Laboratory Dynamos:
From Liquid Metal to Plasmas

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Acknowledgements

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Dedication Roch Kendrick (1963-2009)

Outline

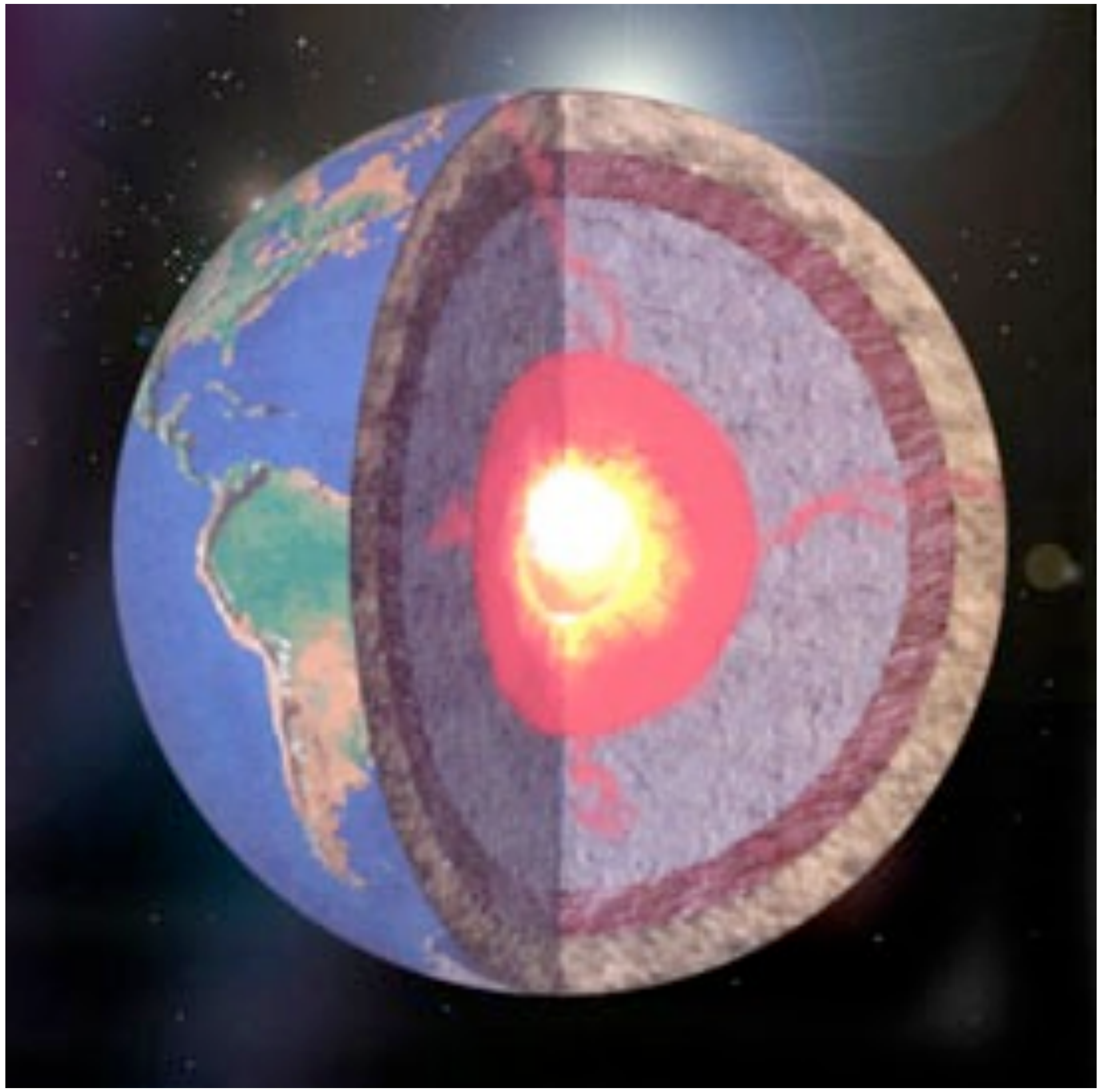
- Introduction
  - Free energy source: Plasma Kinetic (flow) Energy
  - astrophysical examples
- Dynamos: Kinetic energy → Magnetic energy
  - basic models
- Liquid metal experiments
  - self-excitation observations
  - mean-field EMF process
- Future Directions: Plasma Experiments

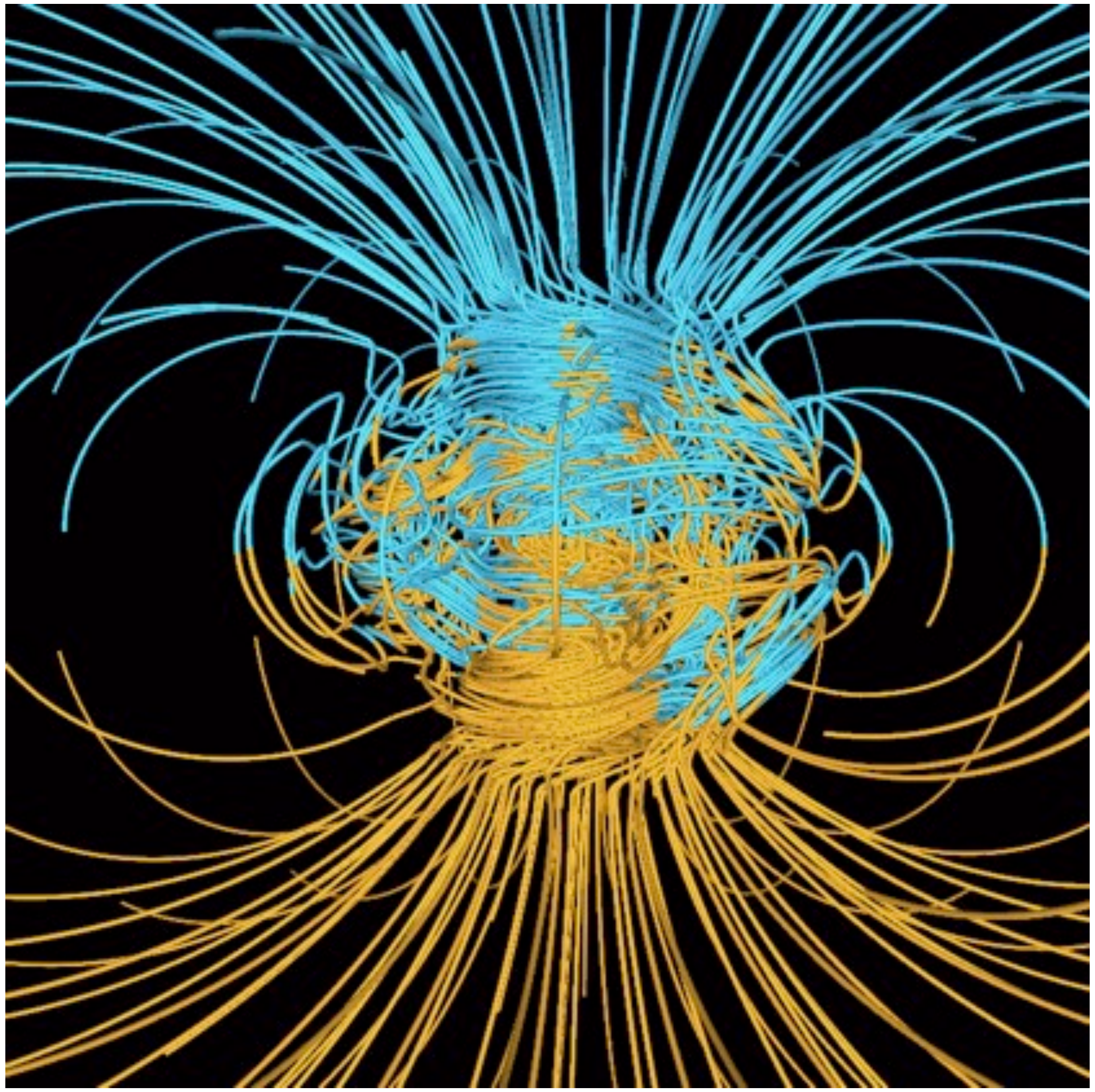
Novel parameter regime: continuous transfer of flow energy → magnetic energy

Cowling	C	$\frac{\frac{B^2}{2\mu_0}}{\frac{1}{2}\rho U^2}$	$C \leq 1$
Magnetic Reynolds	Rm	$\mu_0 \sigma U L$	$100 \leq Rm$
Reynolds	Re	$\frac{UL}{\nu}$	$1 \ll Re$
Magnetic Prandtl	Pm	$\mu_0 \sigma \nu$	$Pm \equiv Rm/Re$
Quasistationary			$T \geq \mu\sigma L^2$

For plasma experiments: steady-state, large, flowing, unmagnetized, hot

The Earth's Dynamo: large scale dipole

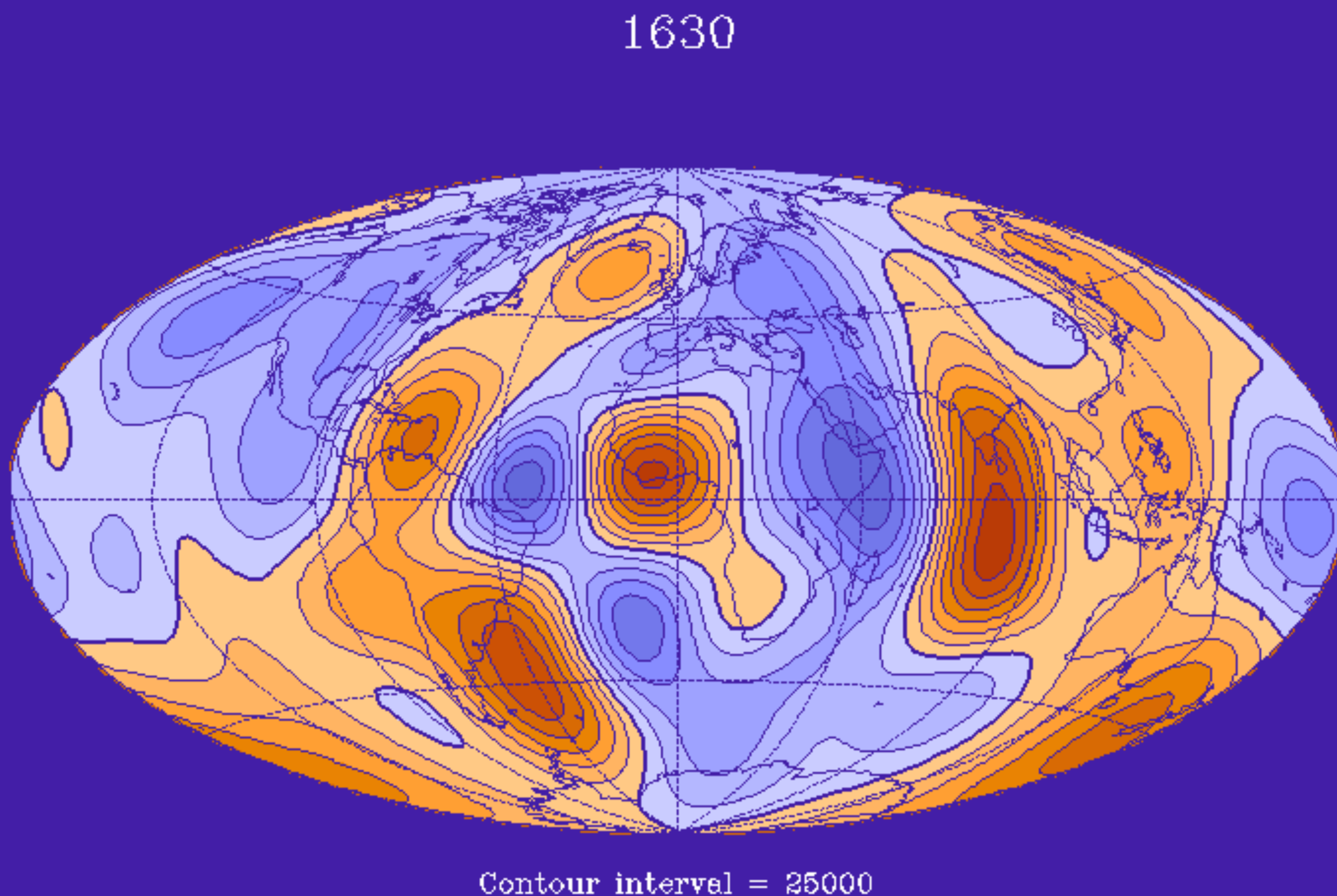




Glatzmaier and Roberts, *A three-dimensional self-consistent computer simulation of a geomagnetic field reversal*, Nature **377** 203 (1995).

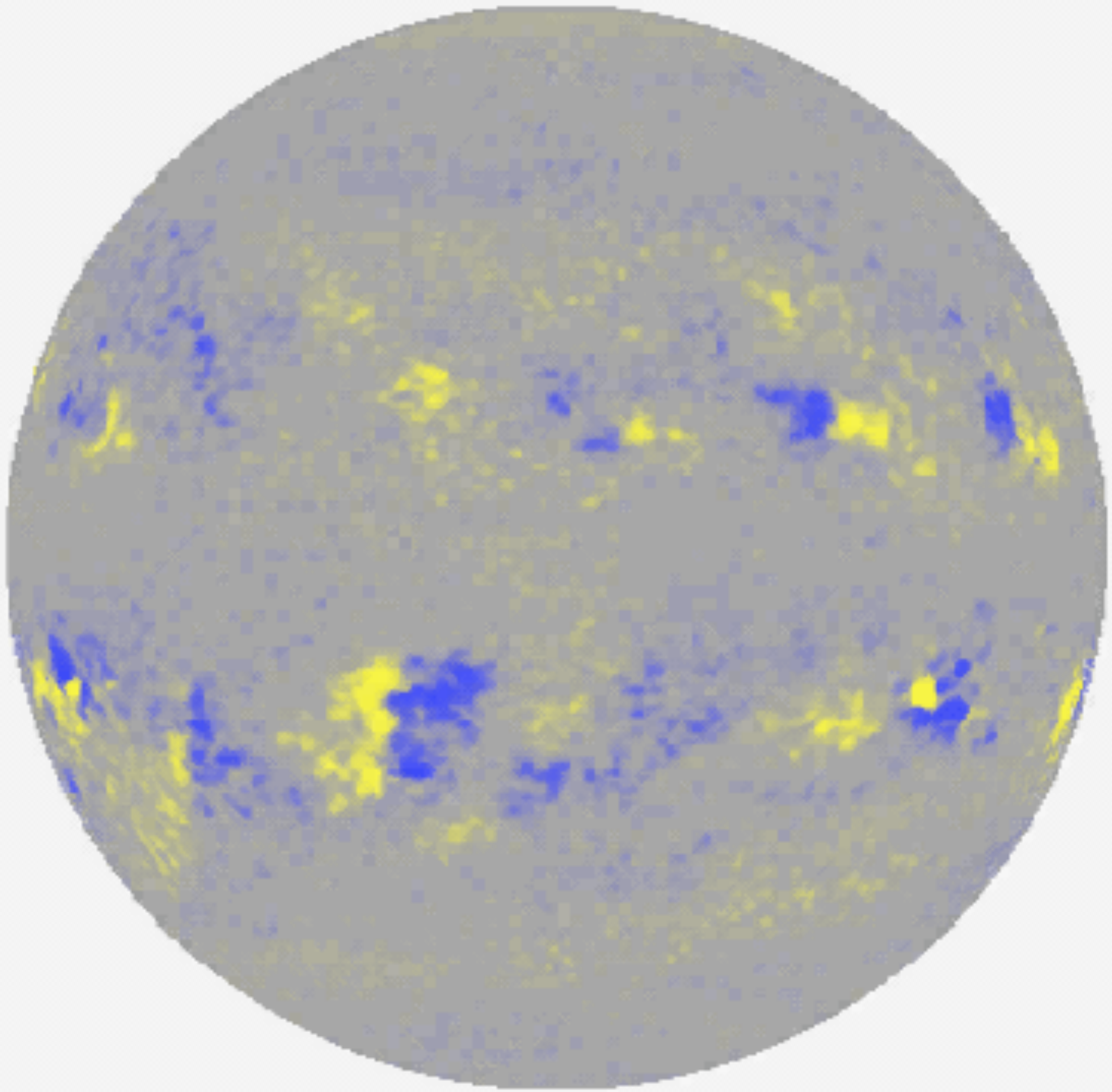
Rm~500-1000, Re=10^9, Pm=10^{-6}, $\tau_\sigma=10^5$ yrs

+ smaller scale fluctuations



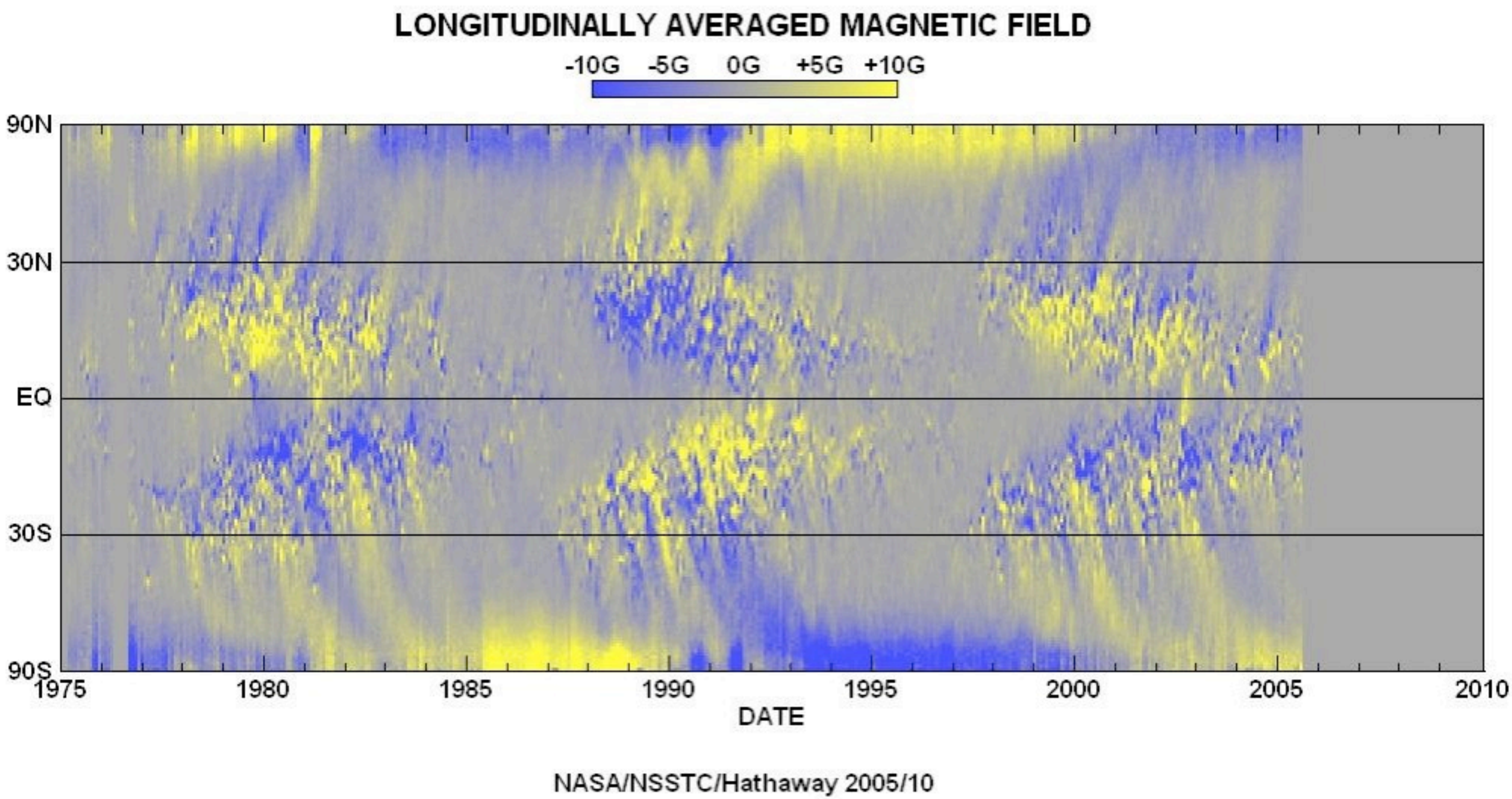
Jackson, Jonkers and Walker, *Four centuries of geomagnetic secular variation*, Phil. Trans. R. Soc. Lond. A **358** 957 (2006).

