

Progress in Vlasov-Fokker-Planck simulations of laser-plasma interactions

C. P. Ridgers, M. W. Sherlock, R. J. Kingham,
A. Thomas, R. Evans

Imperial College London

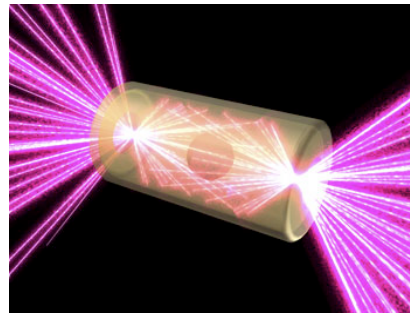
Outline

Part 1 – simulations of long-pulse laser-plasma interactions.

- (1) The interplay of magnetic field dynamics and non-locality.
- (2) Magnetic field generation

Part 2 – Vlasov-Fokker-Planck simulations of short-pulse laser-plasma interactions relevant to fast ignition. The importance of rear-surface effects.

Background – inertial fusion



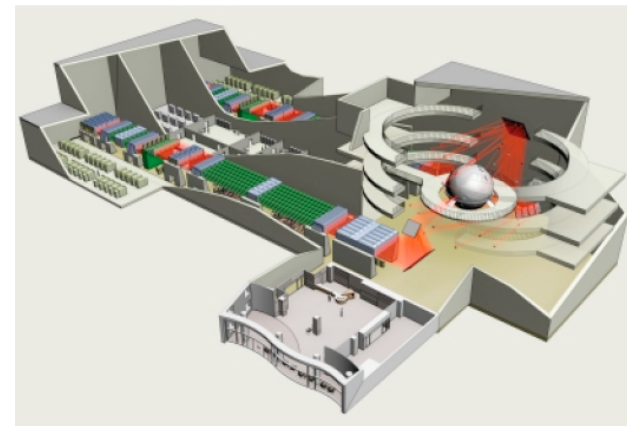
← NIF - National Ignition Facility

- 1.8 MJ in UV based at LLNL
- Partially working now... Ignition in 2011

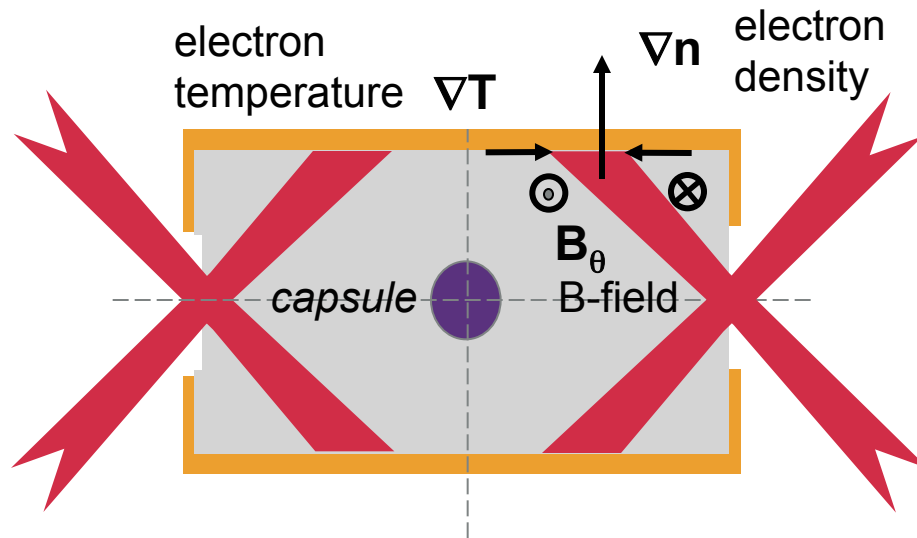
HiPER



- High Power Exper. Research facility
- Proposed European academic facility, with 200--400 kJ
- Site: TBD... UK ?



Indirect drive ICF^{1,2}

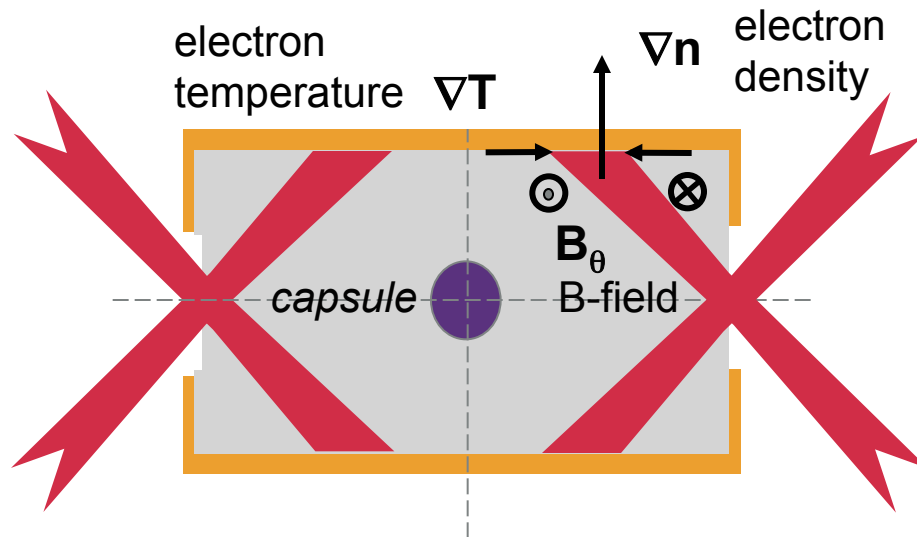


Gradients in T_e , n_e large enough
so that classical diffusive transport
(e.g. Spitzer) not valid.

¹Lindl J.D. et al, *Physics of Plasmas*, **11**, 339, 2004

²Glenzer S.H. et al, *Physics of Plasmas*, **6**, 2117, 1999

Indirect drive ICF^{1,2}



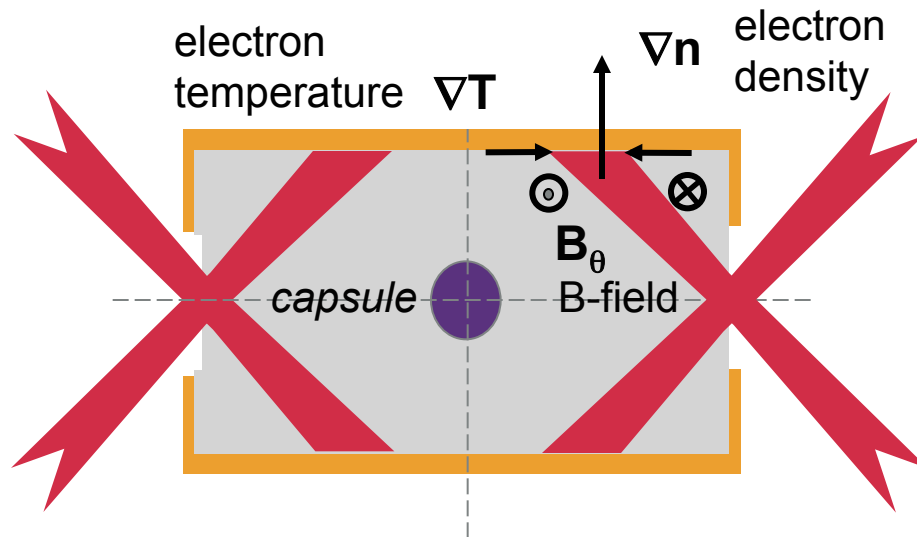
- 1) Does the heat-flow obey classical theory? (non-locality)

Gradients in T_e , n_e large enough so that classical diffusive transport (e.g. Spitzer) not valid.

¹Lindl J.D. et al, *Physics of Plasmas*, **11**, 339, 2004

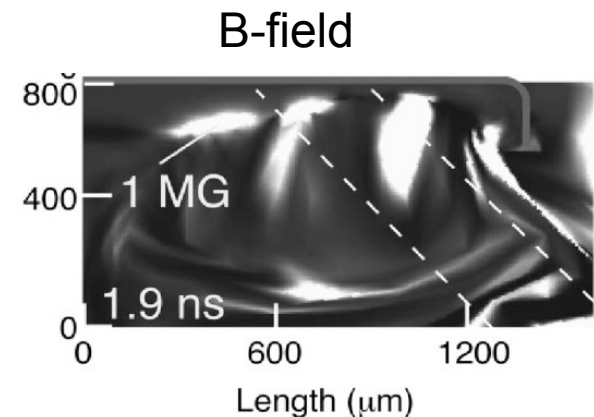
²Glenzer S.H. et al, *Physics of Plasmas*, **6**, 2117, 1999

Indirect drive ICF^{1,2}



Gradients in T_e , n_e large enough so that classical diffusive transport (e.g. Spitzer) not valid.

- 1) Does the heat-flow obey classical theory? (non-locality)
- 2) Interplay between non-locality and B-fields.



¹Lindl J.D. et al, *Physics of Plasmas*, **11**, 339, 2004

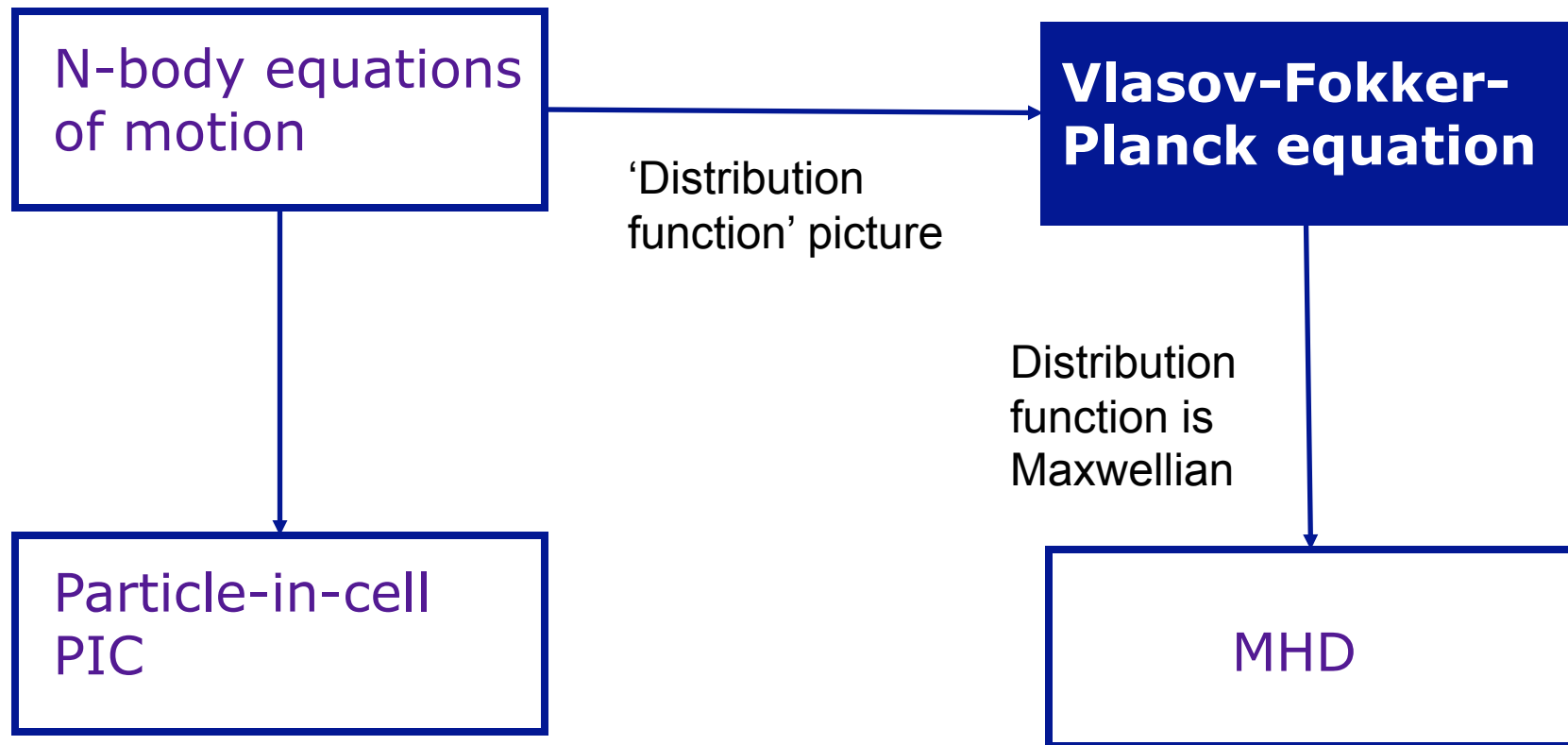
²Glenzer S.H. et al, *Physics of Plasmas*, **6**, 2117, 1999

Indirect drive ICF^{1,2}

First part of talk will examine these three points:

- 1) Does the heat-flow obey classical theory? (non-locality)
- 2) Interplay between non-locality and B-fields.

Modelling long-pulse LPI



The Vlasov-Fokker-Planck equation

Distribution function - density of particles in 6D phase space

$$dN = f(\mathbf{v}, \mathbf{r}, t) d^3\mathbf{r} d^3\mathbf{v}$$

The VFP equation is the appropriate kinetic equation for a plasma:

$$\frac{\partial f}{\partial t} + \nabla_r \cdot (f\mathbf{v}) + \nabla_p \cdot (f\mathbf{F}) = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \mathbf{F} = \frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

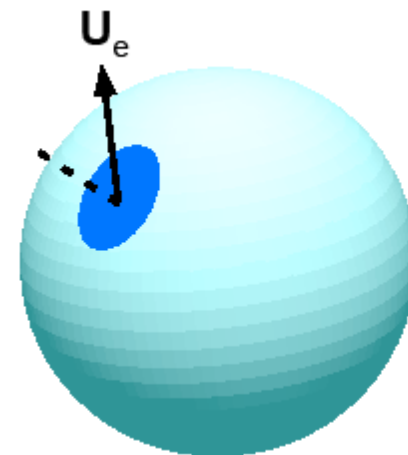
The Vlasov-Fokker-Planck equation

The VFP equation is the appropriate kinetic equation for a plasma:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r (f \mathbf{v}) + \nabla_p \cdot (f \mathbf{F}) = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

VFP equation is continuity is 6D phase space. Compare to:

$$\frac{\partial n_e}{\partial t} + \nabla_r \cdot (n_e \mathbf{U}_e) = 0$$



Modelling long-pulse LPI

Vlasov-Fokker-Planck equation

Distribution
function is
Maxwellian

MHD



Modelling long-pulse LPI

Distribution function is Maxwellian:

$$f(\mathbf{v}, \mathbf{r}, t) = n_e \left(\frac{m_e}{2\pi T_e} \right)^{3/2} \exp\left(-\frac{m_e v^2}{2k_B T_e} \right)$$

Vlasov-Fokker-Planck equation

Distribution
function is
Maxwellian

MHD

Modelling long-pulse LPI

Distribution function is Maxwellian:

$$f(\mathbf{v}, \mathbf{r}, t) = n_e \left(\frac{m_e}{2\pi T_e} \right)^{3/2} \exp\left(-\frac{m_e v^2}{2k_B T_e} \right)$$

MHD requires closure - classical heat flow equation:

$$\mathbf{q} = -\kappa^c \nabla T$$

Vlasov-Fokker-Planck equation

Distribution
function is
Maxwellian

MHD

Modelling long-pulse LPI

Distribution function is Maxwellian:

$$f(\mathbf{v}, \mathbf{r}, t) \neq n_e \left(\frac{m_e}{2\pi T_e} \right)^{3/2} \exp\left(-\frac{m_e v^2}{2k_B T_e} \right)$$

MHD requires closure - classical heat flow equation:

$$\mathbf{q} \neq -\kappa^c \nabla T$$

VFP - Distribution function is **NOT** Maxwellian

$$\mathbf{q} = \frac{m_e}{2} \int d\mathbf{v} \mathbf{v} v^2 f$$

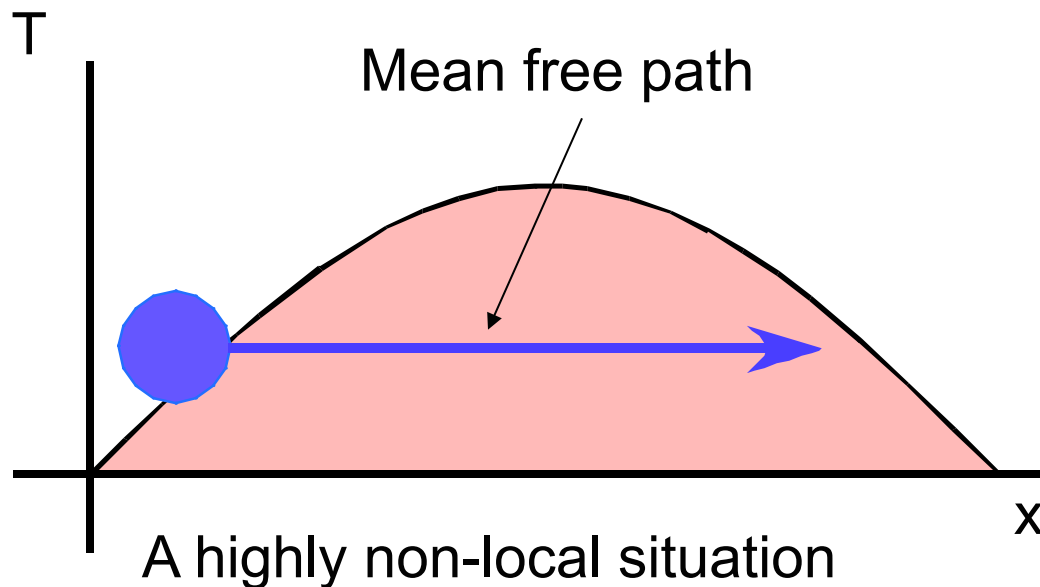
Vlasov-Fokker-Planck equation

Distribution function is Maxwellian

MHD

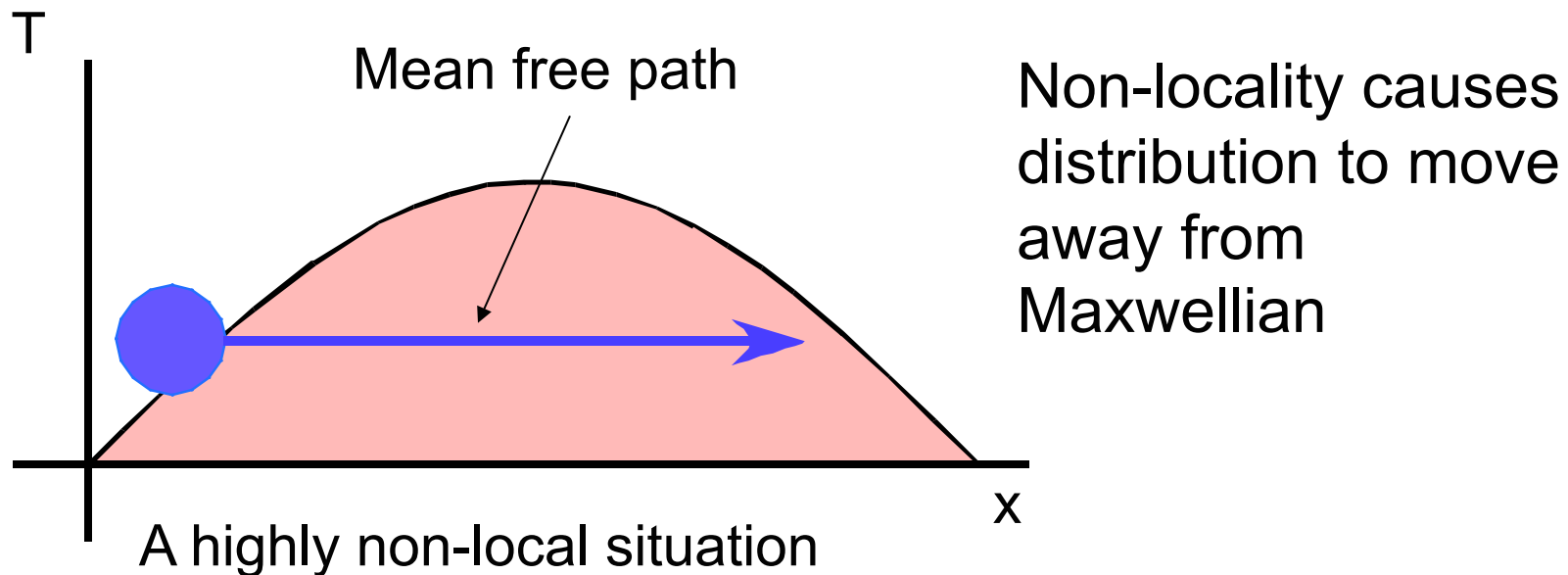
What causes the break-down of classical transport?

