PPS-5000. Credit: Safran

A comparison between kinetic theory and Particle-in-Cell simulations of anomalous electron transport in **E**x**B** plasma discharges

01/28/21

International Online Plasma Seminar (IOPS)

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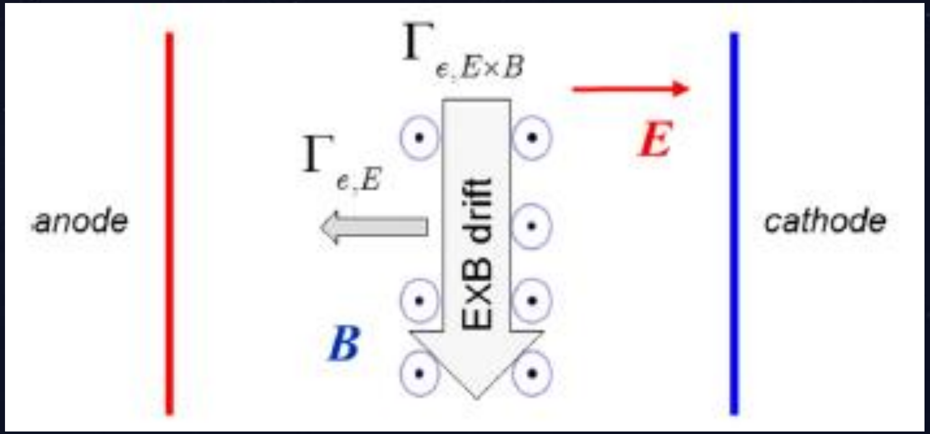
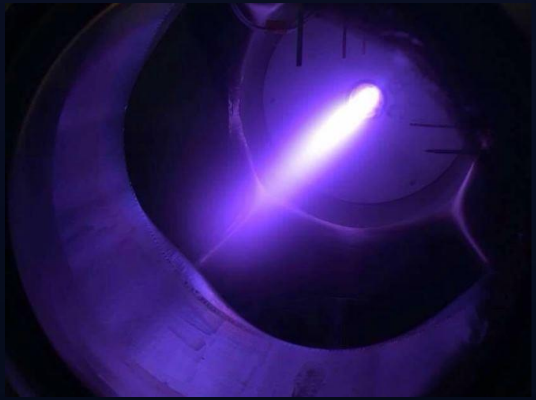
E x B discharges

Main characteristics - Examples

- **Low gas pressure** (typically between 0.1 and 10 mTorr)
- Plasma confinement by a static magnetic field ➔ **Electrons are magnetized – Ions are not magnetized**
- **Closed drift in azimuthal ExB direction ➔ Instabilities** in ExB direction due to large electron drift

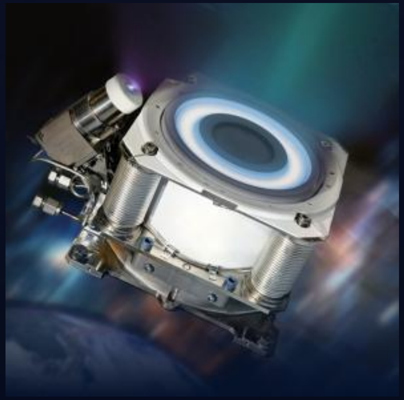
Typical ExB configuration [1]





Penning discharges [Credit: PPPL]

Magnetron discharges [Credit: PT&B]



Hall Thrusters [Credit: Safran]

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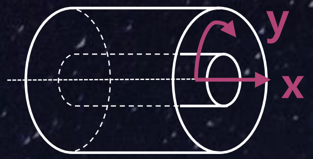
[1] J.-P. Boeuf, "Tutorial: Physics and modeling of Hall thrusters," Journal of Applied Physics, vol. 121, no. 1, p. 011101, 2017

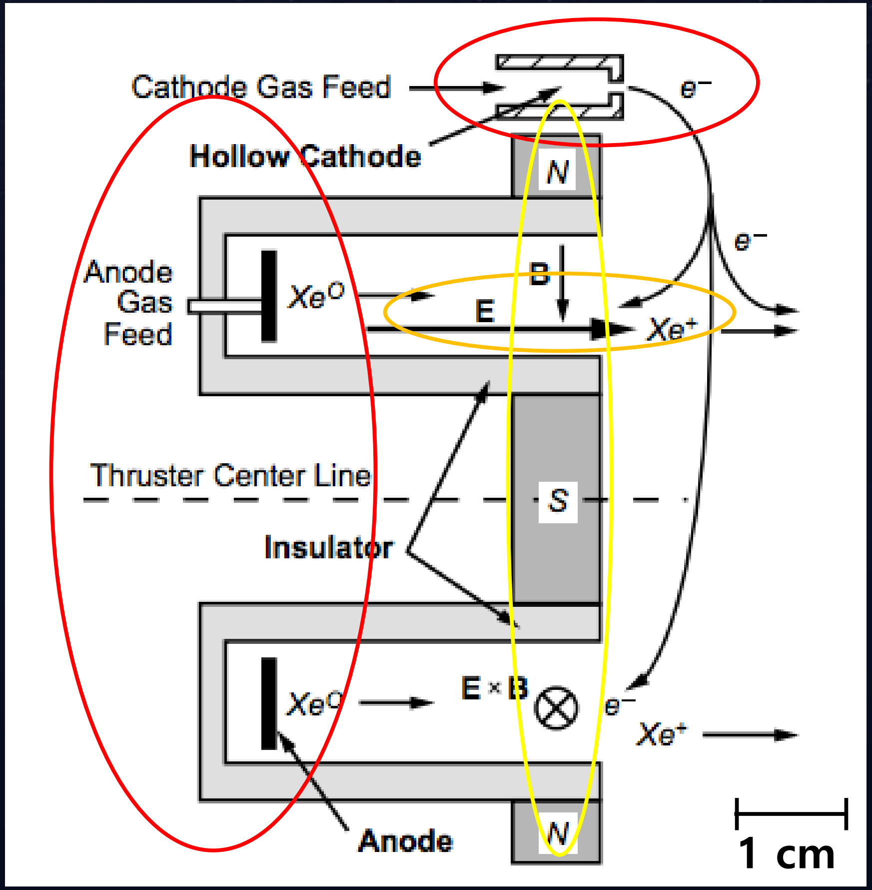


Focus on Hall thrusters (HT)

Working principle

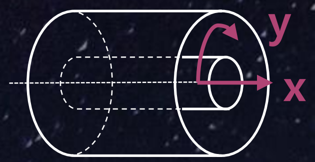
Front view of a HET

Widely use for in-space propulsion



Side view of a HET

- Neutral (anode) and electrons (cathode) injection
- Ions accelerated by axial $\vec{E}$ → Thrust
- Magnetic barrier for electrons (azimuthal drift) → Enhanced ionization



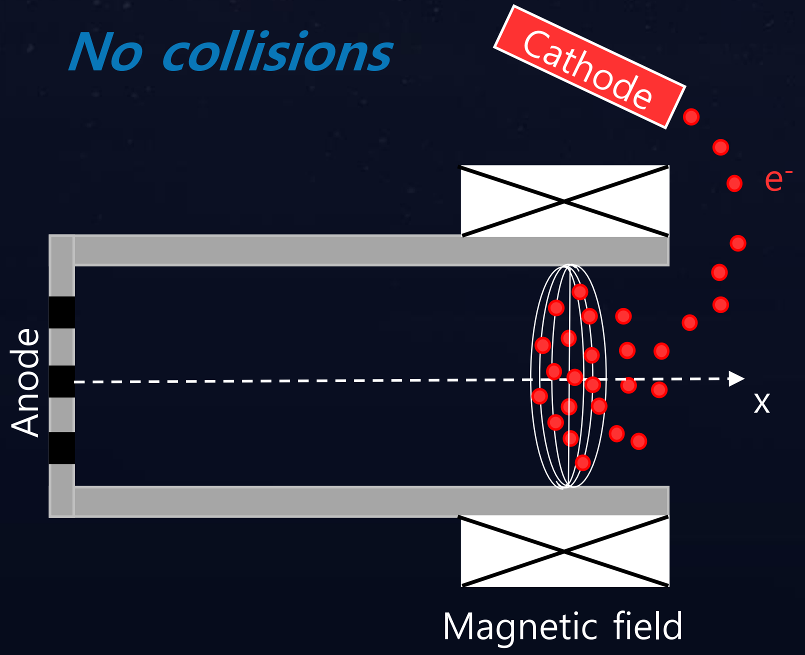
Hall thrusters (HT)

Axial electron mobility μ_e

Axial electron transport: $v_{e,x} = \mu_e E_x$

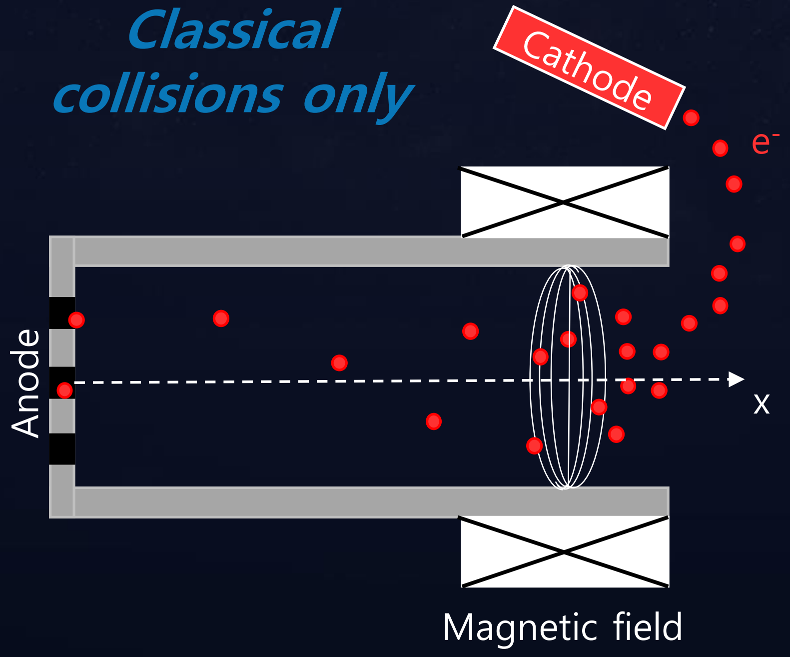
If transport driven by classical collisions: $$\mu_{class} = \frac{\frac{|q|}{m\nu_m}}{1 + \frac{\omega_{ce}^2}{\nu_m^2}}$$