The Sun's dynamo: oscillatory and mostly small-scale

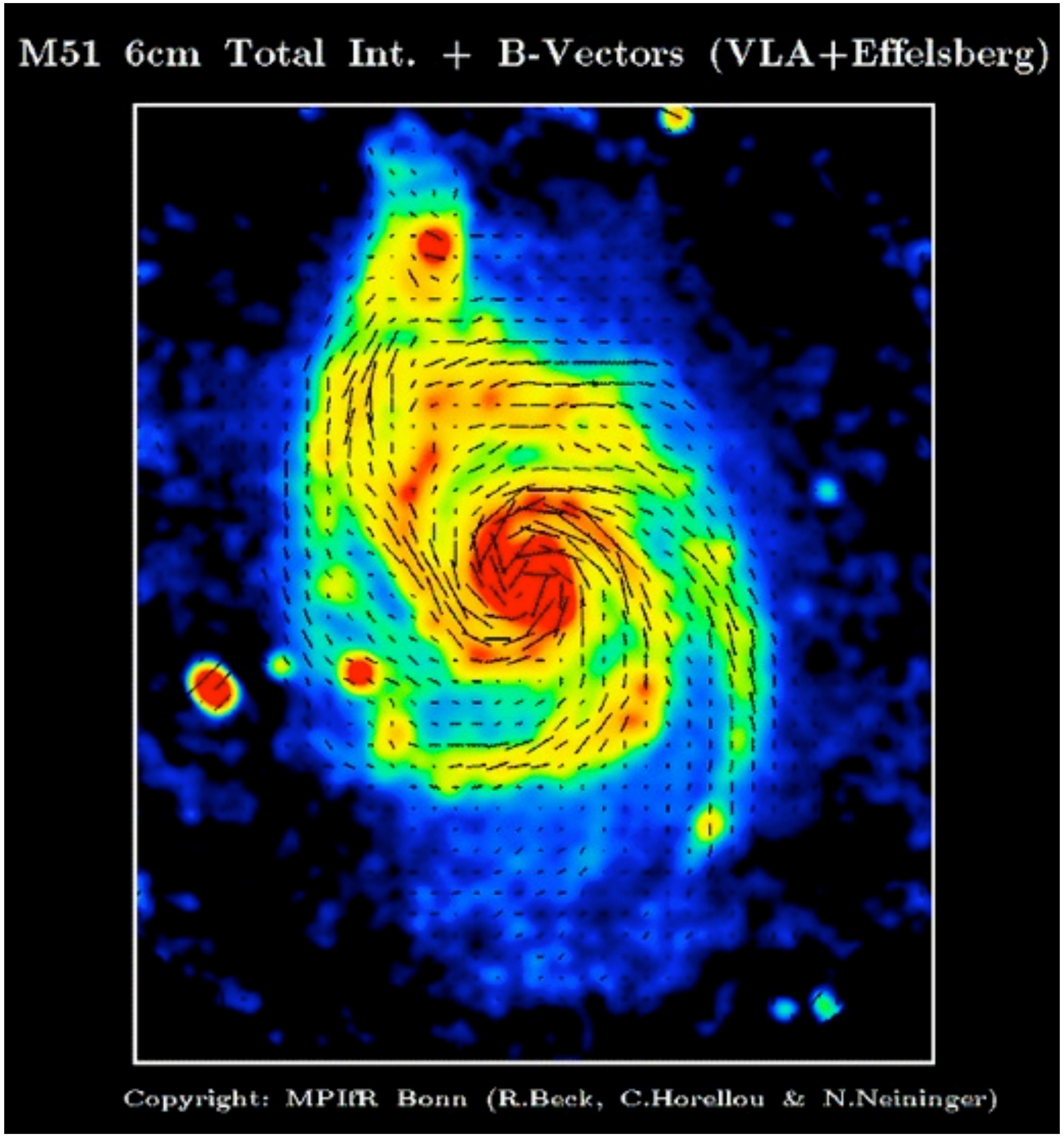


$Rm \sim 10^8$, $Re = 10^{11}$, $Pm = 10^{-3}$, $\tau_\sigma = 10^{11}$ yrs

+ weak large scale field

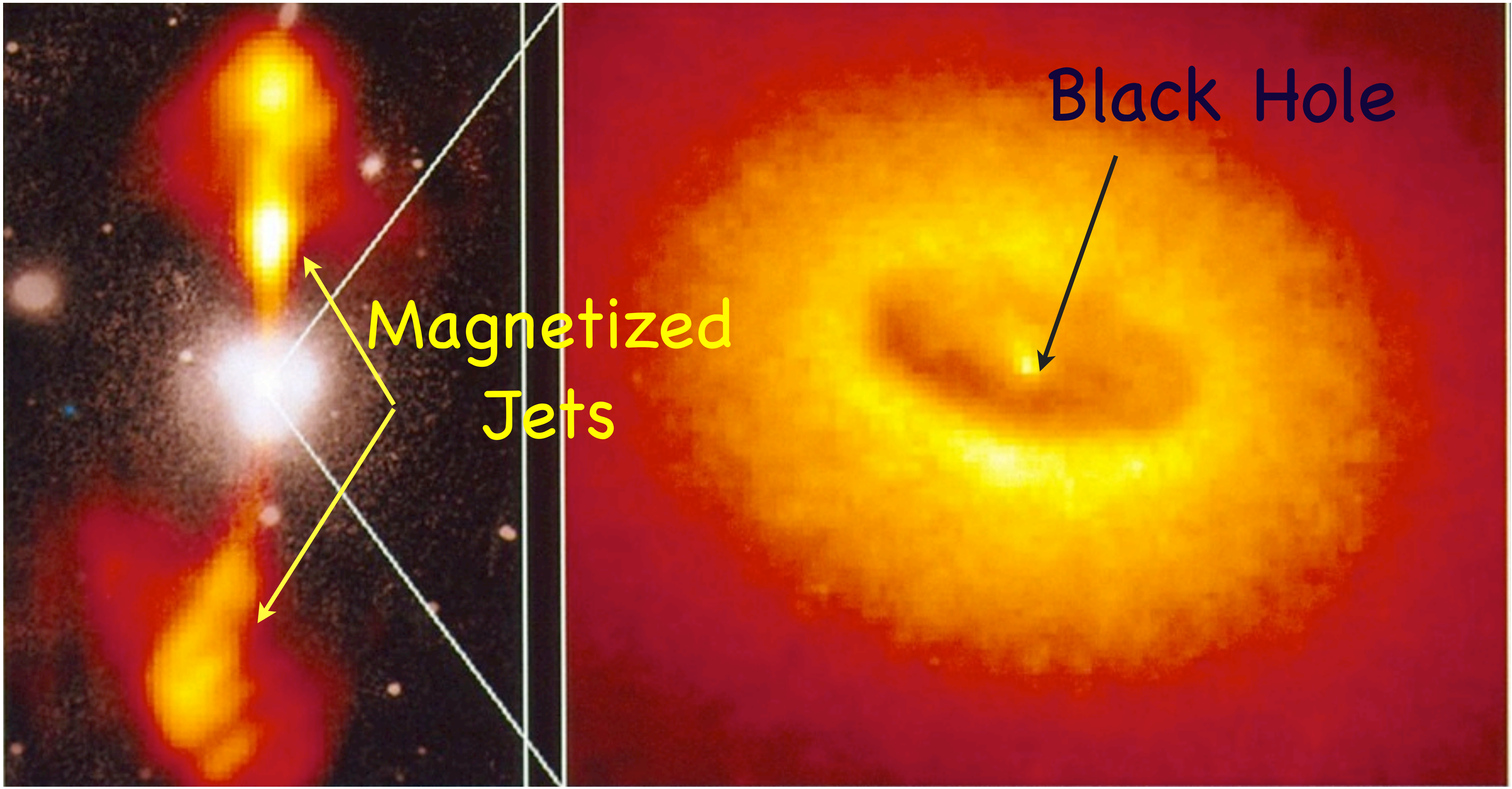


Galactic Magnetic Fields: weak large-scale field + much stronger small-scale



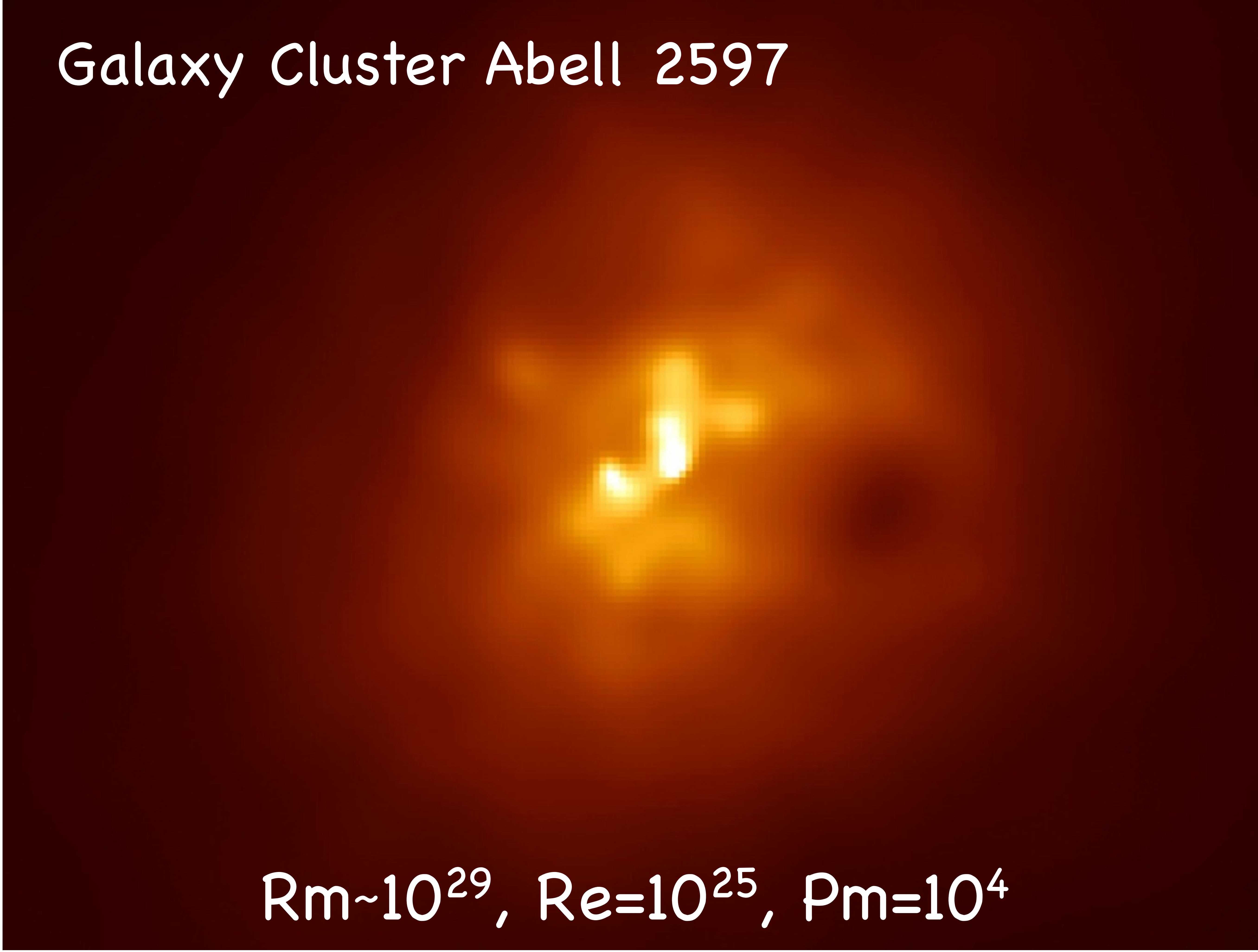
$Rm \sim 10^{14}$, $Re = 10^{9}$, $Pm = 10^{5}$, $\tau_{\sigma} = 10^{23}$ yrs

Disks and Jets around Black Holes



$Rm \sim 10^{19}$, $Re = 10^{14}$, $Pm = 10^{5}$

Galaxy Cluster Abell 2597



$Rm \sim 10^{29}$, $Re = 10^{25}$, $Pm = 10^{4}$

Poorly Understood, Fundamental Plasma and MHD Processes Can Benefit from Experimental Studies

- **Large Scale Dynamo**: What is the size, structure and dynamics of the mean magnetic field created by high magnetic Reynolds number flows—particularly rotating flows?
- **Small Scale Dynamo**: How do turbulent (high Rm) flows create turbulent magnetic fields? What is the nature of plasma turbulence when magnetic fields and velocity fields are in near equipartition?
- **Magnetorotational Instability**: How is angular momentum transport by magnetic instabilities? Can the MRI be a dynamo?
- **Flow Driven Reconnection**: How does plasma flow generate magnetic energy that can accumulate and ultimately be released in explosive instabilities?
- **Plasma Instabilities**: Do plasma instabilities beyond MHD play a role in collisionless, turbulent plasma flows?

Minimum requirements for experimentally addressing each Plasma Process

$\tau_\sigma = \mu_0 \sigma a^2$

Plasma Process	Rm_{crit}	Re	C	$\frac{\lambda}{L}$	β
large scale dynamo					
laminar	$\gtrsim 100$	< 100	$\ll 1$	-	-
with turbulence	$\gtrsim 500$	> 1000	$\ll 1$	-	-
small scale dynamo	$\gtrsim 500$	$\gtrsim 1000$	$\ll 1$	?	?
MHD turbulence	$\gtrsim Re$	$\gtrsim 1000$	~ 1	-	-
MRI					
with mean field	$\gtrsim 10$	—	$\lesssim 1$	?	?
without mean field	$\gtrsim 15000$	—	$\ll 1$	?	?
B field stretching	$\gtrsim 100$	< 100	~ 1	-	-
Plasma Instabilities	$\gtrsim Re$	$\gtrsim 1000$	$\lesssim 1$	$\gtrsim 1$	$\gg 1$

Large, High Te, fast flowing plasmas

Low B, fast flowing plasmas

Dynamos

How could a rotating body such as the Sun become a magnet? [Br. Assoc. Adv. Sci. **159** (1919)].

" . . . possible for the internal cyclic motion to act after the manner of the cycle of a self-exciting dynamo, and maintain a permanent magnetic field from insignificant beginnings, at the expense of some of the energy of the internal circulation."

-J. Larmor

What is a self-exciting dynamo?

Faraday's Law of Induction

$$\vec{J} = \sigma\left(\vec{E} + \vec{V} \times \vec{B}\right)$$

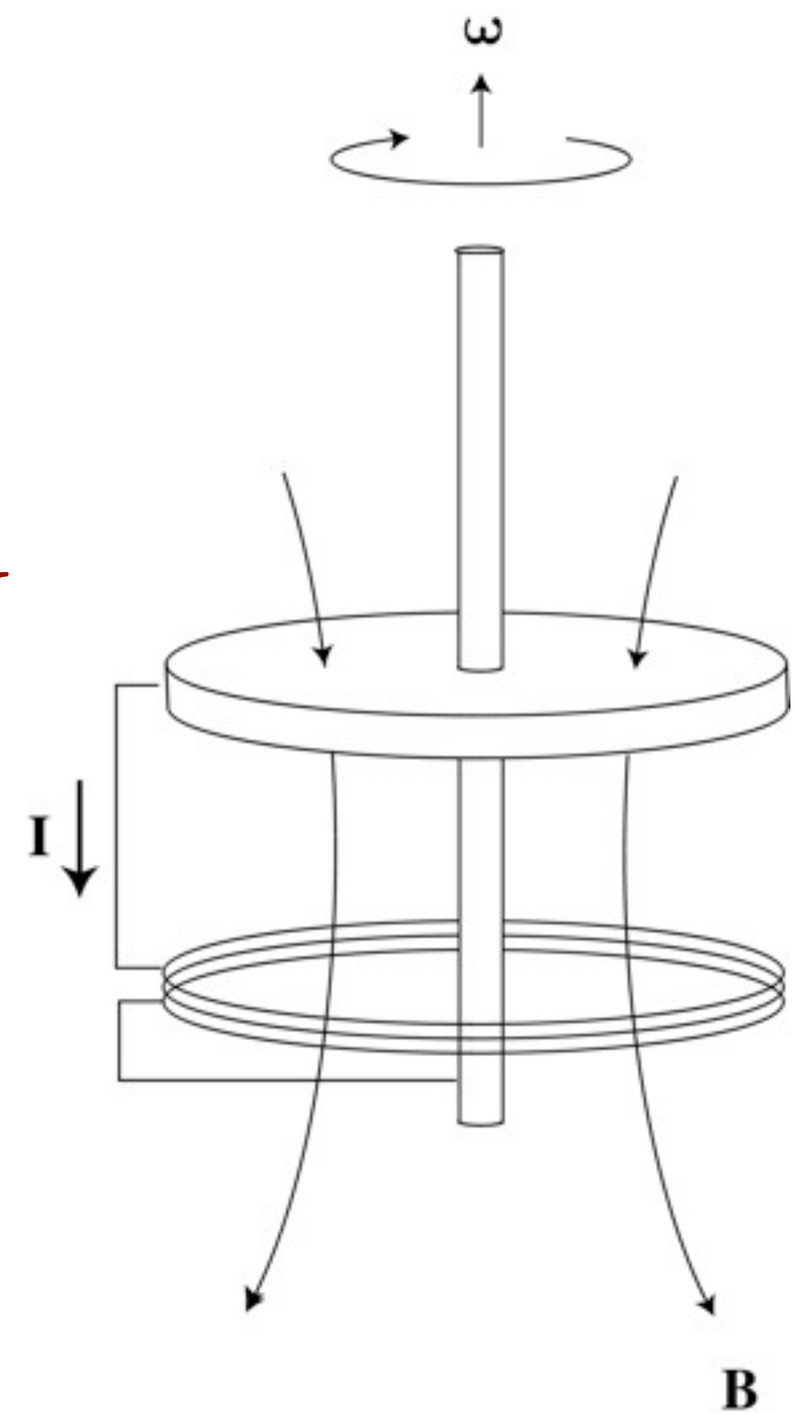
$$\mu_0 \sigma L V$$

Induction Equation

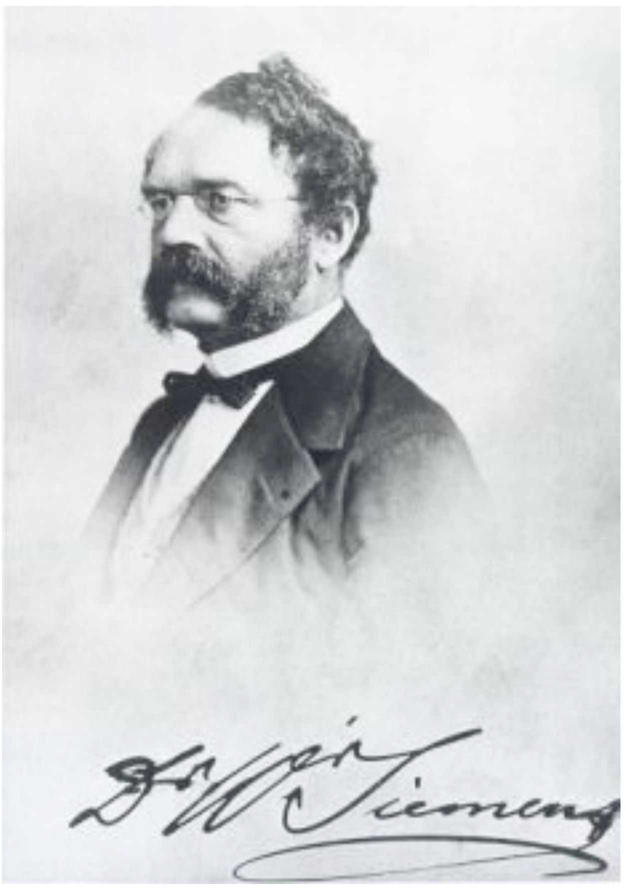
$$\frac{\partial I}{\partial t} = \left(\frac{\omega L}{2\pi N R} - 1\right)\frac{I}{\tau_{L/R}}$$

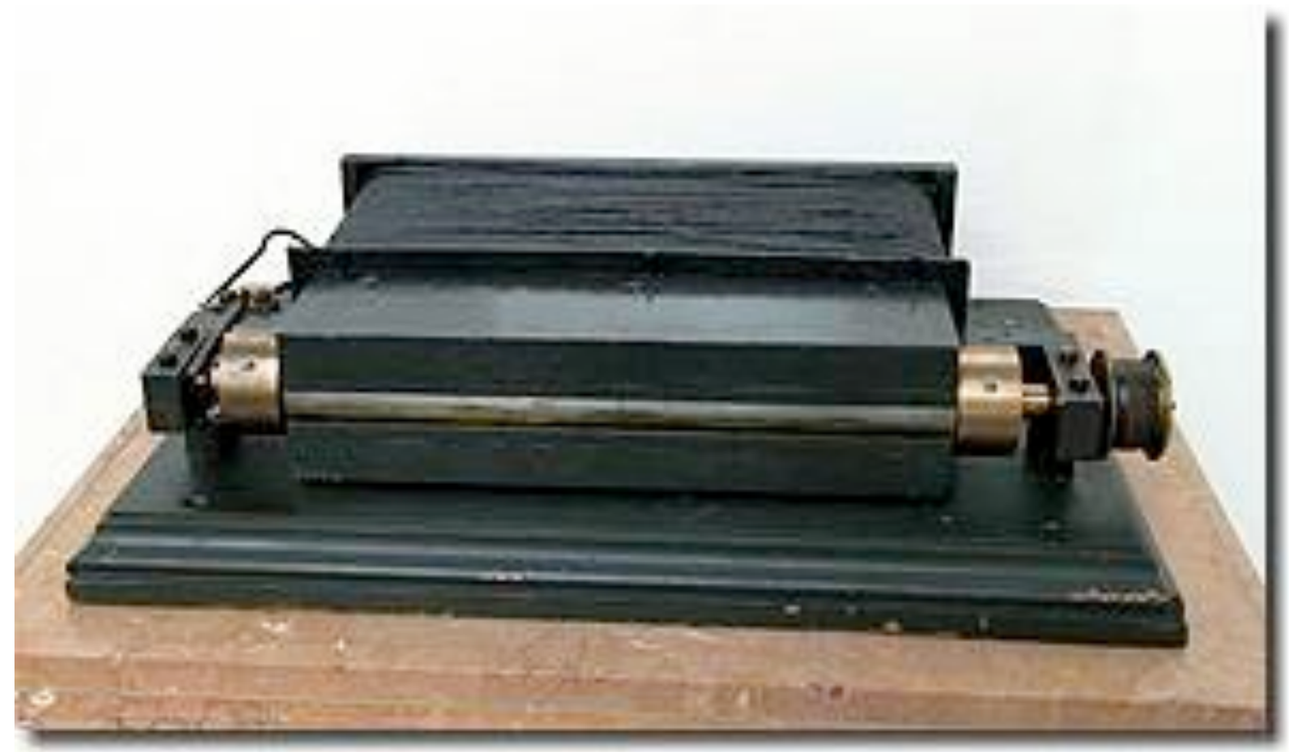
Equation of Motion

$$\frac{\partial \omega}{\partial t} = \frac{T_{ext}}{\mathcal{I}} - \frac{L}{2\pi N R \mathcal{I}} I^2$$



