IOPS (Virtual)
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Cross-field electron diffusion due to the coupling of drift-driven microinstabilities

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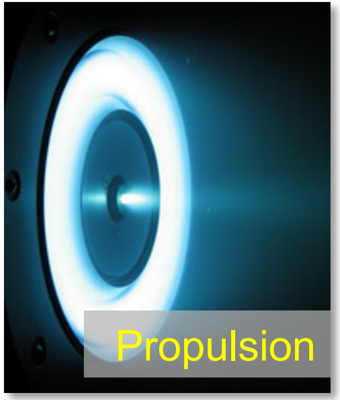
Overview of the research

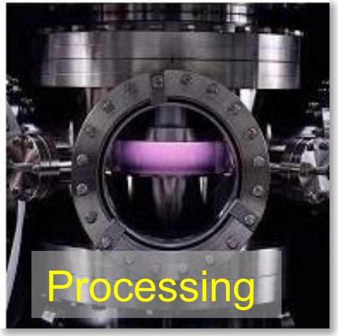
Low-temperature magnetized plasmas are observed in various natural phenomena and engineering devices

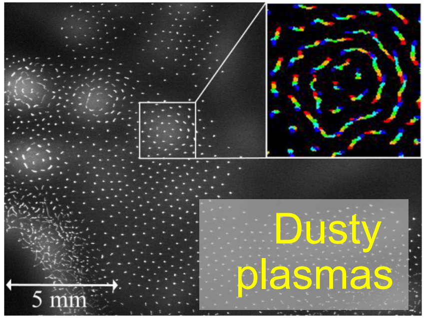
The dynamics is highly **nonlinear**: combination of many different physical (and chemical) processes, including

- Electron gyration (~GHz)
- Instabilities and turbulence (~MHz, μm)
- Self-organization (~kHz, cm)

Need of high-fidelity computational simulations & validation with advanced experiments (e.g. laser diagnostics)



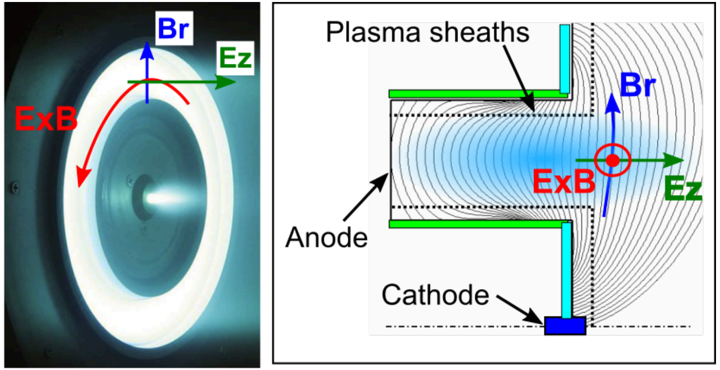




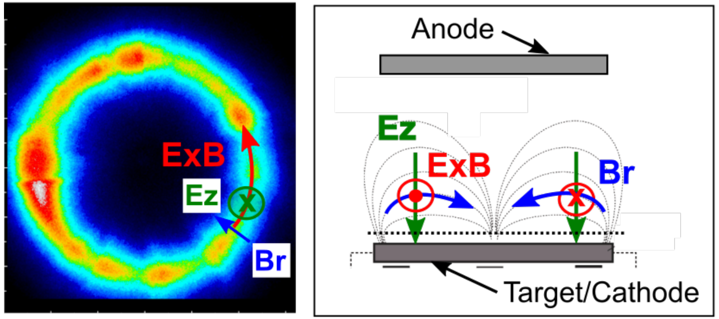
Low temperature magnetized plasmas

Using applied magnetic fields leads to higher electron temperature and density

- **Multiscale** phenomena
  - High-frequency: Instabilities
  - Low-frequency: Self-organization
- **Multiphysics** nature
  - Collisional (intermolecular, walls)
  - Collisionless (non-Maxwellian, kinetic)
- **Power density**: $P_d = j_D E = (j_i + j_e)E$
  - Electric field: E
  - Electron current density: $j_e = e n_e \mu_\perp E$
  - Electron mobility: $\mu_\perp \propto \begin{cases} B^{-2} & \text{(Classical)} \\ B^{-1} & \text{(Anomalous)} \end{cases}$