No collisions

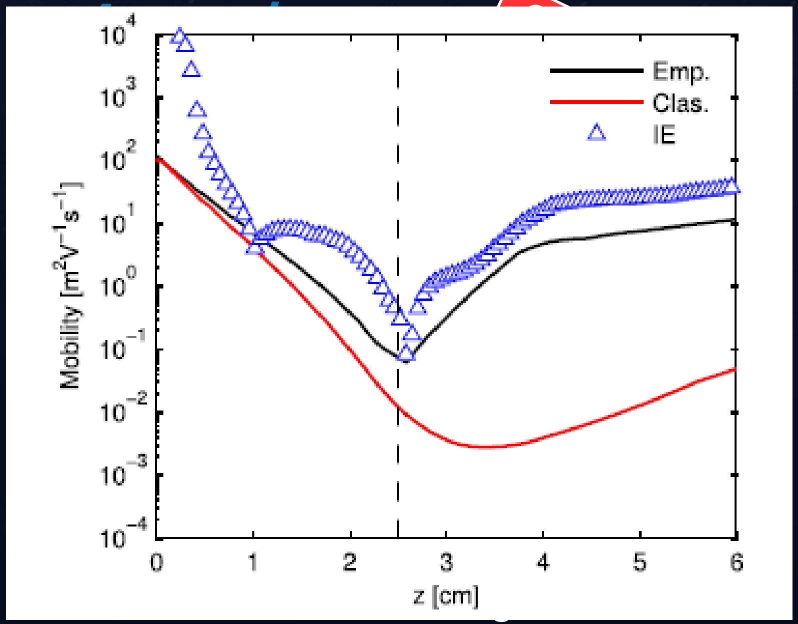


$\nu_m \rightarrow 0 \Rightarrow \mu_{class} \rightarrow 0$

Classical collisions only



$\mu_e = \mu_{class}$

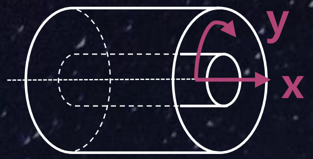


Axial evolution of electron axial mobility μ [1,2]

— $\mu_e = \mu_{class} + \mu_{ano}$

[1] *Adam et al., Plasma Phys. Control. Fusion 50 124041 (2008)*

[2] *Lafleur T, Baalrud S and Chabert P, Phys. Plasmas 23 – 053502 (2016)*

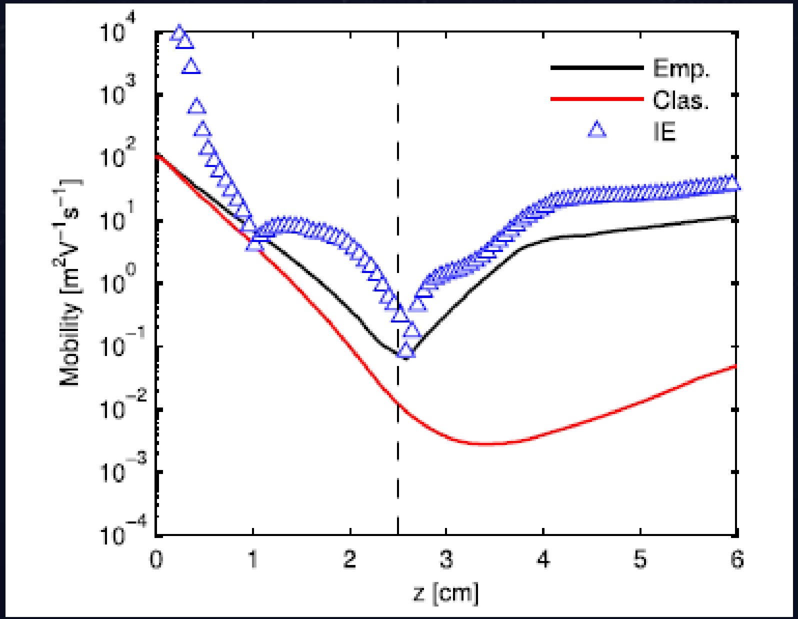


Hall thrusters (HT)

Electron anomalous transport

$$\mu_e = \mu_{class} + \mu_{ano}$$

<u>Different possible causes:</u>



Axial evolution of electron axial mobility μ [1,2]

- Secondary Electron Emission and Electron/wall collisions [3,4]
- Radial sheath instabilities [5,6]
- Gradient-driven fluid instabilities [7,8]
- High-frequency azimuthal oscillations (**Electron Drift Instabilities)** might be the main source of anomalous transport [2,9,10,11]

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[1] Adam et al., Plasma Phys. Control. Fusion 50 124041 (2008)
[2] Lafleur T, Baalrud S and Chabert P, Phys. Plasmas 23 – 053502 (2016)
[3] Sydorenko et al., Phys. Plasmas 13 014501 (2006)
[4] Raitses et al., IEEE Trans. Plasma Sci. 39 995–1006 (2011)
[5] Sydorenko et al., Phys. Plasmas 15 053506 (2008)
[6] Taccogna et al., Appl. Phys. Lett. 94 251502 (2009)
[7] Frias et al., Phys. Plasmas 19 072112 (2012)
[8] Smolyakov et al., Plasma Physics and Controlled Fusion 59, 14041 (2017).
[9] Adam et al., Phys. Plasmas 11, 295 (2004)
[10] I. Katz et al, IEPC-2015-402 (2015)
[11] Lafleur, T. and Chabert, P., Plasma Sources Sci. Technol., 27, 015003 (2018).

Hall thrusters (HT)

A broad range of instabilities

Breathing-mode (BM)

Strong axial oscillations
$\lambda \sim \mathrm{cm}$
High fluctuations of densities and discharge current

Ion Transit-Time Instability (ITTI)

Mid-range axial oscillations
$\lambda \sim \mathrm{cm}$
Ion wave-riding and cold trapped ions

Electron Drift Instability (EDI)

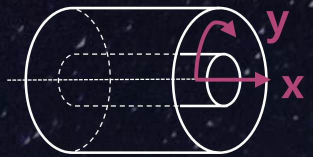
Strong azimuthal oscillations
$\lambda \sim \mathrm{mm}$
Significant enhancement of axial electron transport

Rotating spokes

Strong azimuthal oscillations
$\lambda \sim \mathrm{cm}$
High fluctuations of plasma density close to the anode

Modified Two-Stream Instability (MTSI)

Strong azimuthal oscillations
$\lambda \sim \mathrm{cm}$
Observed in radial-azimuthal simulations



Hall thrusters (HT)

Electron anomalous transport

EDI contribution to the anomalous transport can be described by an **electron-ion friction force** $R_{ei,y} = -q < \delta n_e \delta E_y >$ with $\mu_{ano} = \mu_{class} \frac{\omega_{ce}}{\nu_m} \frac{R_{ei,y}}{|q| n_e E_x}$

(electron drift velocity >> ion drift velocity)

✓ Kinetic instabilities (need EVDF)

✓ Azimuthal propagation (mainly)

✓ Axial convection

✓ Interacts with low-frequency instabilities (e.g. Breathing Mode)

Particle-In-Cell code [1,2,3]

Axial-azimuthal (x,y) plane [4,5,6,7,8]

Long simulation time [4,5,8]

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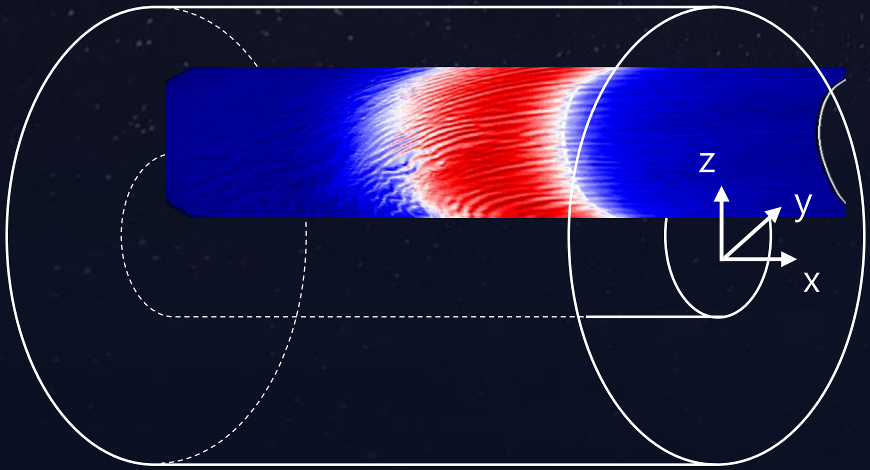
[1] Bœuf J-P, Journal of Applied Physics 121, 011101 (2017)
[2] Hara K., Plasma Sources Sci. Technol. 28 044001 (2019)
[3] Taccogna F., Garrigues L., Rev. Mod. Plasma Phys. 3, 12 (2019)
[4] Adam et al., Phys. Plasmas 11, 295 (2004)
[5] P. Coche, L. Garrigues, Phys. Plasmas 21, 023503 (2014)
[6] J.P. Boeuf, L. Garrigues, Phys. Plasmas 25, 061204 (2018)
[7] T. Lafleur, P. Chabert, Plasma Sources Sci. Technol. 27, 015003 (2018)
[8] T. Chernyshev et al, J. Phys. D: Appl. Phys. 52 444002 (2019)

Our main tool, *LPPic*

A versatile 2D Particle-In-Cell code



PIC Stability conditions

$$\begin{cases} \Delta t \leq \dfrac{0.2}{w_p} \quad \text{with} \quad w_p = \sqrt{\dfrac{n_e.e^2}{m.\varepsilon_0}} \\ \Delta x \leq \lambda_d \quad \text{with} \quad \lambda_d = \sqrt{\dfrac{\varepsilon_0.k_B.T_e}{n_e q^2}} \end{cases}$$

High plasma density ($\sim 10^{18}\ m^{-3}$)

$$\Delta t \sim 2\,.\,10^{-12}\ s$$
$$\Delta x \sim 2\,.\,10^{-5}\ m$$

- Simulate the axial-azimuthal (x,y) plane
- Highly-parallelized (MPI - domain decomposion) scalable up to 1500 CPUs
- Typical simulation time (360-1500 CPU): 2–60 days

Complex code (> 10 000 code lines)

Verified against 6 other codes (2D ExB benchmark) [1]

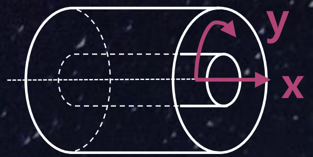
Same for other 2 cases [1]

[1] T. Charoy et al., ***2D axial-azimuthal particle-in-cell benchmark for low-temperature partially magnetized plasmas***., Plasma Sources Sci. Technol., 28(10):105010, 2019.

1

Theoretical model for anomalous transport

T. Charoy, T. Lafleur, A. Tavant, P. Chabert, and A. Bourdon.
A comparison between kinetic theory and particle-in-cell simulations of anomalous electron transport in ExB plasma discharges.
Phys. Plasmas, 27(6):063510, 2020.