The self-excited generator of Werner von Siemens (1866)





The "dynamo electric principle"

MHD equations describe well the magnetic field evolution in a liquid metal or plasma

Ohm's law and Pre-Maxwell's Equations

$$\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{V} \times \mathbf{B} \right)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Induction Equation

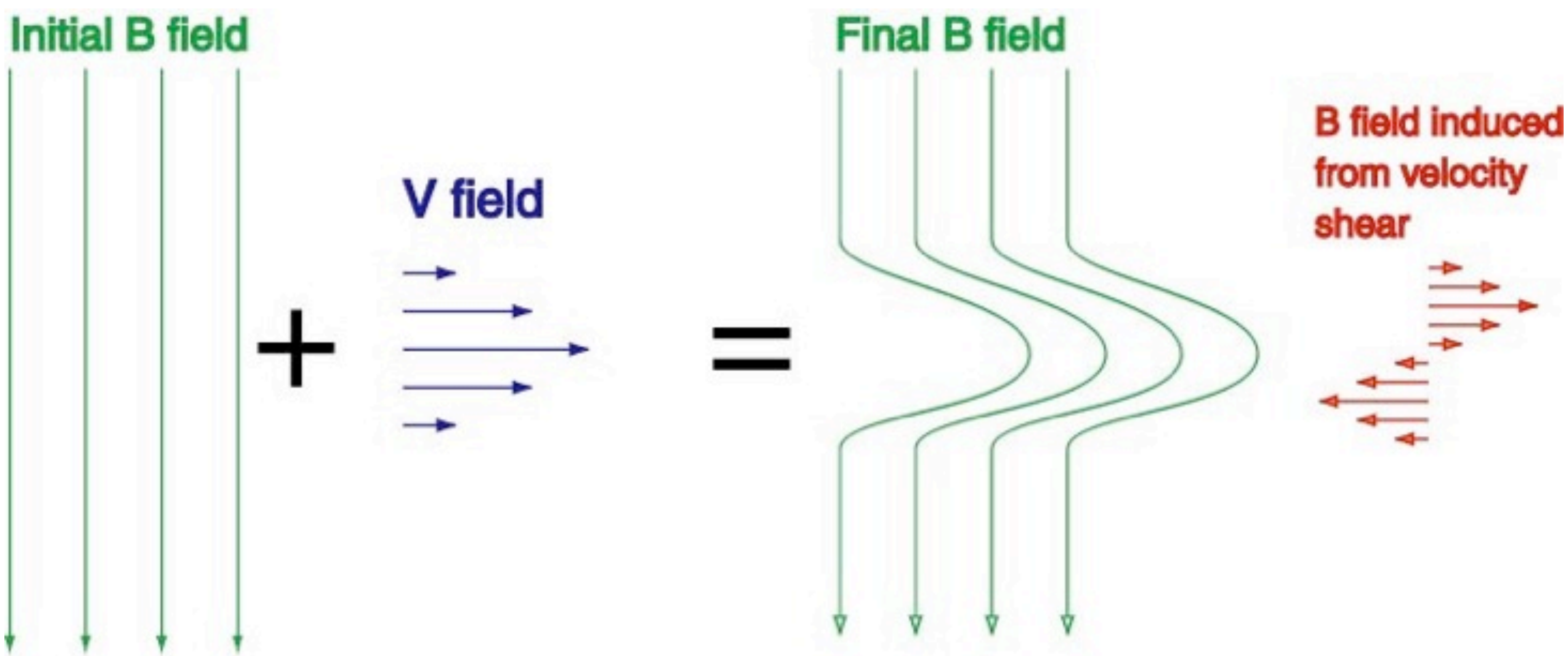
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\frac{\nabla \times V \times B}{\frac{1}{\mu_o \sigma} \nabla^2 B} \sim \mu_O \sigma L V_0 \equiv Rm$$

Equation of Motion

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{V} + F_{prop}$$

Fluid flow can amplify and distort magnetic fields when the magnetic Reynolds number is large

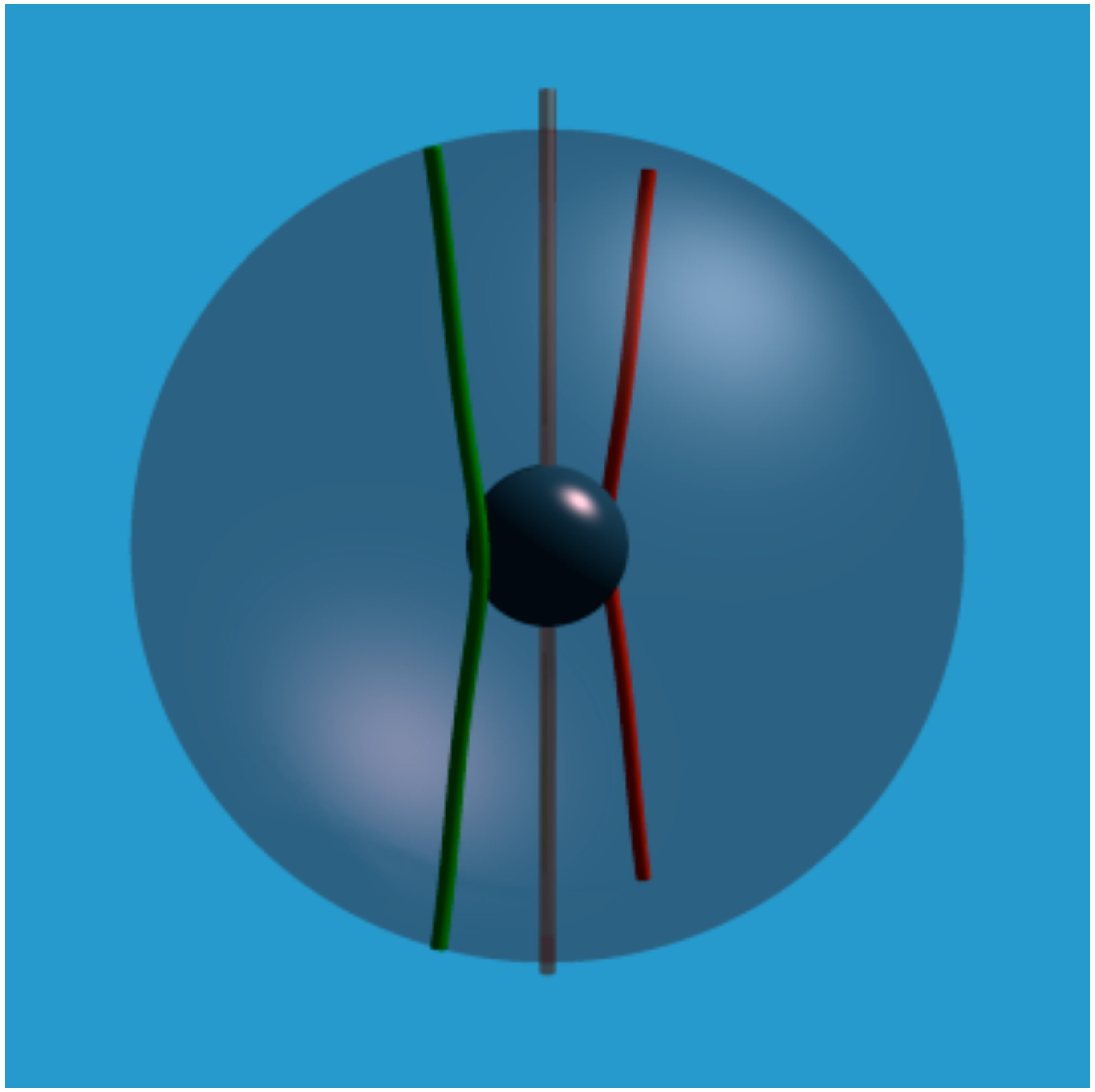


- transverse component of field is generated, amplifying the initial field
- finite resistance leads to diffusion of field lines

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \quad 0$$

Standard Model of an MHD dynamo

Step 1: dipole field can be converted into strong toroidal field



The "Ω effect"

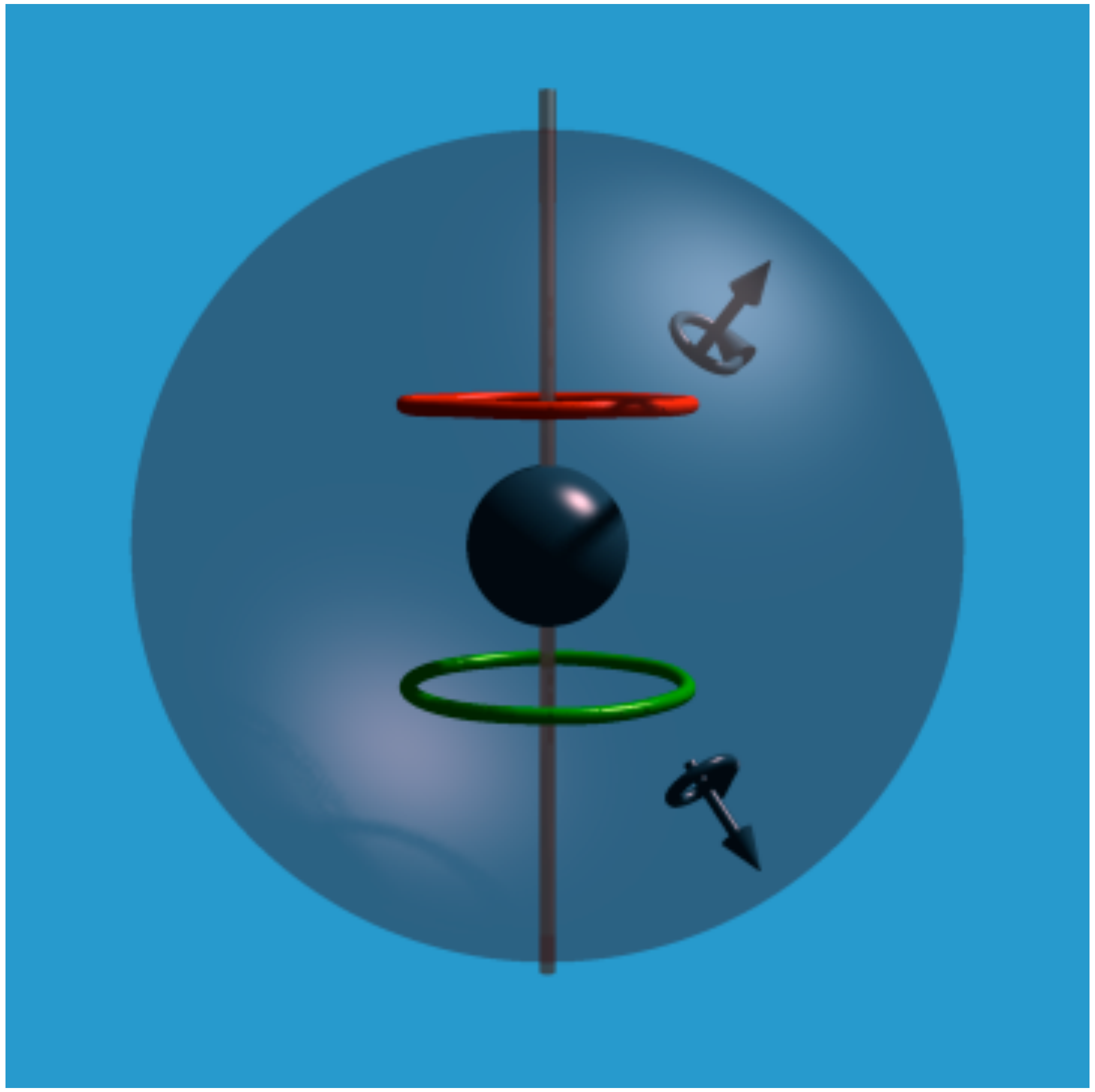
Cowling's Anti-Dynamo Theorem

When the magnetic field and the fluid motions are symmetric about an axis...no stationary dynamo can exist.

T.G. Cowling, *The magnetic fields of sunspots*, Monthly Notices Roy. Astron. Soc. **94** 39 (1933).

Standard Model of an MHD dynamo

Step 2: Nonaxisymmetric, helical flows convert toroidal field back into dipole



The "α effect"

Mean Field Electrodynamics

$$\boldsymbol{B} = \langle \boldsymbol{B} \rangle + \widetilde{\boldsymbol{b}}, \quad \boldsymbol{V} = \langle \boldsymbol{V} \rangle + \widetilde{\boldsymbol{v}}$$

$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

$$\mathcal{E} = \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle = \alpha \langle \boldsymbol{B} \rangle + \beta \nabla \times \langle \boldsymbol{B} \rangle$$

E.N. Parker, *Hydromagnetic dynamo models*, Astrophys. J. **122** 293 (1955)

Why do Experiments?

...in magnetohydrodynamics one should not believe the product of a long and complicated piece of mathematics if it is unsupported by observation.

Enrico Fermi

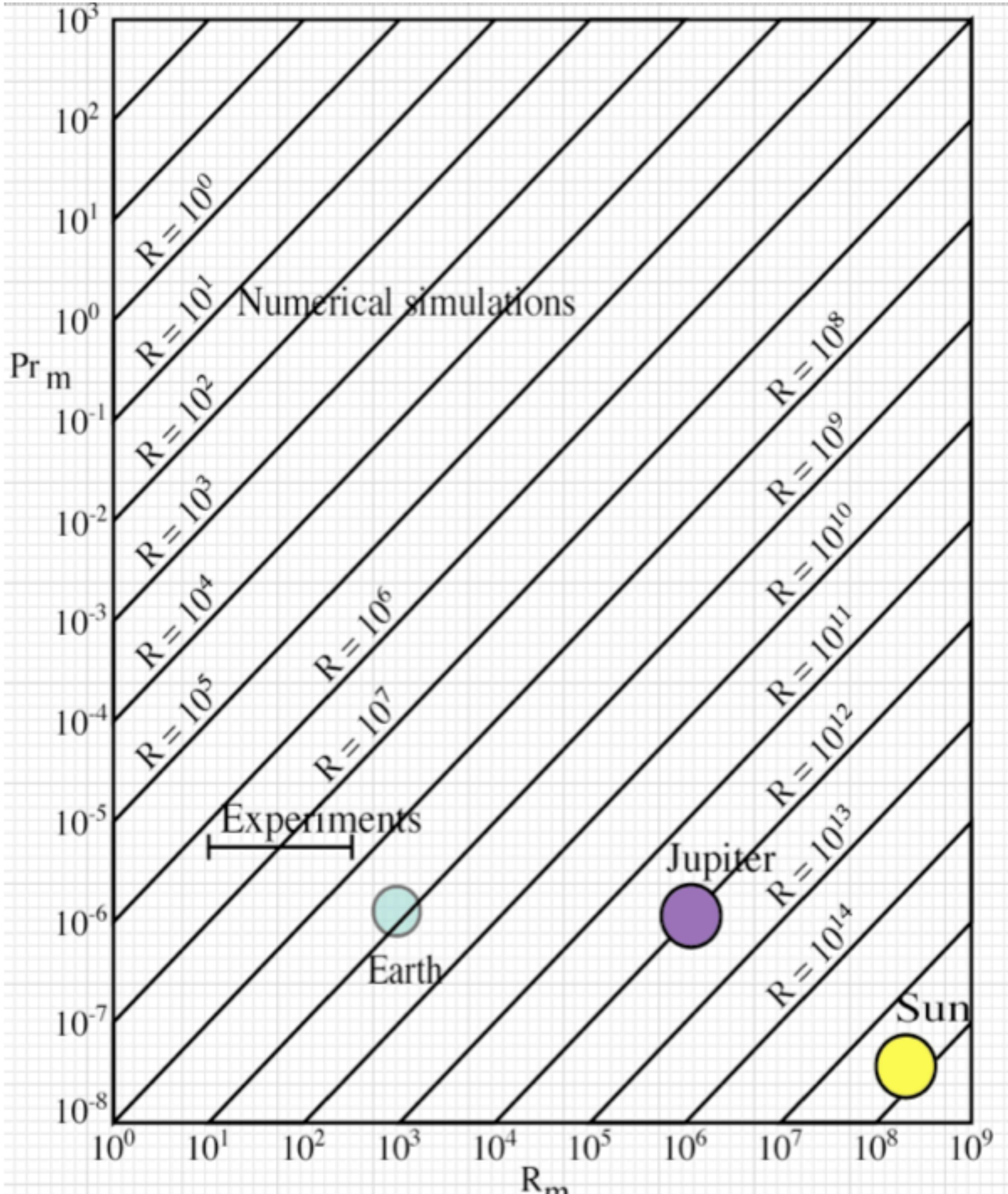
Why not Direct Numerical Simulations?

- viscous dissipation scale : $\ell_\nu = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$
- magnetic dissipation scale : $\ell_\eta = \ell_\nu \, P_m^{-3/4}$
- dynamical dynamo : $R_m \sim 100 \quad (Re \sim 10^8)$
- scale range : $\frac{L}{\ell_\nu} = Re^{3/4} \sim 10^6$

=> (10,000,000)^3 DNS,
that will run for 100 magnetic diffusion times, i.e. 10,000 large eddy turnover times.

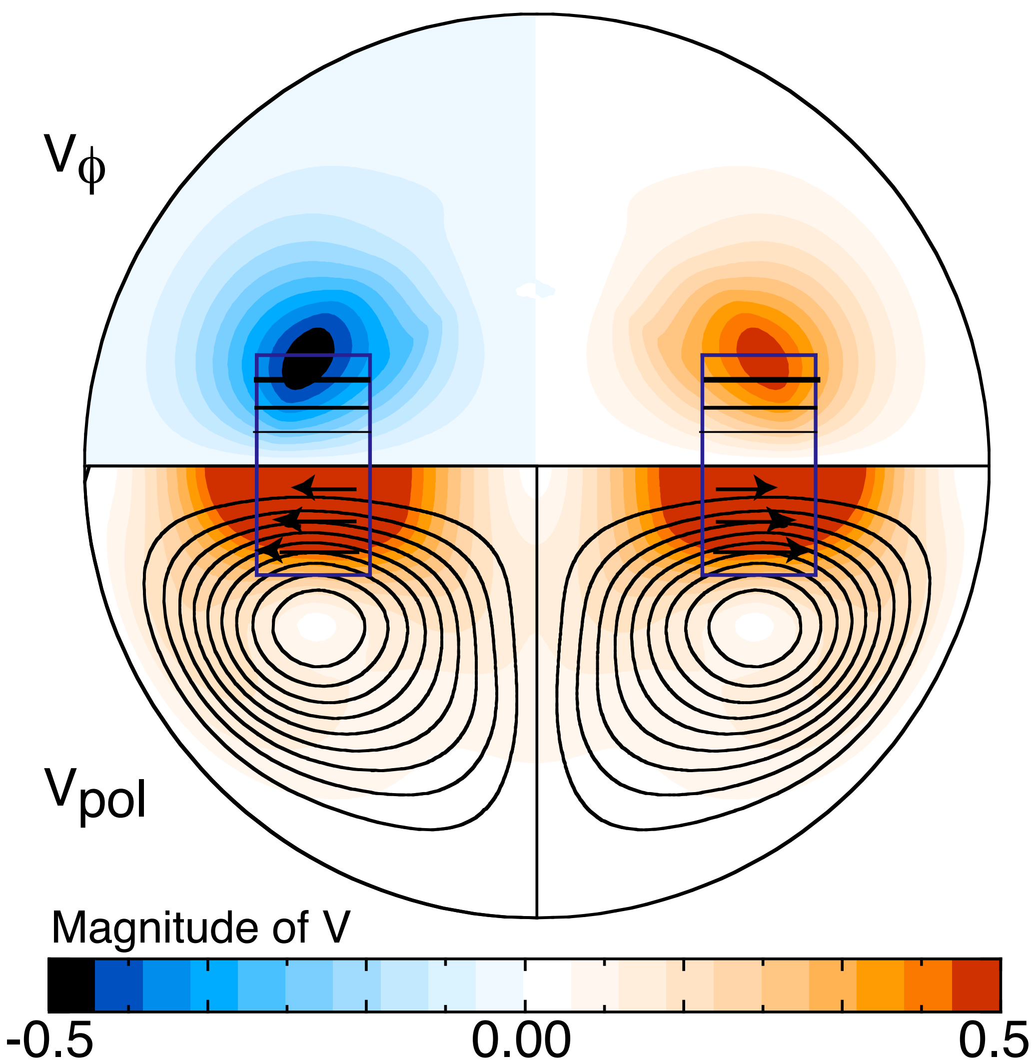
$$T_B = L^2/\eta = Rm(L/U)$$

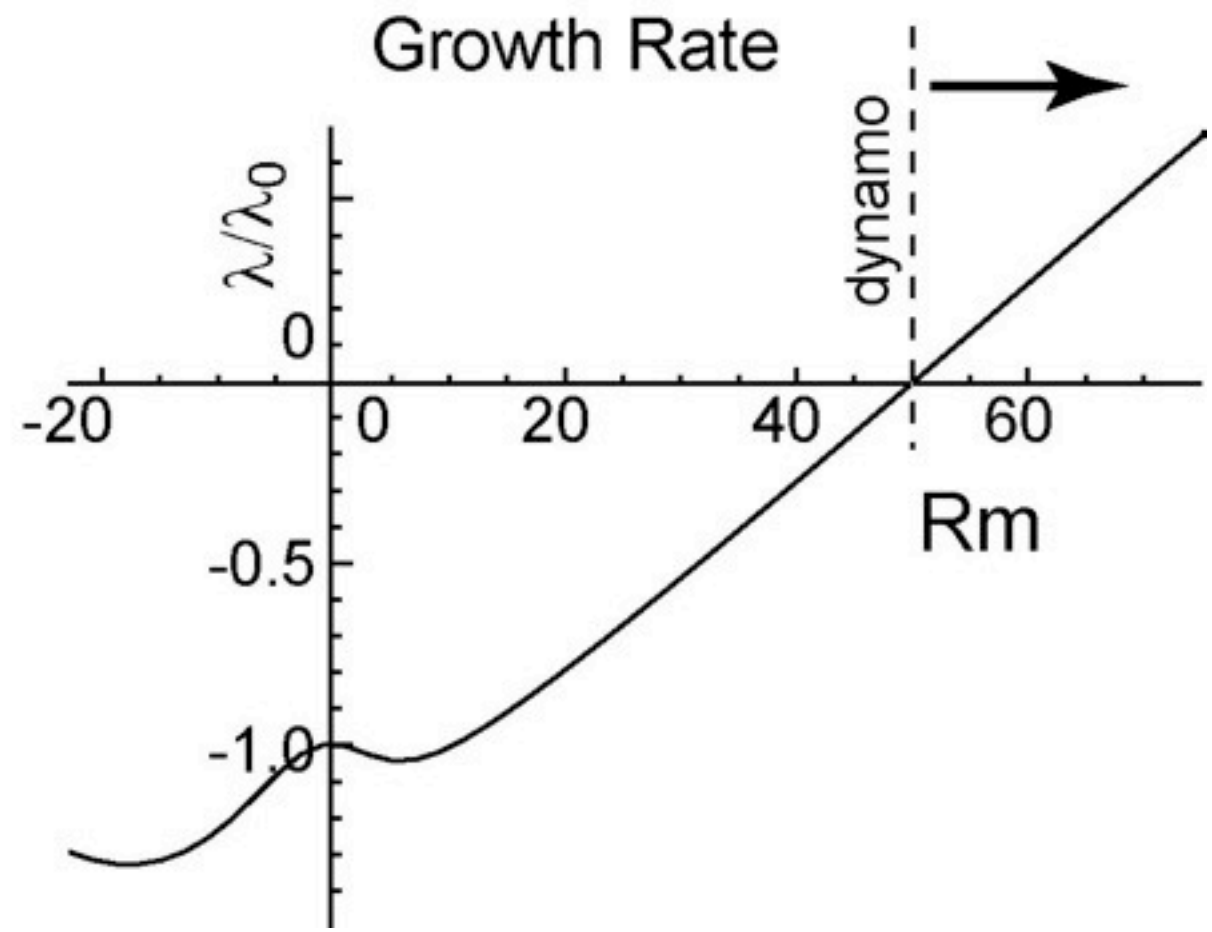
\+ boundary conditions



Pr_m
R_m
$R = 10^0$
$R = 10^1$
$R = 10^2$
$R = 10^3$
$R = 10^4$
$R = 10^5$
$R = 10^6$
$R = 10^7$
$R = 10^8$
$R = 10^9$
$R = 10^{10}$
$R = 10^{11}$
$R = 10^{12}$
$R = 10^{13}$
$R = 10^{14}$
Numerical simulations
Experiments
Earth
Jupiter
Sun