Non-Locality: Scale-length of physical variables becomes comparable to mean-free-path



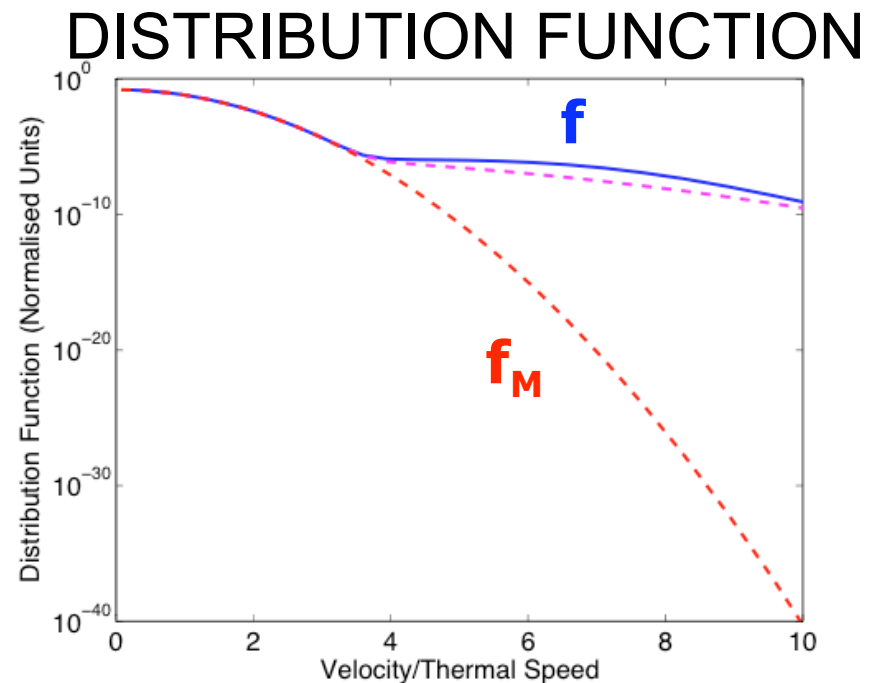
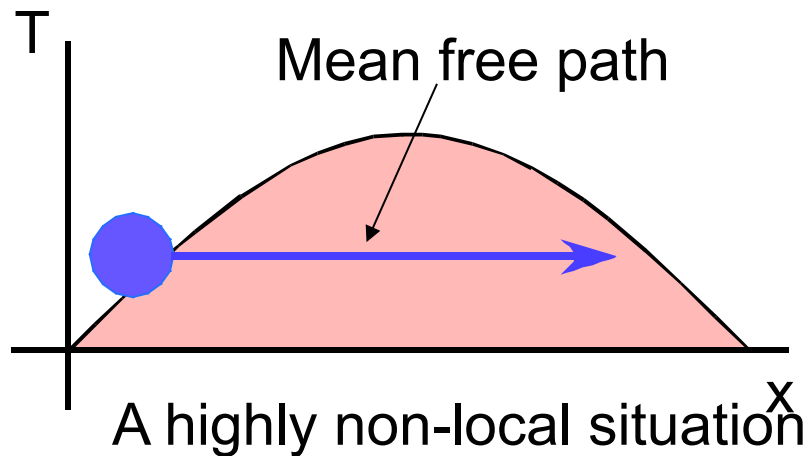
What causes the break-down of classical transport?

Non-Locality: Scale-length of physical variables becomes comparable to mean-free-path



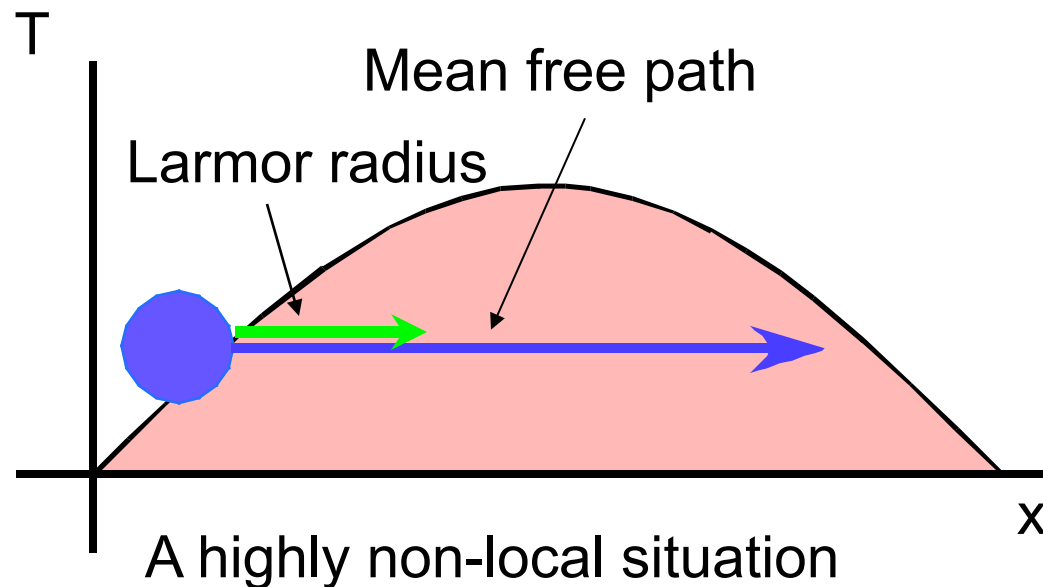
What causes the break-down of classical transport?

Non-Locality: Scale-length of physical variables becomes comparable to mean-free-path



What causes the break-down of classical transport?

Non-Locality: Scale-length of physical variables becomes comparable to mean-free-path



Effect on B-field on non-locality?

IMPACT - Implicit Magnetised Plasma And Collisional Transport⁵

- Solves the VFP Equation (and Maxwell's equations) with hydrodynamic ion flow
- Expands distribution function in spherical harmonics so can only deal with weakly anisotropic distributions
- Implicit finite-differencing - very robust + large Δt (e.g. $\sim \text{ps}$ for $\Delta x \sim 1 \mu\text{m}$ vs **3fs**)
- Valid when collisions important (unlike PIC)
- 2D (3V) VFP code with self consistent B_z -fields

⁵Kingham R.J. & Bell A.R., *Journal of Computational Physics* **194**, 1, 2004

IMPACT - Implicit Magnetised Plasma And Collisional Transport⁵

What's new?

FIRST 2D VFP code to be able to simulate long-pulse laser-plasma interactions with self-consistent magnetic fields

Ideal for studying interaction between non-local transport and magnetic fields

⁵Kingham R.J. & Bell A.R., *Journal of Computational Physics* **194**, 1, 2004

Indirect drive ICF^{1,2}

First part of talk will examine these three points:

- 1) Does the heat-flow obey classical theory? (non-locality)**
- 2) Interplay between non-locality and B-fields.

B-Field and Non-Locality: Experimental Measurement⁶

Gas jet:

Initial temperature - 20eV

Number density - $1.5 \times 10^{19} \text{cm}^{-3}$

Laser:

Energy - 100J

Spot size - $100 \mu\text{m}$

Intensity - $1 \times 10^{15} \text{W/cm}^2$

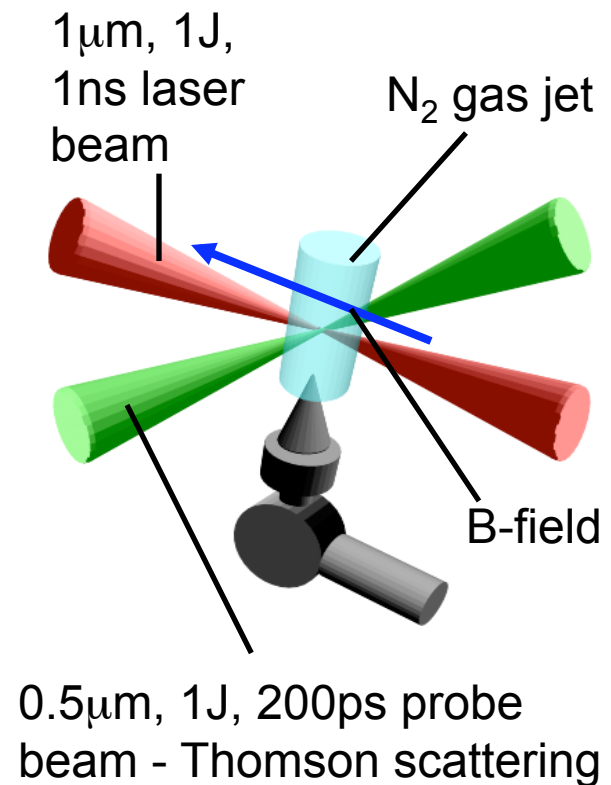
Wavelength - $1.053 \mu\text{m}$

Pulse duration - 1ns

Magnetic field:

Non-local case - 0T

Local case - 12T

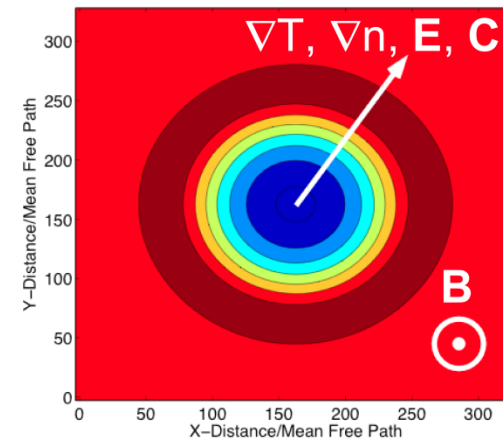
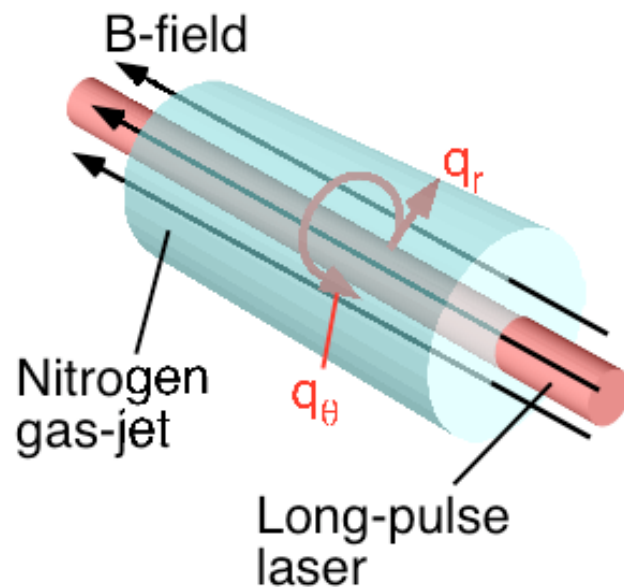


**Strong B-field expected to
“localize” transport**

⁶Froula D.H. et al, *Physical Review Letters*, **98**, 135001, 2007

Using IMPACT with MHD to model magnetized transport experiments

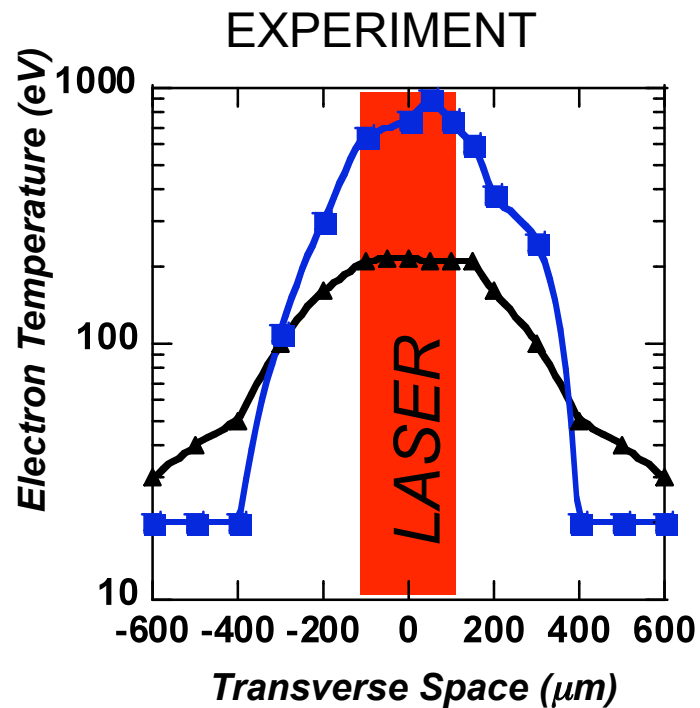
Simulation geometry:



**IMPOSE A RANGE OF FIELDS
 $B=0-12T$**

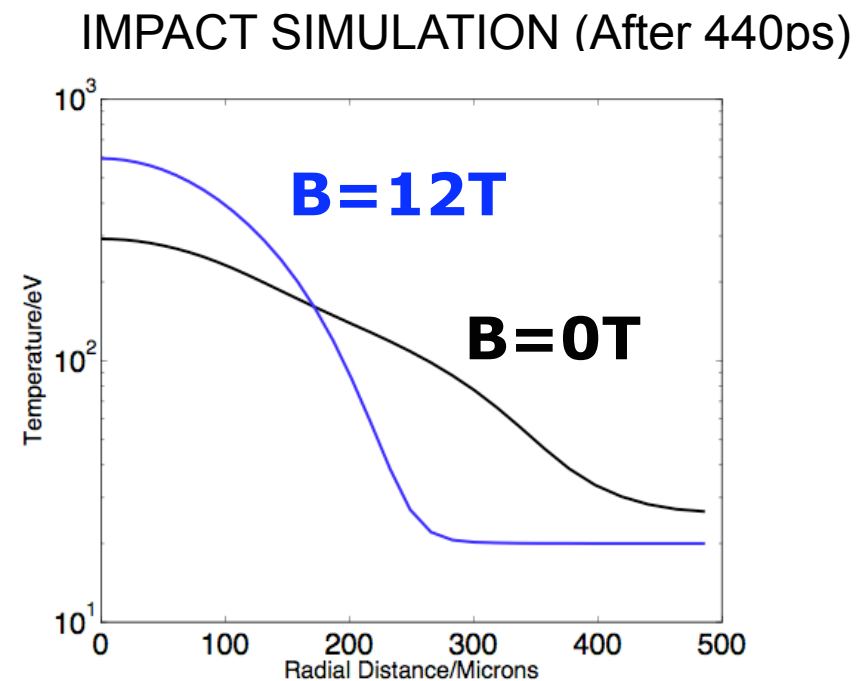
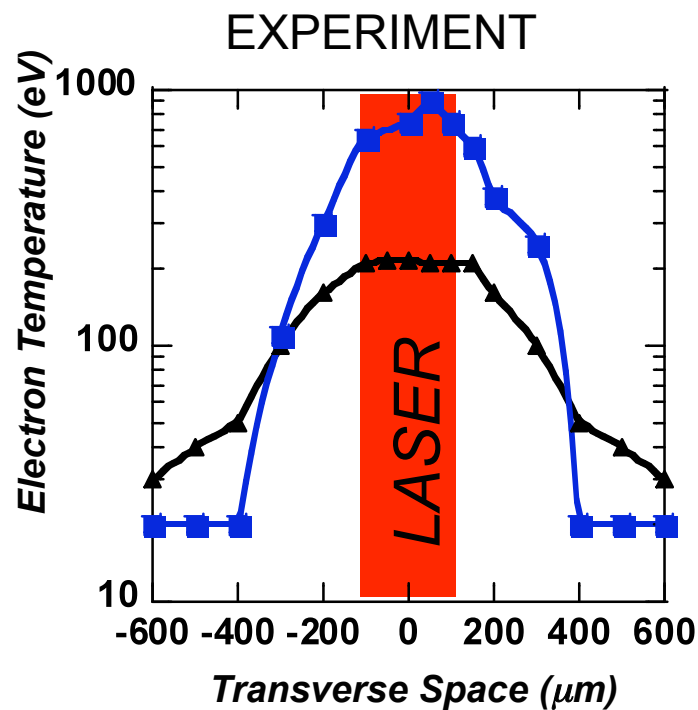
Simulation results - pre-heat

TEMPERATURE PROFILE ACROSS LASER BEAM



Simulation results - pre-heat⁷

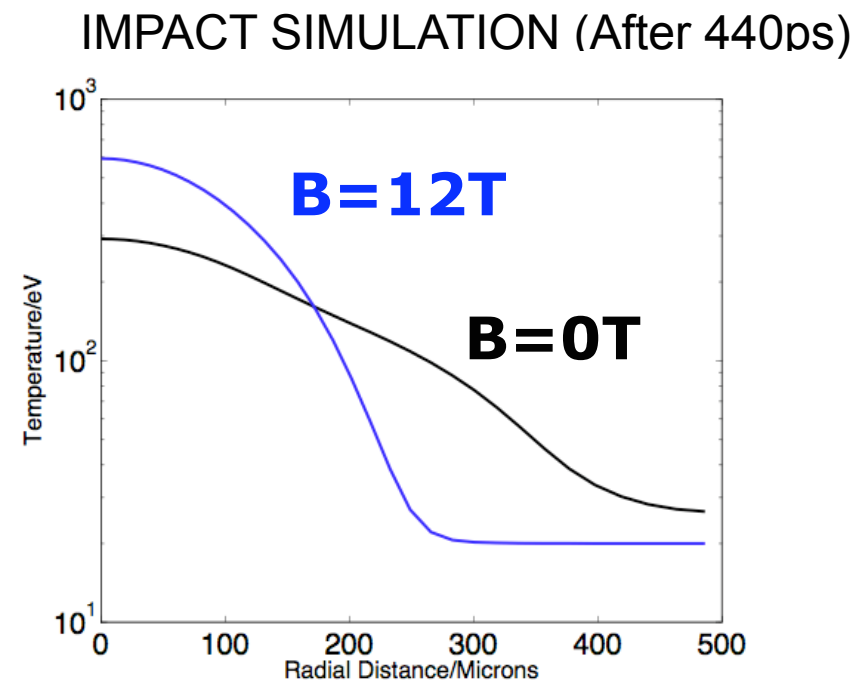
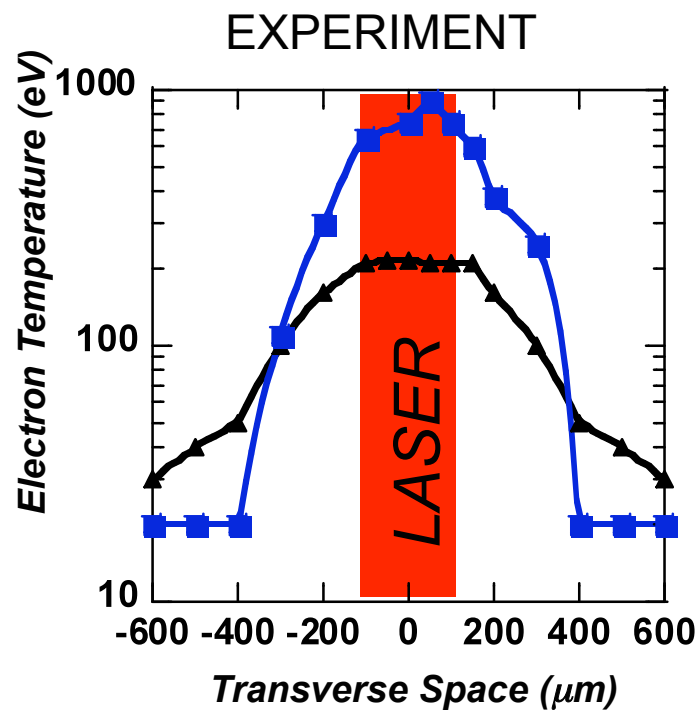
TEMPERATURE PROFILE ACROSS LASER BEAM



⁷Ridgers C.P., Kingham R.J. & Thomas A.G.R., *Phys Rev Lett*, **100**, 075003, 2008

Simulation results - pre-heat⁷

TEMPERATURE PROFILE ACROSS LASER BEAM

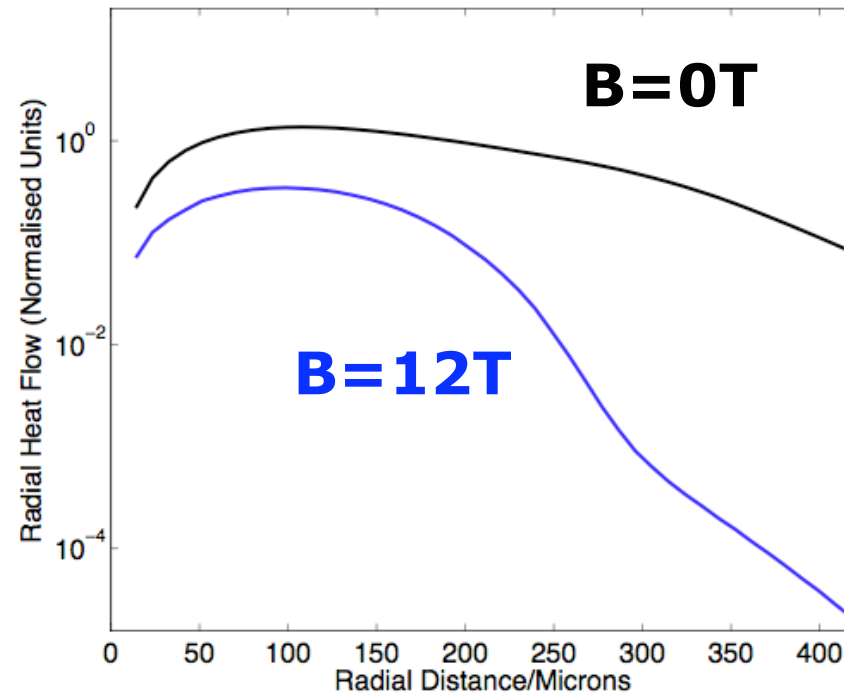


We do see less pre-heat in un-magnetized case - but is it non-local?