Quasi-linear kinetic theoretical model

Derived for the first time in [1]

$$R_{ei,y} = q < \delta n_e \delta E_y >$$

If we consider:

- Only a single dominant wave mode
- Instability only in azimuthal direction
- $k_y \lambda_d = \frac{1}{\sqrt{2}}$ (ion-acoustic behaviour)
- $v_\varphi \approx c_s \ll v_d$

We can obtain (see [2] for full derivation)

$$\boxed{R_{ei,quasi-linear} = 2\sqrt{2}\pi\omega_{pe}^2\lambda_d\varepsilon_{wave}\frac{dF_{e,0}}{dv_{e,y}}|_{v_{e,y}=0}}$$

$\varepsilon_{\mathrm{wave}} = \frac{1}{4}\varepsilon_0|\delta\mathbf{E}|^2$: wave energy density

$\frac{\mathrm{dF_{e,0}}}{\mathrm{dv_{e,y}}}|_{\mathrm{v_{e,y}}=0}$: derivative of the azimuthal EVDF at $\mathrm{v_{e,y}} = 0$

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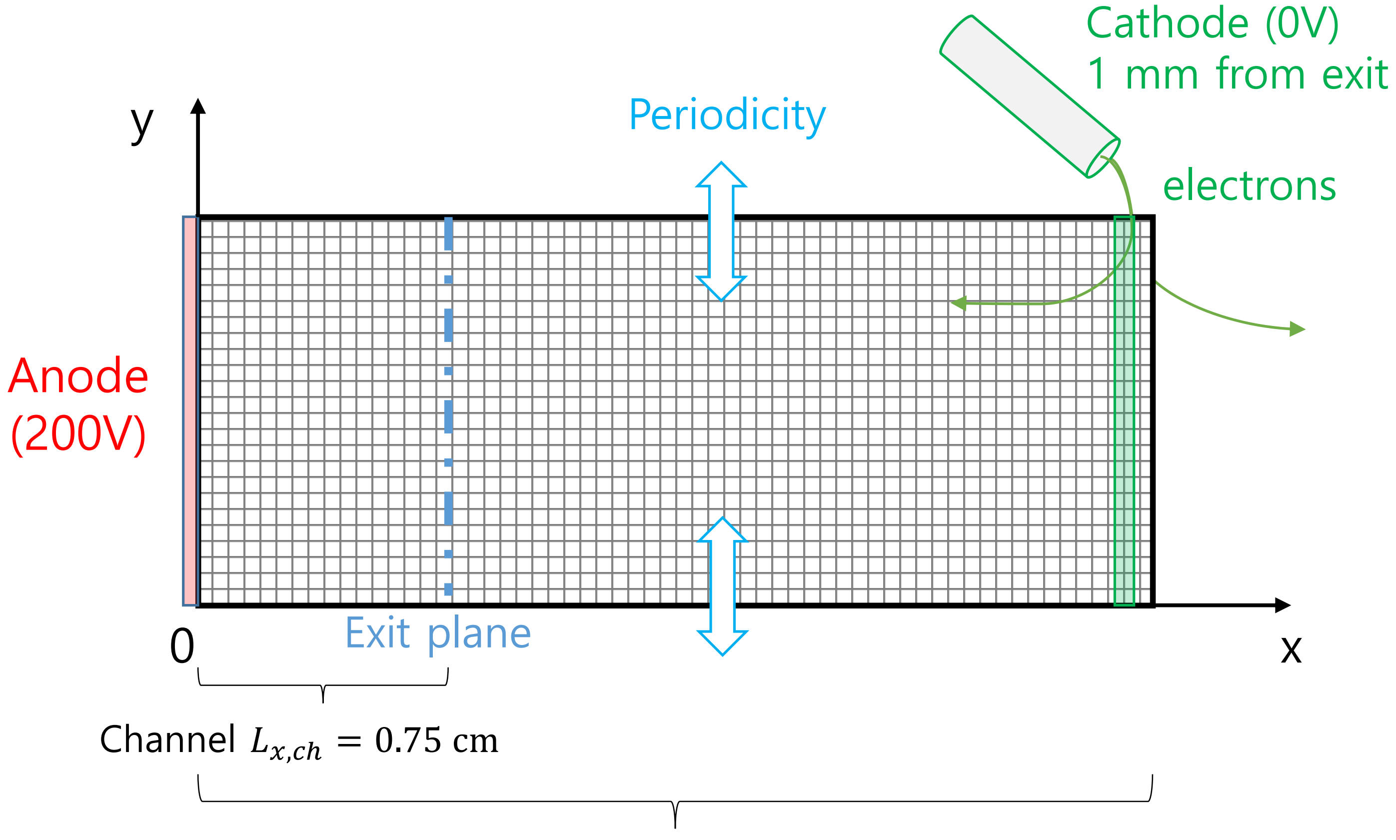
[1] T. Lafleur, R. Martorelli, P. Chabert, and A. Bourdon. Phys. Plasmas, 25(6):061202, 2018

[2] T. Charoy, T. Lafleur, A. Tavant, P. Chabert, and A. Bourdon. Phys. Plasmas, 27(6):063510, 2020.

2D PIC axial-azimuthal simulations

Simplified configuration case

Simplified case used as a benchmark [1]



Cathode (0V)
1 mm from exit

Periodicity

electrons

Anode
(200V)

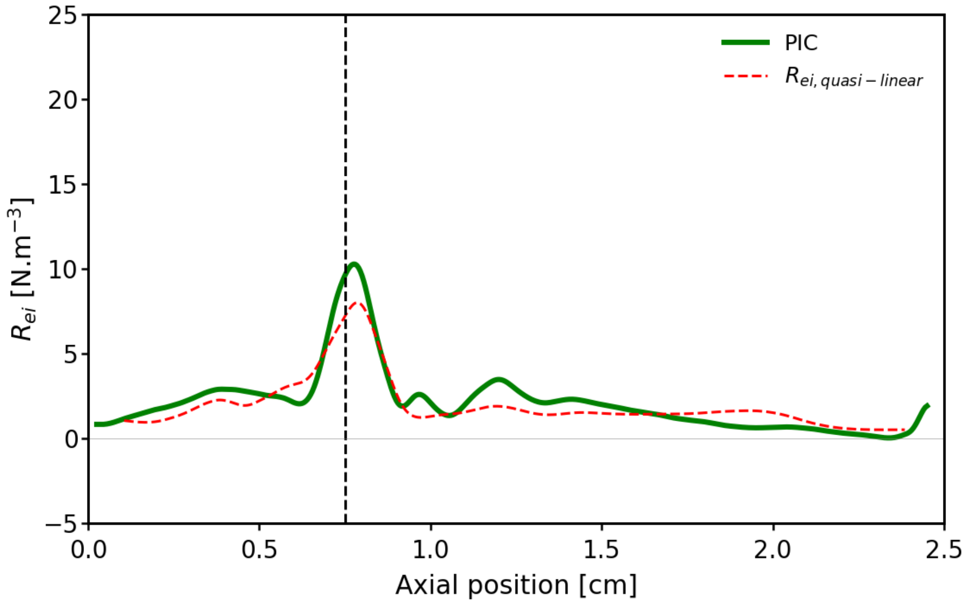
Exit plane

Channel $L_{x,ch} = 0.75\ \mathrm{cm}$

Simulation domain $L_{x,tot} = 2.5\ \mathrm{cm}$

- Fixed magnetic field
- No collision
- Fixed ionization profile

➔ **Stationary state**



Comparison of electron-ion friction force calculated from the PIC (green) or approximated (red)

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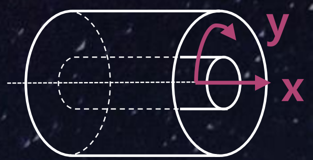
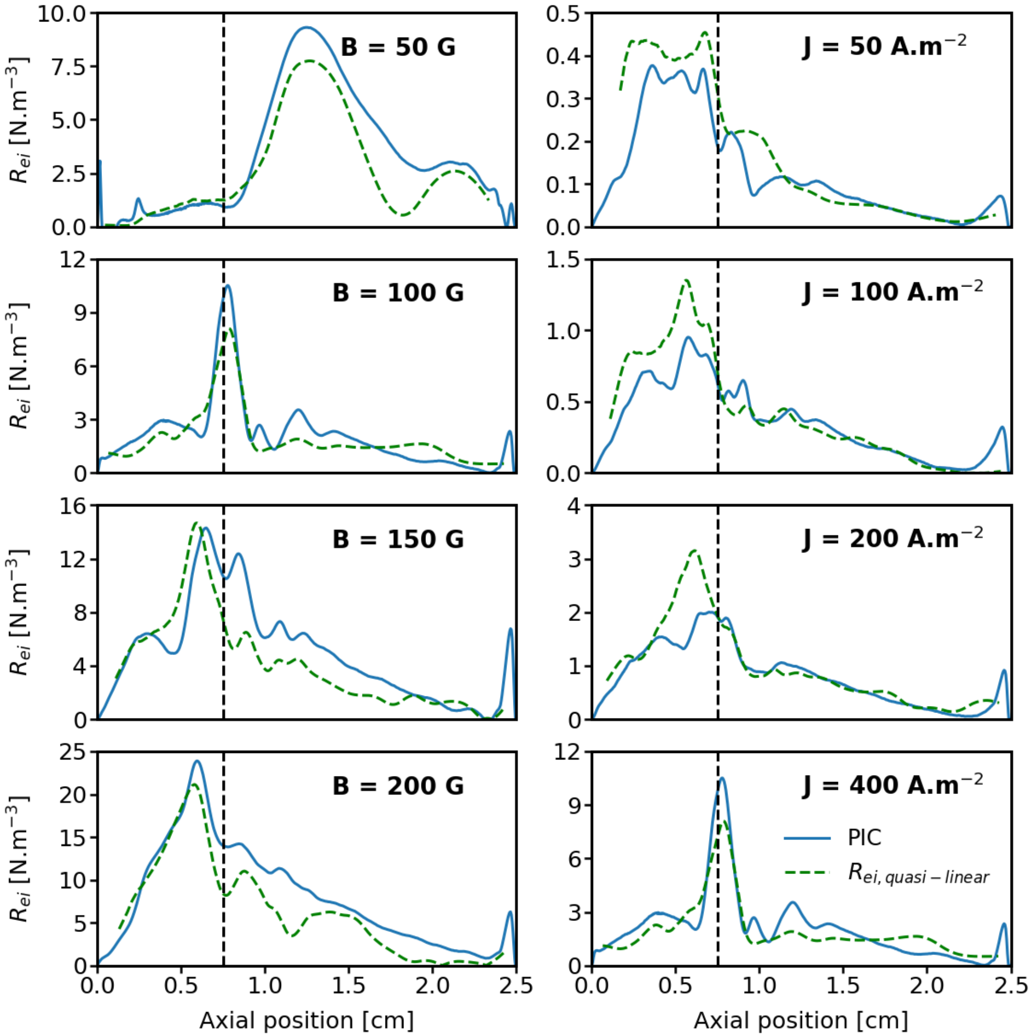
[1] T. Charoy et al., Plasma Sources Sci. Technol., 28(10):105010, 2019.