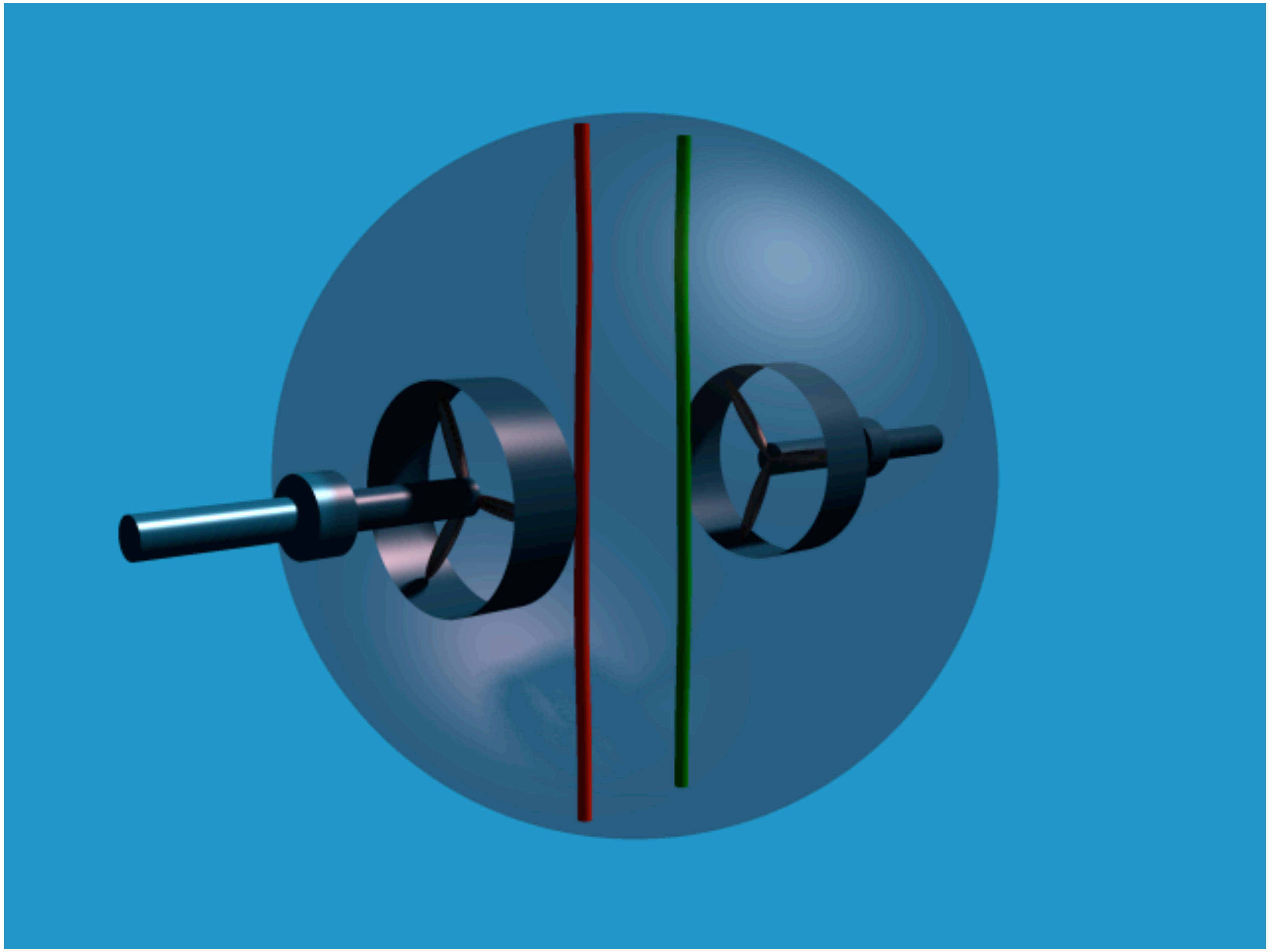
This simplest possible self-exciting flow: a two vortex flow with Rm_{crit}~50

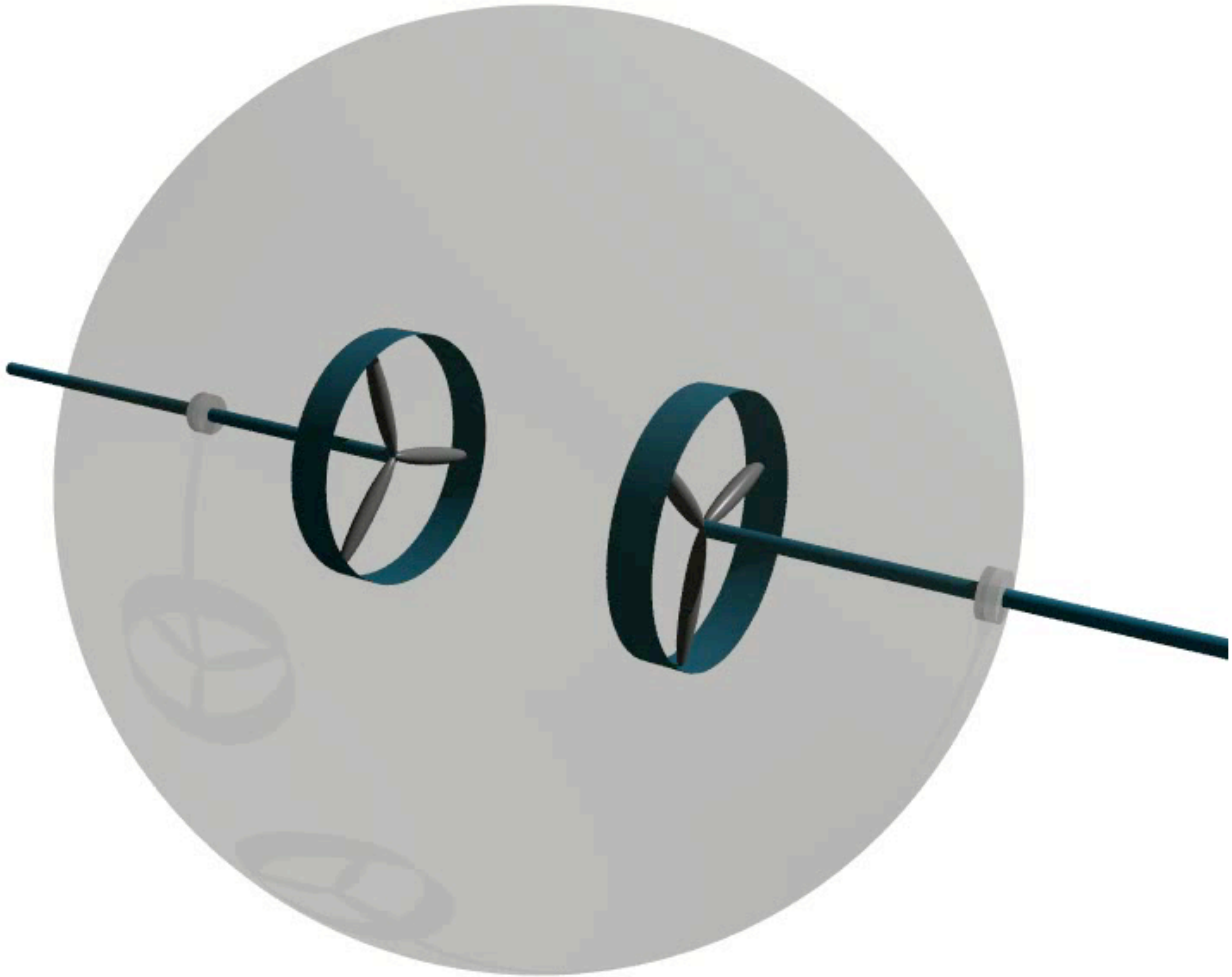




Dudley and James, *Time-dependent kinematic dynamos with stationary flows*, Proc. Roy. Soc. Lond. A. **425** 407 (1989).

Dynamo is of the stretch-twist-fold type: field line stretching and reinforcement leads to dynamo





Laboratory Constraints

- Power requirement

$$P = \frac{\delta E_V}{\delta t} \sim \frac{(\rho L^3) U^2}{L/U} = \rho L^2 U^3$$

$$R_M = \mu_0 \sigma L U$$

- In terms of control parameter

$$R_M = \mu_0 \sigma \left(\frac{PL}{\rho} \right)^{1/3}$$

- Consequences:

  No convective dynamo in the lab

  R_M=1 is a large number

  Fluid = liquid sodium

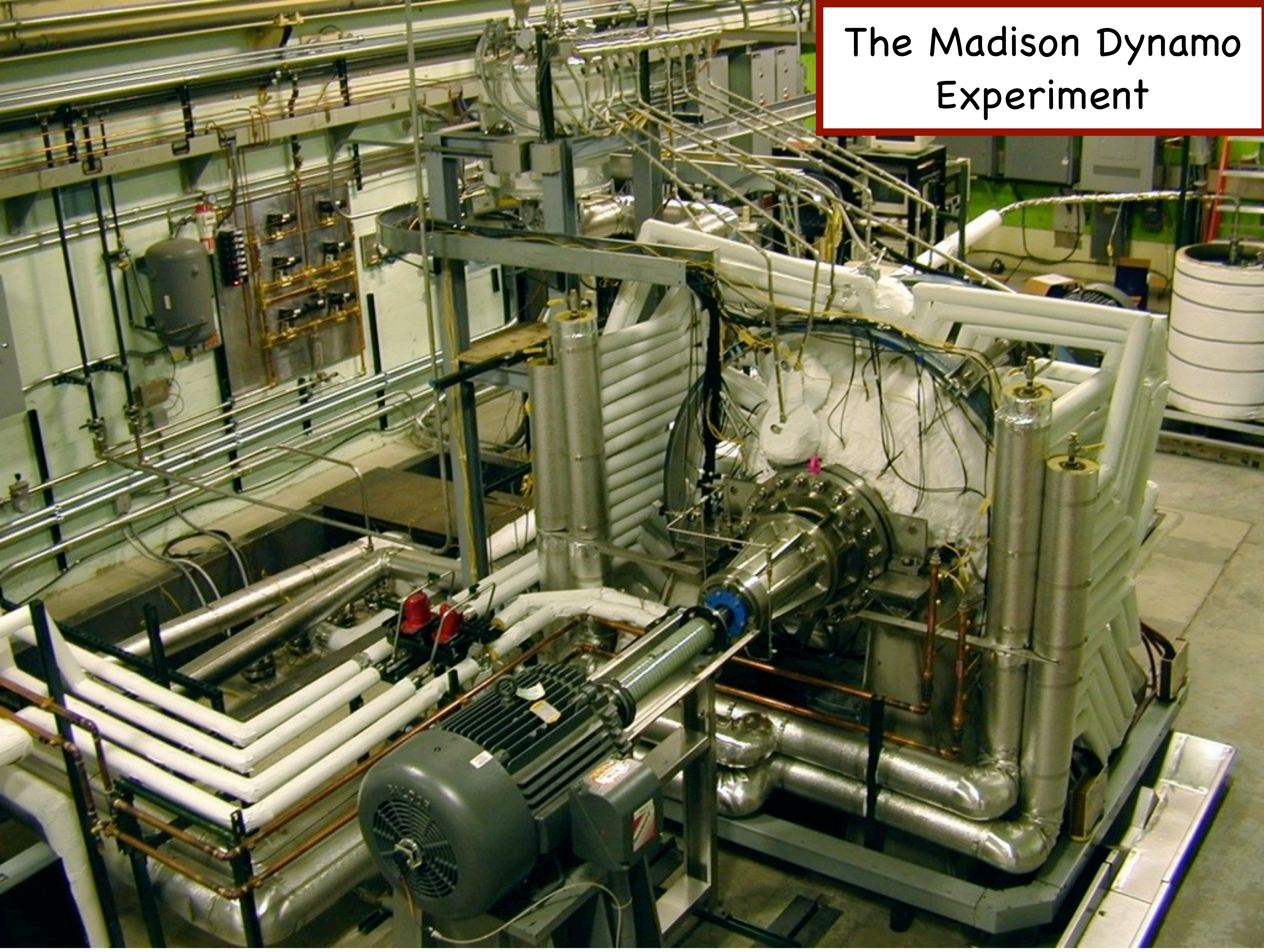
  Large power input (x 100 kW) and cooling

Dynamo and MRI Process

1. Begin with small magnetic field ($C \ll 1$)
2. Stir until $Rm > Rm_{crit}$
3. Magnetic field spontaneously created

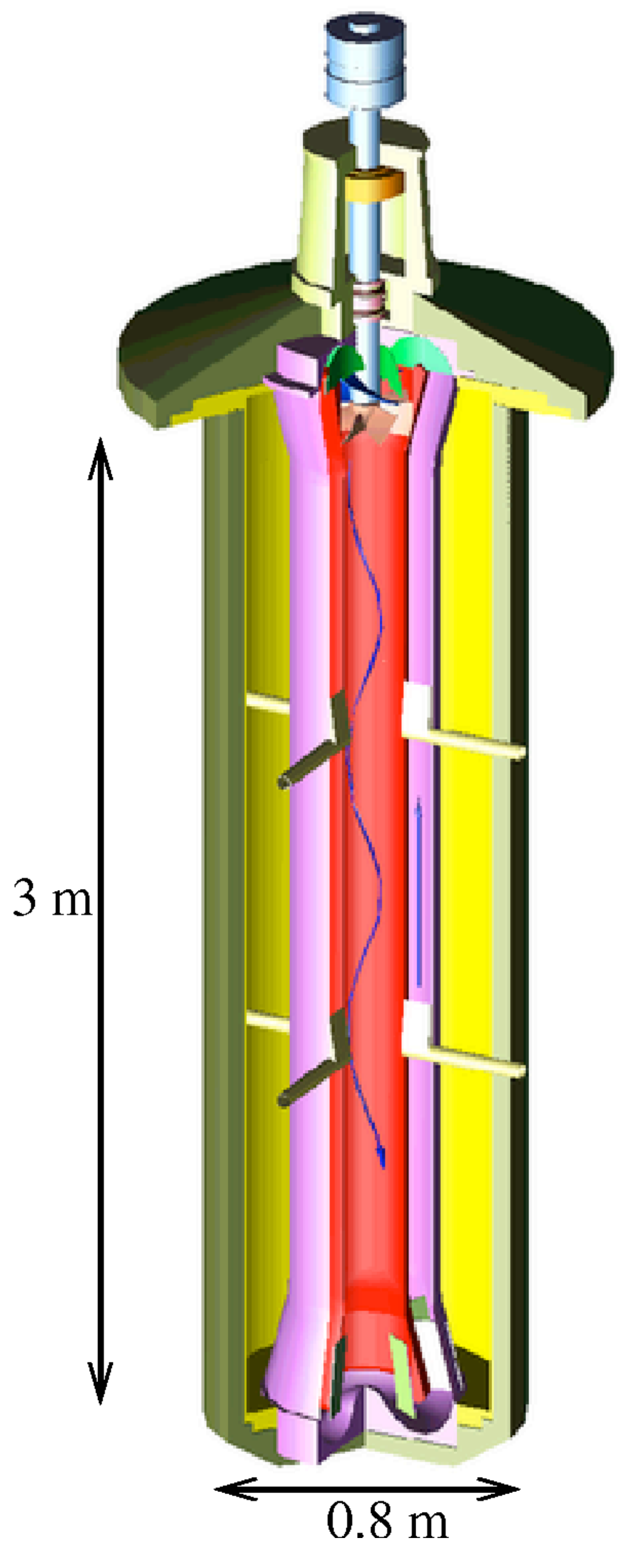
Challenge: to create a large, highly conducting, unmagnetized, fast flowing laboratory plasma for study

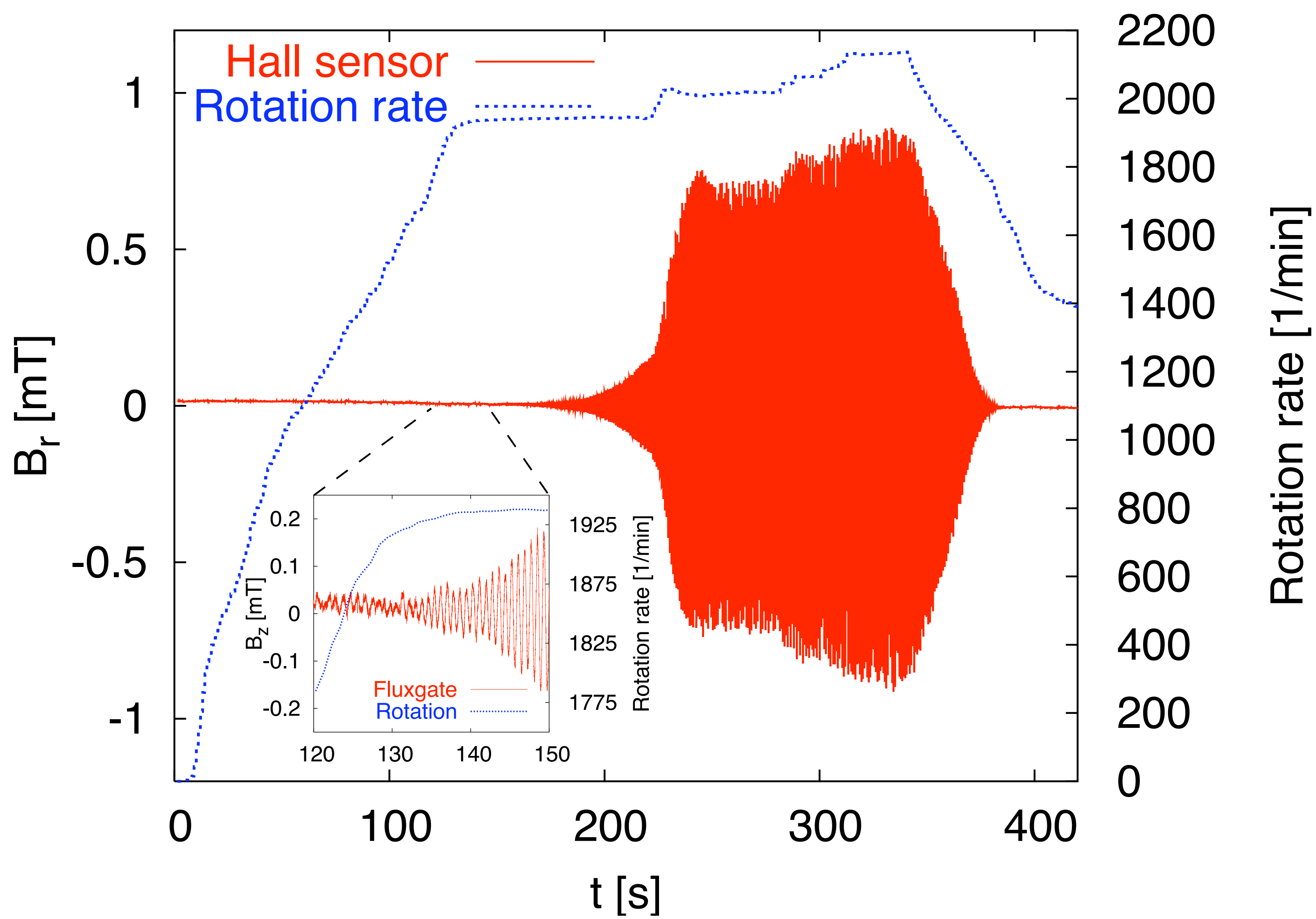
- -difficult to stir a plasma
- -need some confinement for plasma to be hot