⁷Ridgers C.P., Kingham R.J. & Thomas A.G.R., *Phys Rev Lett*, **100**, 075003, 2008

Simulation results - is it non-local?⁷

RADIAL HEAT FLOW



⁷Ridgers C.P., Kingham R.J. & Thomas A.G.R., *Phys Rev Lett*, **100**, 075003, 2008

Simulation results - is it non-local?⁷

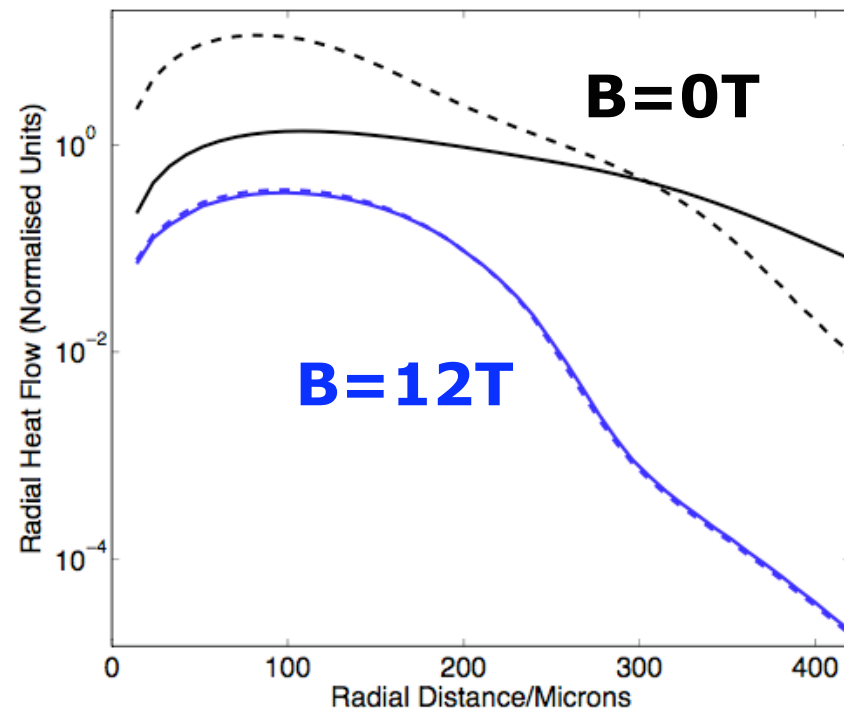
Braginskii heat flow:

$$\mathbf{q} = -\kappa^c \nabla T$$

$\mathbf{q}$ from instantaneous
temperature profile.

12T field suppresses
non-locality

RADIAL HEAT FLOW



⁷Ridgers C.P., Kingham R.J. & Thomas A.G.R., *Phys Rev Lett*, **100**, 075003, 2008

Indirect drive ICF^{1,2}

First part of talk will examine these three points:

1) Does the heat-flow obey classical theory? (non-locality)

**A: Non-locality causes classical theory to break down
BUT transport can be relocalised by a strong B-field,
in which case classical theory applies.**

2) Interplay between non-locality and B-fields.

Indirect drive ICF^{1,2}

First part of talk will examine these three points:

1) Does the heat-flow obey classical theory? (non-locality)

2) Interplay between non-locality and B-fields.

The Nernst effect^{8,9}

First moment of VFP equation gives Ohm's law:

$$\frac{e}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\nabla P_e}{n_e m_e} + \underline{\underline{\alpha^c}} \cdot \frac{\mathbf{j}}{n_e e} - \underline{\underline{\beta^c}} \cdot \frac{\nabla T_e}{m_e}$$

⁸Nishiguchi A. et al, *Physics of Fluids*, **28**, 3683, 1985

⁹Haines M.G., *Plasma Physics and Controlled Fusion*, **28**, 1705, 1986

The Nernst effect^{8,9}

First moment of VFP equation gives Ohm's law:

$$\frac{e}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\nabla P_e}{n_e m_e} + \underline{\underline{\alpha^c}} \cdot \frac{\mathbf{j}}{n_e e} - \underline{\underline{\beta^c}} \cdot \frac{\nabla T_e}{m_e}$$

Put this into Faraday's law (and simplify!): $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$

⁸Nishiguchi A. et al, *Physics of Fluids*, **28**, 3683, 1985

⁹Haines M.G., *Plasma Physics and Controlled Fusion*, **28**, 1705, 1986

The Nernst effect^{8,9}

First moment of VFP equation gives Ohm's law:

$$\frac{e}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\nabla P_e}{n_e m_e} + \underline{\underline{\alpha}}^c \cdot \frac{\mathbf{j}}{n_e e} - \underline{\underline{\beta}}^c \cdot \frac{\nabla T_e}{m_e}$$

Put this into Faraday's law (and simplify!): $\frac{\partial B}{\partial t} = -\nabla \times \mathbf{E}$

$$\frac{\partial B}{\partial t} + \nabla \cdot [(\mathbf{U} + \mathbf{v}_N)B + \eta \nabla B] = 0 \quad \mathbf{v}_N = \frac{\mathbf{q}_e}{5/2 n_e T_e}$$

⁸Nishiguchi A. et al, *Physics of Fluids*, **28**, 3683, 1985

⁹Haines M.G., *Plasma Physics and Controlled Fusion*, **28**, 1705, 1986

The Nernst effect^{8,9}

First moment of VFP equation gives Ohm's law:

$$\frac{e}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\nabla P_e}{n_e m_e} + \underline{\underline{\alpha}}^c \cdot \frac{\mathbf{j}}{n_e e} - \underline{\underline{\beta}}^c \cdot \frac{\nabla T_e}{m_e}$$

Put this into Faraday's law (and simplify!): $\frac{\partial B}{\partial t} = -\nabla \times \mathbf{E}$

Frozen-in flow

Nernst effect

Resistive diffusion

$$\frac{\partial B}{\partial t} + \nabla \cdot [(\mathbf{U} + \mathbf{v}_N)B + \eta \nabla B] = 0$$

$$\mathbf{v}_N = \frac{\mathbf{q}_e}{5/2 n_e T_e}$$

⁸Nishiguchi A. et al, *Physics of Fluids*, **28**, 3683, 1985

⁹Haines M.G., *Plasma Physics and Controlled Fusion*, **28**, 1705, 1986

The Nernst effect^{8,9}

First moment of VFP equation gives Ohm's law:

$$\frac{e}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\nabla P_e}{n_e m_e} + \underline{\alpha}^c \cdot \frac{\mathbf{j}}{n_e e} - \underline{\beta}^c \cdot \frac{\nabla T_e}{m_e}$$

Put this into Faraday's law (and simplify!): $\frac{\partial B}{\partial t} = -\nabla \times \mathbf{E}$

Nernst effect

$$\frac{\partial B}{\partial t} + \nabla \cdot [(\mathbf{U} + \mathbf{v}_N)B + \eta \nabla B] = 0$$

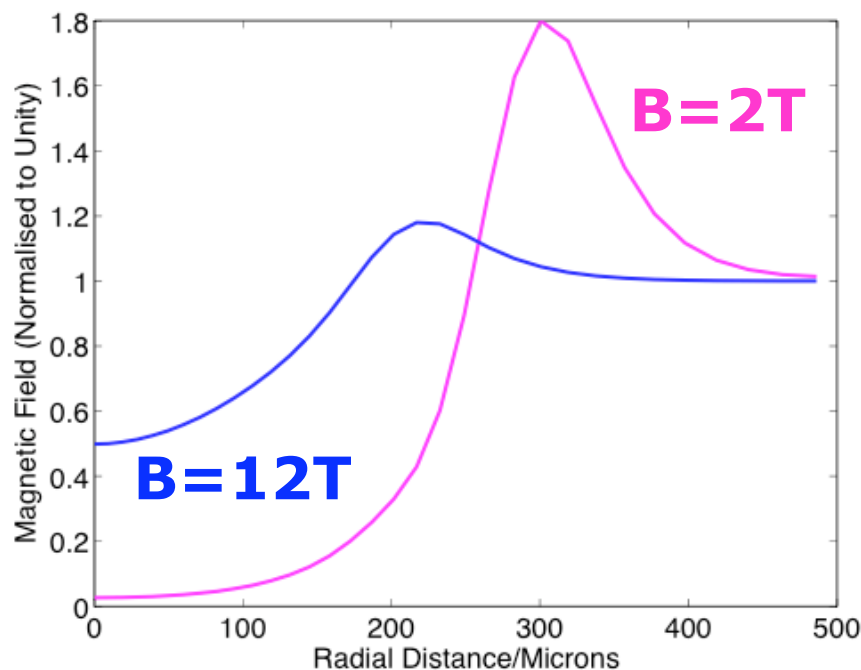
$$\mathbf{v}_N = \frac{\mathbf{q}_e}{5/2 n_e T_e}$$

⁸Nishiguchi A. et al, *Physics of Fluids*, **28**, 3683, 1985

⁹Haines M.G., *Plasma Physics and Controlled Fusion*, **28**, 1705, 1986

Nernst Advection dominates over Frozen-in-Flow⁷

B-FIELD ACROSS BEAM

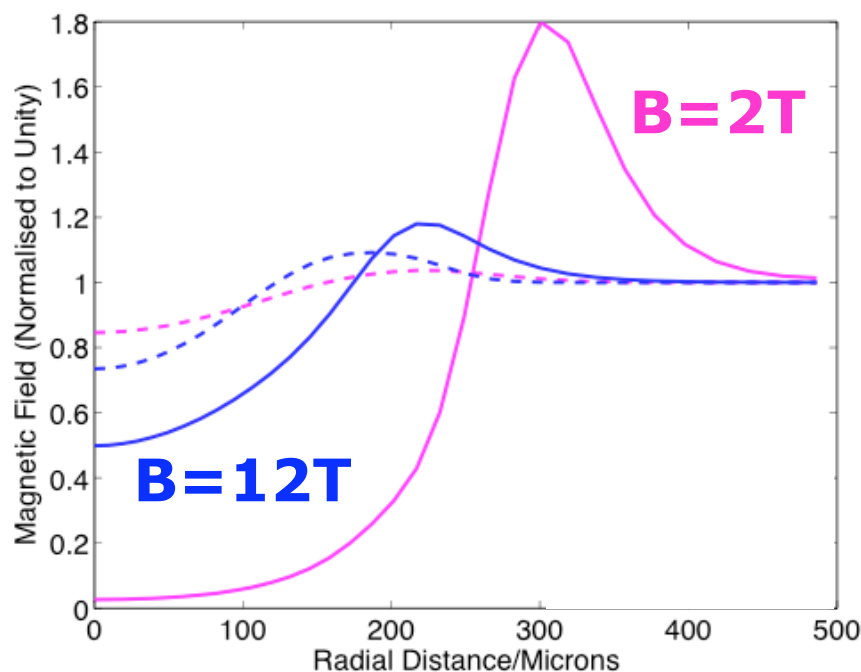


Solid lines -
full B-field calculation

Dashed lines -
frozen-in flow only

Nernst Advection dominates over Frozen-in-Flow⁷

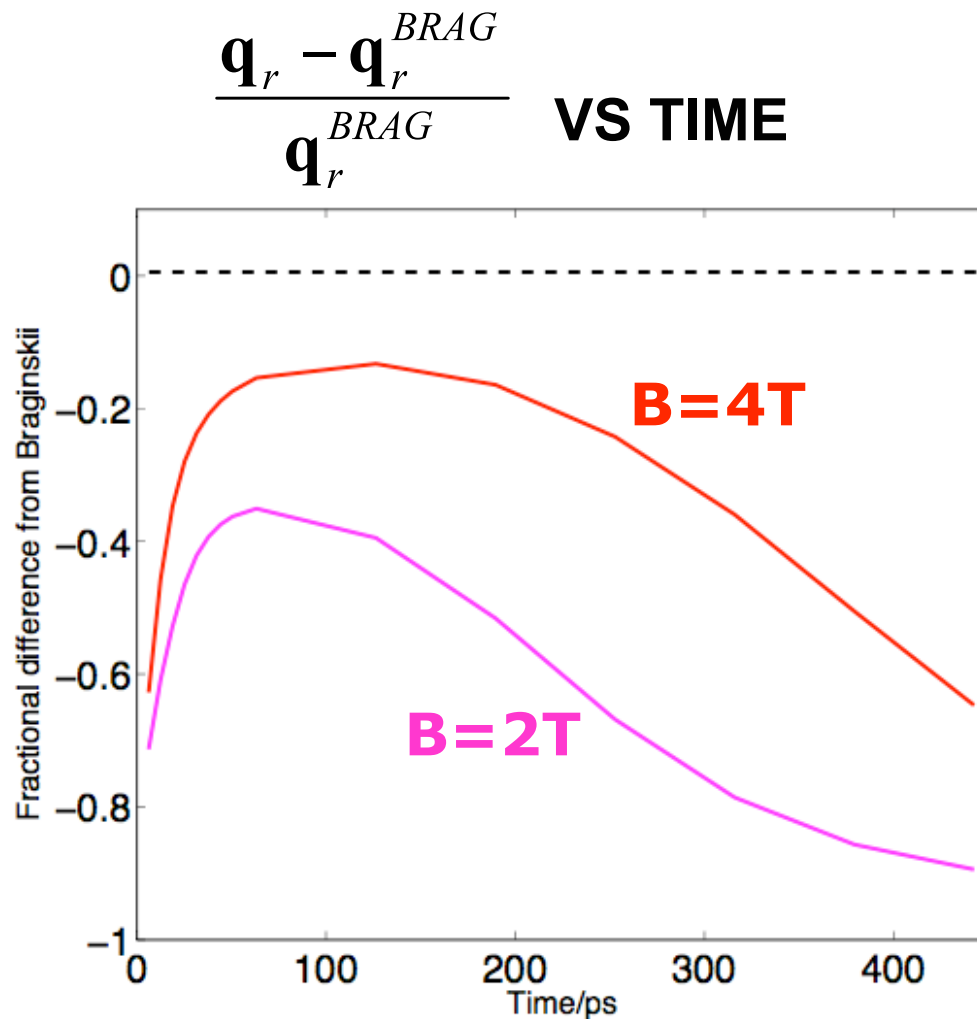
B-FIELD ACROSS BEAM



Solid lines -
full B-field calculation

Dashed lines -
frozen-in flow only

Evolution of non-locality



Degree of non-locality
increases with time

Indirect drive ICF^{1,2}

First part of talk will examine these three points:

- 1) Does the heat-flow obey classical theory? (non-locality)
- 2) Interplay between non-locality and B-fields.

A: Enhanced B-field advection, caused by the Nernst effect, causes B to drop so that even if it were initially sufficient to localise transport, non-locality can re-emerge in time

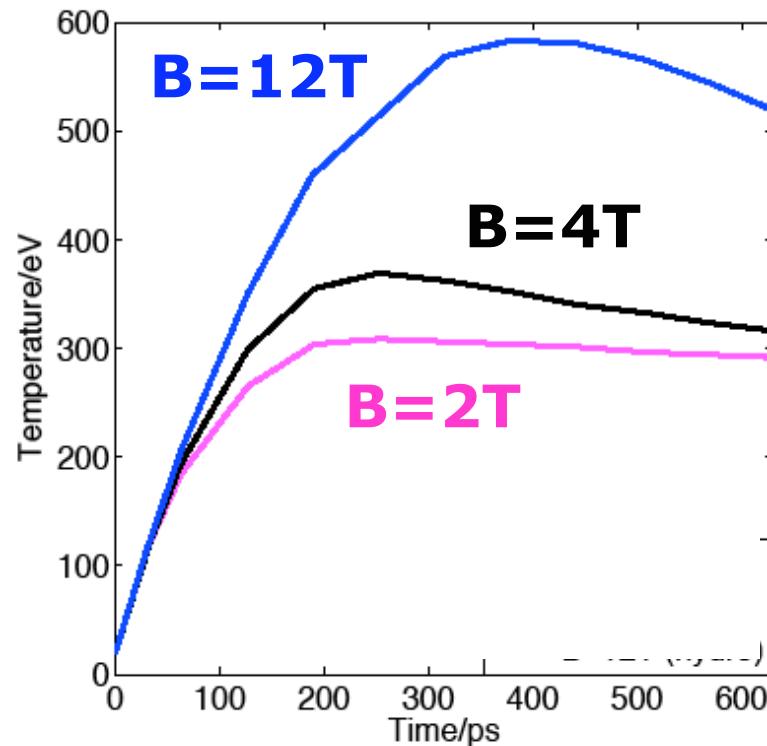
Summary - non-locality & B-fields

- Magnetic field advection can be dominated by the Nernst effect which therefore should be included.
- Enhanced B-field advection can lead to the re-emergence of non-locality, even if the initial B-field suppresses it.
- MAGNETIC FIELD EVOLUTION AND NON-LOCALITY COUPLED

Can these effects be measured experimentally?