Simplified simulation case to stress-test the models [2]

Extensive parametric study: Variation of magnetic field maximum value and discharge current density



Axial evolution of electron-ion friction force for different values of magnetic field maximum value B (left) and current density J (right)

┄┄┄ $$R_{ei,quasi-linear} = 2\sqrt{2}\pi\omega_{pe}^2\lambda_d\varepsilon_{wave}\frac{dF_{e,0}}{dv_{e,y}}|_{v_{e,y}=0}$$

── **PIC**

- Various shapes and amplitudes for electron-ion friction force.
- Enhancement of the friction force when increasing the current density: instability growth rate increases.

$$\gamma_{IAI} = \sqrt{\frac{\pi m}{8M}}\frac{\boldsymbol{k.v_{de}}}{(1+k^2\lambda_{de}^2)^{3/2}}$$

Quasi-linear theoretical model approximates very well the friction force

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[2] T. Charoy et al. Phys. Plasmas, 27(6):063510, 2020.

LIMITATIONS

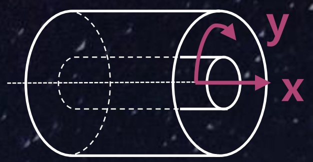
- Imposed ionization source term
- No collisions
- Smaller discharge channel length
- Limited azimuthal length

✖ No axial instabilities (Breathing Mode or Ion Transit-Time Instabilities)
✖ Lack of thermalization processes
✖ No long-wavelength azimuthal instabilities

2

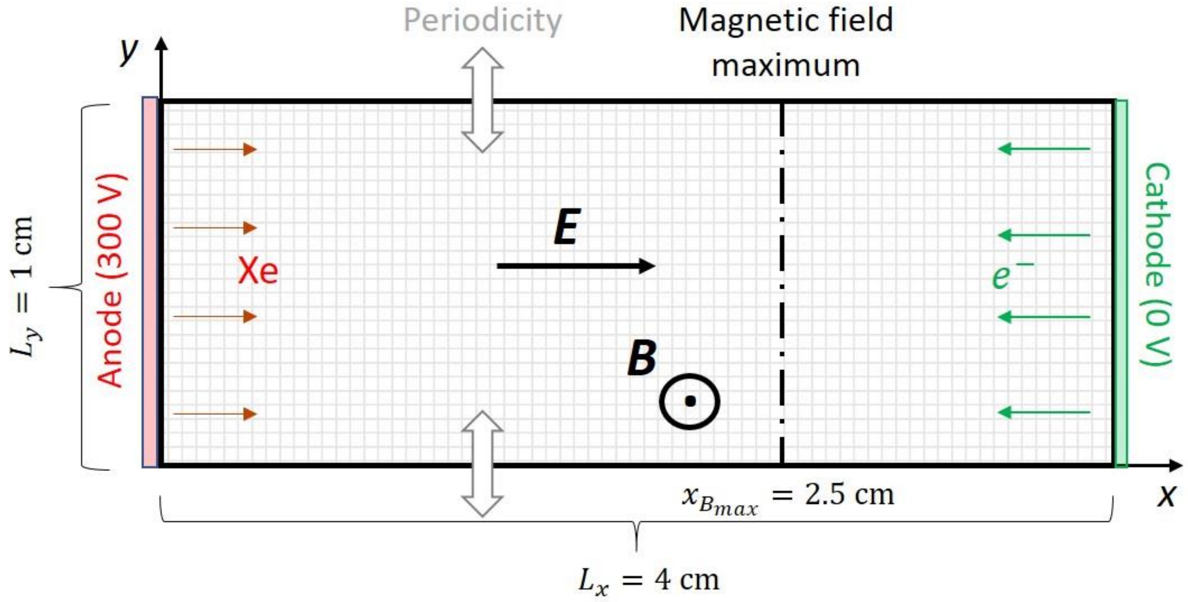
Towards self-consistent axial-azimuthal simulations

T. Charoy, T. Lafleur, A. Alvarez Laguna, A. Bourdon and P. Chabert, *The interaction between ion transit-time and electron drift instabilities and their effect on anomalous electron transport in Hall thrusters,* Submitted to Plasma Sources Sci. Technol.



Typical simulation case

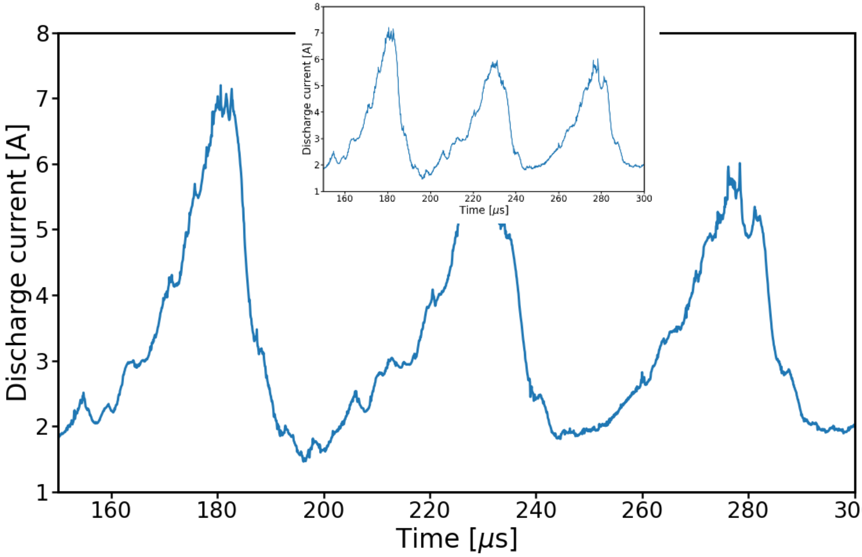
Main parameters



Chosen models

Cathode model	Quasi-Neutrality
Collisions	Yes (LXCat database)
Neutral dynamics	Euler equations (HLLC)
Anode ion recombination	Yes

➔ Self-consistent simulations: $I_d = f(t)$



PIC Stability conditions

$$\left\{\begin{array}{l}\Delta t \le \frac{0.2}{w_p} \quad \text{with} \quad w_p = \sqrt{\frac{n_e.e^2}{m.\varepsilon_0}} \\ \Delta x \le \lambda_d \quad \text{with} \quad \lambda_d = \sqrt{\frac{\varepsilon_0.k_B.T_e}{n_e q^2}}\end{array}\right.$$

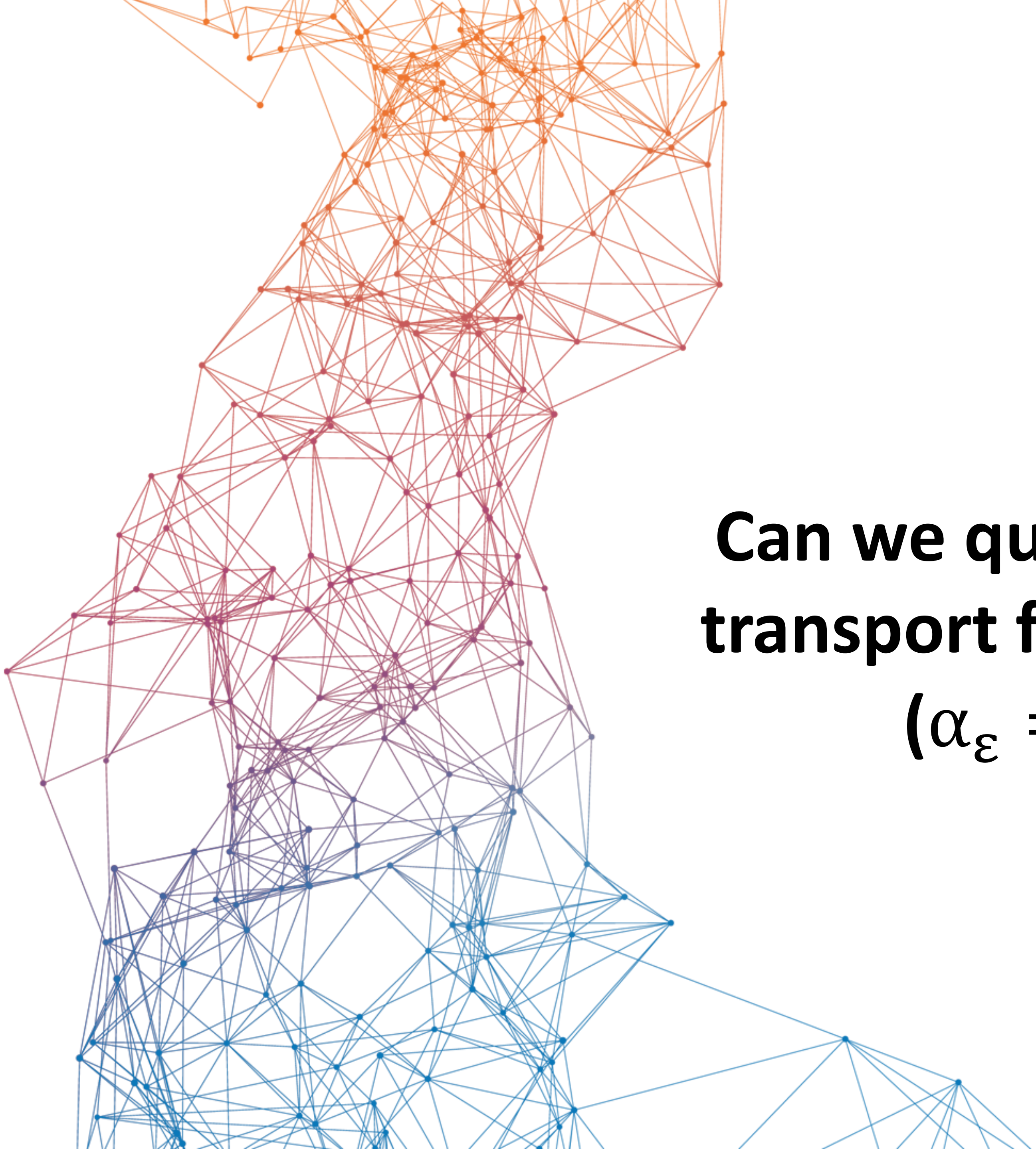
$\varepsilon_0' = \alpha_\varepsilon \times \varepsilon$

