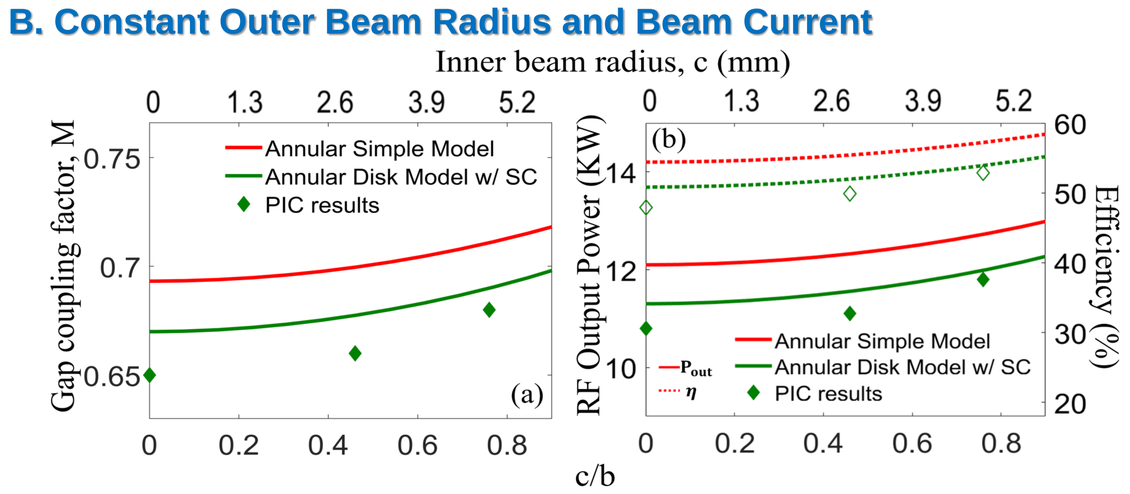

Fig. 6. Comparison of (a) M and (b) P_{RF} (solid lines) and η (dotted lines) from simple and disk model with space charge for different *c and compared with* PIC results (XOOPIC).

- Increasing c increases the charge density as cross-sectional area of beam is decreased due to the fixed b and total current.
- For a larger c, simple and disk models show a higher M
- Space charge effects show a reduction of up to 1.17% in M at c =5mm, which is lower than solid beam.
- P_{RF} and η increase with a larger c, confirmed with PIC results (XOOPIC).

C. Electron Velocity and Kinetic Energy

- The green line (when c/b =0) indicates a solid beam.
- In Fig. 7a and Fig 8a, beam cross-sectional area is constant and b is used as 6.5mm, 7.5mm and 9.5 mm for c/b = 0, 0.5, and 0.7
- In Fig. 7b, and Fig 8b, b is fixed and c varies as 0, 5 and 6.49mm
- Increasing c causes a reduction in final electron velocity, indicates that more energy is transferred from beam to the microwave field.
- Normalized $KE(=(\sum KE/N_D)/KE_0)$ decreases when c increases, where KE_0 and $\sum KE$ are initial and total kinetic energy.
- A reduction in KE indicates more efficient energy transfer from the electron beam to the RF fields.

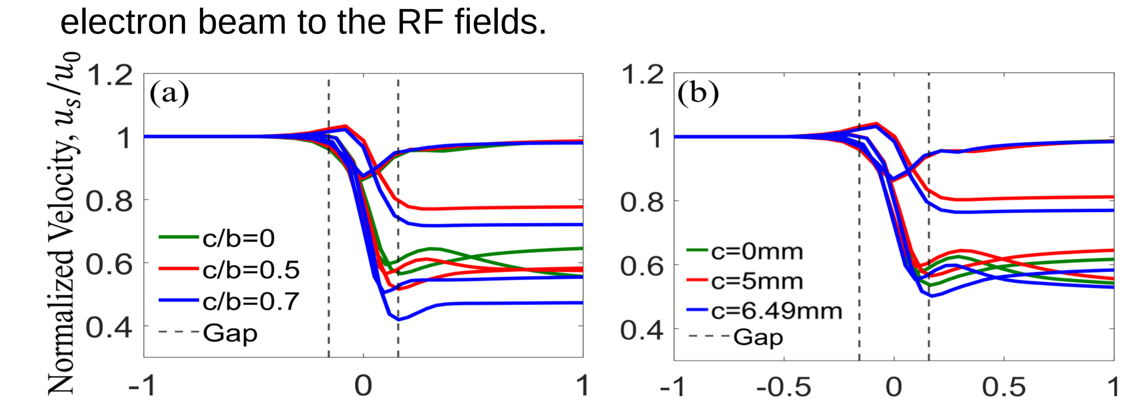

Fig. 7. Normalized velocity of disk model with space charge for different c/b of fixed beam current (a) with constant charge density and (b) with fixed b