The Nernst effect leads to cooling in the laser heated region when the laser is still on at full power

TEMPERATURE VS TIME



Nernst cooling

Equilibrium: Heating = Cooling by thermal
 from laser conduction

Nernst cooling

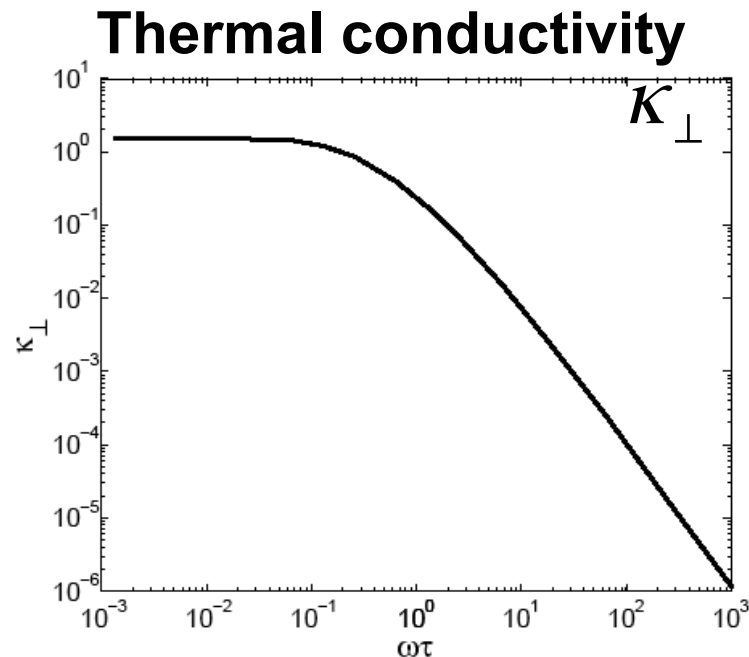
Equilibrium: Heating = Cooling by thermal
 from laser conduction

Thermal conduction rate controlled by Hall parameter.

$$\omega\tau \propto \frac{B}{n_e} T_e^{3/2}$$

Thermal conduction rate controlled by Hall parameter.

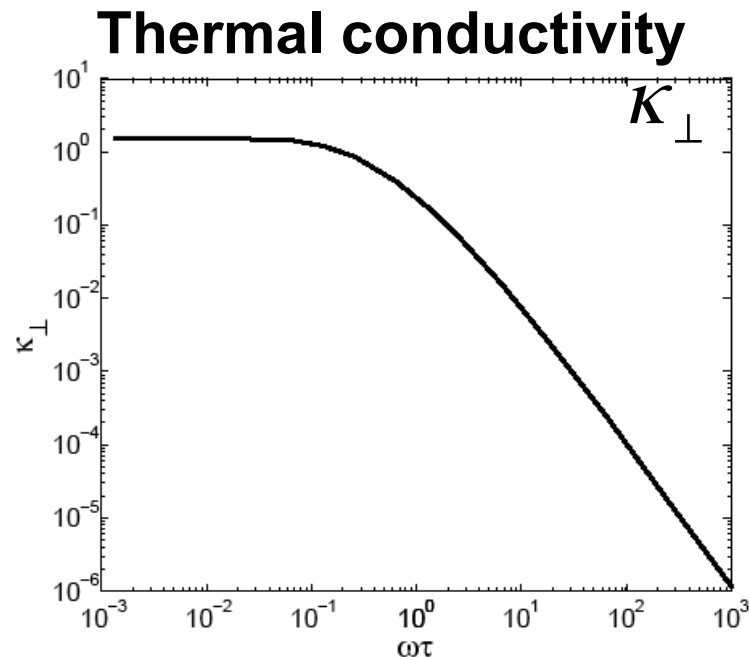
$$\omega\tau \propto \frac{B}{n_e} T_e^{3/2}$$



Thermal conduction rate controlled by Hall parameter.

$$\omega\tau \propto \frac{B}{n_e} T_e^{3/2}$$

**Nernst drives out B-field faster
than central hot-spot drives
out plasma hydrodynamically,
Hall parameter decreases**

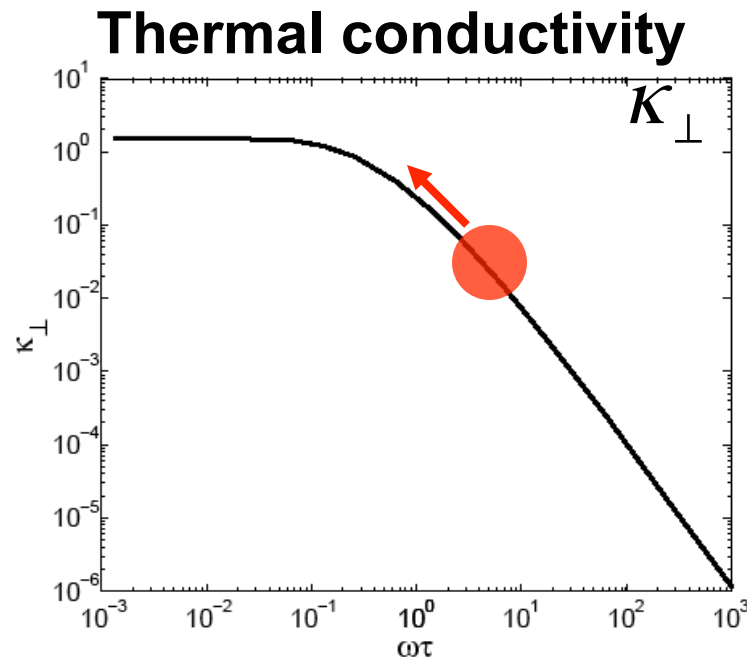


Thermal conduction rate controlled by Hall parameter.

$$\omega\tau \propto \frac{B}{n_e} T_e^{3/2}$$

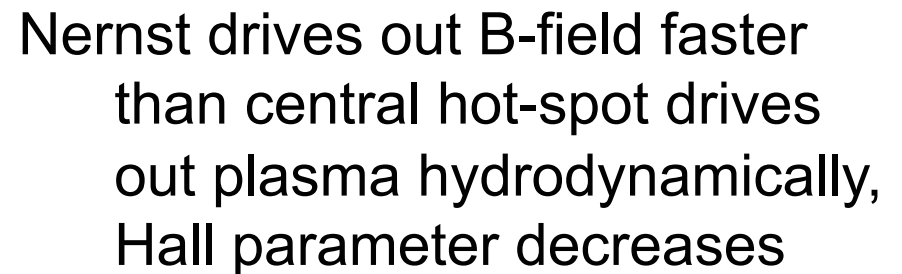
Nernst drives out B-field faster
than central hot-spot drives
out plasma hydrodynamically,
Hall parameter decreases

Conductivity increases



Thermal conduction rate controlled by Hall parameter.

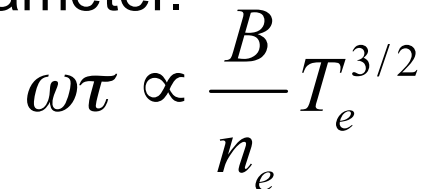
$$\omega\tau \propto \frac{B}{n_e} T_e^{3/2}$$



Conductivity increases

**Central temperature decreases
– Nernst cooling**

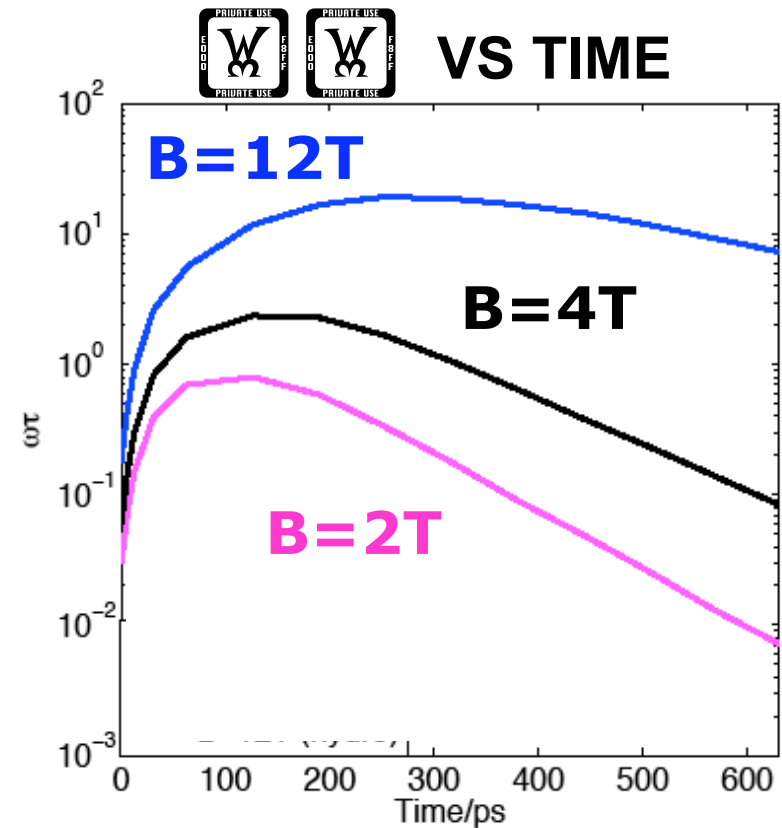
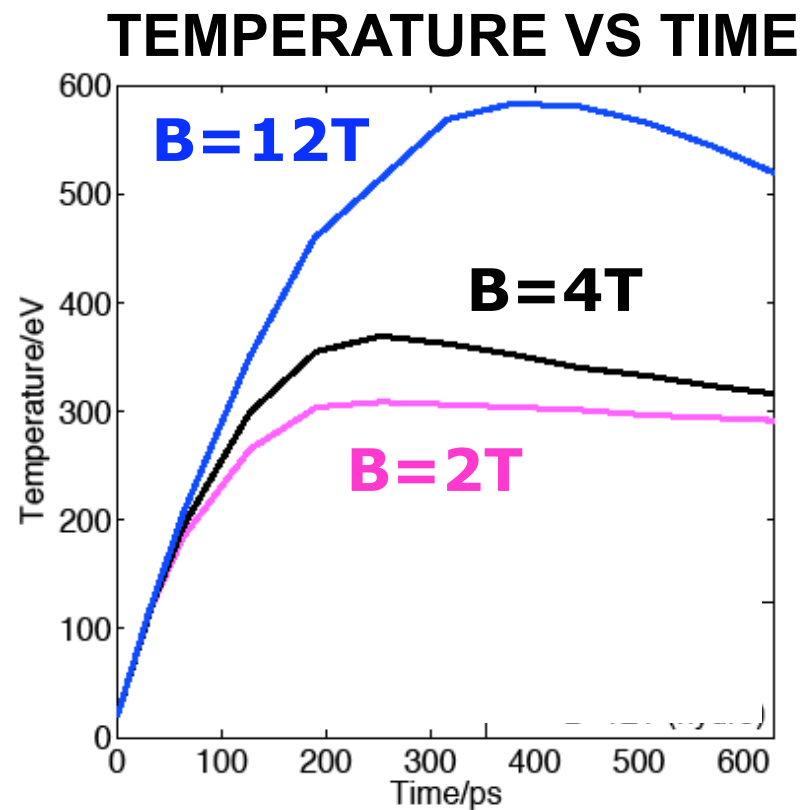
Thermal conduction rate controlled by Hall parameter.



Conductivity increases

Central temperature decreases – Nernst cooling

Can these effects be measured experimentally?



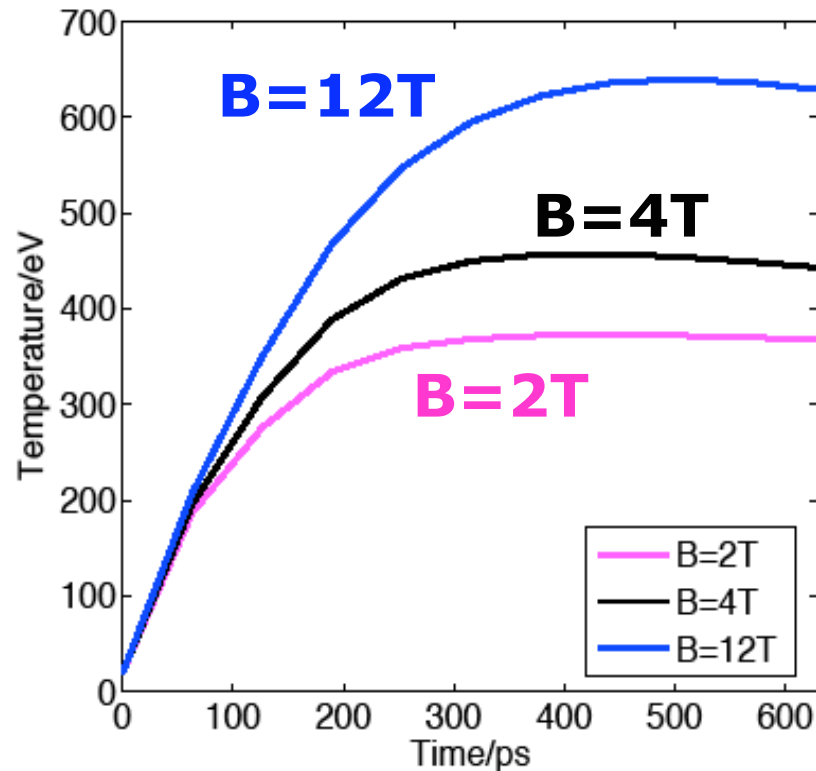
Thermal conduction rate controlled by Hall parameter.

$$\omega\tau \propto \frac{B}{n_e} T_e^{3/2}$$

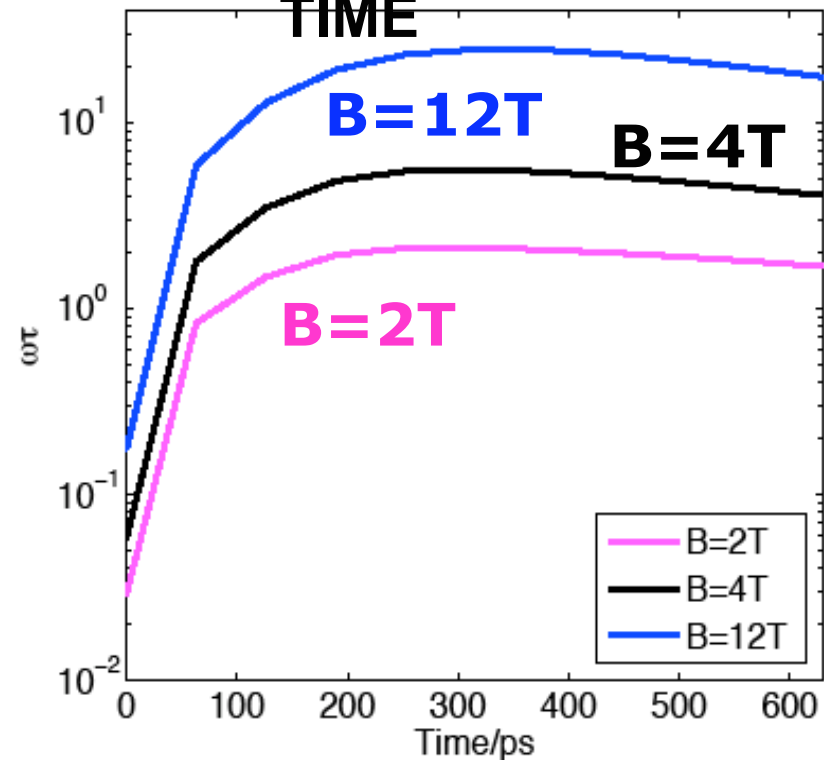
COOLING OF CENTRAL LASER-HEATED REGION DOES NOT OCCUR

Cooling can't happen for frozen-in-flow

TEMPERATURE VS TIME



  VS
TIME



Outline

Part 1 – simulations of long-pulse laser-plasma interactions.

- (1) The interplay of magnetic field dynamics and non-locality.
- (2) Magnetic field generation

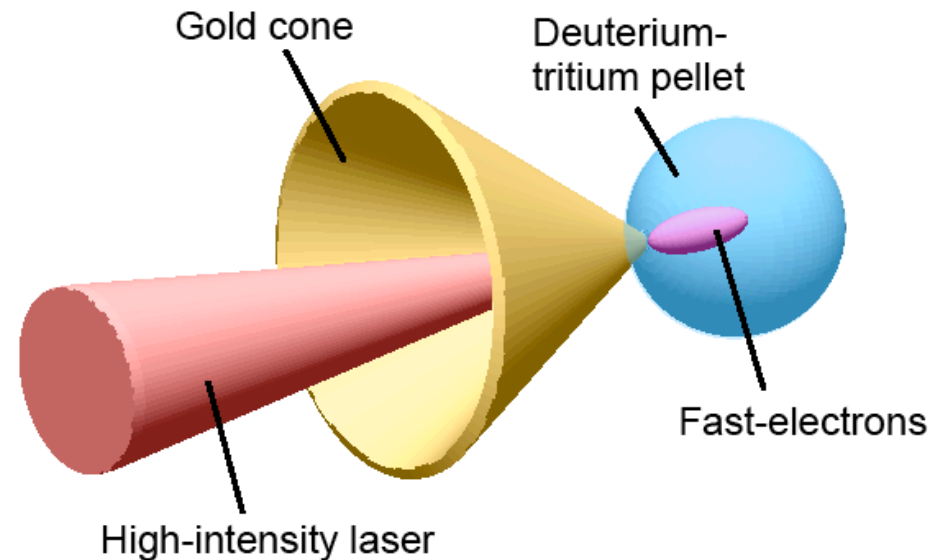
Part 2 – Vlasov-Fokker-Planck simulations of short-pulse laser-plasma interactions relevant to fast ignition. The importance of rear-surface effects.

Short-pulse modelling

Fast ignition¹³ – decouple compression and heating phase.