s

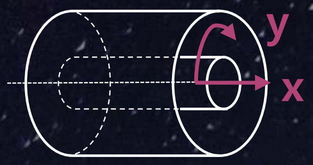
e BM

-12 s

ate

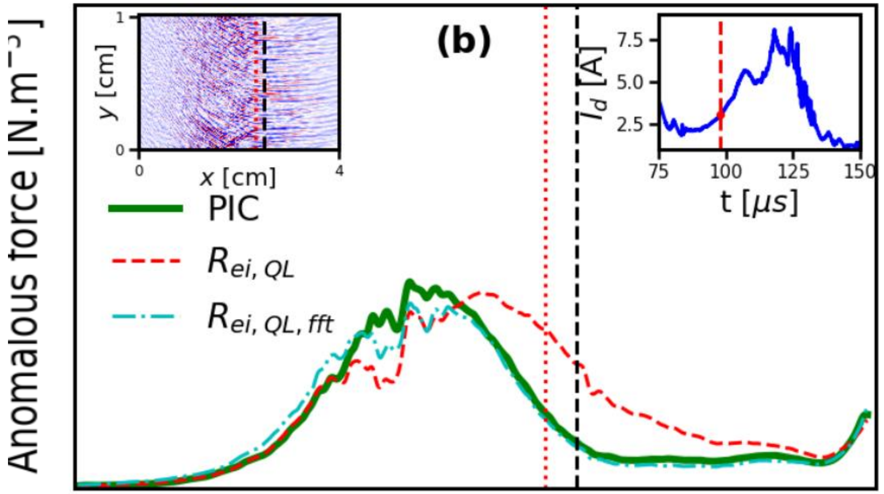


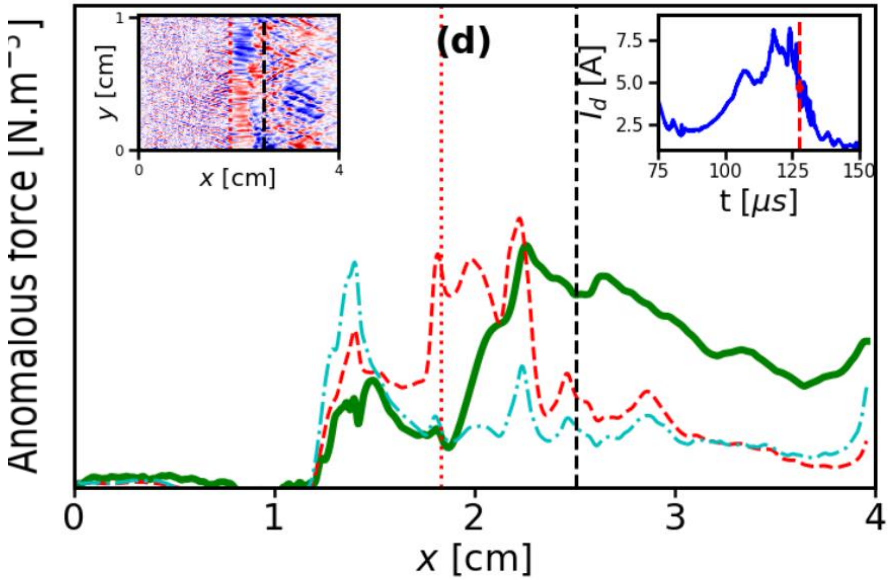
Can we quantified the anomalous transport for a case close to reality ($\alpha_{\varepsilon} = 4, L_y = 1\ \mathrm{cm}$)?



Case closer to reality ($\alpha_\varepsilon = 4, L_y = 1$ cm)

Anomalous force approximation





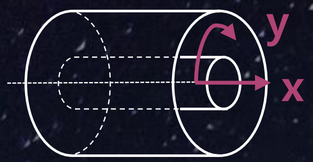
When I_d ↗

- ✓ $R_{ei,quasi-linear}$ matches well everywhere.
- ✓ Perfect match for $R_{ei,quasi-linear,fft}$ (no ion-acoustic approximation).

When I_d ↘

- ✓ Good matching inside the thruster for $R_{ei,quasi-linear}$
- ✖ Under-prediction in the plume.

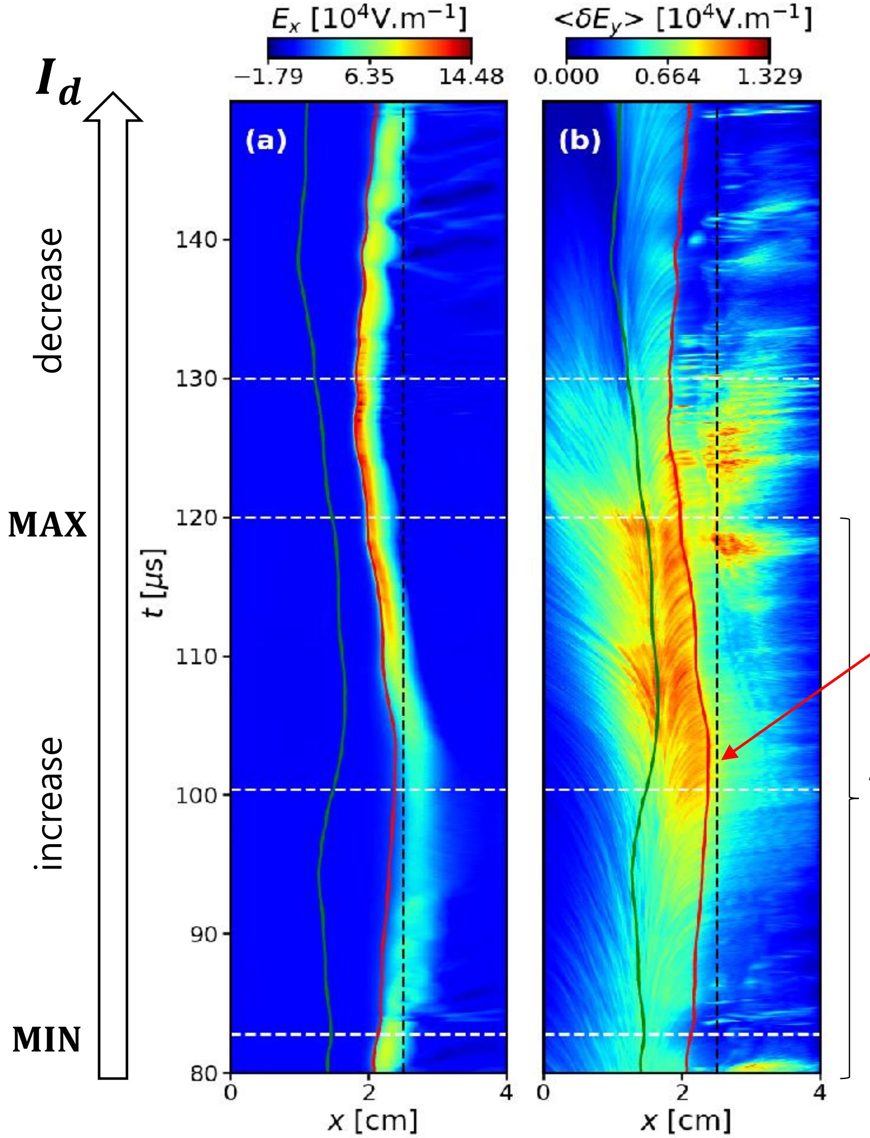
One of the hypothesis for $R_{ei,quasi-linear}$ might not be valid



Case closer to reality ($\alpha_\varepsilon = 4, L_y = 1$ cm)

I_d - discharge current / E_x- axial electric field
δE_y- azimuthal electric field fluctuations

Axial and azimuthal electric field fluctuations (axial-time)

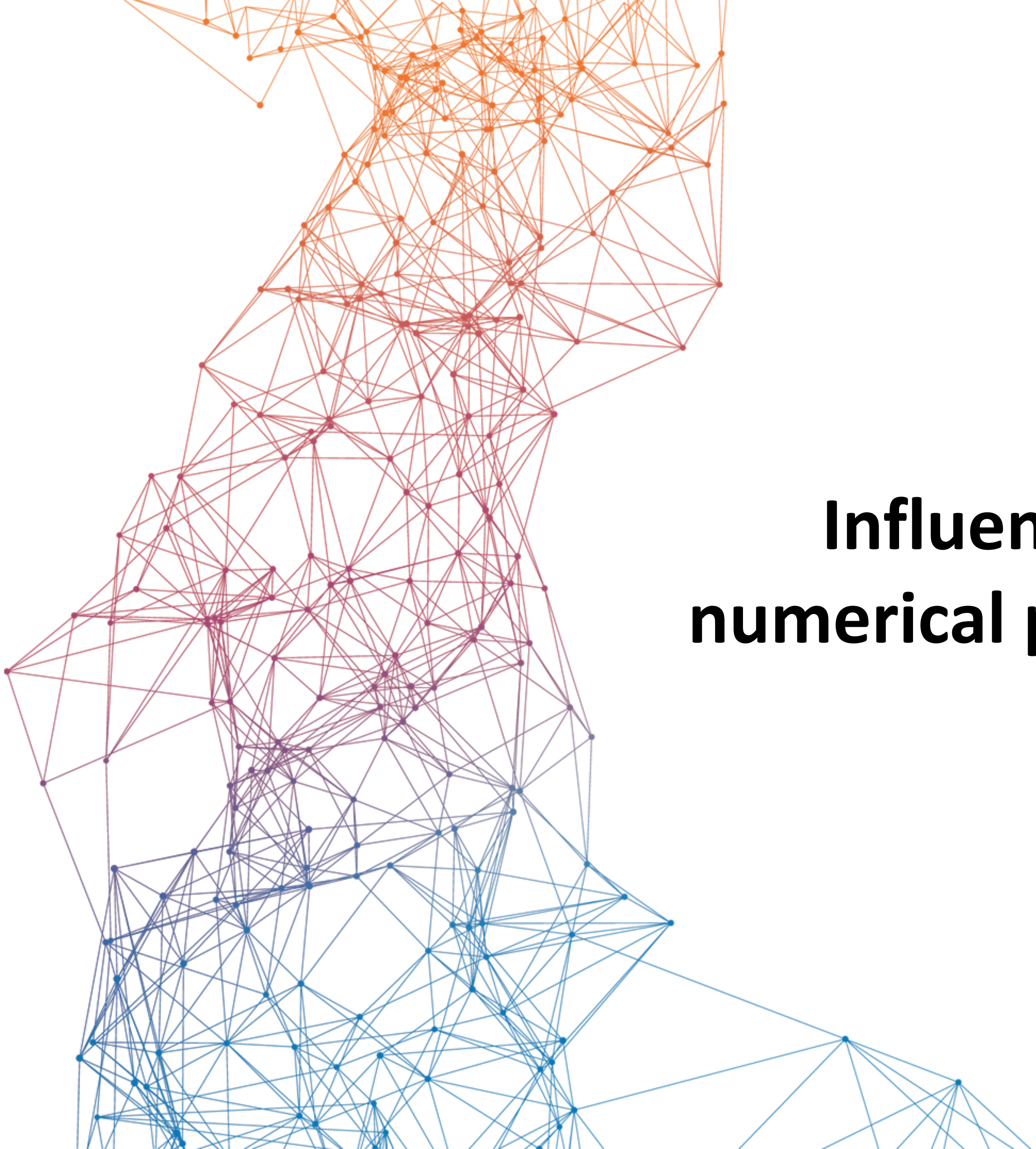


- $< \delta E_y >$ ↗ when I_d ↗

- Axial position of ion sonic point (red curve) marks a separation between two zones for the azimuthal instabilities