The Madison Dynamo Experiment

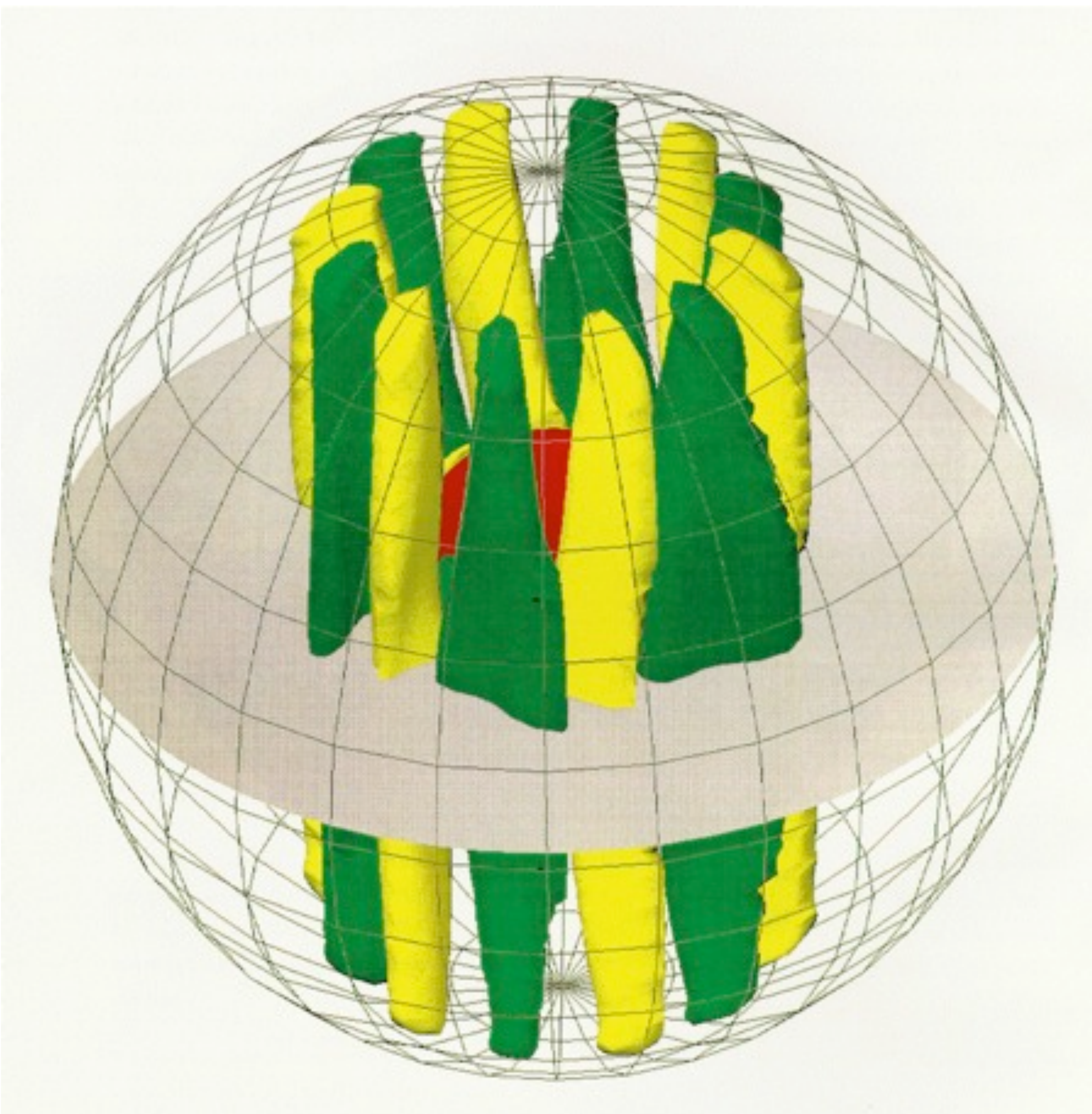
Riga experiment successfully self-generated a dynamo in 2001 (single helical vortex)





A. Gailitis, et al.,*Magnetic Field Saturation in the Riga Dynamo Experiment*, Phys. Rev. Lett. **86** 3024 (2001)

The Karlsruhe Dynamo: small-scale helical flows generate large-scale magnetic field

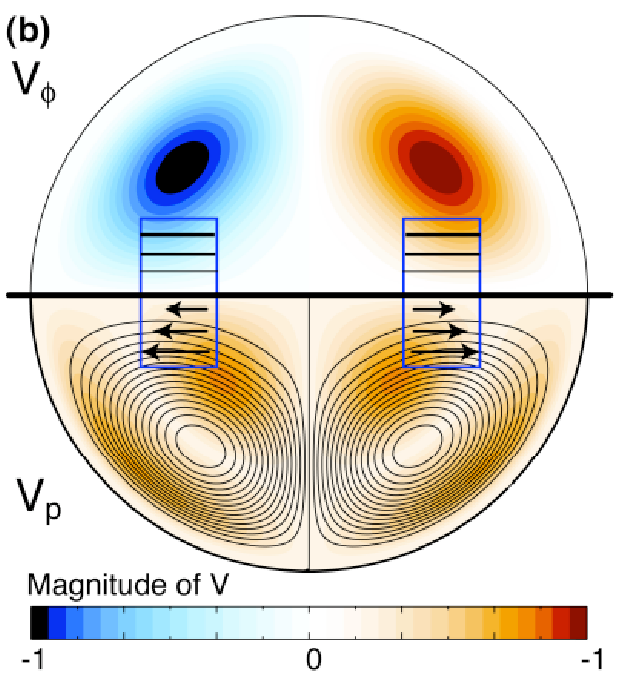


Muller and Stieglitz, *Experimental demonstration of a homogeneous dynamo*, Phys. Fluids **13** 561 (2001).

Small Prandtl number of sodium (Rm<<Re) implies turbulence

- Direct Numerical Simu
  of MHD equations witl
  mechanical forcing

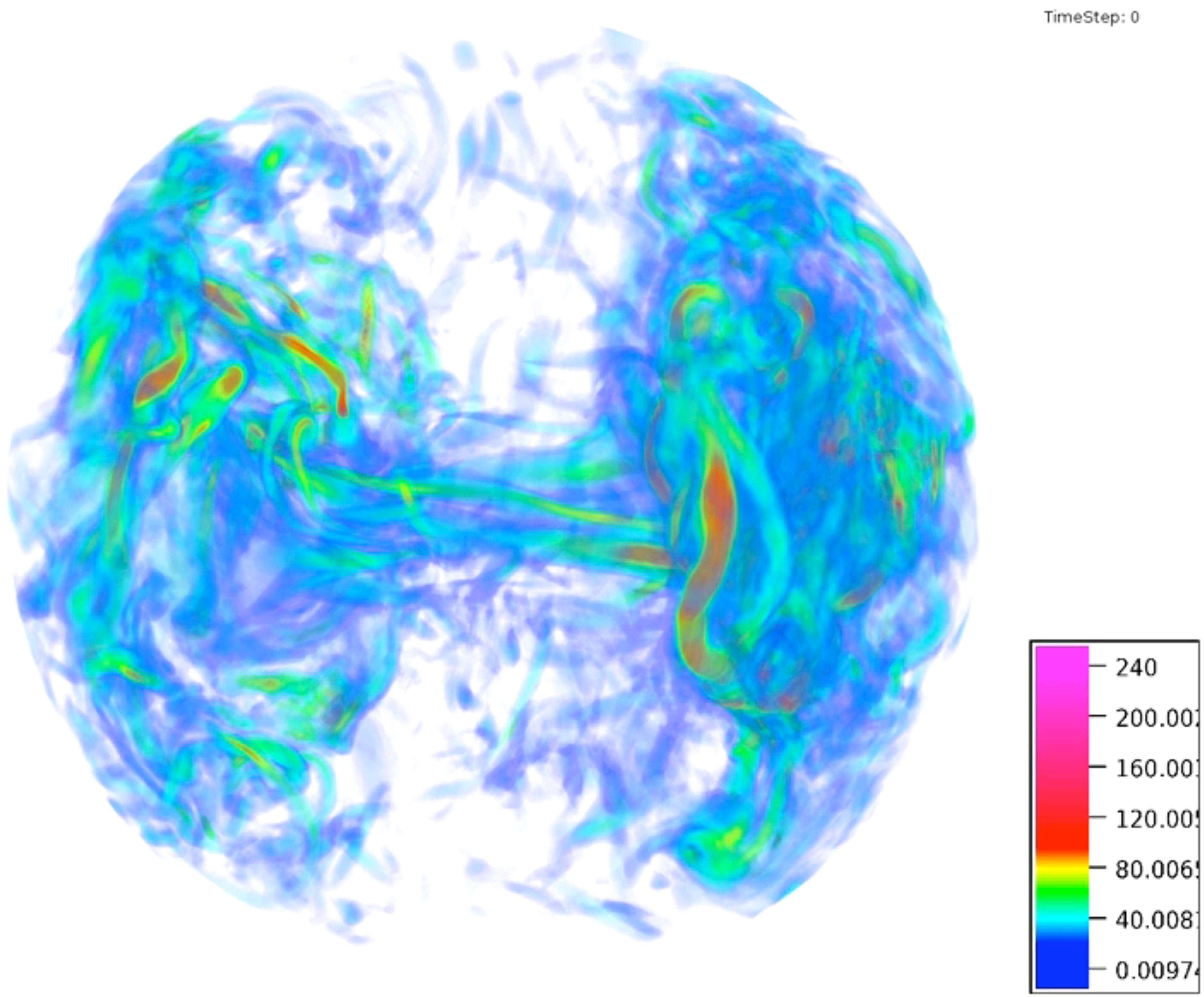
- Re=2200; turbulence
  Re>450



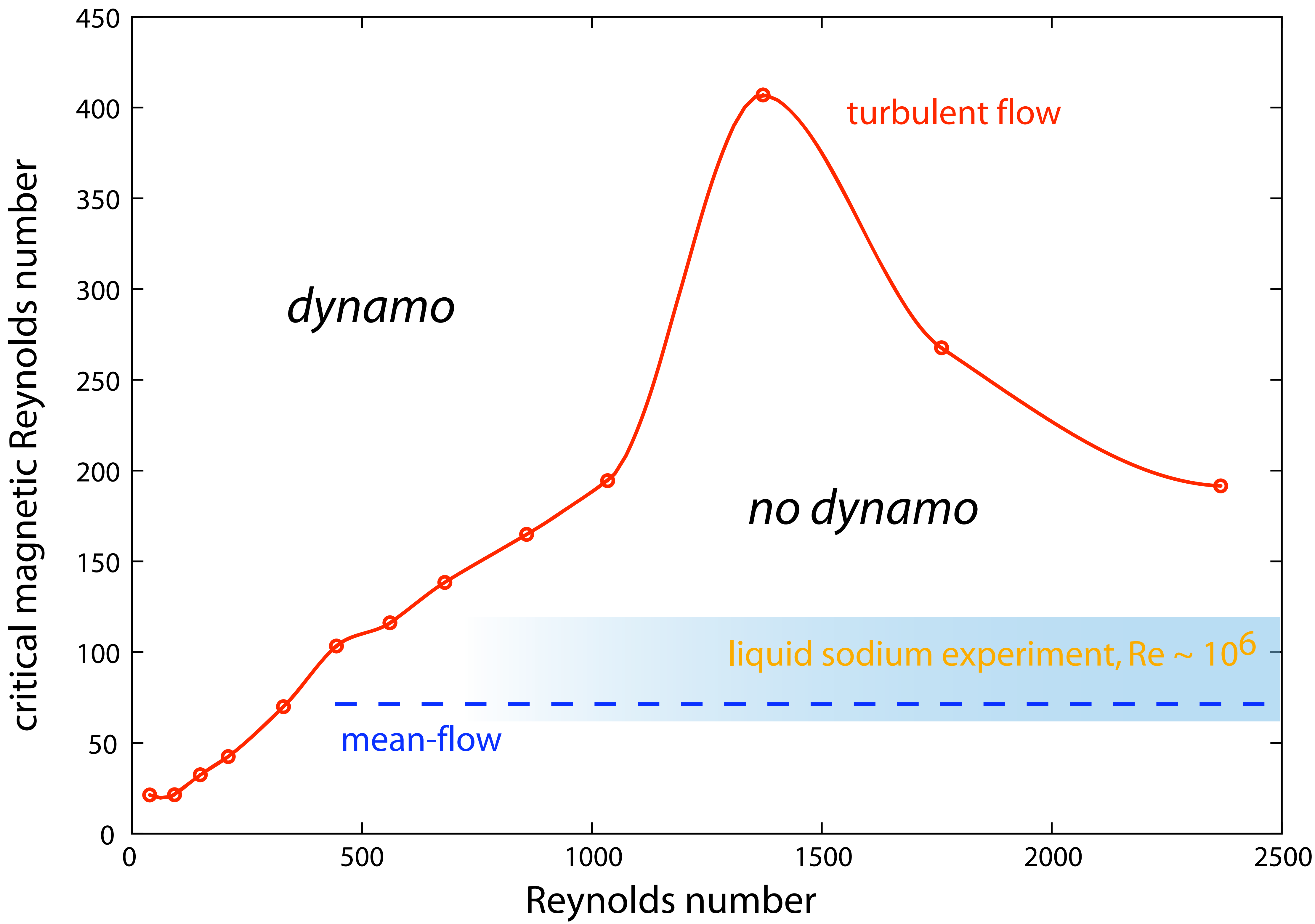
$$F_\phi(\rho, z) = \rho^2 \sin(\pi \rho b)$$

$$F_z(\rho, z) = -\epsilon \sin(\pi \rho c)$$

$$0.25a < |z| < 0.55a,\ \rho < 0.3$$

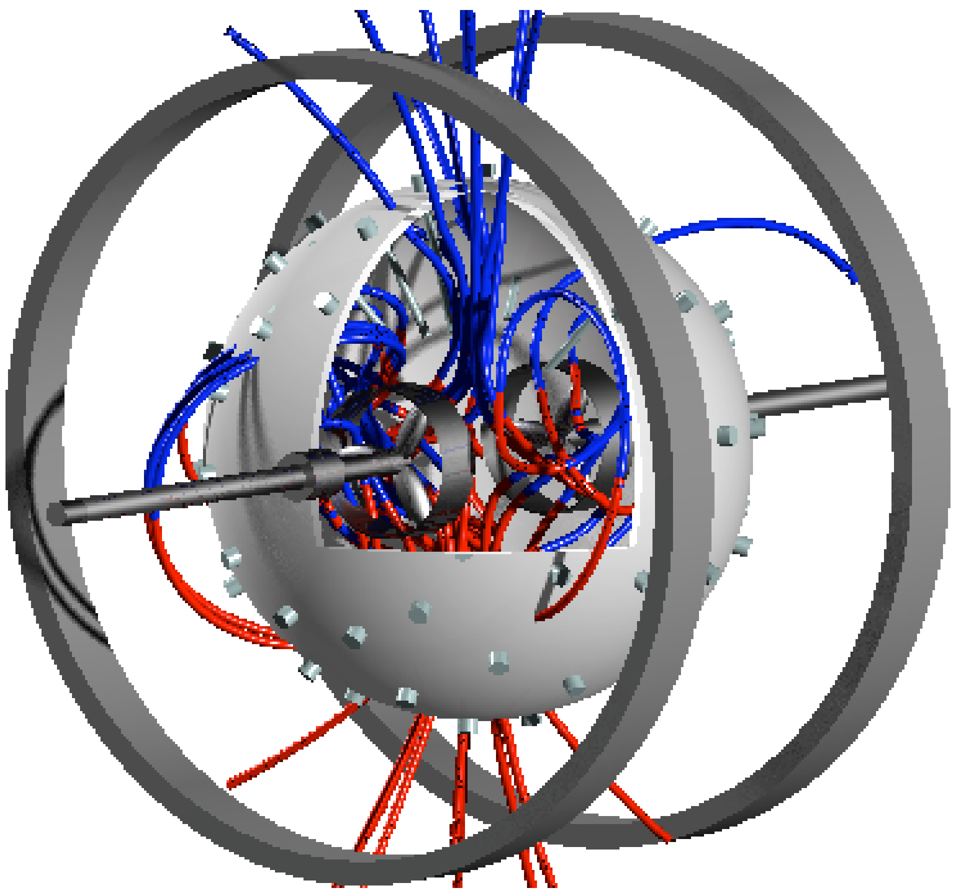


Turbulence, in the two-vortex dynamo, increases Rm_{crit} by factor of 5 (DNS)

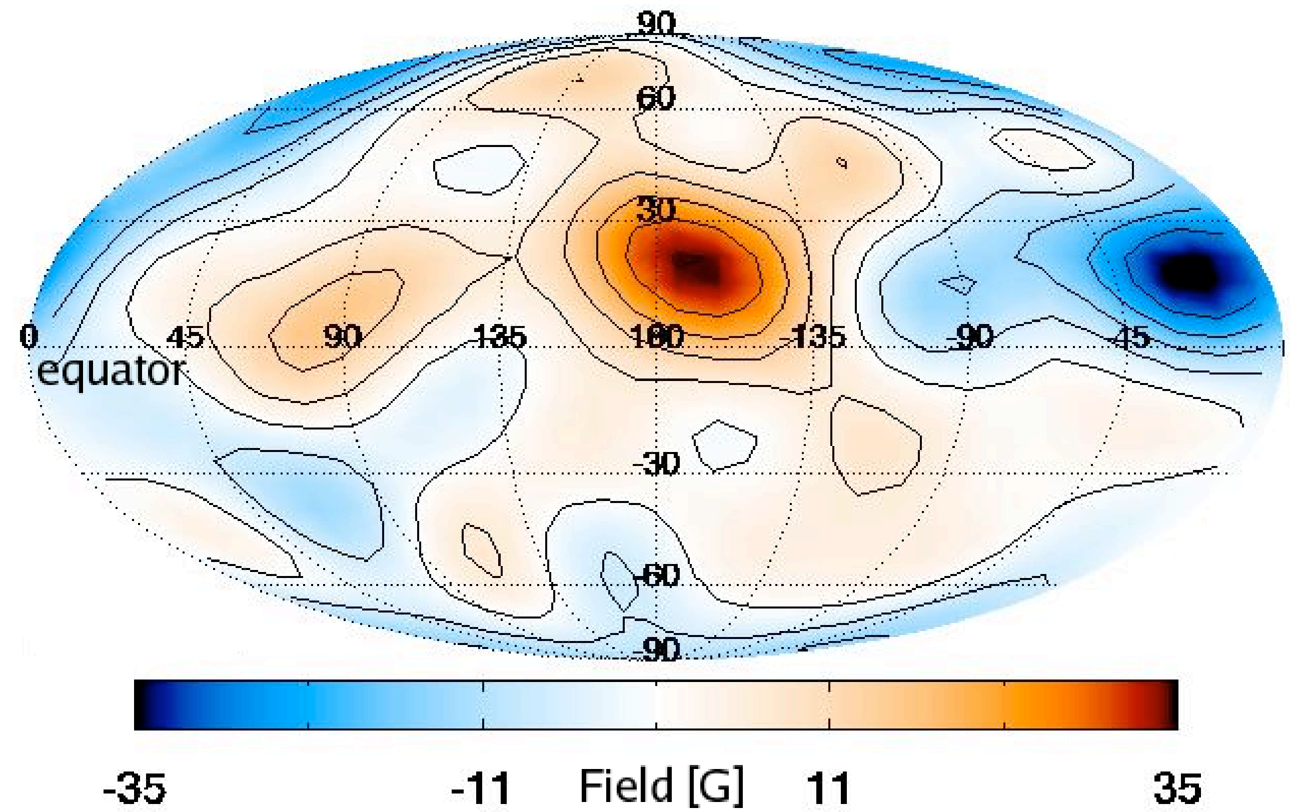


Excited eigenmode has structure similar to that predicted for the mean-flow, self-generated dynamo