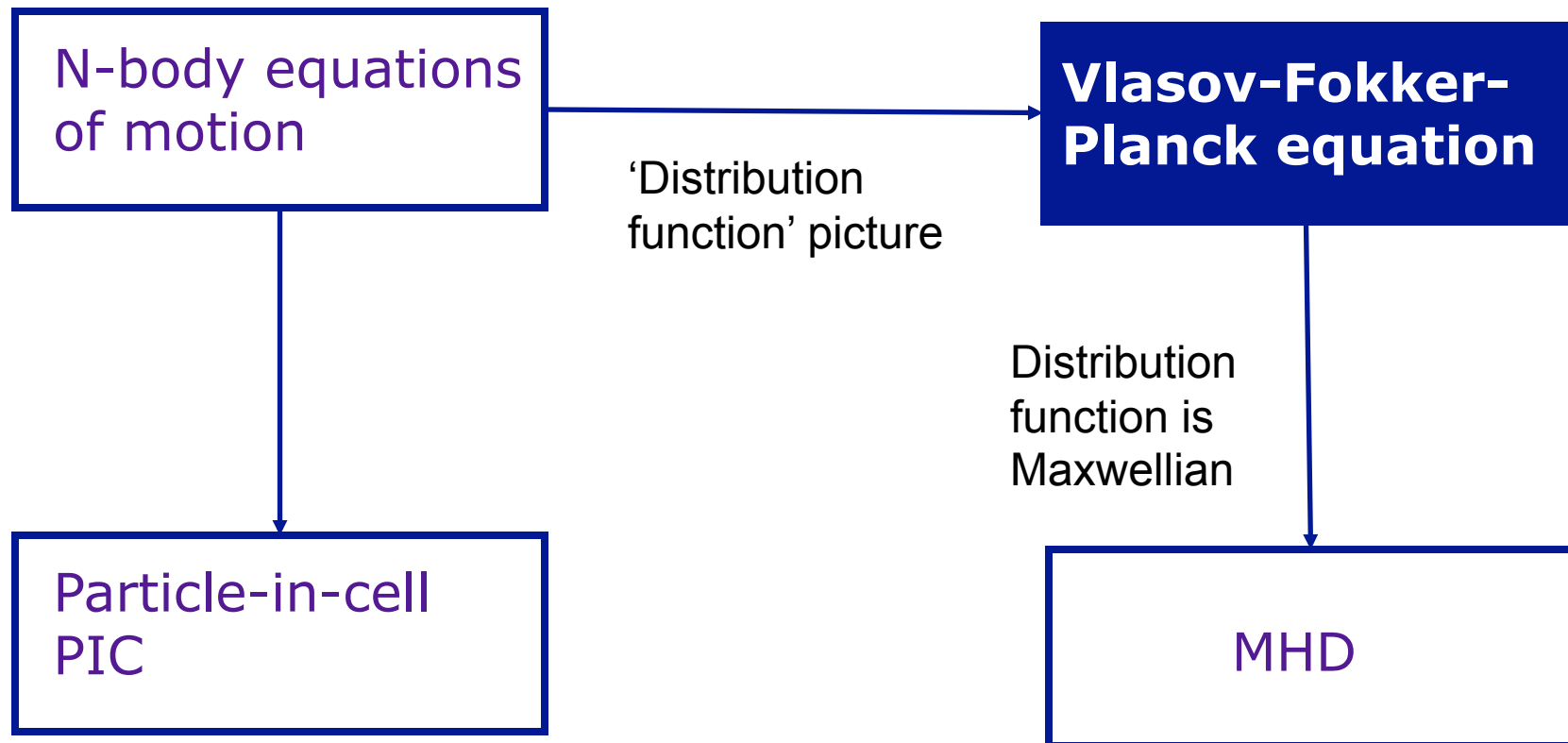
Long-pulse laser
compresses target

Short-pulse laser interacts
with cone and
generates fast
electrons, whose
absorption in the core
heats target like a
spark-plug

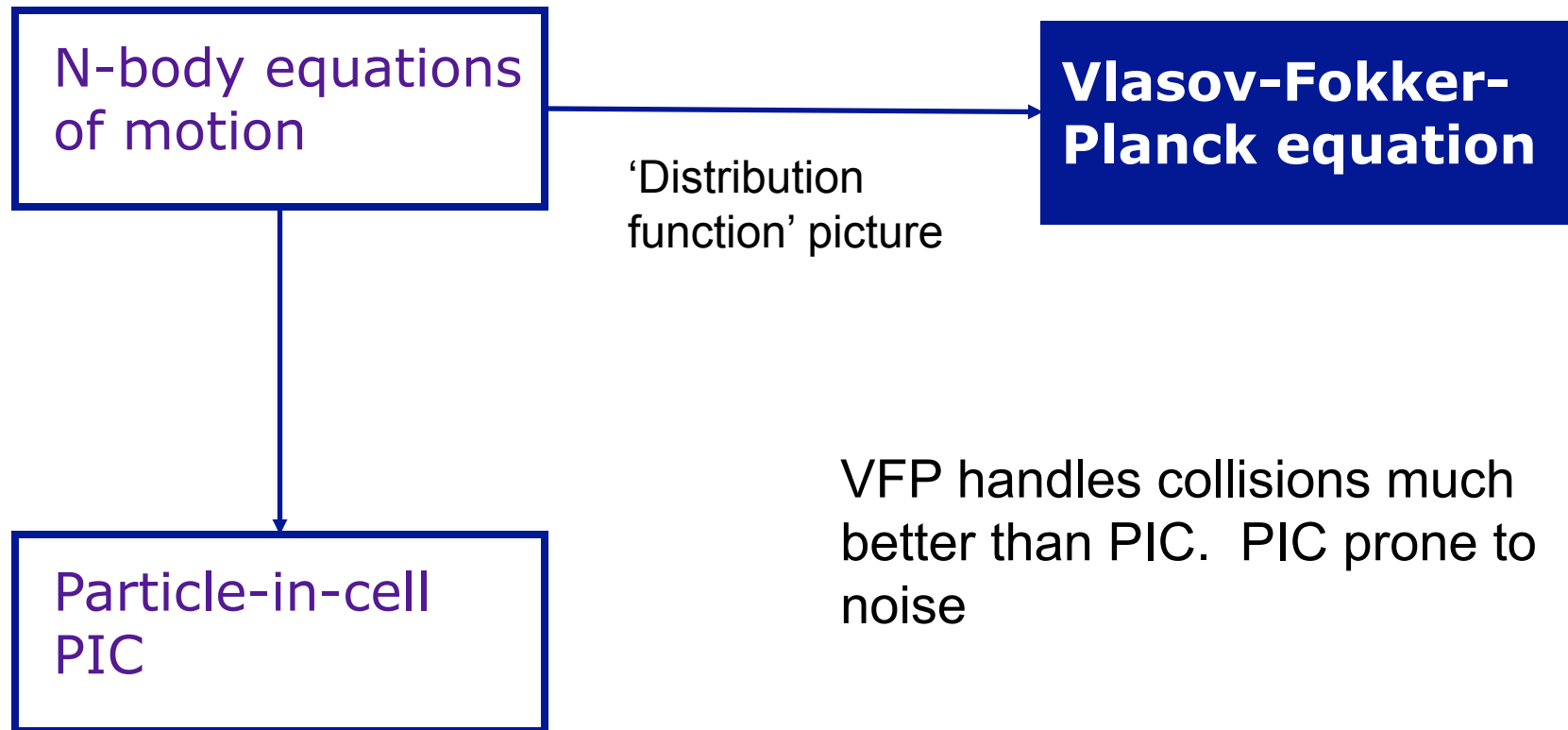


¹³M. Tabak et. al., *Physics of Plasmas*, **1**, 1626 (1971)

Modelling long-pulse LPI

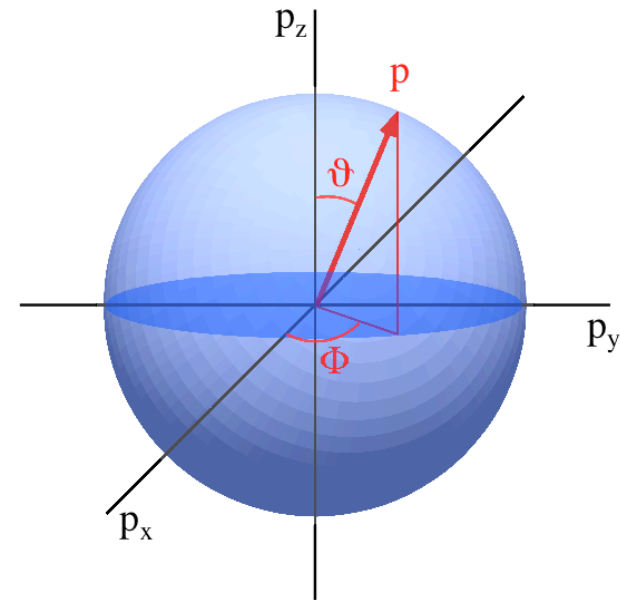


Modelling long-pulse LPI



FIDO¹⁴

- Solves the VFP Equation (and Maxwell's equations) without expanding distribution function
- Implicit EM solve but explicit momentum space solve limits Δt to 3fs
- Can deal with very anisotropic distributions - beams
- Runs in hybrid mode or full VFP mode
- 2D (3V) VFP code with self consistent E_x , E_y and B_z -fields



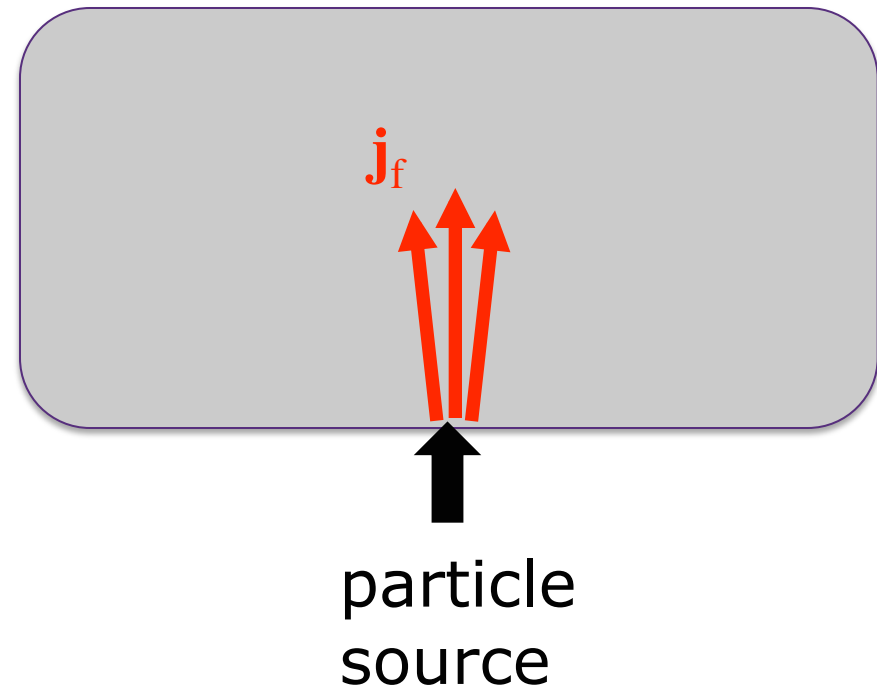
¹⁴Sherlock M.W. submitted to PRL

Short-pulse modelling

Usual model:

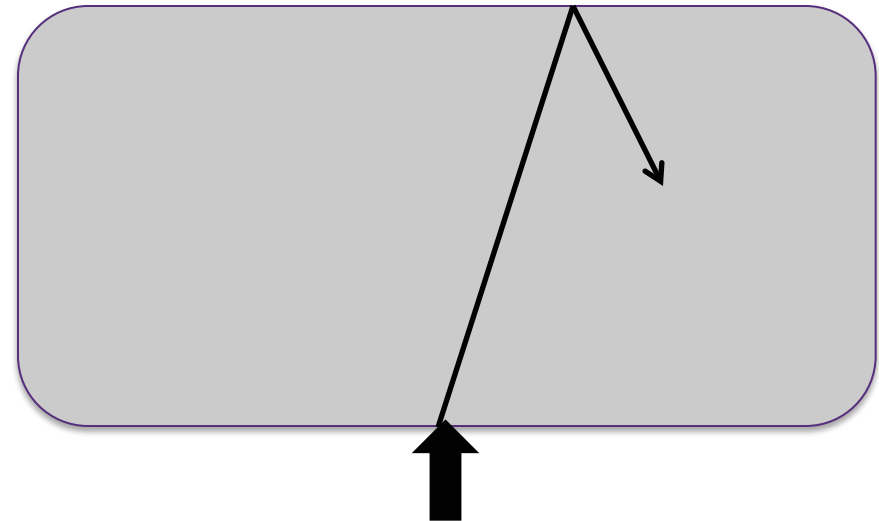
Fast electrons treated
kinetically

Background electrons
treated as fluid



'Standard' hybrid mode

Standard hybrid
usually uses reflective
boundary conditions

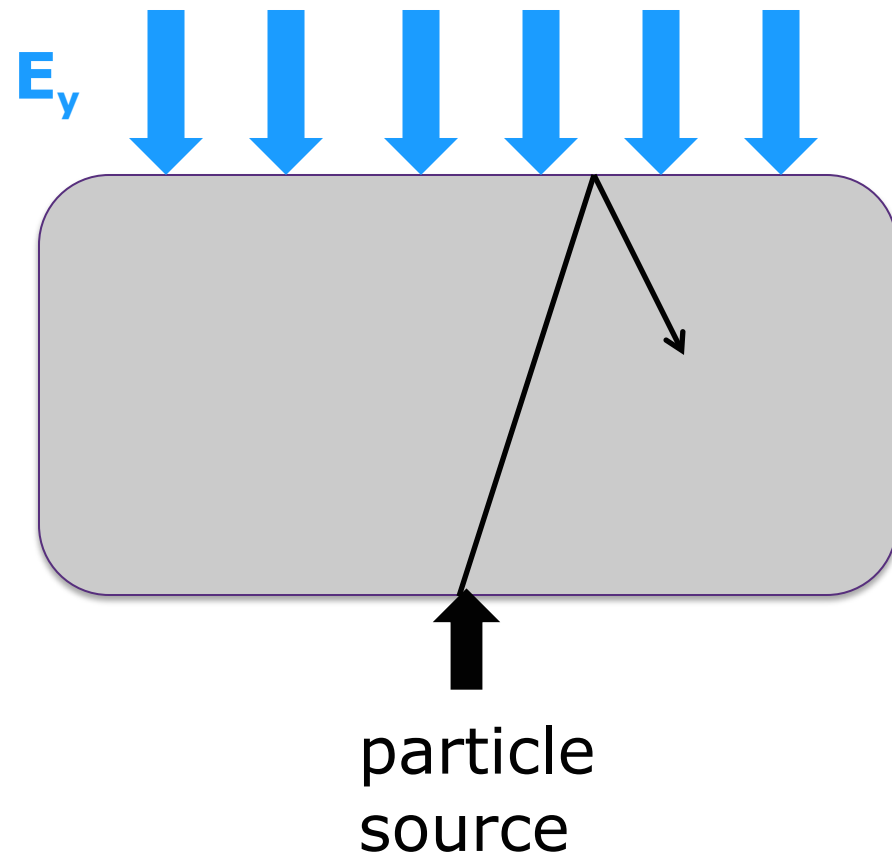


particle
source

‘Standard’ hybrid mode

Standard hybrid
usually uses reflective
boundary conditions

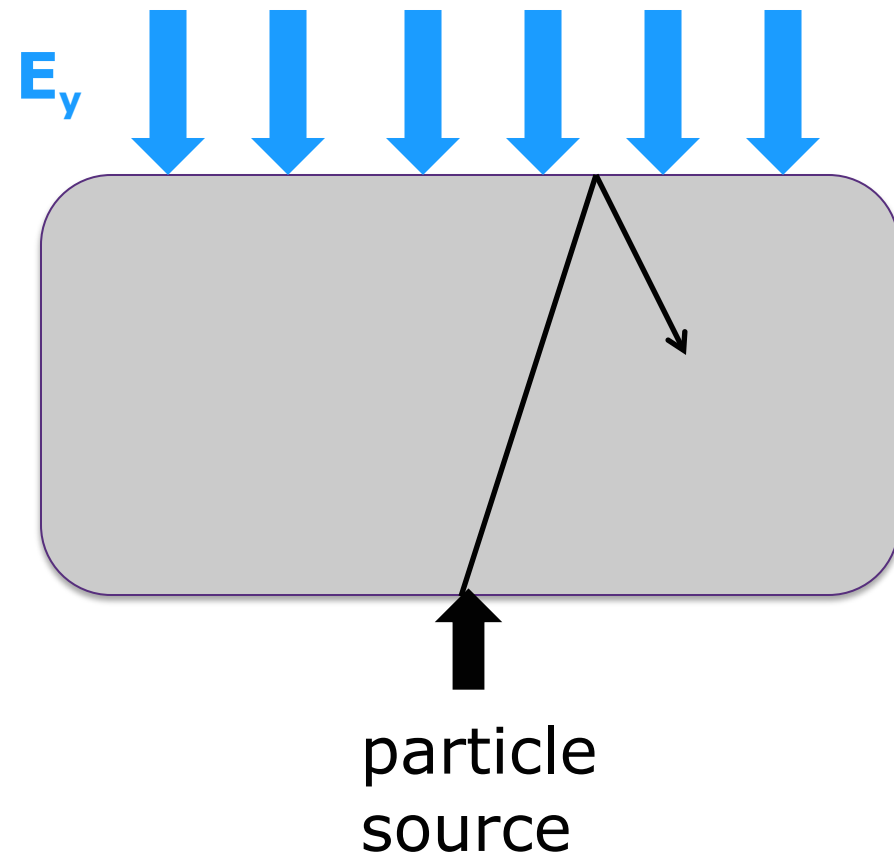
Justified by large
sheath field at rear
surface – reflects
particles. But



'Standard' hybrid mode

Standard hybrid
usually uses reflective
boundary conditions

Justified by large
sheath field at rear
surface – reflects
particles. But

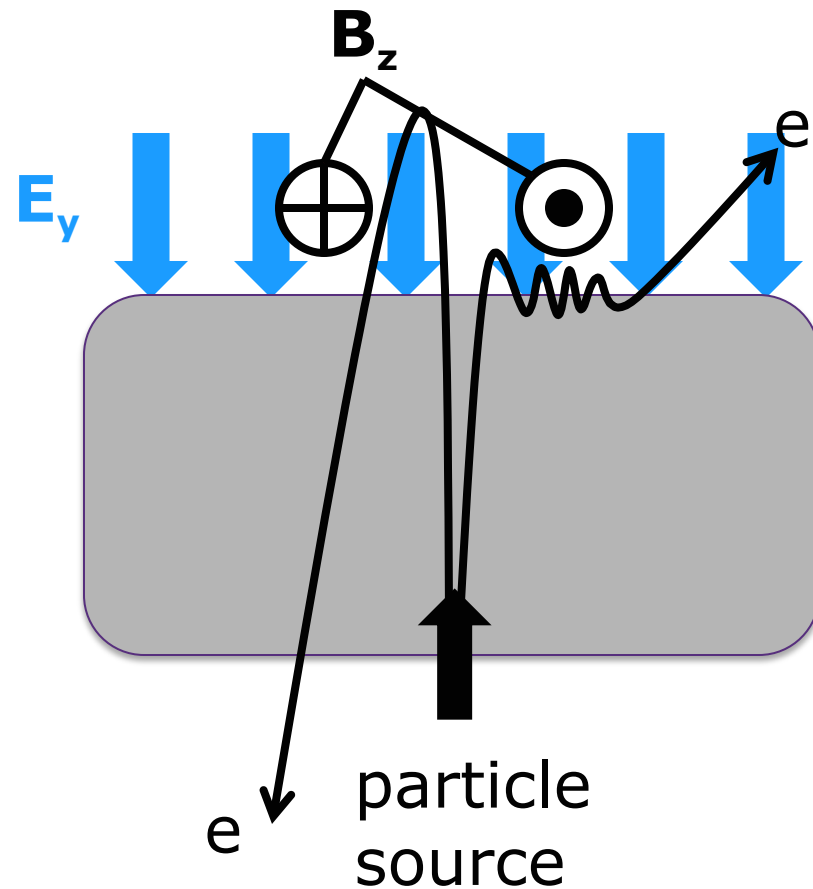


... only if there is no magnetic field

'Improved' hybrid mode

Improved hybrid:

B-fields lead to more complicated motion of electrons at rear-surface.

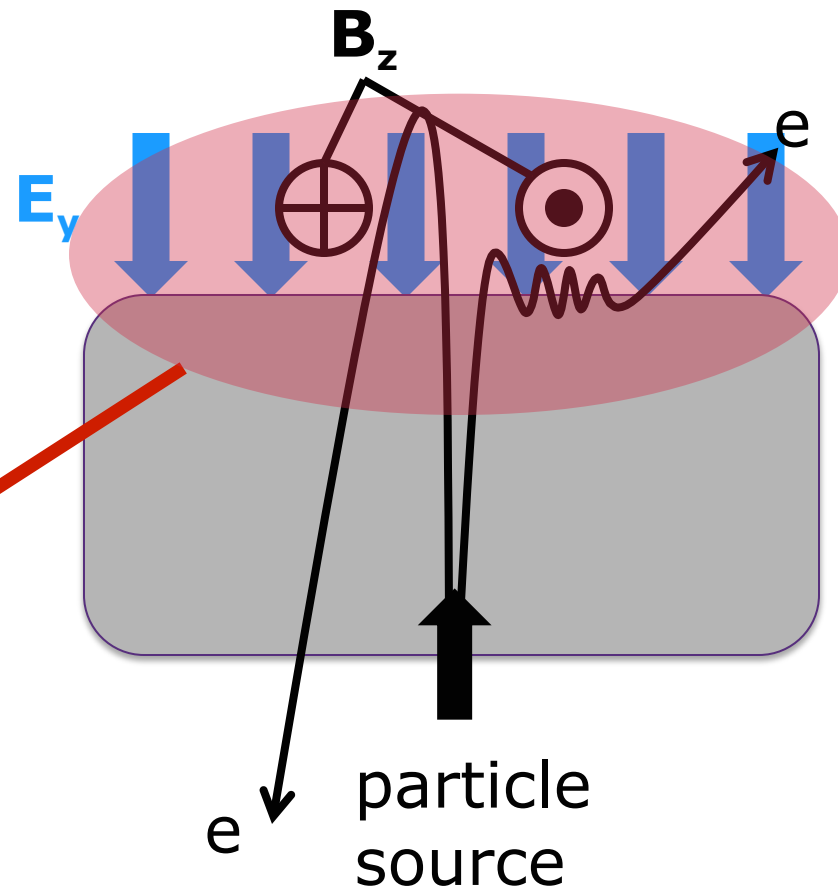


'Improved' hybrid mode

Improved hybrid:

B-fields lead to more complicated motion of electrons at rear-surface.