Change of azimuthal instabilities downstream of the ion sonic point

Explain the under-estimation of the friction force

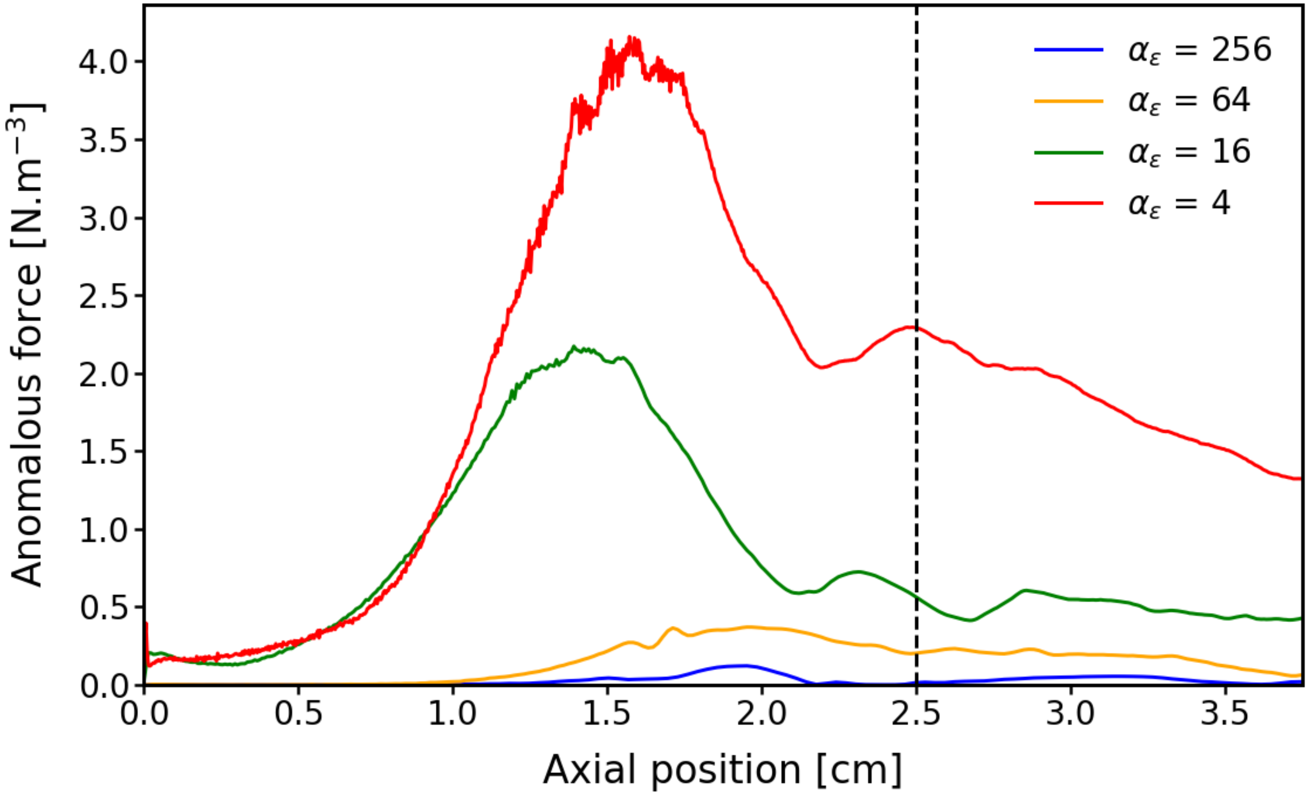


Influence of the imposed numerical parameters α_{ε} and L_y?

Influence of scaling factor α_ε

On the macroscopic transport

Anomalous force averaged over several BM oscillations



α_ε ↘ ⇒ R_{ei} ↗ ⇒ enhanced electron axial transport

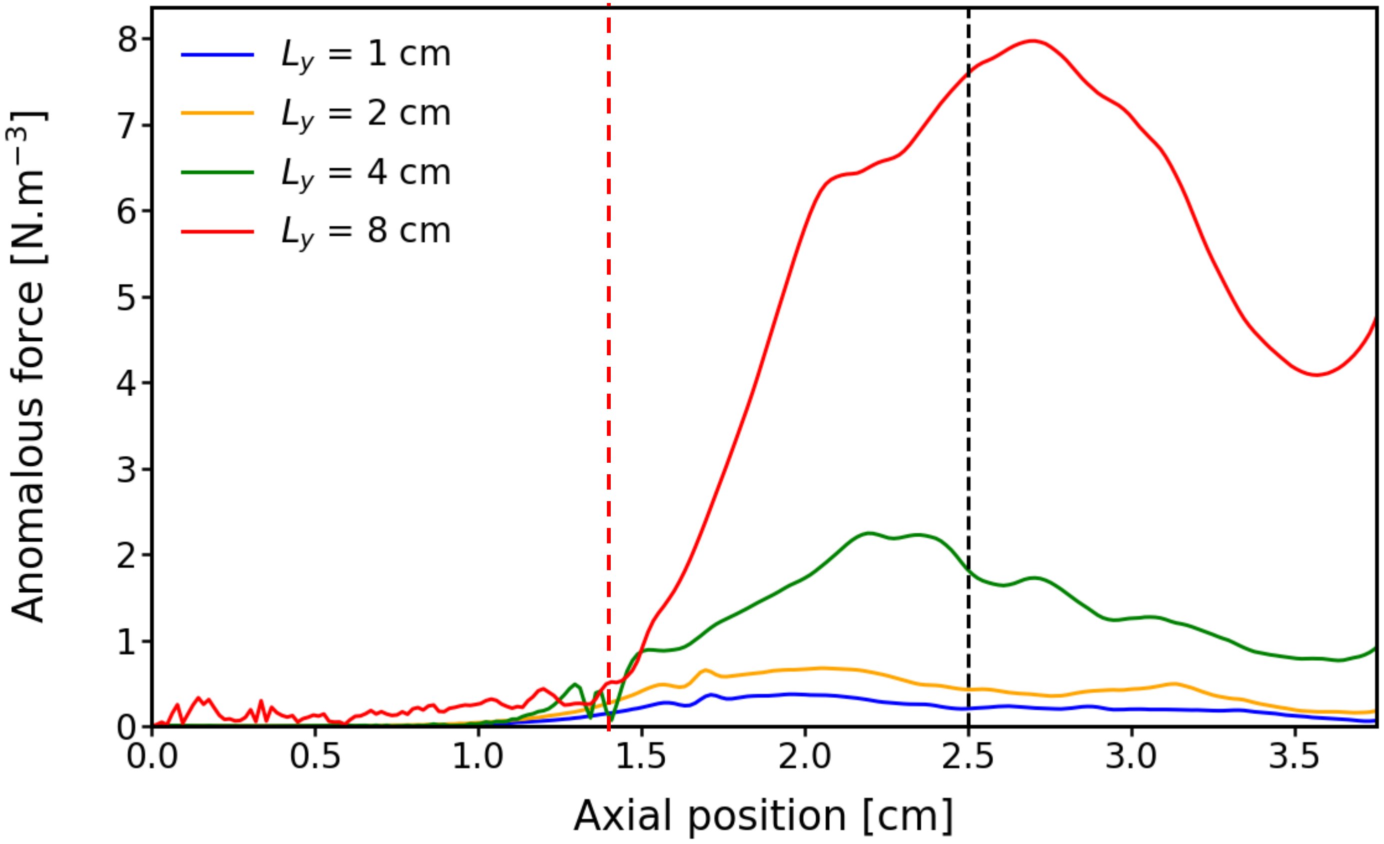
$$\gamma_{\mathrm{IAI}} = \sqrt{\frac{\pi m}{8M}} \frac{\mathbf{k}.\mathbf{v_{de}}}{\left(1 + k^2 \lambda_{de}^2\right)^{3/2}} \propto \left(\alpha_\varepsilon \varepsilon_0\right)^{-3/2}$$

Using α_ε damps the EDI

Influence of azimuthal domain length L_y (α_ε = 64)

Macroscopic effetcs

Anomalous force averaged over several BM oscillations

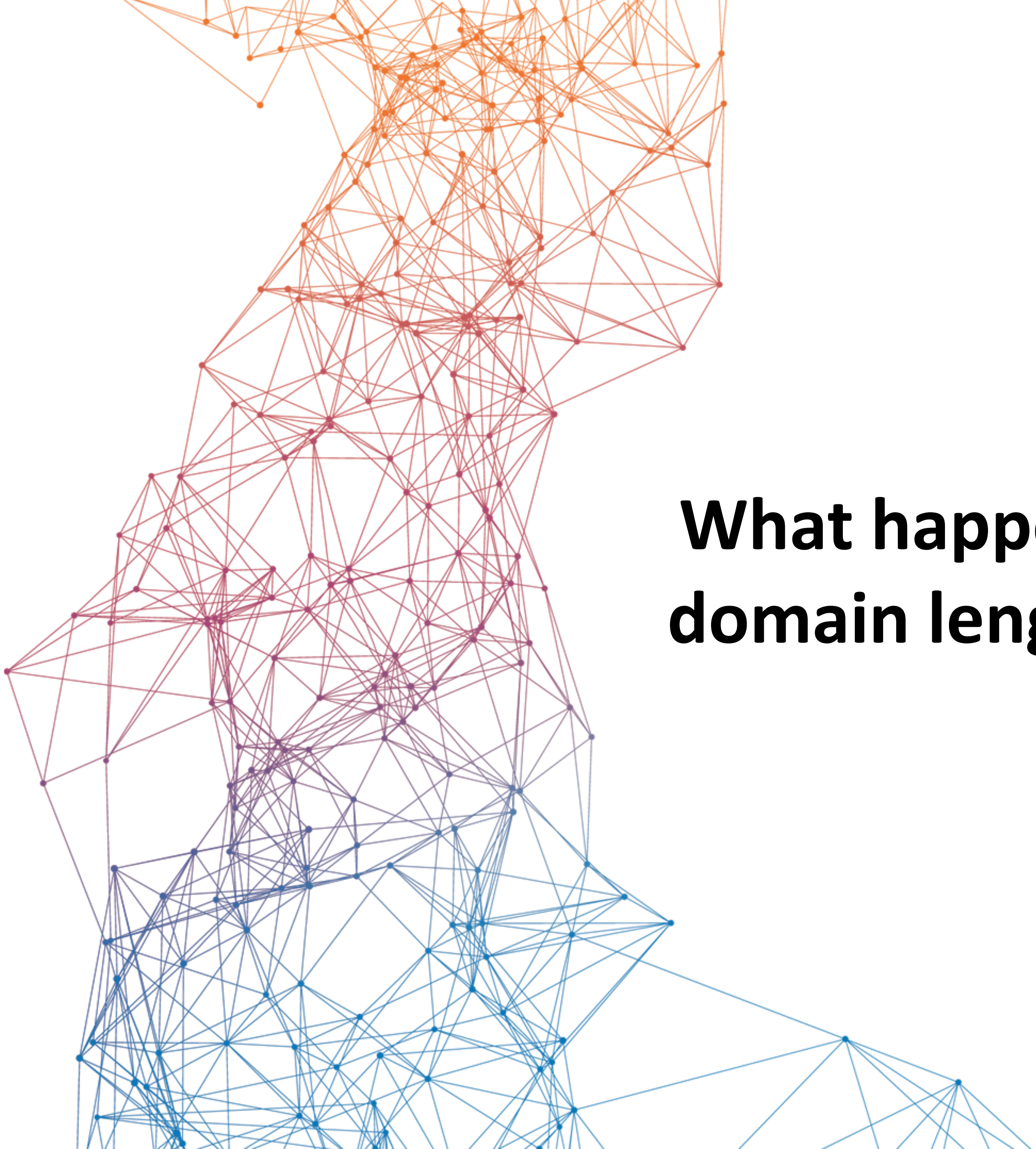


- <u>Upstream of ion sonic point</u>:

$$R_{ei} \sim constant$$

- <u>Downstream of ion sonic point</u>:

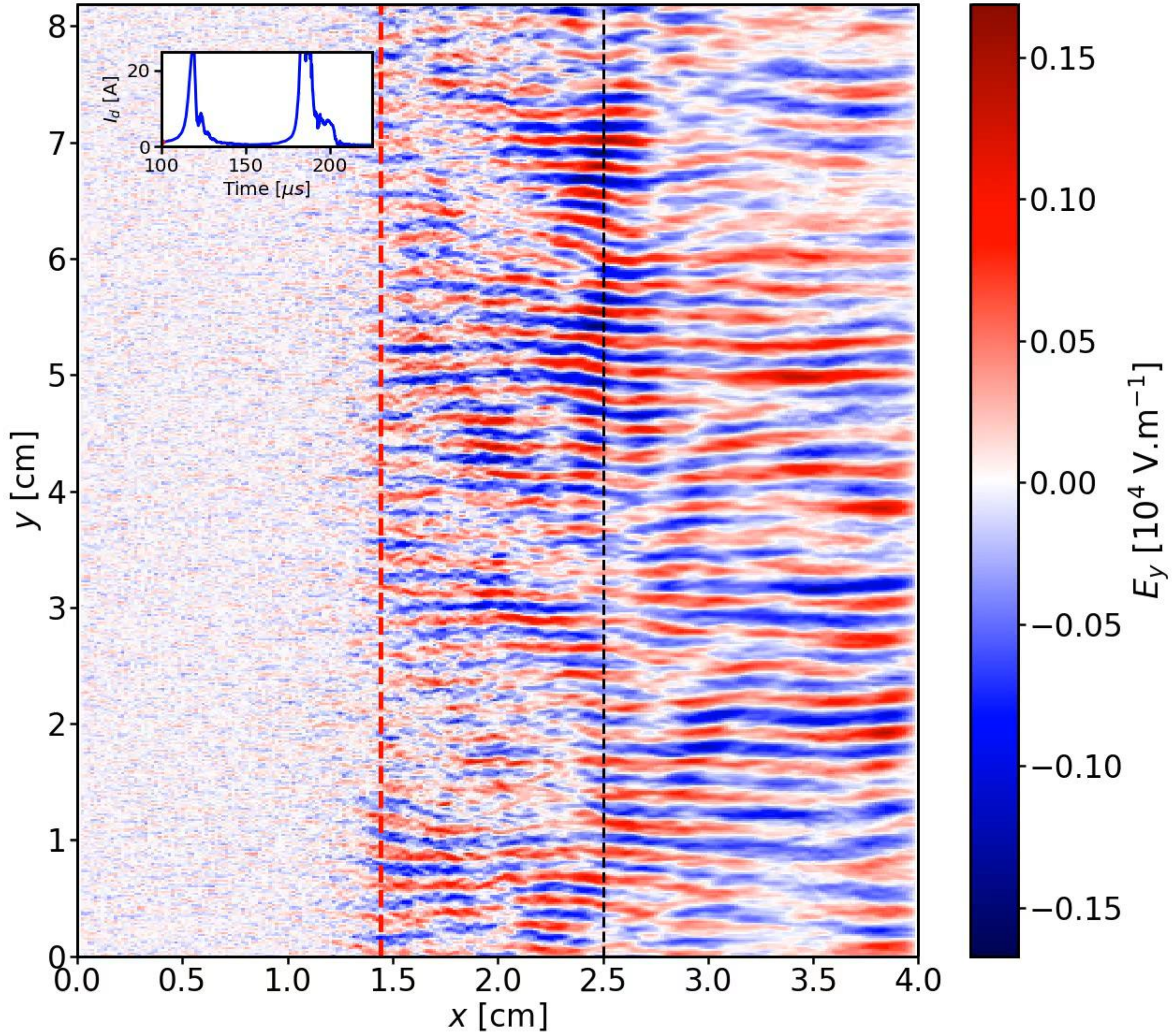
R_{ei} ↗ when L_y ↗ (EDI stronger)



What happens when the azimuthal domain length is long i.e. L_y= 8 cm?

Case with a long azimuthal length ($\alpha_{\varepsilon} = 64$, $L_y = 8$ cm)

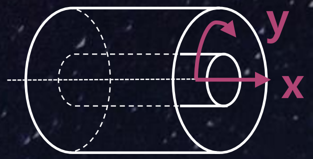
Video - 2D maps of azimuthal electric field



- Upstream of ion sonic point, short-wavelength modes

- Downstream of ion sonic point, transition to long-wavelength modes when I_d ↘

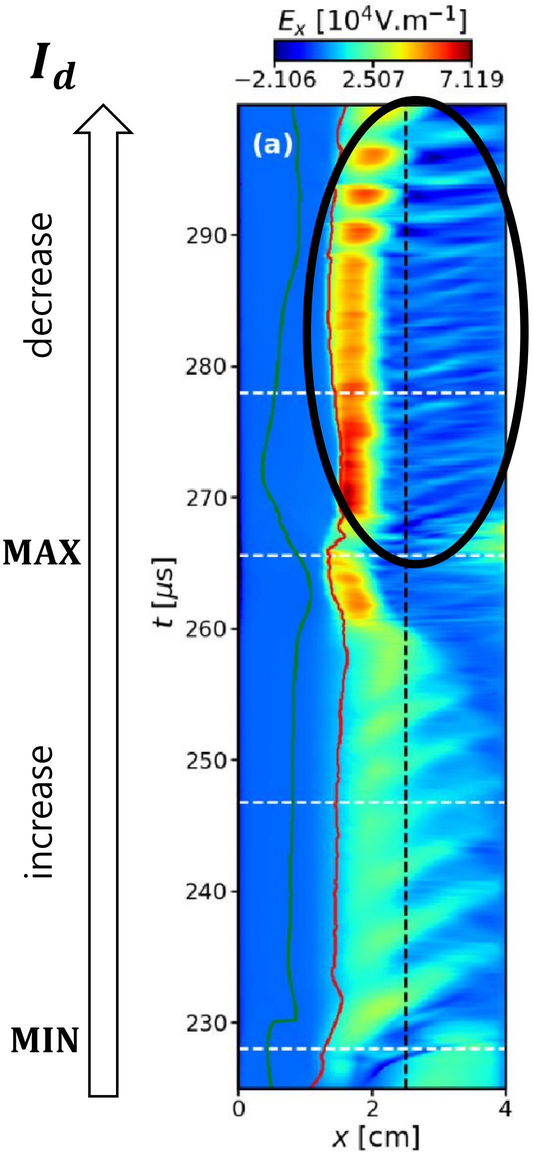
Signature of inverse energy cascade?



Case with a long azimuthal length ($\alpha_\varepsilon = 64$, $L_y = 8$ cm)

I_d - discharge current
E_x - axial electric field

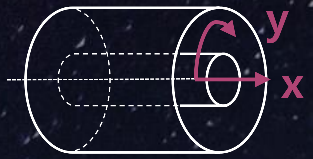
Evidence of another instability: Ion Transit-Time Instability (ITTI)



- Downstream of the ion sonic point (when I_d ↘): axial low-frequency (~100 kHz) fluctuations of E_x
- Low-velocity ion group in the plume

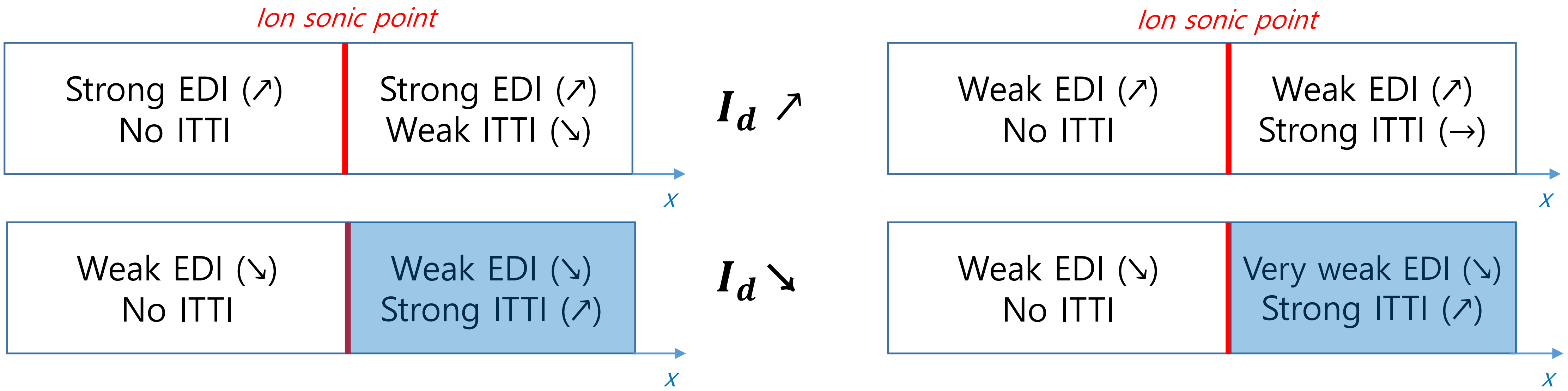
Evidence of Ion Transit-Time Instability (ITTI)

- Origin at the ion sonic point
- Seems to be damped when α_ε ↘ (EDI stronger)