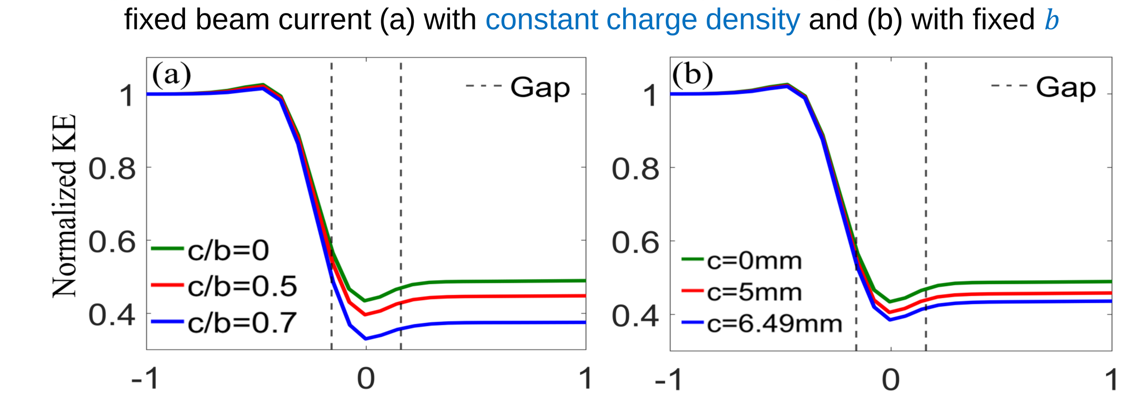

Fig. 8. Normalized KE of disk model with space charge for different c/b of fixed beam current (a) with constant charge density and (b) with fixed b

Scaling to High Frequency

- At higher frequencies, P_{RF} (solid lines) and η of the single gap-cavity device drop significantly
- At 5GHz, using c = 5mm and I_{RF} =1.9 A, P_{RF} (η) for annular beam calculated from disk model with space charge is 16.9kW (56.9%) which are higher compared to solid beam of 11.49kW (38.3%).
- At 18GHz, we vary c while setting V_a=50kV, b = 7.2mm, RF current I_{RF}=1.9A.
- For c =7.15mm, P_{RF} (η) is 20.58kW (34.6%) for disk model with space charge, which are significantly higher than solid beam (c = 0) of 1.93kW (3.03%).

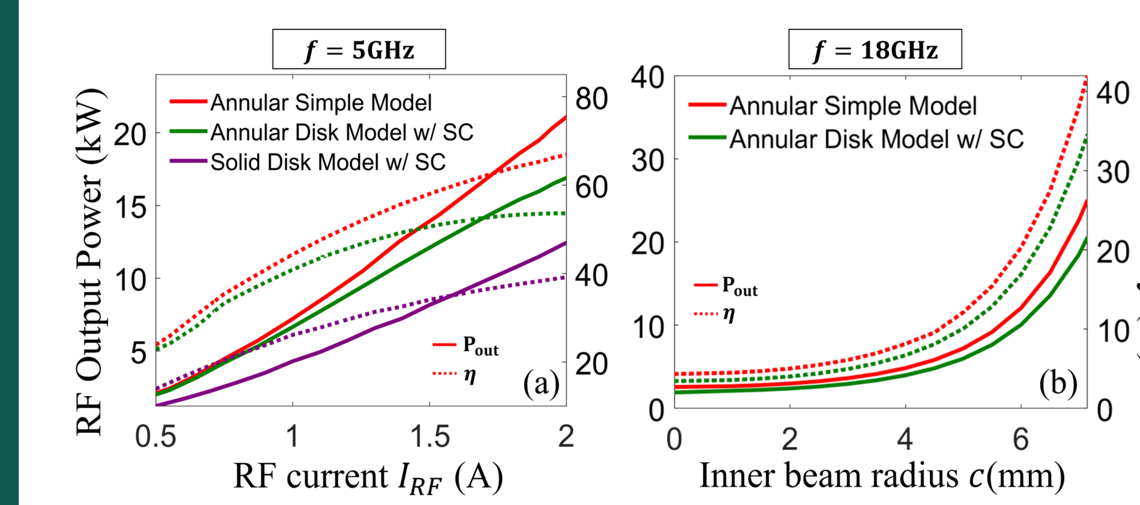

Fig. 9. Comparison of P_{RF} (solid lines) and η (dotted lines) for a single gap-cavity device (a) between annular and solid [2] beam for different RF current at f = 5GHz (C-band) (b) for different c when b=7.2mm at f =18GHz (K-band).

Conclusion

- We have analysed the gap coupling factor for annular beam-gap interaction, using both large signal analytical model and disk model.
- We provide parametric scaling analyses and quantitative assessment on how increasing the ratio of inner and outer beam radius increases the gap coupling factor.

Future Works

- Future studies may extend the investigation to other beam profiles, including sheet and multi-beam configurations.
- Examine electron bunching dynamics to enable more efficient and compact high-frequency amplifiers.
- Our method may also be extended for multiple cavity devices, such as CC-TWTs.

References

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Acknowledgements

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