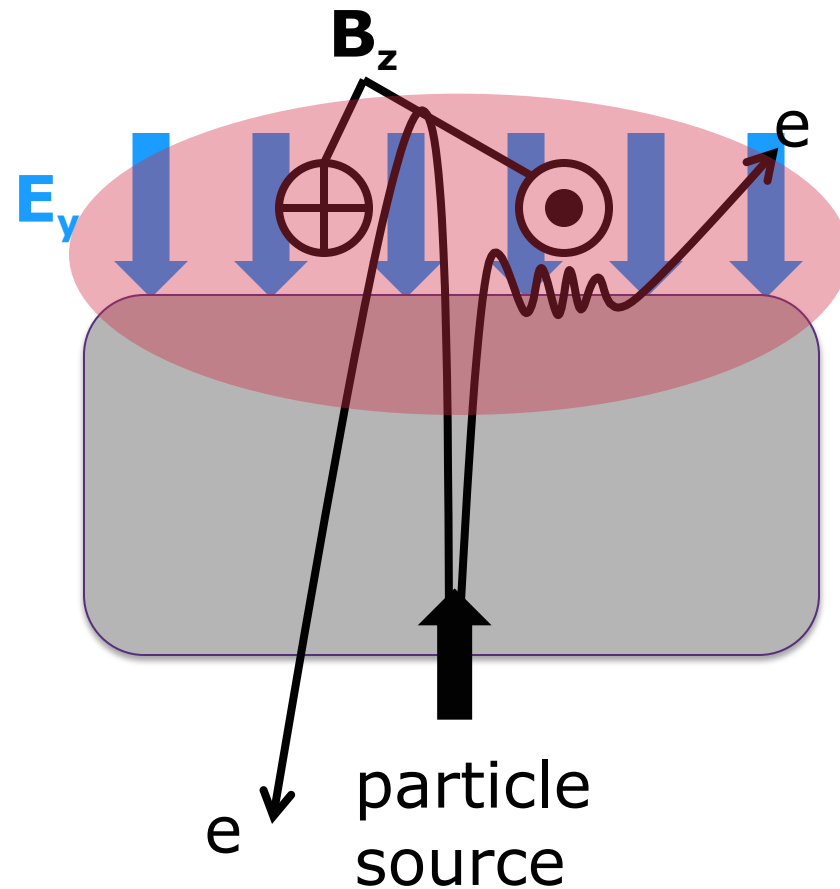
I will simulate rear-surface effects



‘Improved’ hybrid mode

What’s new?

FIRST continuum VFP
hybrid simulation of
fast-electron transport
in solids to include
target edge effects



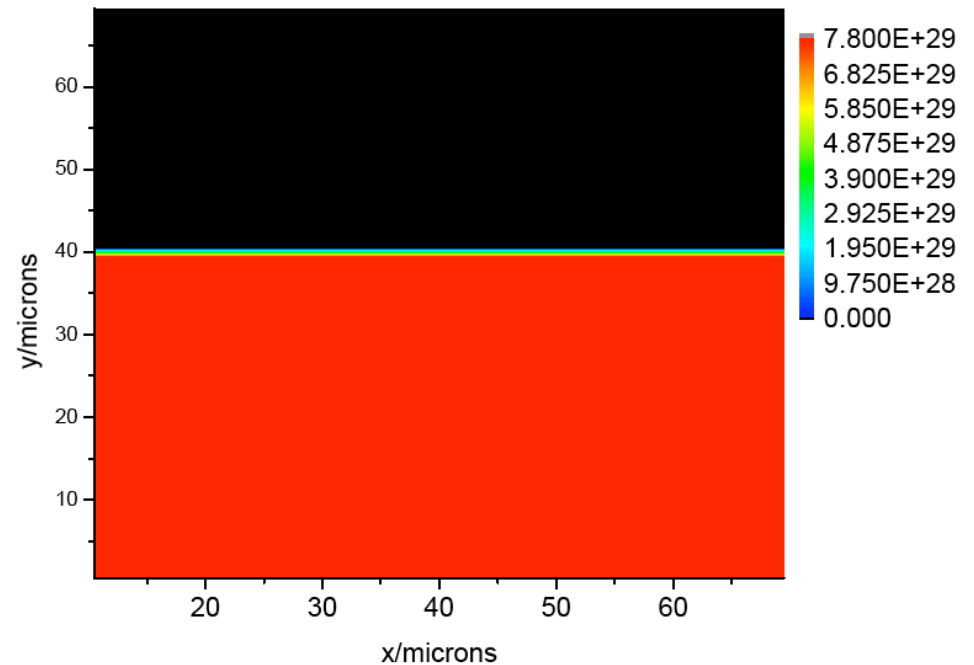
Solid-target simulations

BACKGROUND PROPERTIES:

Background electron
number density
 $n_e = 7.8 \times 10^{23} \text{cm}^{-3}$

Ionic charge – $Z=13$
(Aluminium)

Initial temperature
 $T_e = 50 \text{eV}$ (somewhat
arbitrary)



BACKGROUND ELECTRON NUMBER DENSITY

Solid-target simulations

LASER AND ELECTRON INJECTION:

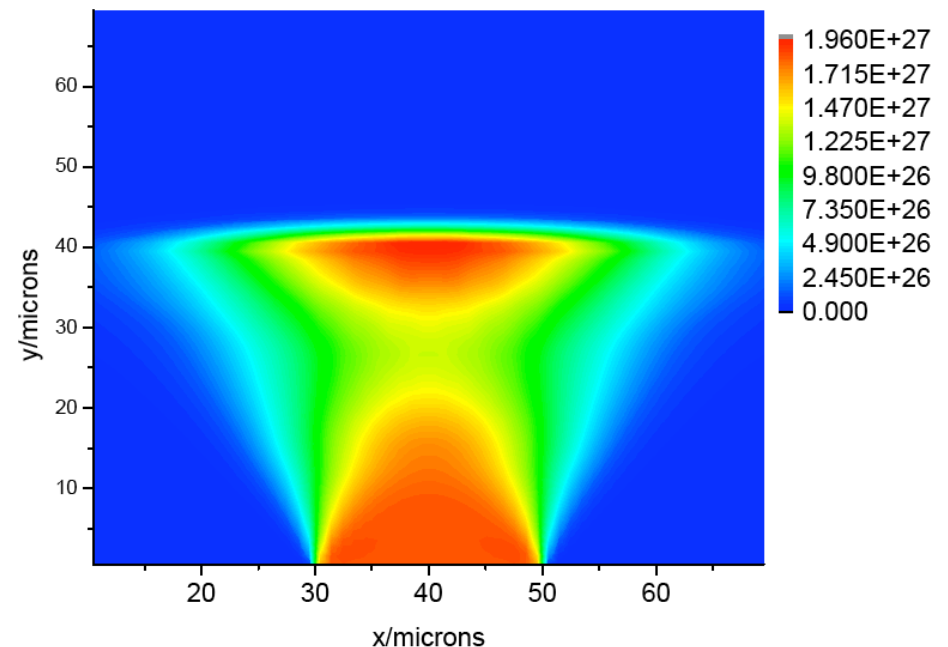
Laser intensity
 $I = 5 \times 10^{19} \text{ W cm}^{-2}$

Absorption fraction
 $\eta = 0.3$

Injection region $8 \mu\text{m} \times 1 \mu\text{m}$ using square mask

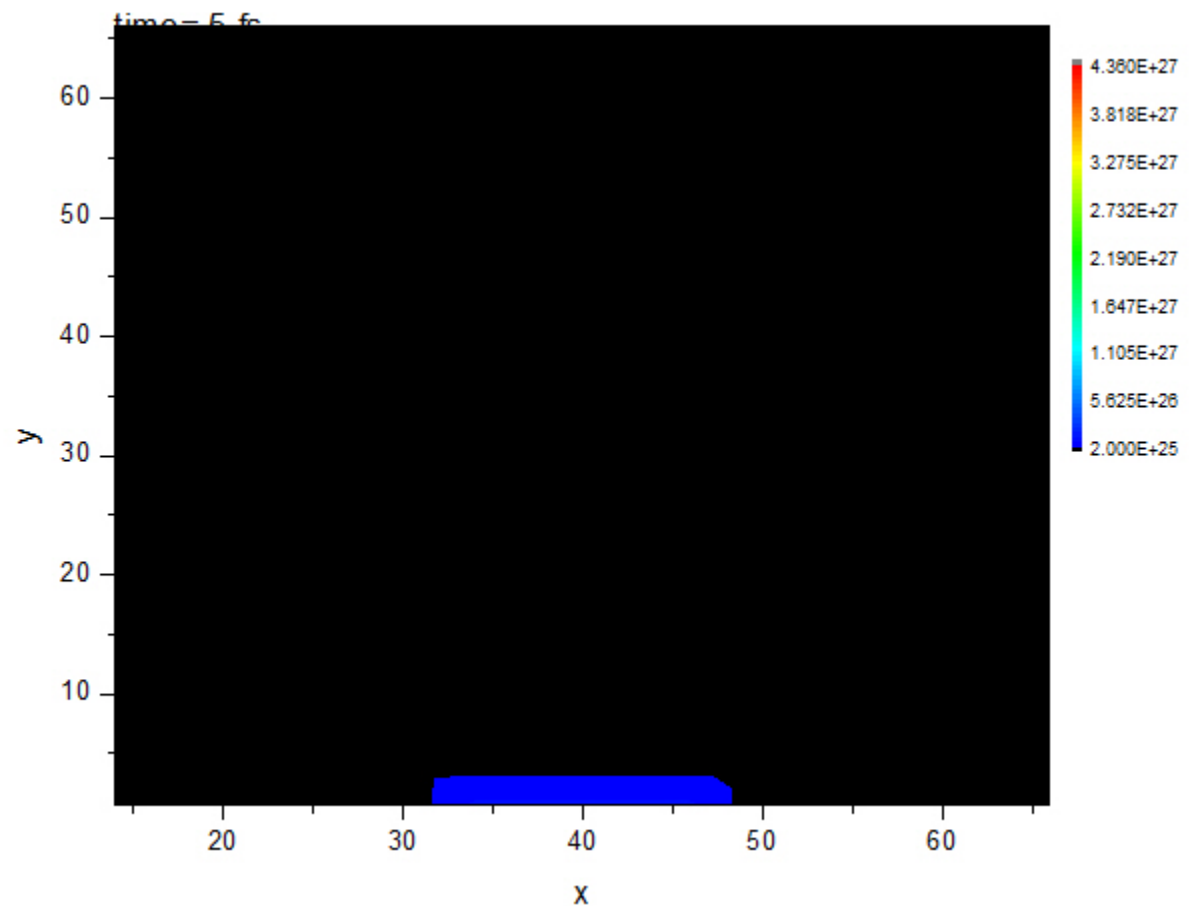
Max injection angle
 $\theta = 28^\circ$

INJECTED ELECTRON DENSITY (after 200fs)