Predicted

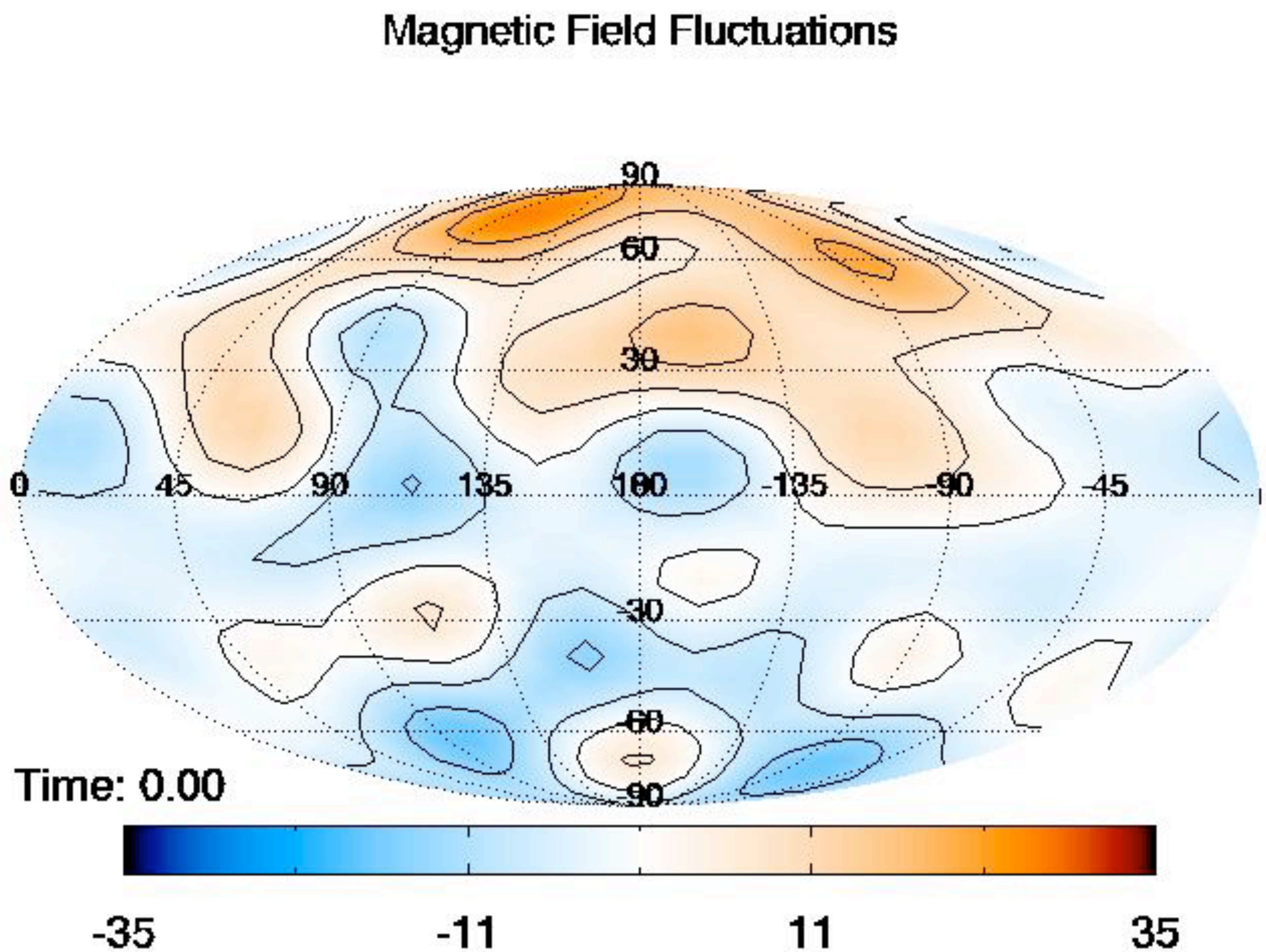


Observed



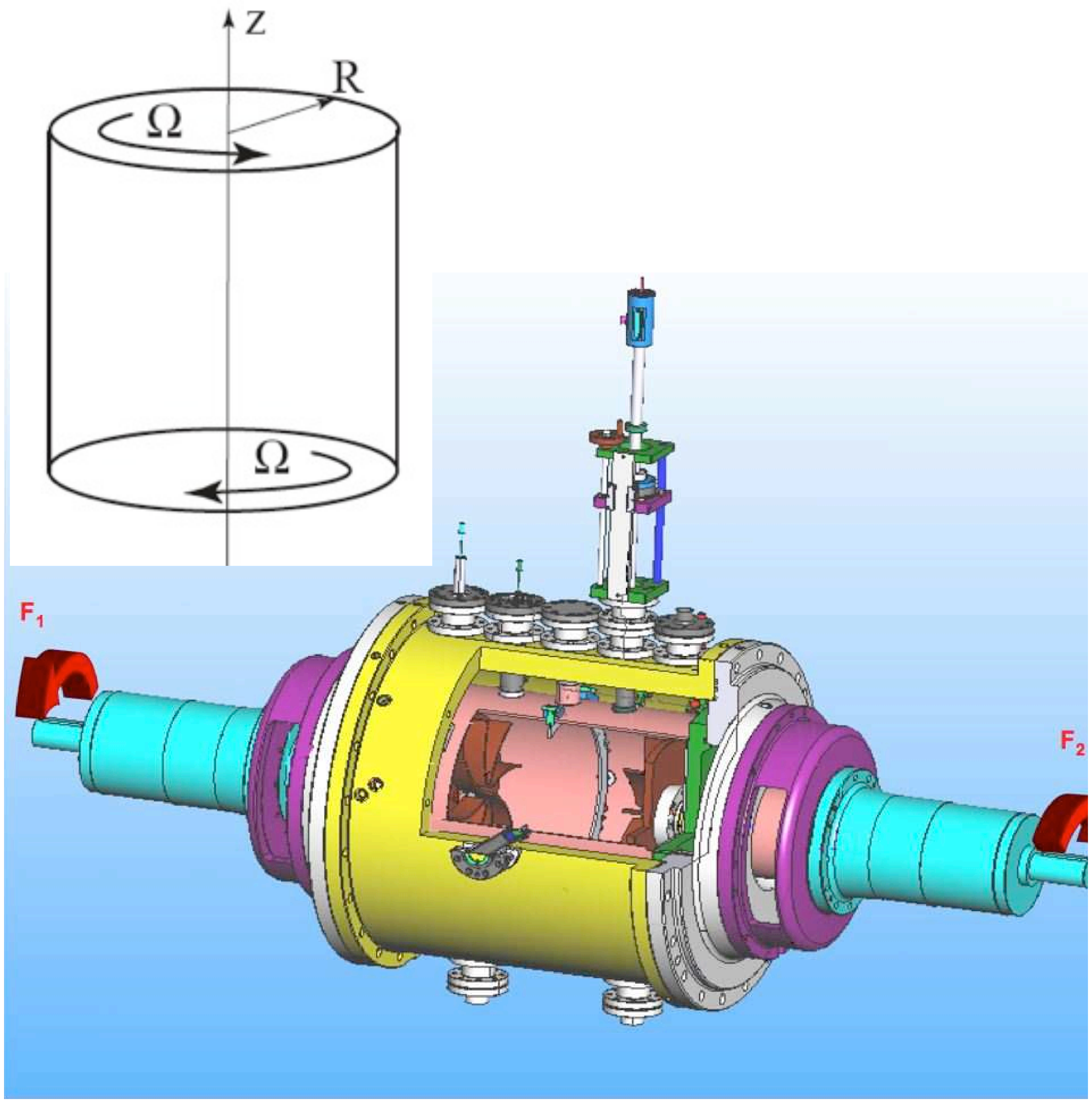
Nornberg, Spence, Jacobson, Kendrick, and Forest, *Intermittent magnetic field excitation by a turbulent flow of liquid sodium*, Phys. Rev. Lett. **97** 044503 (2006).

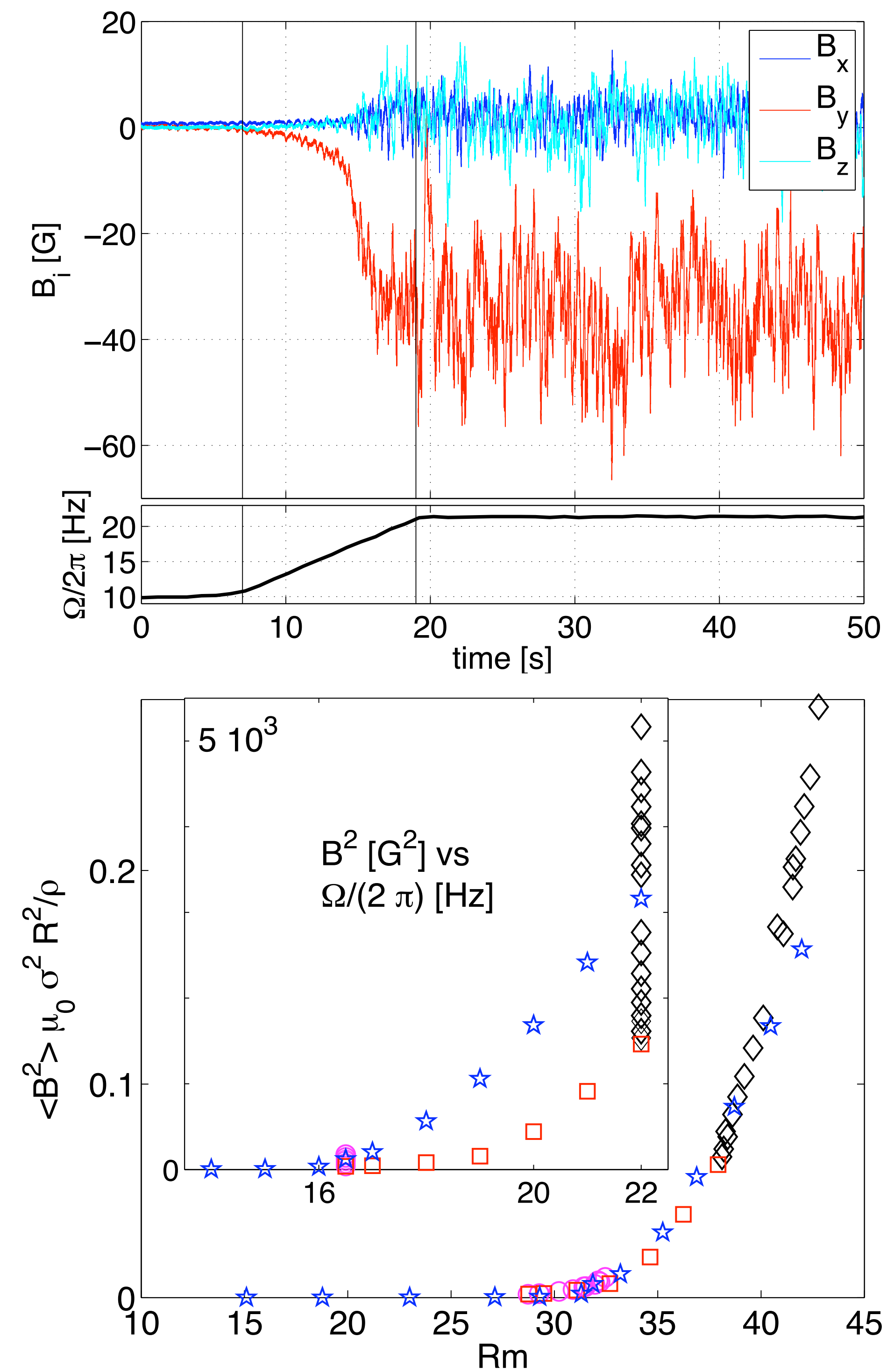
Intermittent equatorial dipole is observed on surface of sphere



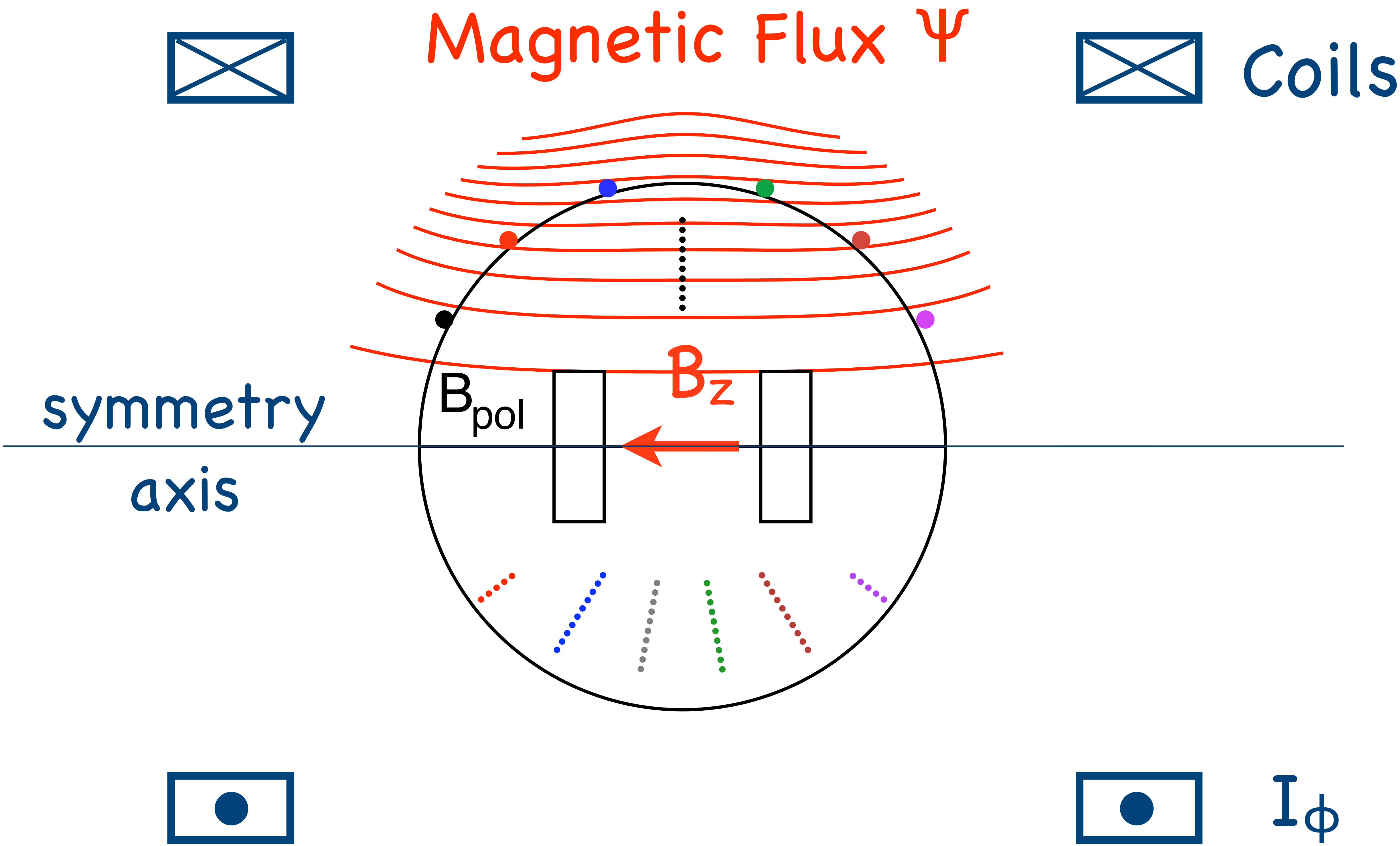
Equator

France: The VKS-II Experiment in Cadarache
France recently self-excited using iron impellers

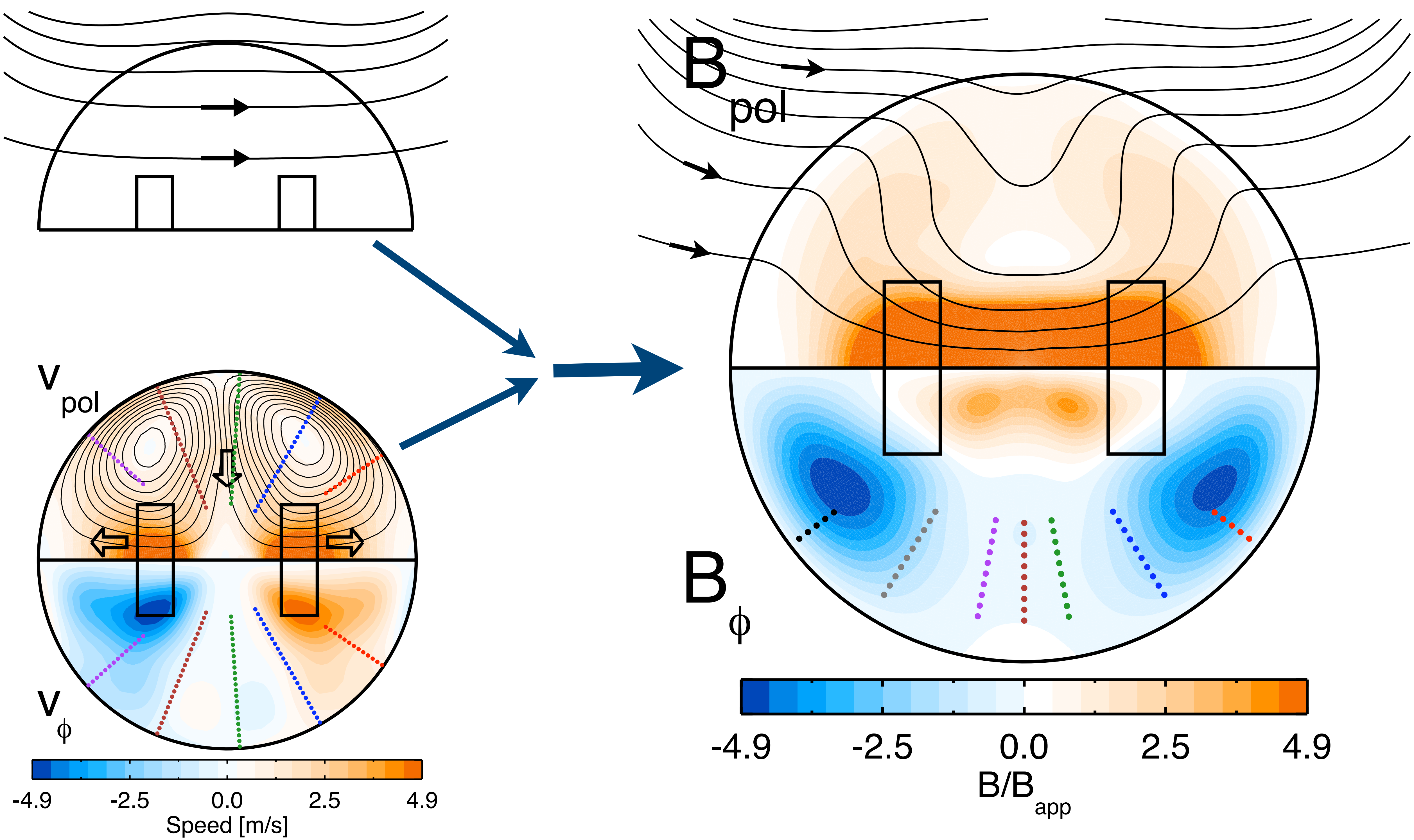




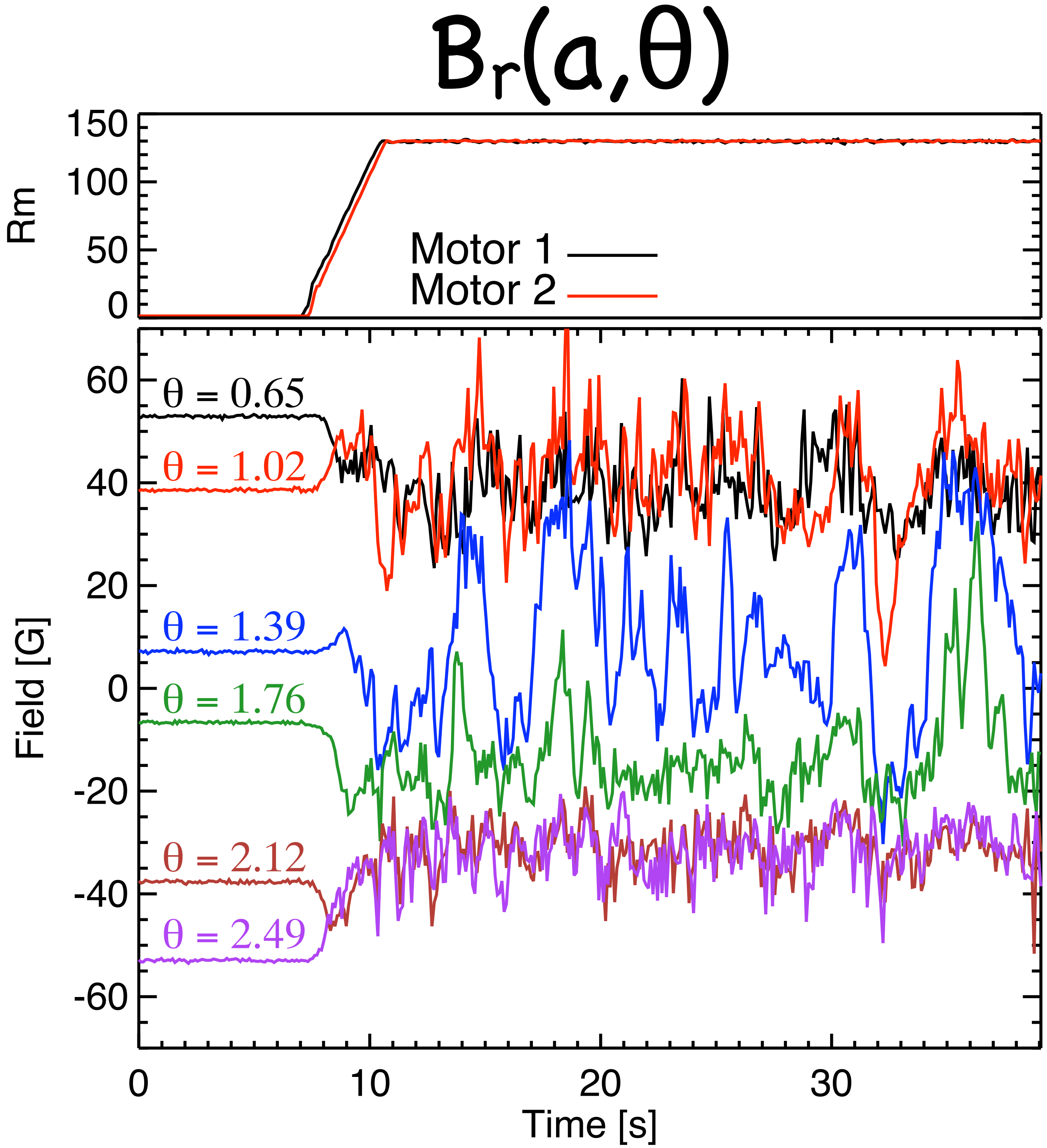
Experiment: apply axisymmetric poloidal seed field to sphere and measure induced magnetic fields