Interaction ITTI/EDI

Summary of the 2 main cases



$\boldsymbol{\alpha_\varepsilon} = \mathbf{4}$ - $L_y = 1$ cm

Ion sonic point

Strong EDI (↗)
No ITTI

Strong EDI (↗)
Weak ITTI (↘)

x

Weak EDI (↘)
No ITTI

Weak EDI (↘)
Strong ITTI (↗)

x

$\boldsymbol{I_d}$ ↗

$\boldsymbol{I_d}$ ↘

$\alpha_\varepsilon = 64$ - $\boldsymbol{L_y} = \mathbf{8}$ **cm**

Ion sonic point

Weak EDI (↗)
No ITTI

Weak EDI (↗)
Strong ITTI (→)

x

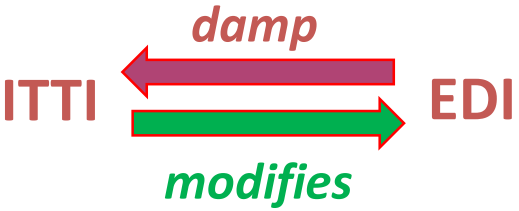
Weak EDI (↘)
No ITTI

Very weak EDI (↘)
Strong ITTI (↗)

x

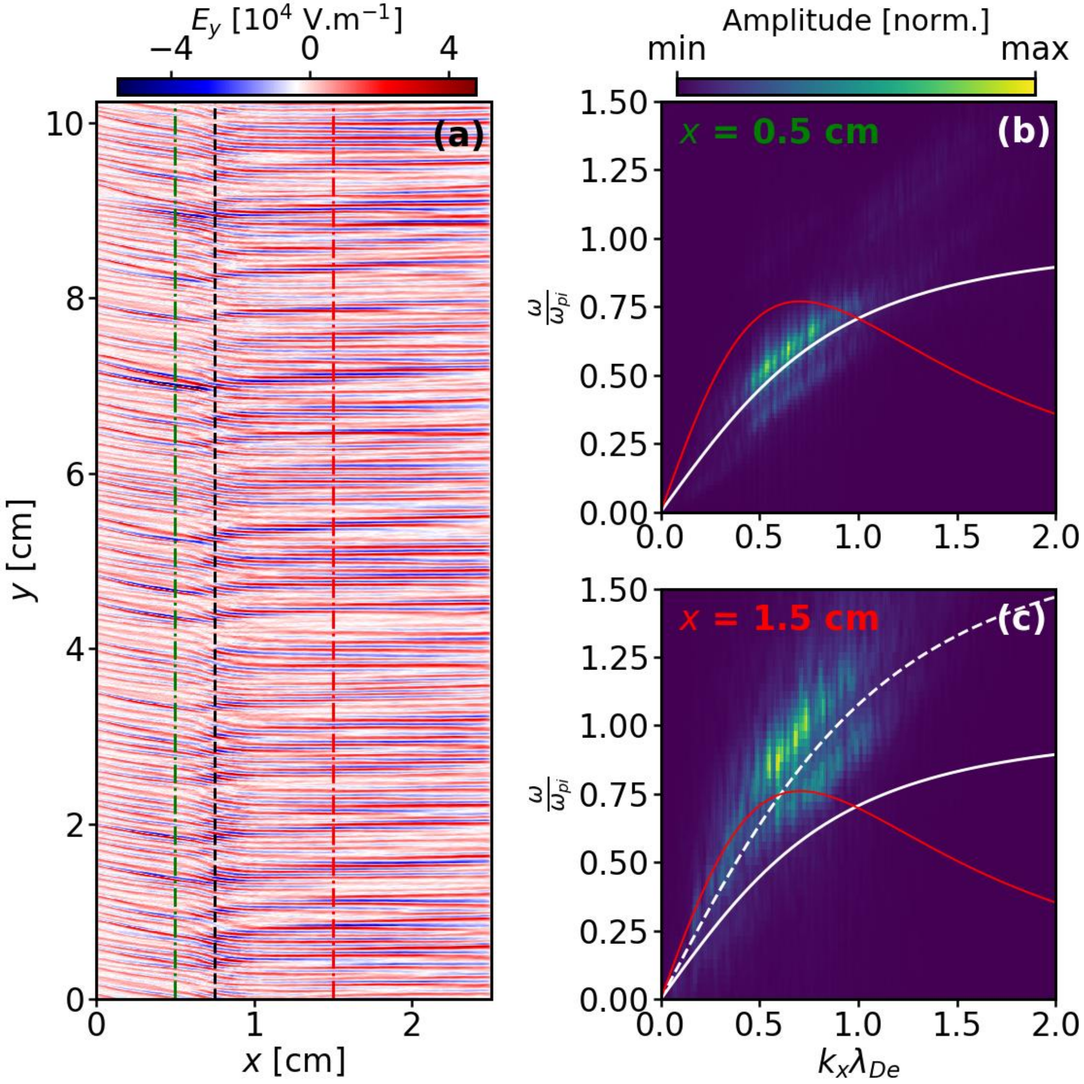
➔ $R_{ei,quasi-linear}$ too low

➔ Long-wavelength azimuthal modes



Insights from simplified benchmark case with $L_y = 10$ cm

2D maps of E_y and corresponding FFT at 2 axial positions



- **No ITTI** (because of: constant ionization source term? Ion sonic point in the plume?)

?

- **No long-wavelength structures**

Conclusion

Electron anomalous transport

- ✓ Strong contribution of EDI through the electron-ion friction force
- ✓ Excellent agreement of the theoretical model except in the plume, when I_d ↘

Downstream of the ion sonic point

- ✓ Transition from short to long-wavelength structures
- ✓ Probably due to the interaction between ITTI and EDI

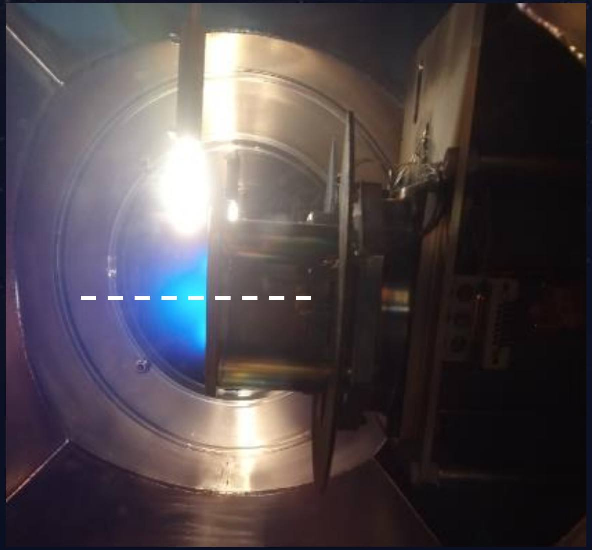
Prospects

- ☐ Improve code performances to simulate lower α_ε and longer L_y
- ☐ Theoretical analysis of EDI/ITTI interaction

We are hiring!

Hall thruster experimental diagnostics

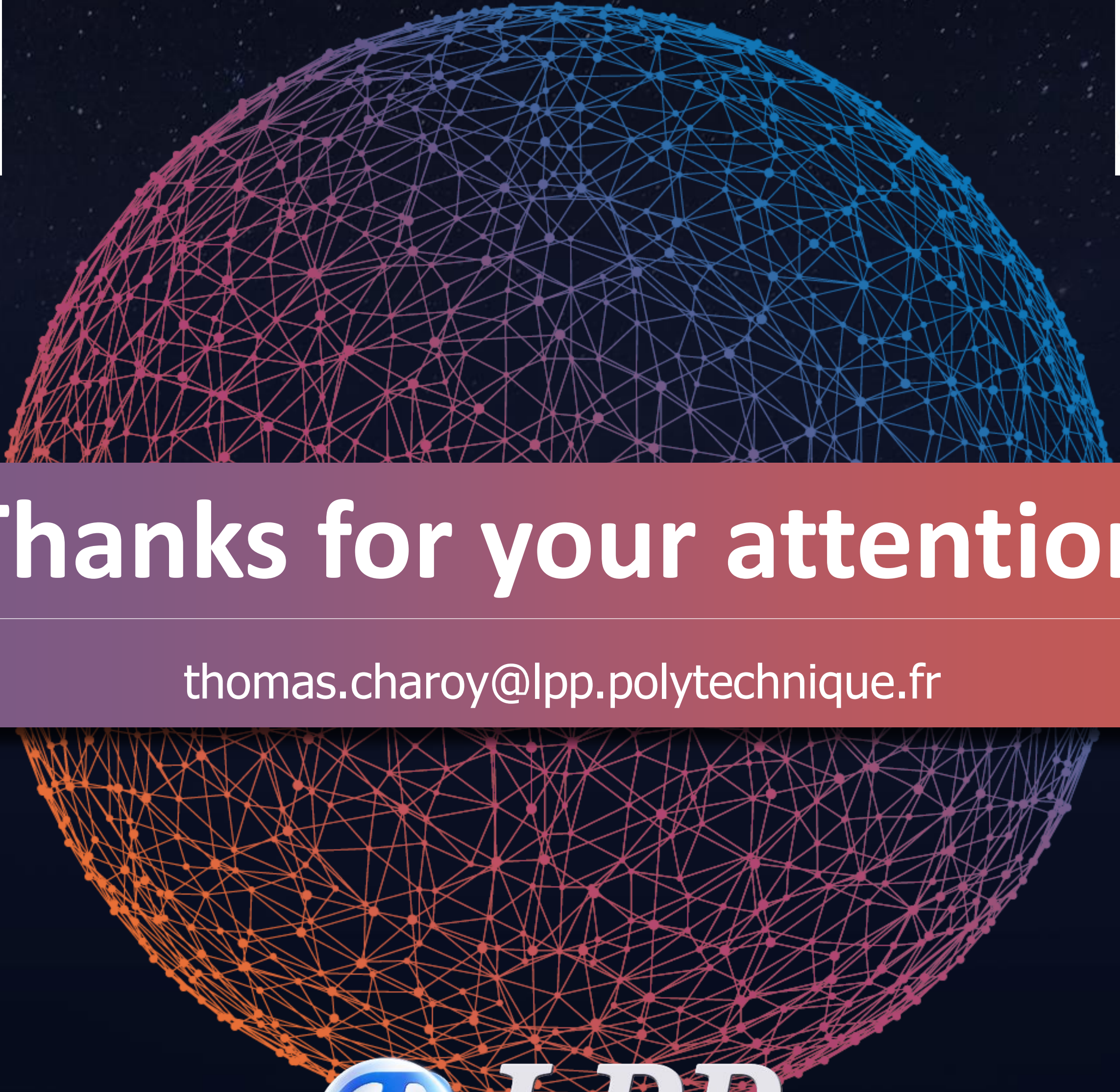
Side view



Front view

Open PostDoc position (24 months)

- ✓ Time and space resolved optical emission spectroscopy and imaging of the HT channel
- ✓ Comparison with PIC simulations via an in-house Collisional Radiative Model
- ✓ Development of a miniaturized and space flight proven imaging system that will be embarked on a flying satellite within the EU CHEOPS LOW POWER project.



Thanks for your attention

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