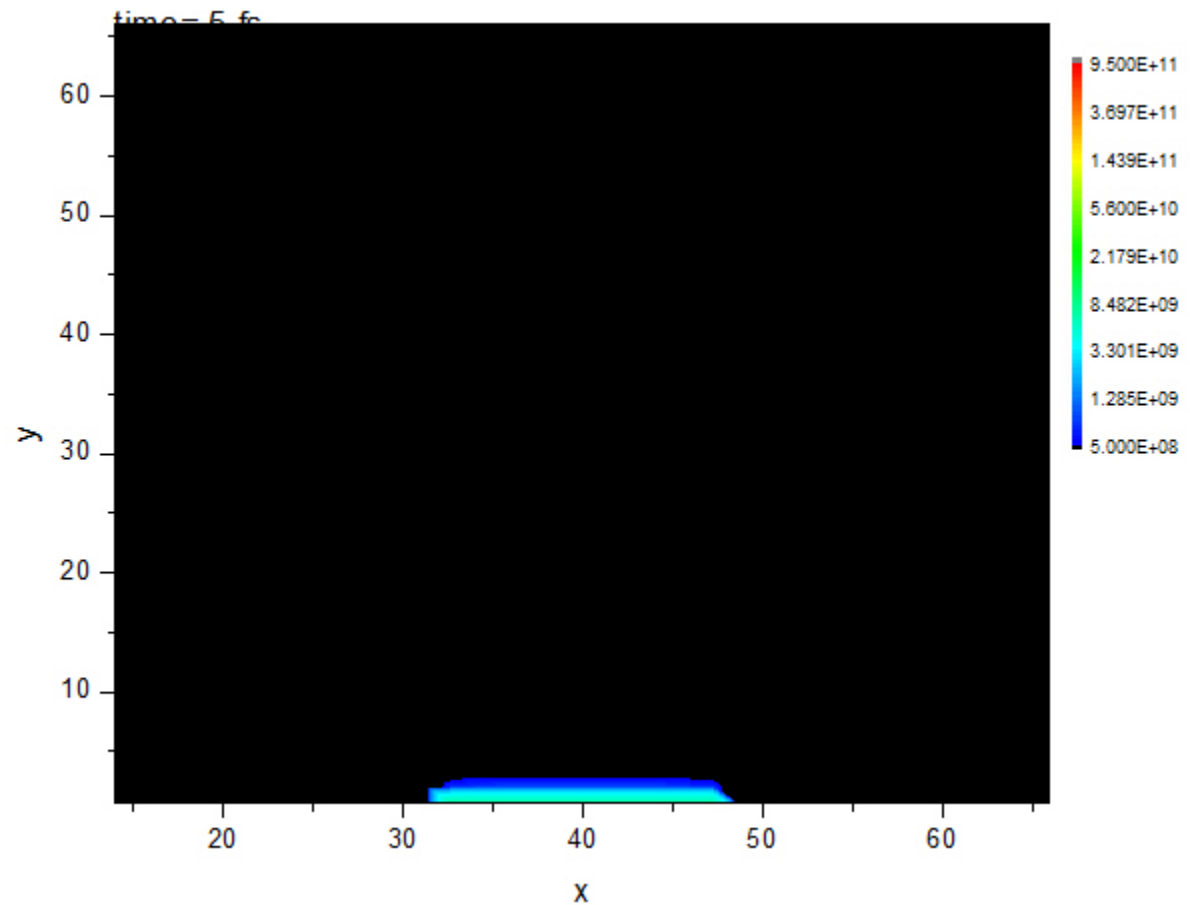
Solid-target simulations

Fast electron
number
density



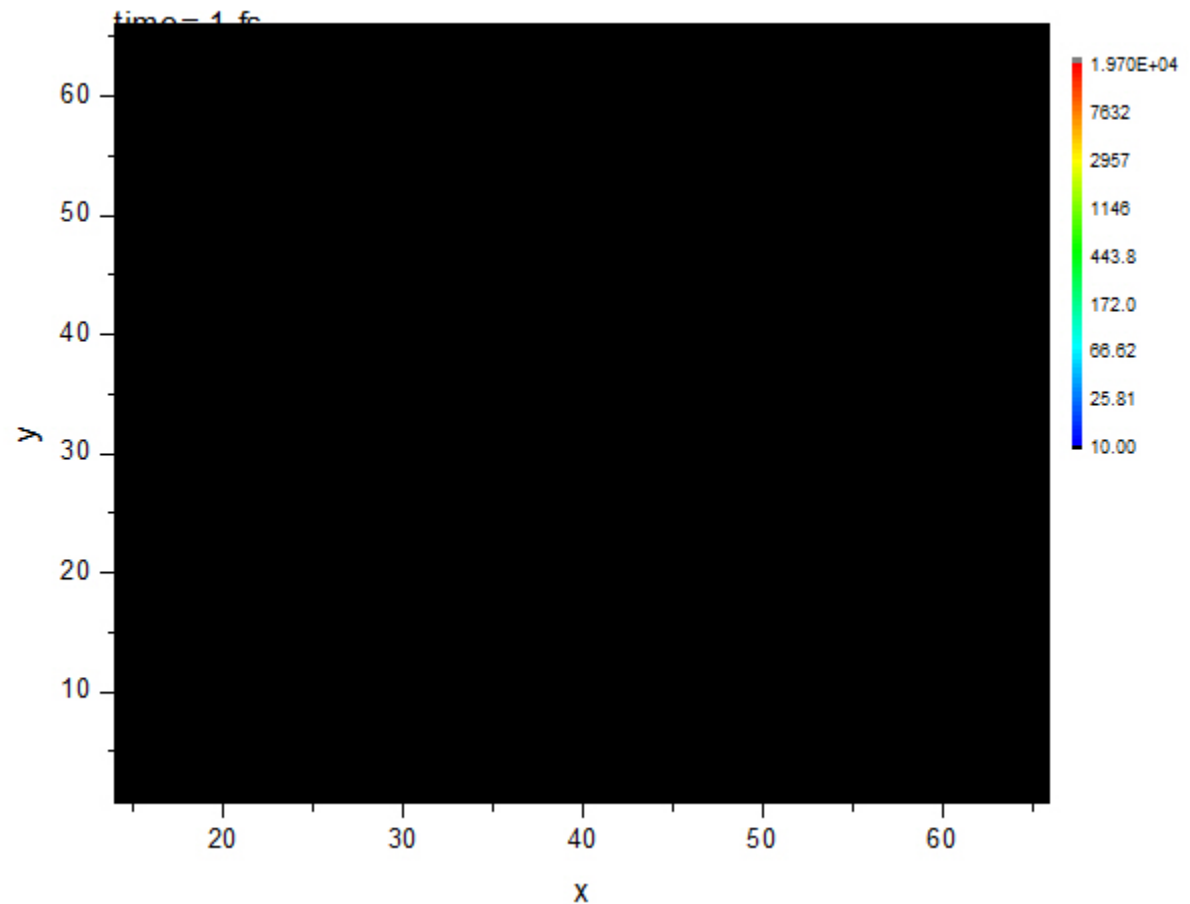
Solid-target simulations

Electric
field



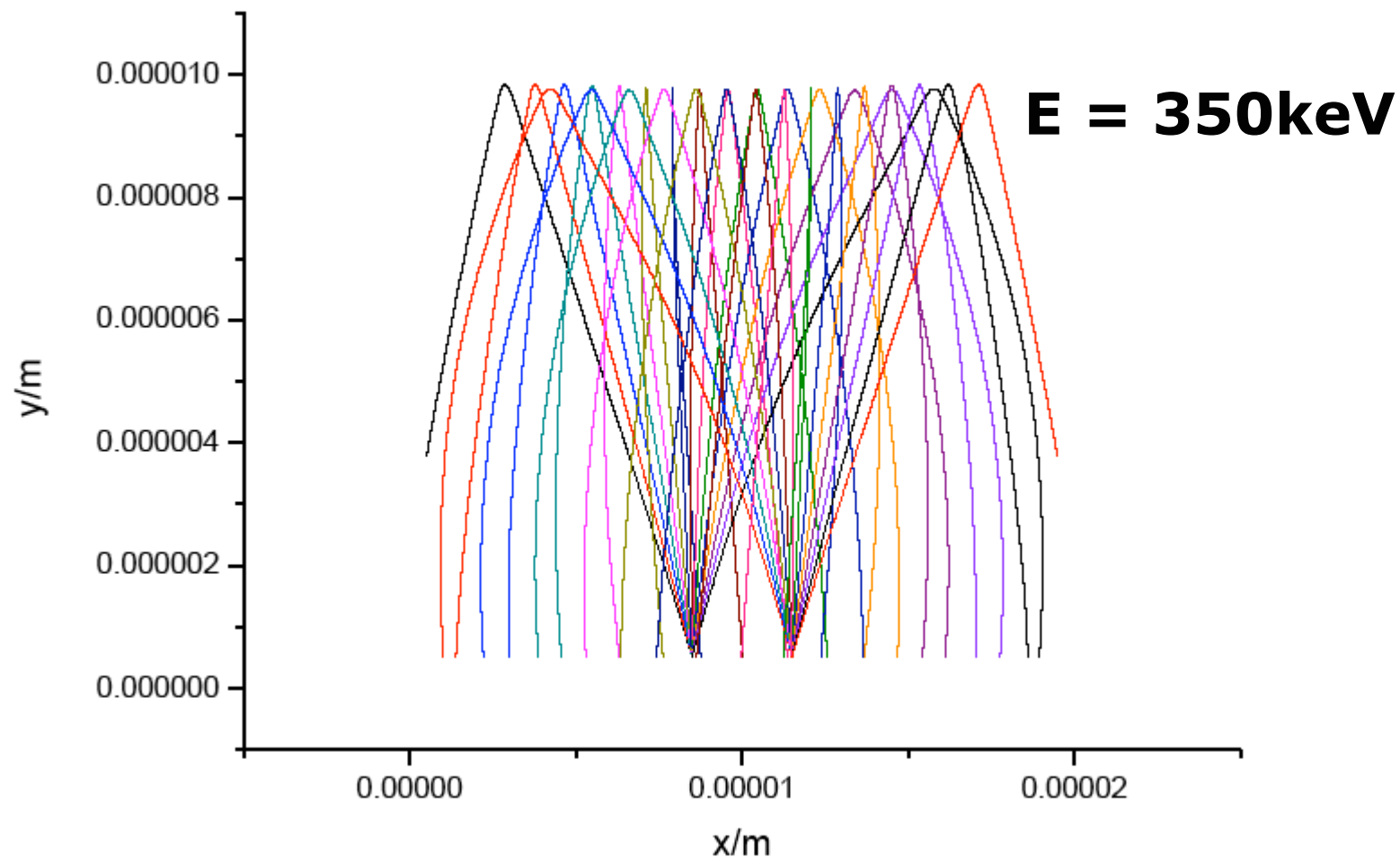
Solid-target simulations

Magnetic
field



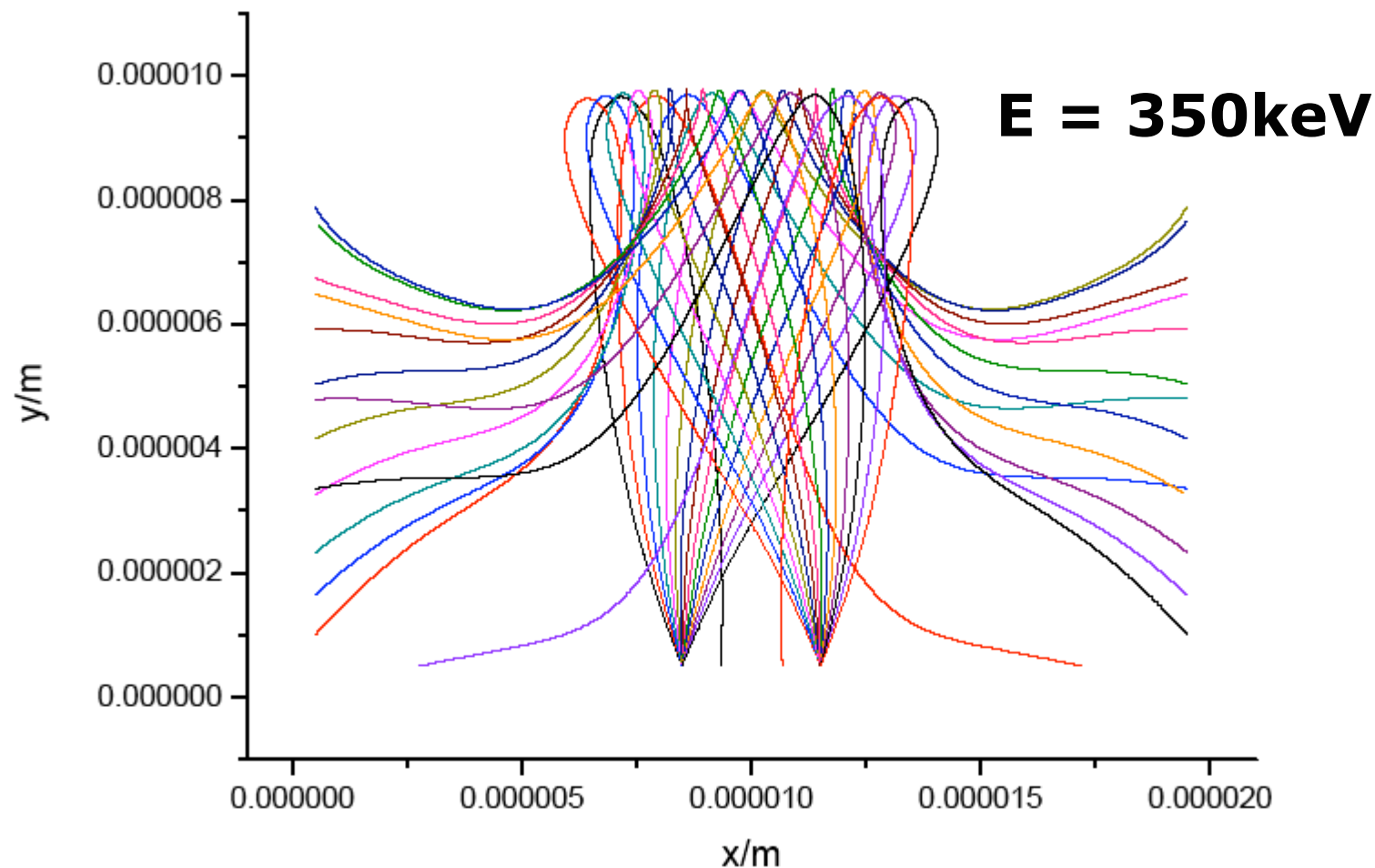
Particle-tracking

Particles injected at 350keV affected by **E** fields ONLY



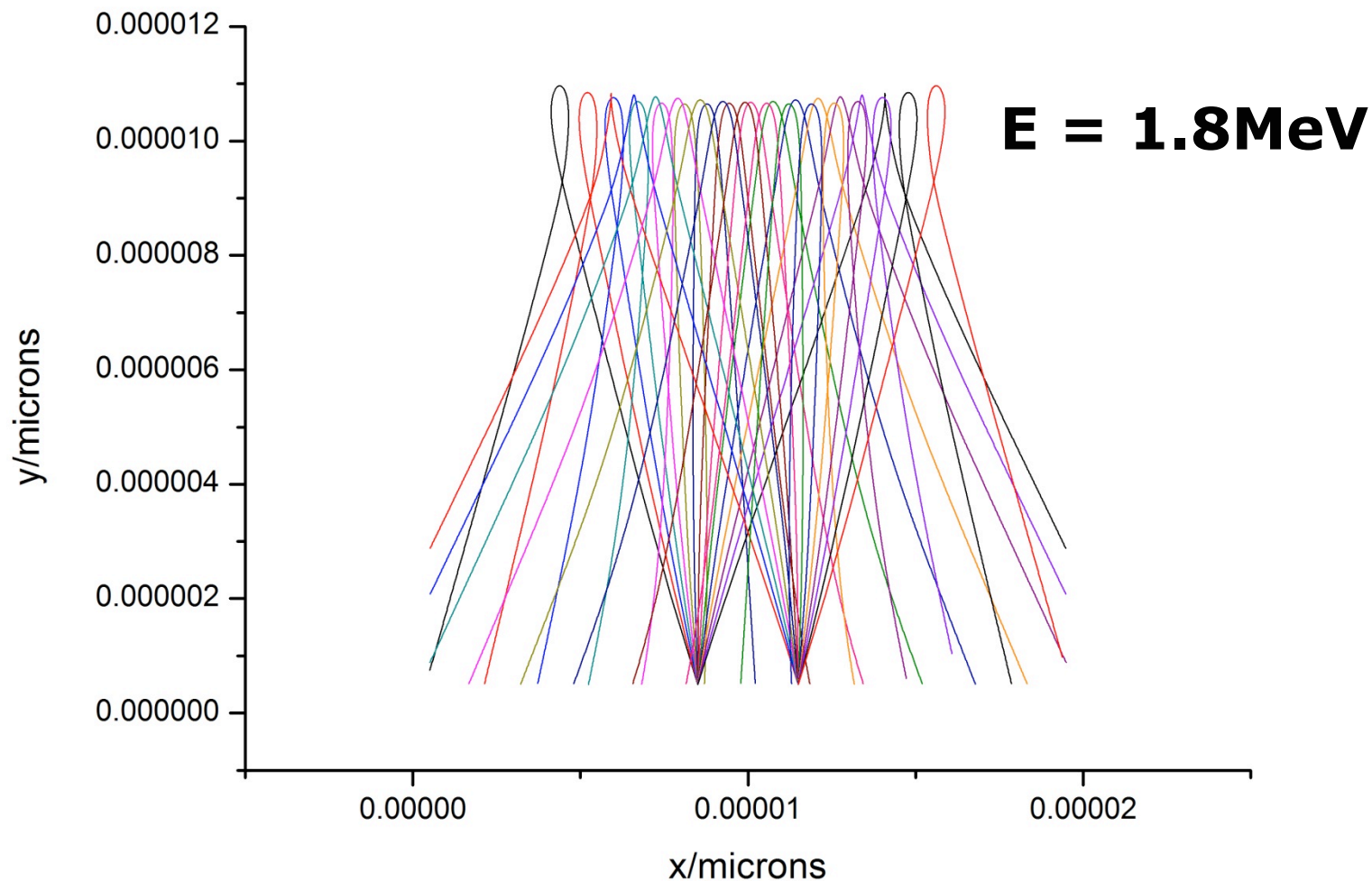
Particle-tracking

Particles injected at 350keV affected by **E** and **B** fields



Particle-tracking

Particles injected at 1.8MeV affected by **E** and **B** fields



Experiments?

Proposals for experiments:

- (1) Is filamentation inside the target real? – use tracer layers and rear surface measurements to see where it happens
- (2) Measure rear surface B and look at effects of scale-length?

Summary

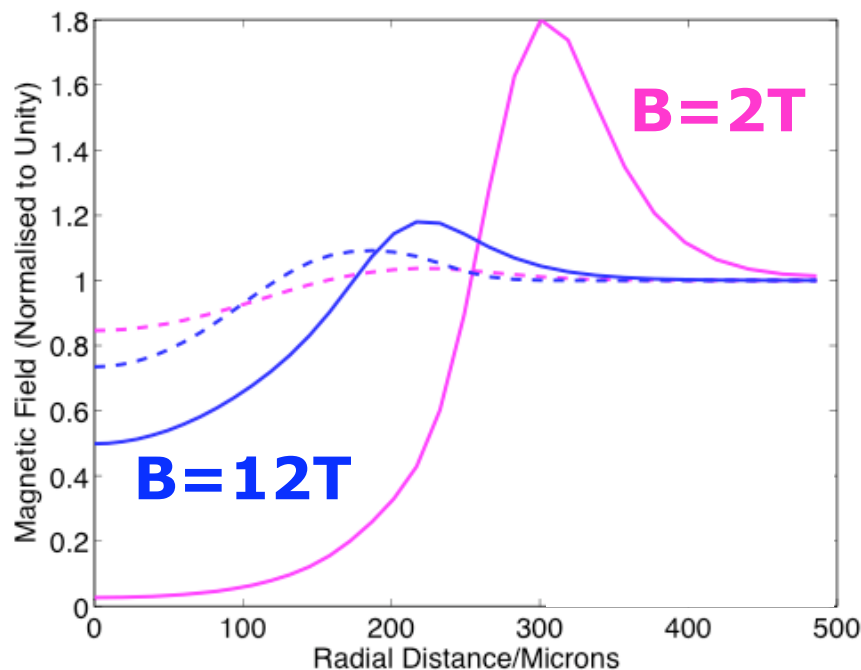
- Realistic inclusion of rear surface effects is important for the explanation of experiments.
- Large fields develop at the rear surface of the target and dominate the behaviour of electrons as they exit the target (and head towards experimental diagnostics)
- Large magnetic fields develop, necessitating a two dimensional treatment of the rear surface

Conclusions

- Kinetic modelling gives useful insight into both long and short-pulse laser-plasma interactions
- Magnetic field dynamics can only be modelled correctly by using the appropriate Ohm's law...
- ...In long-pulse gas-jet interactions this means the inclusion of the Nernst effect...
- ...In short-pulse solid-target interactions this means including edge effects by blending the Ohm's law with a full solution of Maxwell's equations

Nernst Advection dominates over Frozen-in-Flow

B-FIELD ACROSS BEAM

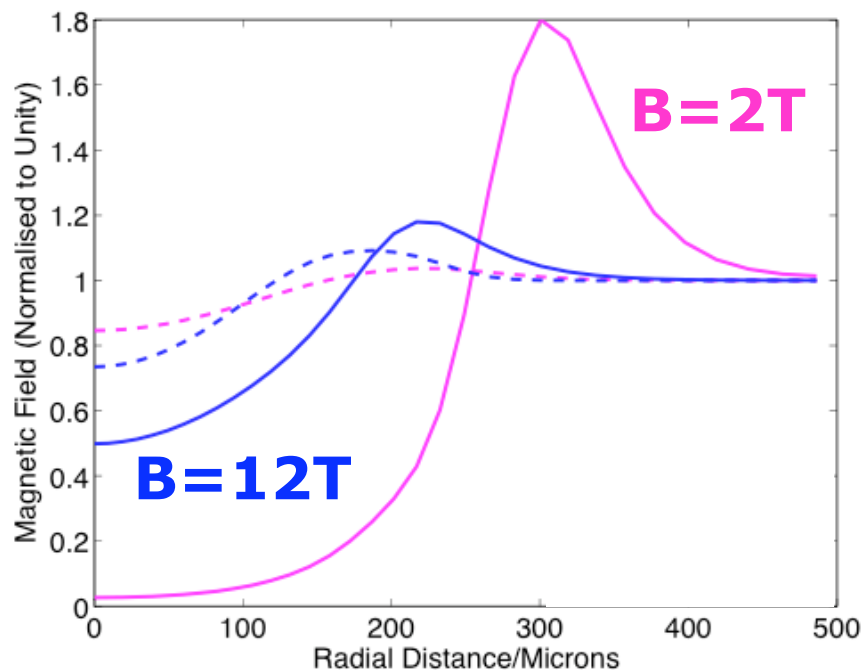


Dimensionless ratio quantifying
Nernst vs. Frozen-In-Flow...

$$R_N = \frac{v_N}{U} = \frac{3 \Delta T_e}{n_e (\omega_e \tau_e)^2 L_T U}$$

Nernst Advection dominates over Frozen-in-Flow

B-FIELD ACROSS BEAM



Dimensionless ratio quantifying
Nernst vs. Frozen-In-Flow...

$$R_N = \frac{v_N}{U} = \frac{3 \Delta T_e}{n_e (\omega_e \tau_e)^2 L_T U}$$

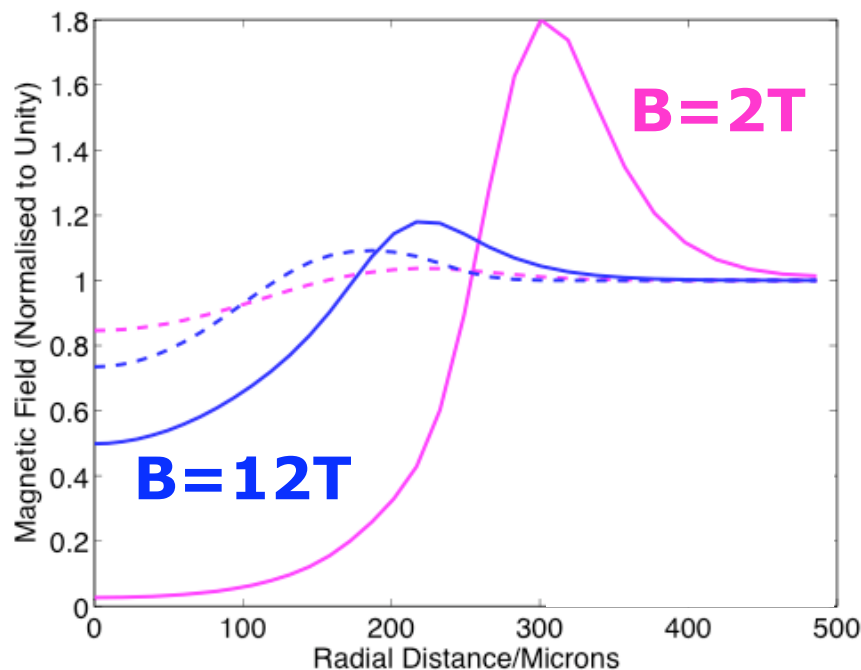
In sims. $R_N = 8$ **B = 2T**

$R_N = 3$ **B = 4T**

$R_N = 0.3$ **B = 12T**

Nernst Advection dominates over Frozen-in-Flow

B-FIELD ACROSS BEAM



Dimensionless ratio quantifying
Nernst vs. Frozen-In-Flow...

$$R_N = \frac{v_N}{U} = \frac{3 \Delta T_e}{n_e (\omega_e \tau_e)^2 L_T U}$$

In sims. $R_N = 8$ **B = 2T**

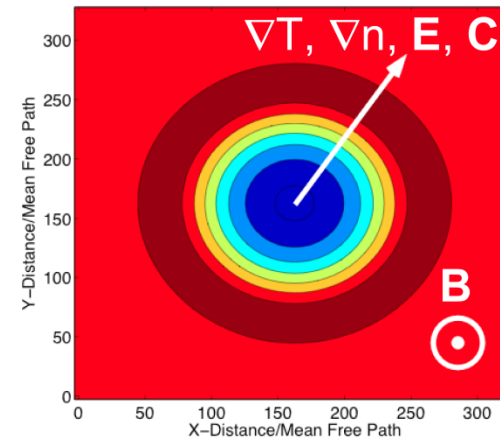
$R_N = 3$ **B = 4T**

$R_N = 0.3$ **B = 12T**

Importance of Nernst is reduced with larger $\omega\tau$

The Nernst effect^{8,9}

$$\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial r} \left(\frac{q_r}{2.5 n_e T_e} B_z \right)$$

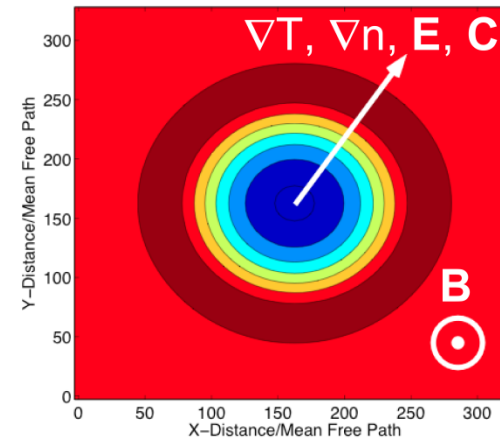


⁸Nishiguchi A. et al, *Physics of Fluids*, **28**, 3683, 1985

⁹Haines M.G., *Plasma Physics and Controlled Fusion*, **28**, 1705, 1986

The Nernst effect^{8,9}

$$\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial r} \left(\frac{q_r}{2.5 n_e T_e} B_z \right)$$