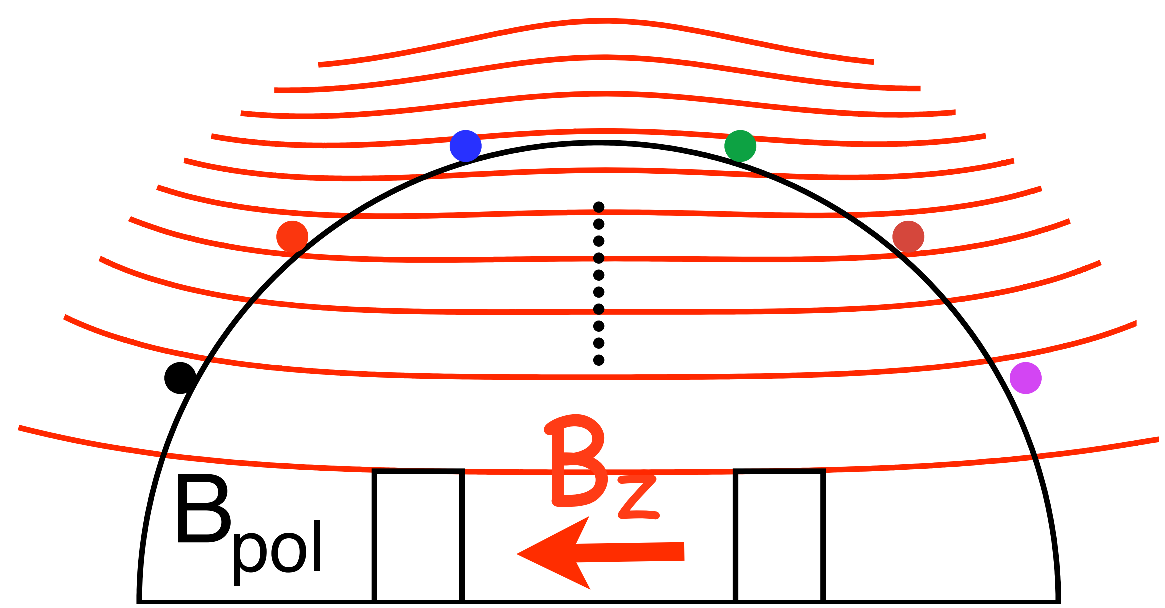


Predicted total magnetic fields



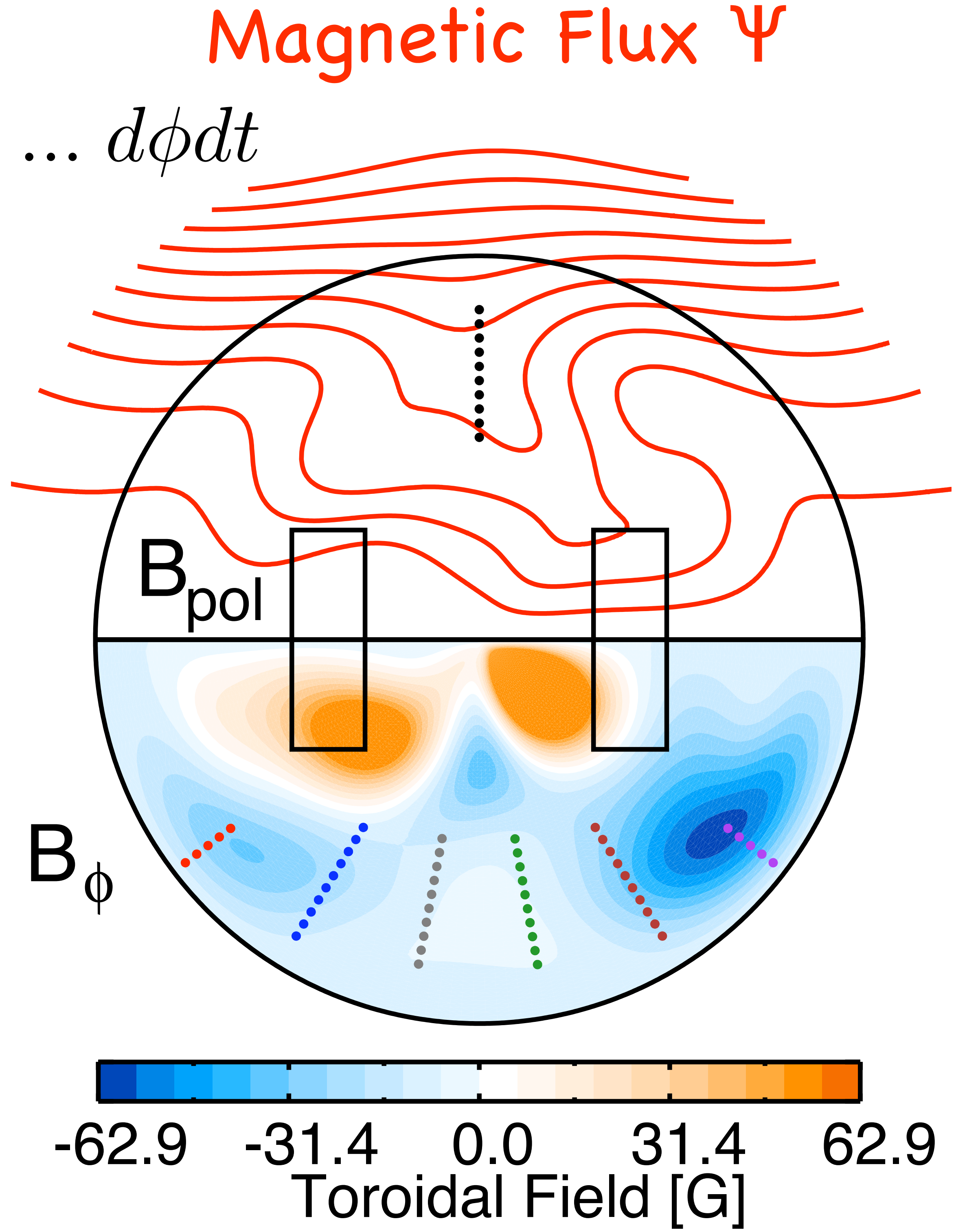
Large scale (mean) and small scale (turbulent) magnetic fields are generated by liquid sodium flows





The time-averaged, axisymmetric part of the magnetic field shows poloidal flux expulsion and a strong Ω effect

$$\langle \ldots \rangle \equiv \frac{1}{2\pi T} \int^{T} \int_{0}^{2\pi} \ldots \, d\phi dt$$



Question: Does a simple Ohm's law make sense?

Measured in Sodium Experiment

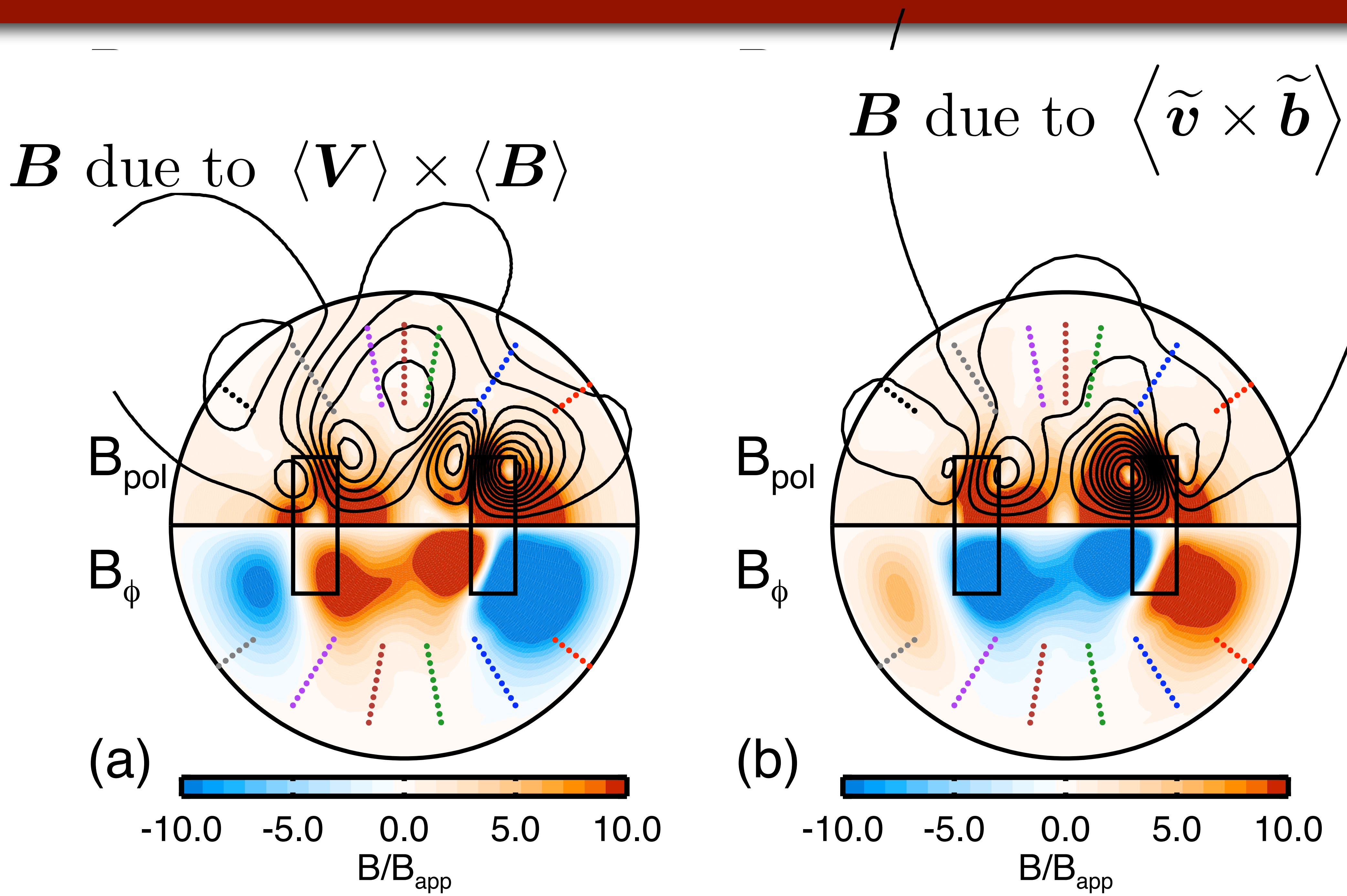
$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

Measured by LDV

Fluctuation Driven Currents

$$\langle ... \rangle \equiv \frac{1}{2\pi T} \int^{T} \int_{0}^{2\pi} ... \, d\phi dt$$

Field can be separated into mean-flow, mean-field driven currents and fluctuation generated currents



Spence, Nornberg, Jacobson, Parada, Kendrick, and Forest, *Turbulent Diamagnetism in Flowing Liquid Sodium*, Phys. Rev. Lett. **98** 164503 (2007).

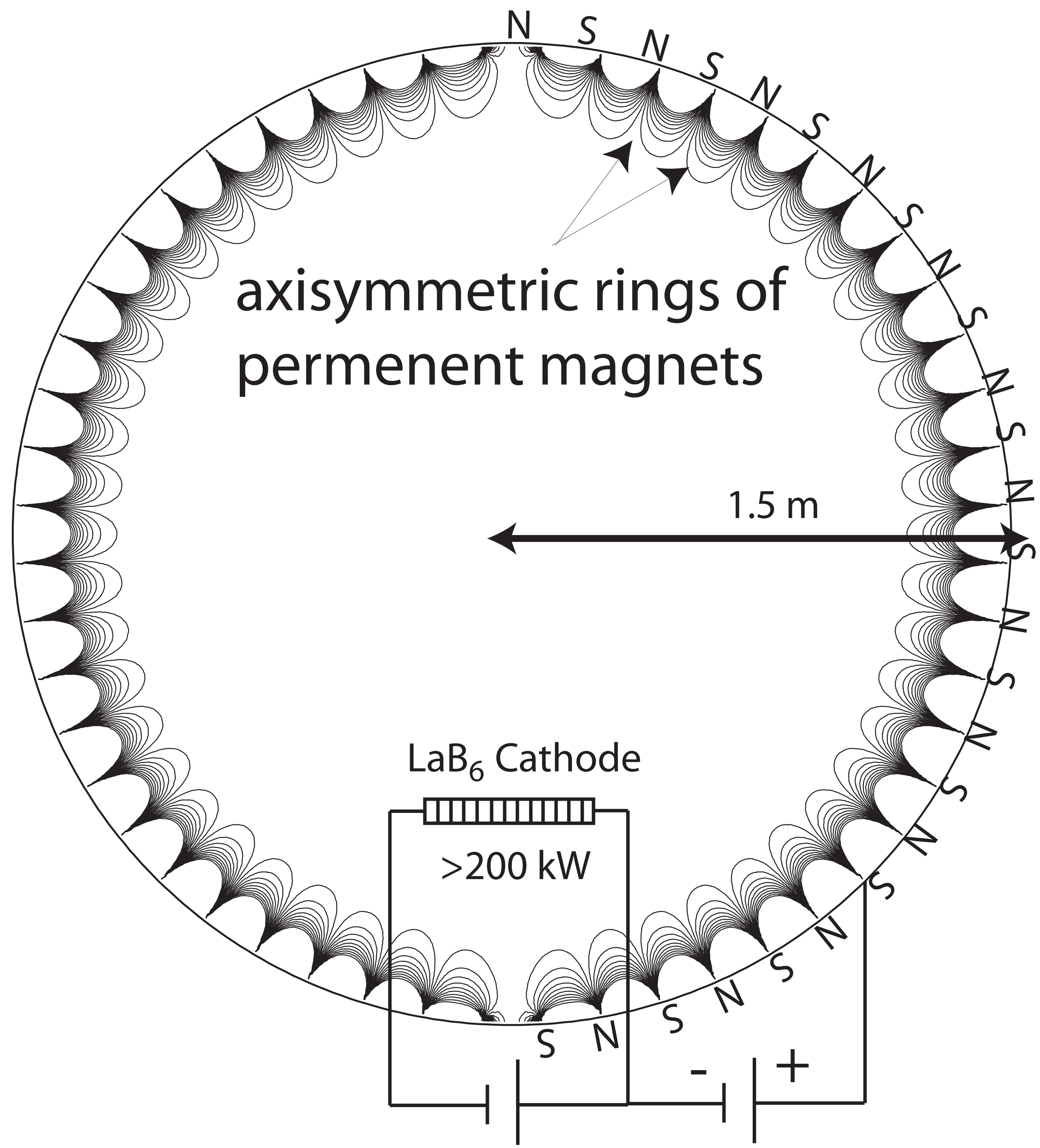
Future Directions

Liquid Metal Experiments are limited: the next frontier for experimental dynamo studies should be plasma based

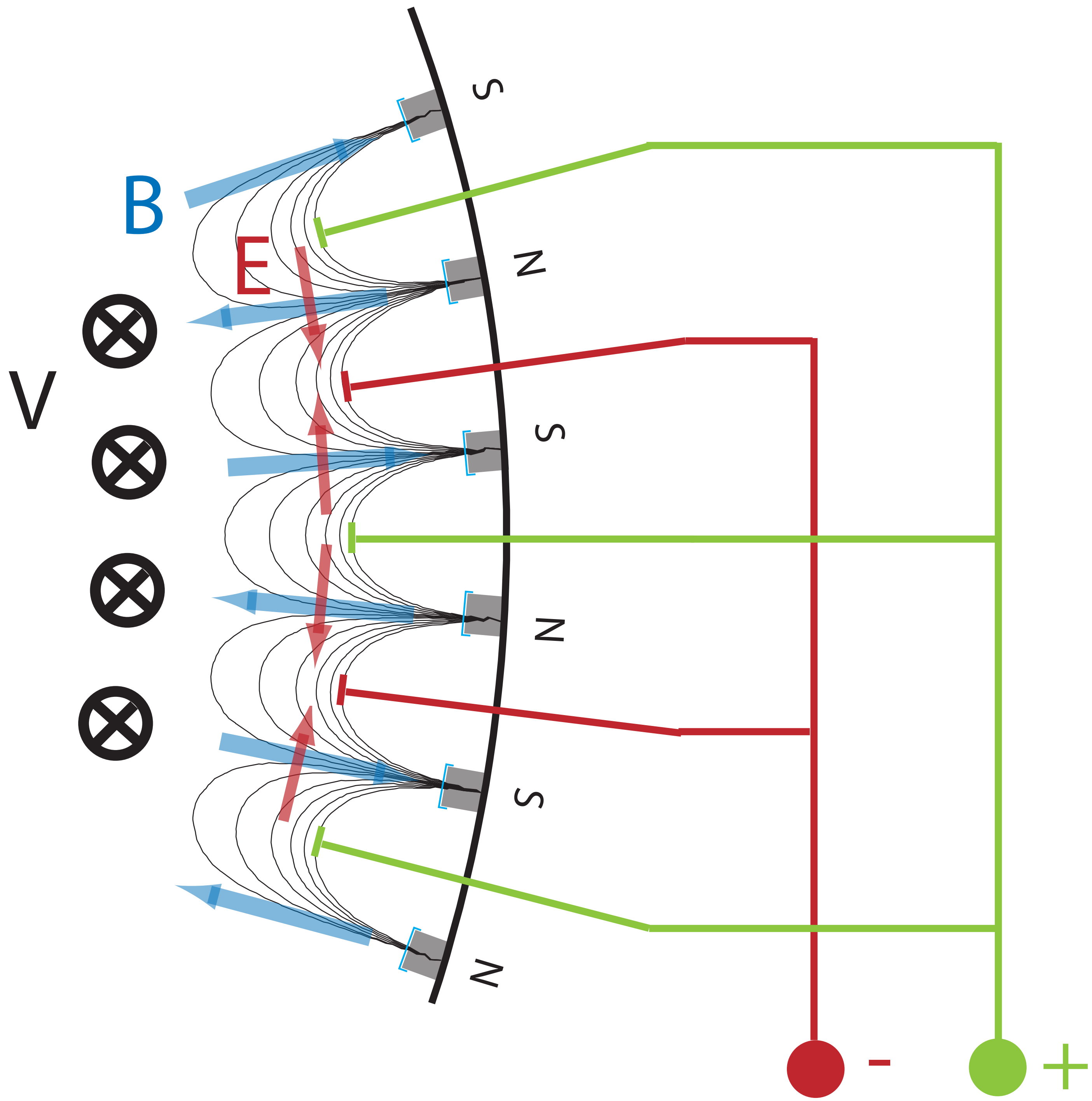
- Liquid metals have advantage that confinement is free and conductivity is independent of confinement, BUT:
  - ➡ Unfortunate Power Scaling Limitation: $P_{mech} \sim Rm^3 / L$
  - ➡ Prandtl Number is always very small: $Rm \ll Re$
- Plasmas have the potential for
  - Variable Pm
  - $Rm \gg 100$
  - intrinsically include "plasma effects" important for astrophysics (compressibility, collisionality)
  - broader class of available diagnostics

Plasma Dynamo Facility is under development

- Axisymmetric Ring Cusp
- edge confinement provided by 1.5 T, NdFeB Magnets
- high power plasma source using LaB_6
- Challenges
  - cooling of magnets
  - insulators