ONLY see Nernst if collision frequency depends on velocity

Hotter (faster) particles carry B-field further

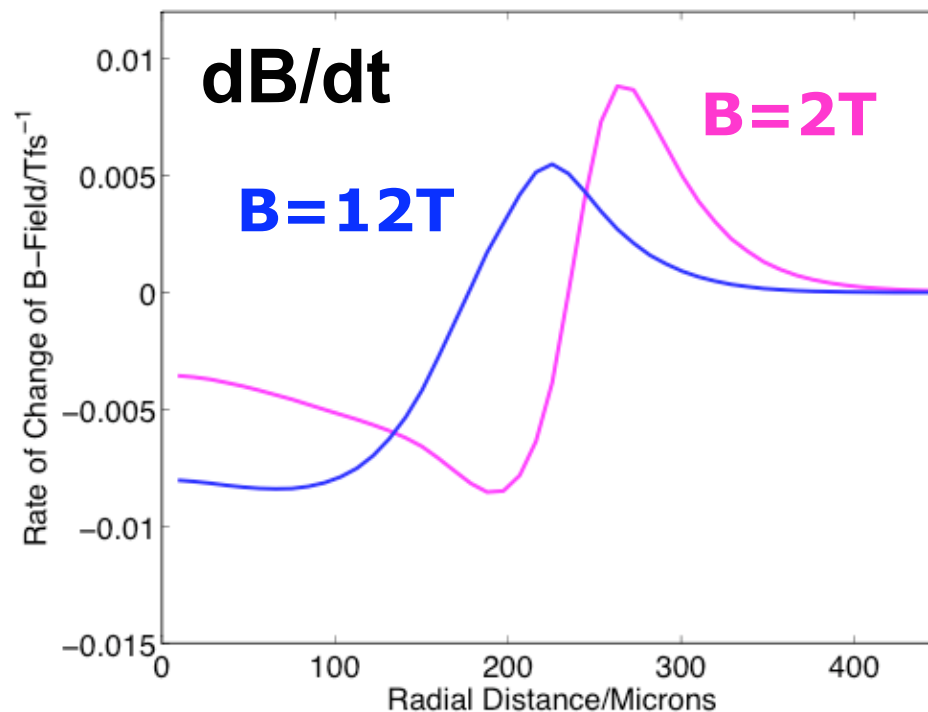
Magnetic field 'frozen' to heat flow

⁸Nishiguchi A. et al, *Physics of Fluids*, **28**, 3683, 1985

⁹Haines M.G., *Plasma Physics and Controlled Fusion*, **28**, 1705, 1986

Non-Local Nernst¹⁰

The Nernst effect is modified by non-locality



Calculate $\frac{dB}{dt}$ from:

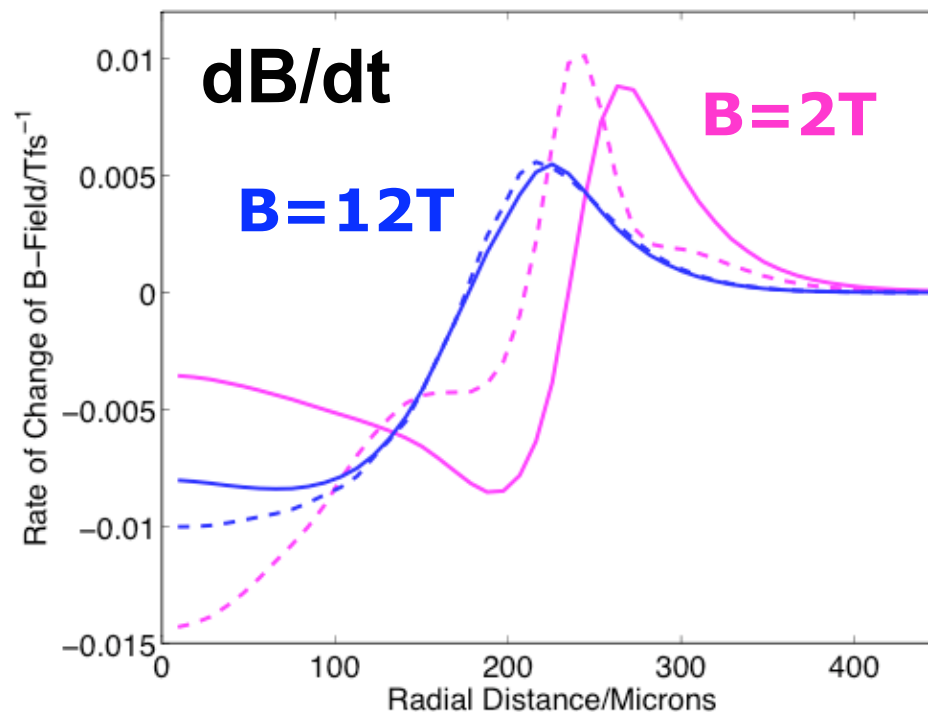
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Solid lines – $\mathbf{E}$ from
VFP code

¹⁰T.H. Kho & M.G. Haines, *Physics of Fluids*, Vol. 29, No. 8, 1986

Non-Local Nernst¹⁰

The Nernst effect is modified by non-locality



Calculate $\frac{dB}{dt}$ from:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Solid lines – $\mathbf{E}$ from
VFP code

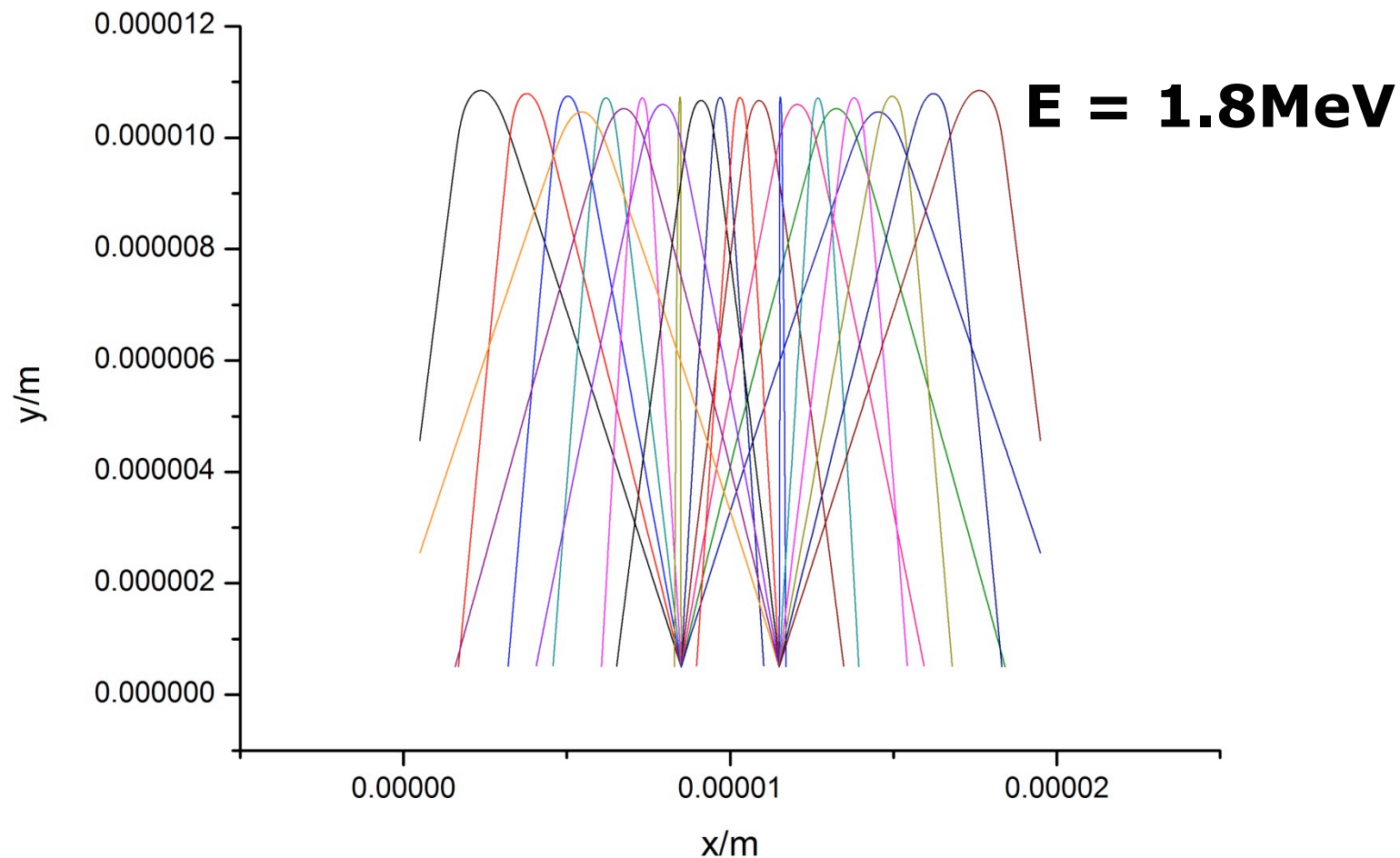
Dashed lines – $\mathbf{E}$ from
classical Ohm's law

Nernst not classical

¹⁰T.H. Kho & M.G. Haines, *Physics of Fluids*, Vol. 29, No. 8, 1986

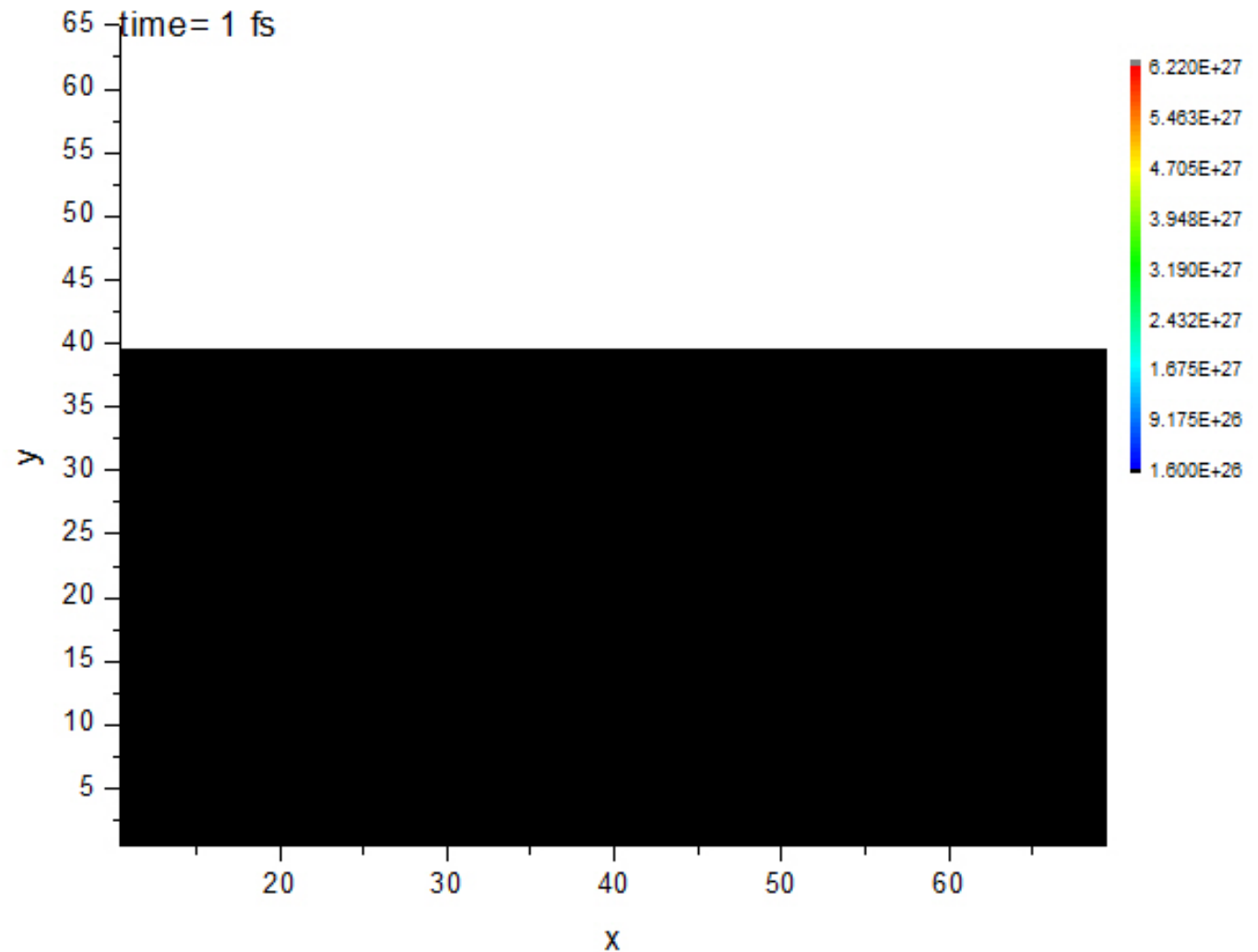
Particle-tracking

Particles injected at 350keV affected by **E** fields ONLY



Reflective boundary conditions

Fast
electron
number
density



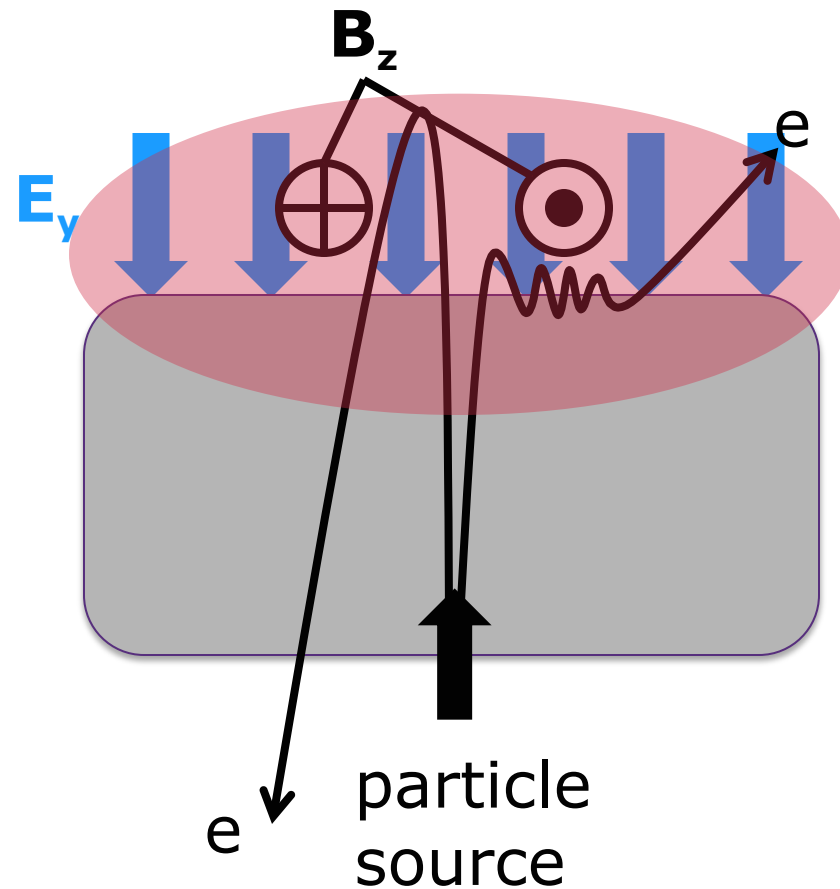
‘Improved’ hybrid mode

Need to solve
Maxwell’s equations
and Ohm’s law.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \mu_0 \mathbf{j}$$

**VFP code must
supply current**



'Improved' hybrid mode

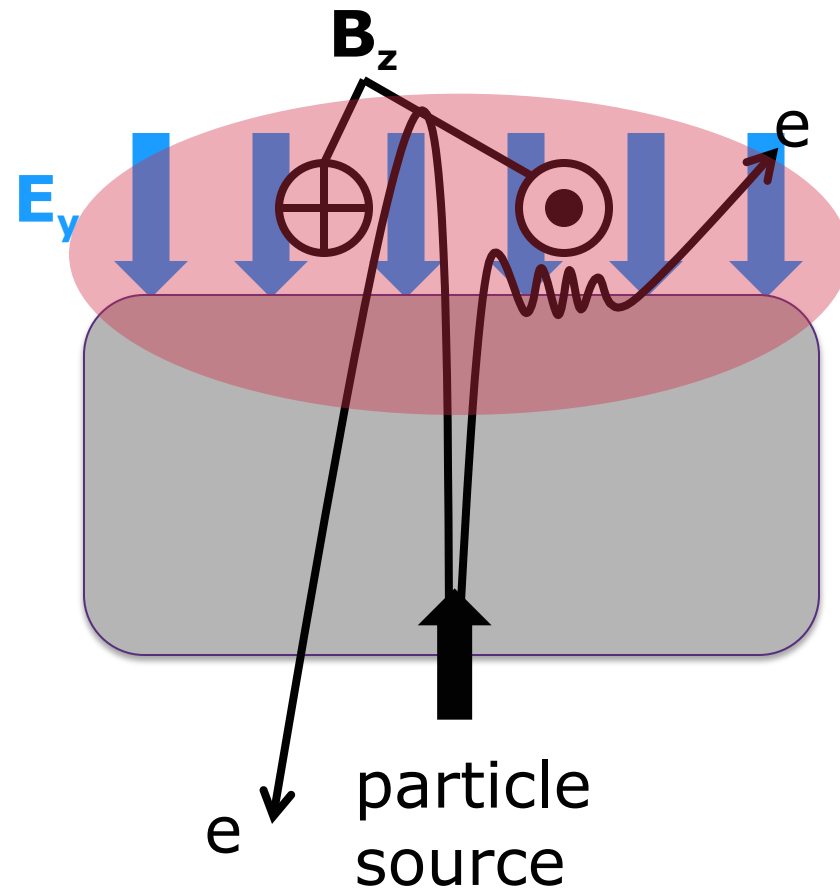
Need to solve
Maxwell's equations
and Ohm's law.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \mu_0 \mathbf{j}$$

Standard

Ohm's law: $\mathbf{E} = \eta \mathbf{j}_b$



'Improved' hybrid mode

Need to solve
Maxwell's equations
and Ohm's law.

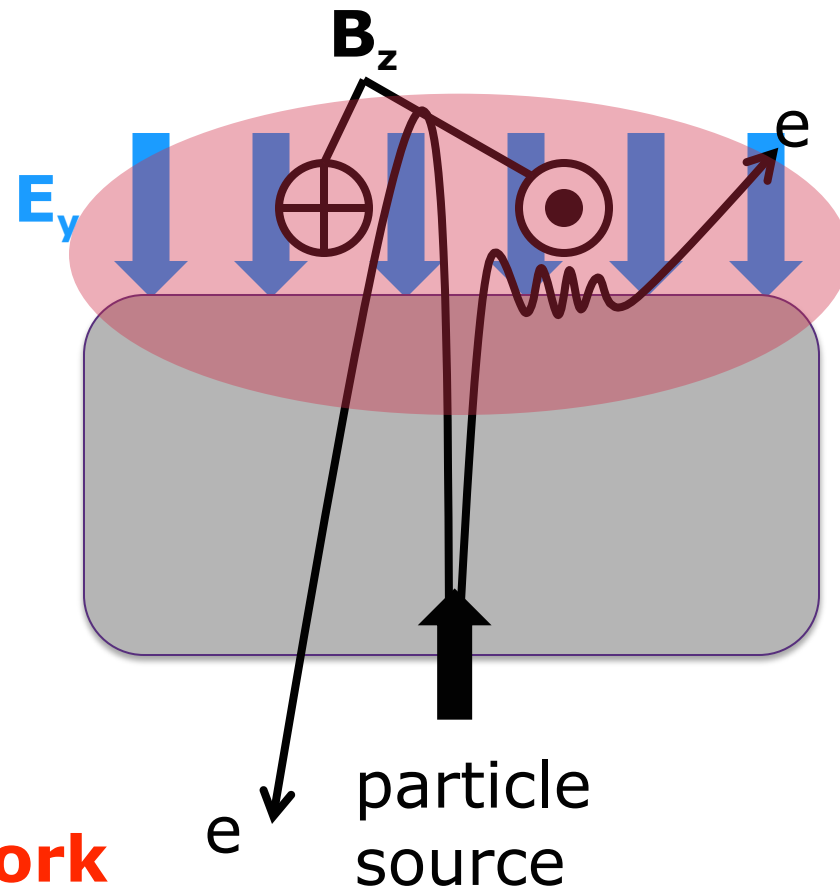
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \mu_0 \mathbf{j}$$

Standard

Ohm's law: $\mathbf{E} = \eta \mathbf{j}_b$

**BUT... This doesn't work
for very low density**



'Improved' hybrid mode

Need to solve
Maxwell's equations
and Ohm's law.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \mu_0 \mathbf{j}$$

Standard

Ohm's law: $\mathbf{E} = \eta \mathbf{j}_b + \dots$

**Include more terms and
ensure realistic resistivity
in vacuum**