Multipole Magnetic Field can be used to drive flow at edge

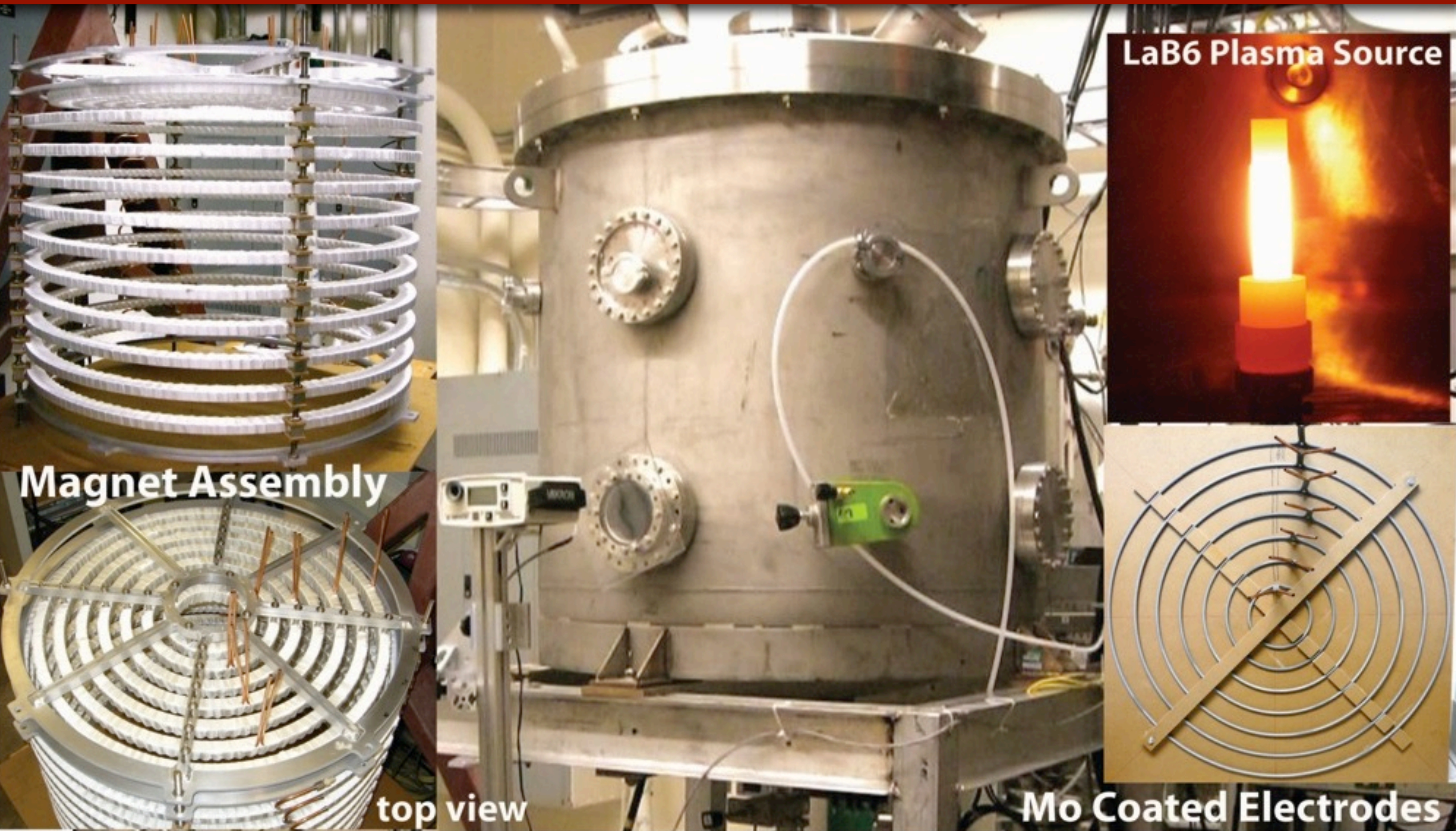


Arbitrary $V\varphi\ (r = a, \theta)$

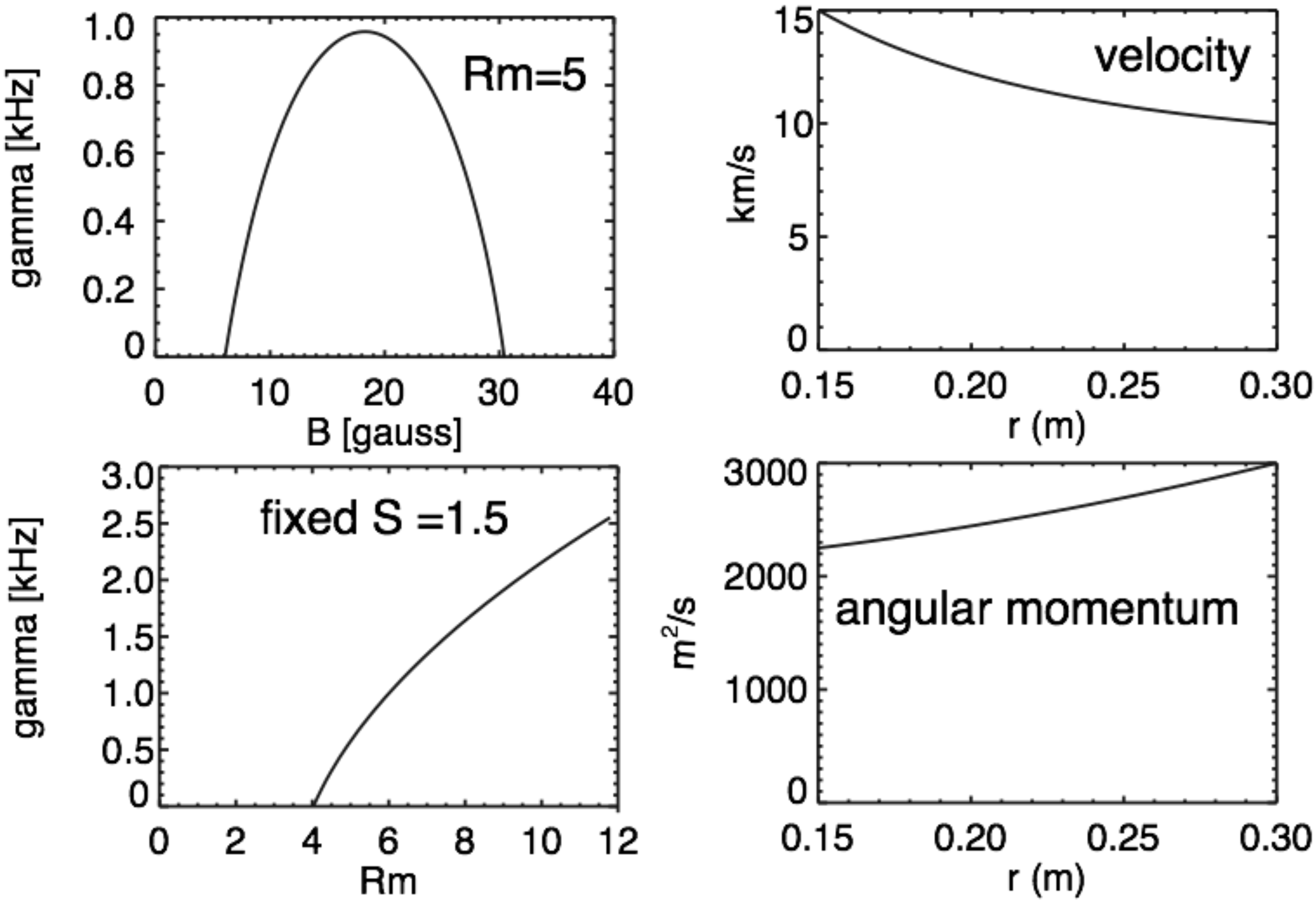
Formulary of Key Dimensionless Parameters

Magnetic Reynolds Number	Rm	$\mu_0 \sigma U L$	1.5	$\frac{T_{\mathrm{e,eV}}^{3/2} U_{\mathrm{km/s}} L_{\mathrm{m}}}{Z}$
Reynolds Number	Re	$\frac{UL}{\nu}$	8	$\frac{a_{\mathrm{m}} U_{km/s} \mu^2 n_{18}}{T_{\mathrm{i,eV}}^{5/2}}$
Magnetic Prandtl Number	Pm	$\mu_0 \sigma \nu$	0.18	$\frac{T_{\mathrm{e,eV}}^{3/2} T_{i,eV}^{5/2}}{\mu^2 n_{18}}$
Cowling Number	C	$\frac{\frac{B^2}{2\mu_0}}{\frac{1}{2}\rho U^2}$	4.75	$\frac{B_{\mathrm{G}}^2}{\mu n_{18} U_{km/s}^2}$

Plasma Couette Flow Experiment is a prototype for dynamo experiment and will be used to study MRI



MRI dispersion in Plasma Couette Flow

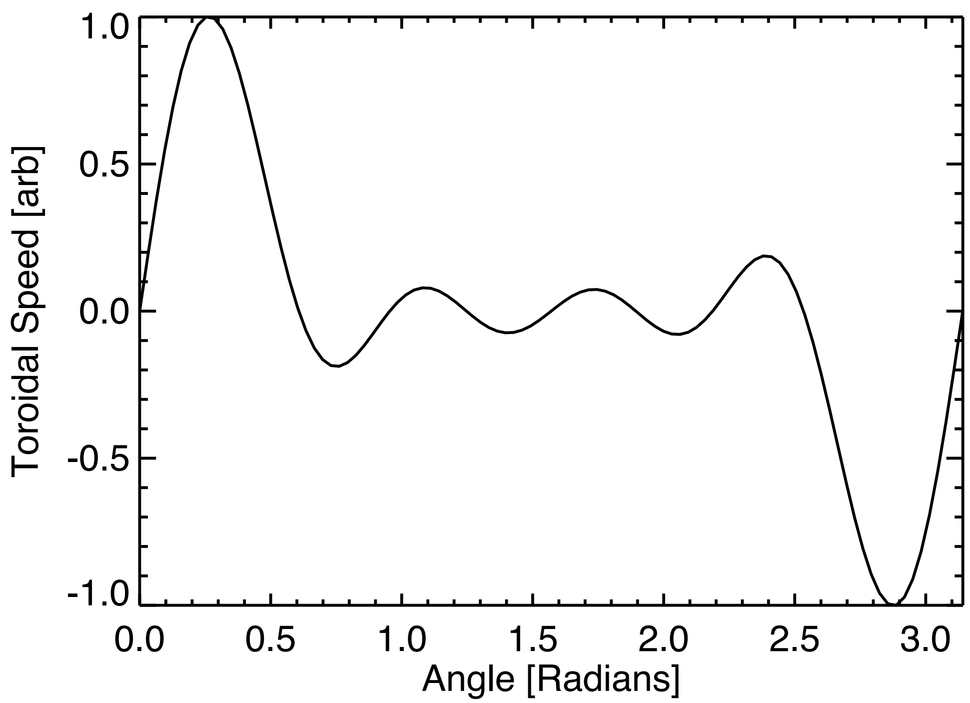


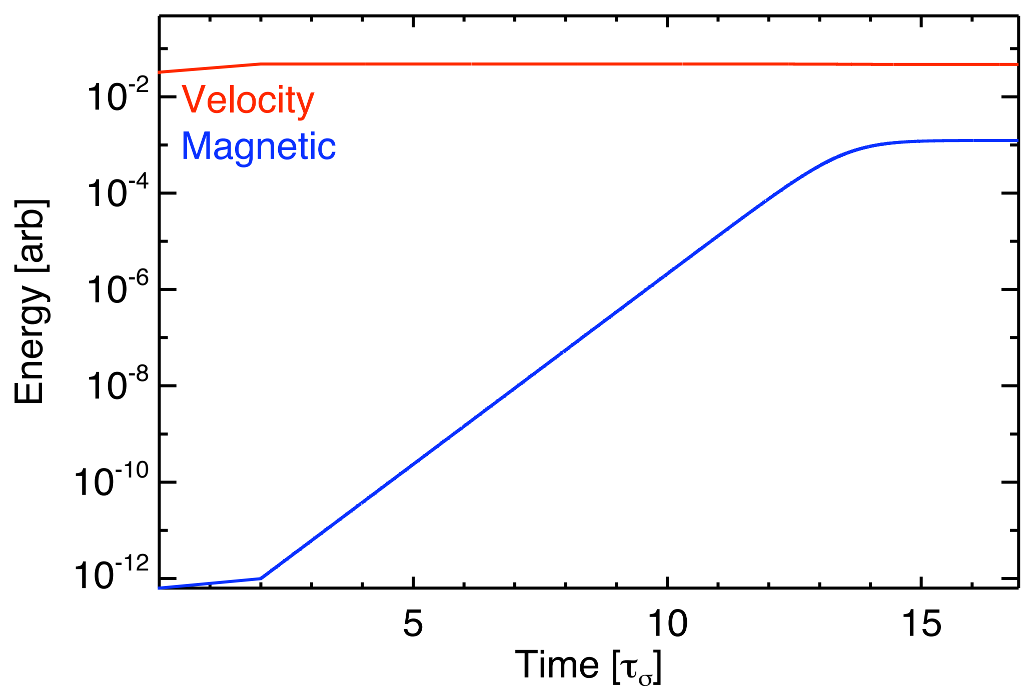
Plasma Parameters

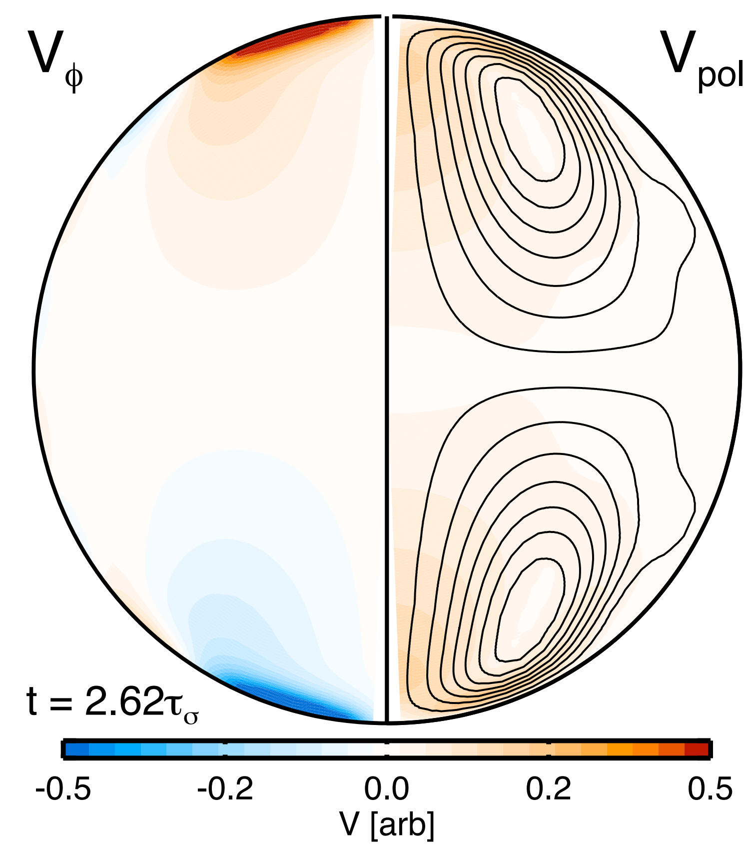
plasma radius	a	1.5	m
density	n	10^{17}—10^{19}	m^{-3}
electron temperature	T_e	2—20	eV
ion temperature	T_i	0.5—2	eV
peak flow speed	U_{max}	0—20	km/s
ion species	H, He, Ne, Ar	1, 4, 20, 40	amu
magnetic field	$r < 1.2$ m	<0.1	gauss
magnetic field	at cusp	$>10^4$	gauss
current diffusion time	$\mu_0 \sigma a^2$	50	msec
pulse length	τ_{pulse}	5	sec
heating power	P	< 0.5	MW

Rm_{max}	> 1000
Re	24—3.8×10^6
Pm	3×10^{-4}—56
C	10^{-4}
β	10^4

Two Vortex Plasma Dynamo Flow can be driven at boundary (spherical Von Karman Flow)



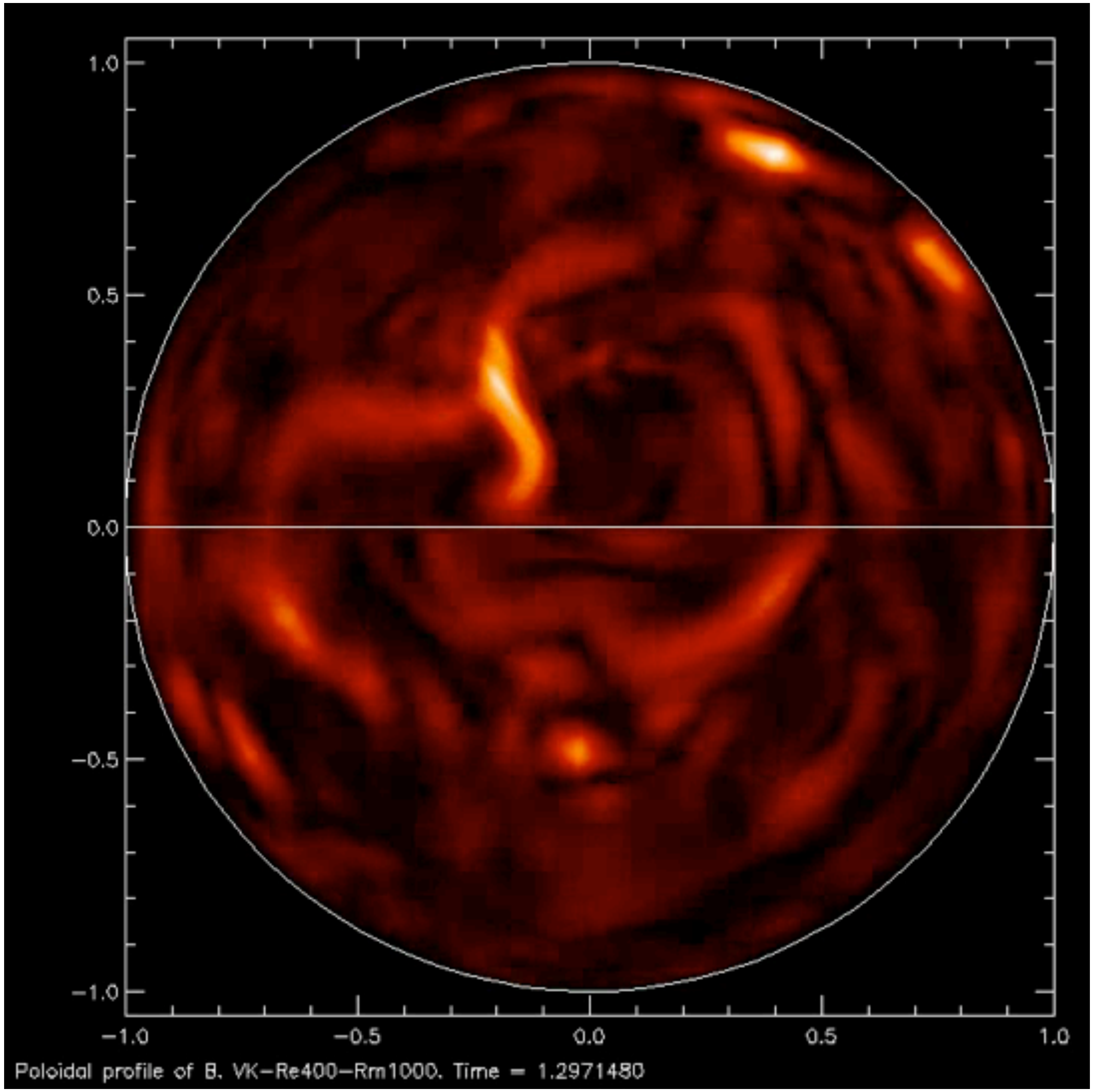




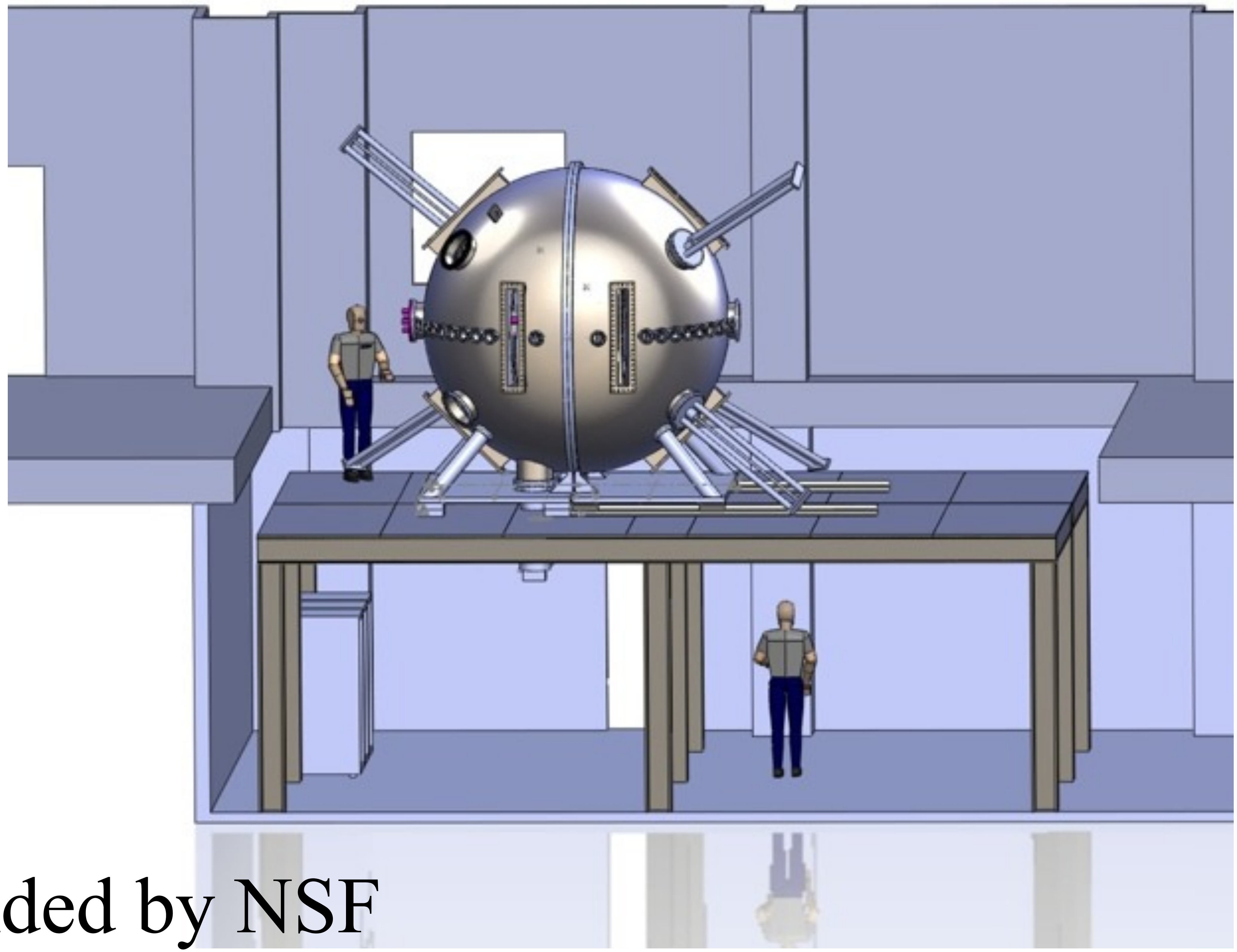
- Plasma Rm=300, Re=100
  - Te=10 eV
  - U=10 km/s,
  - n=10^{18} m^{-3}
  - Hydrogen

Small Scale Dynamo at Pm>1

- Rm=1000
- Re=400
- Plasma
  - Te = 13 eV
  - Ti = 1 eV
  - deuterium
  - U = 15 km/s
  - n = 10^{18} m^{-3}



New Plasma Dynamo Facility is under construction at UW to investigate flow driven instabilities in plasmas



Funded by NSF

Summary

- High Rm, weak magnetized flows define a unique parameter regime for experiments
  - Free energy source: flow energy
- Dynamos: Kinetic energy → Magnetic energy
  - liquid metal experiments demonstrate effect
- The next frontier may be plasma-based experiments