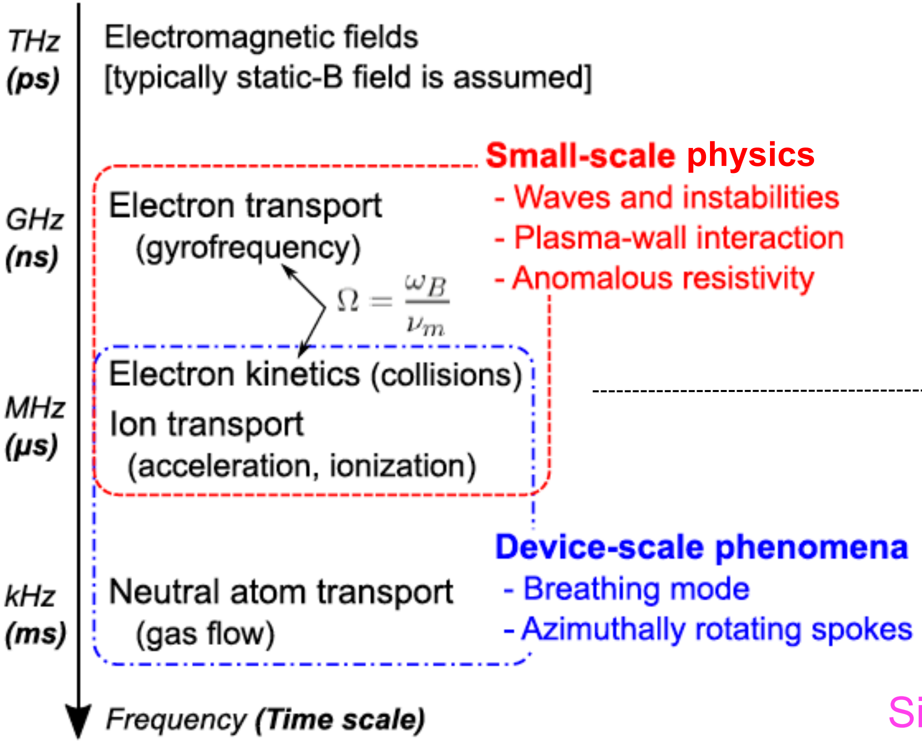


(a) Hall Effect Thruster



(b) Magnetron Discharge

Multiscale phenomena in partially magnetized plasmas



Extremely challenging to obtain reliable dynamic experimental data at >1-10 MHz (for now)

Next decade!

- Laser Thomson scattering (plasma waves)

- Fast camera (optical)
- High-speed probe system
- Laser induced fluorescence

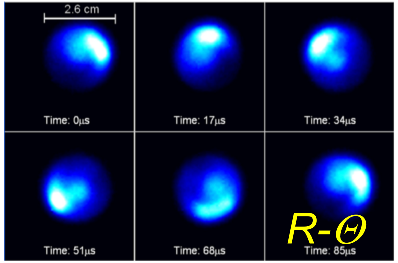
Significant progress in past decade

Validation

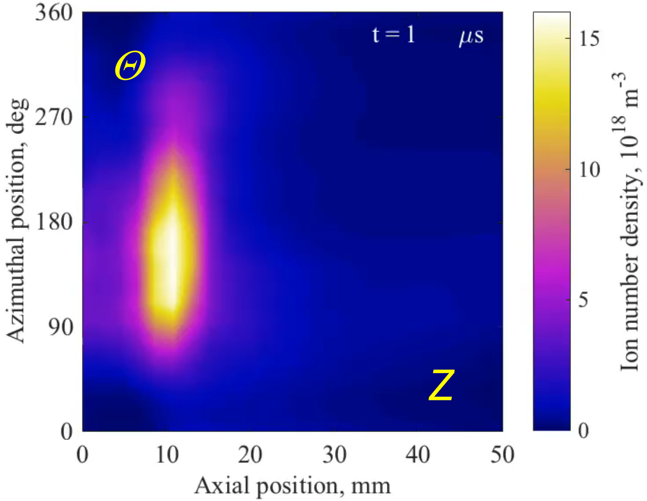
[Hara, PSST **28**, 044001 (2019)]

Low-frequency (10-30 kHz) plasma oscillations

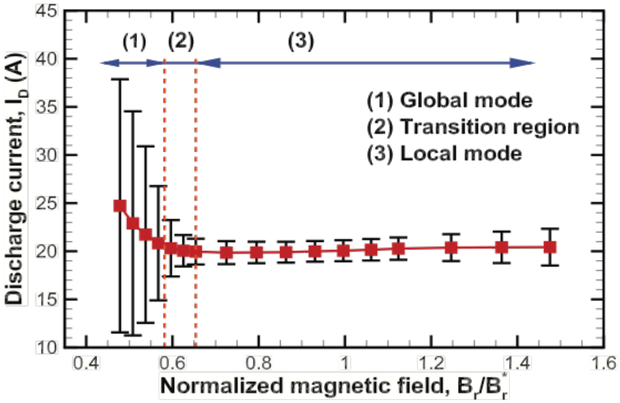
Movie



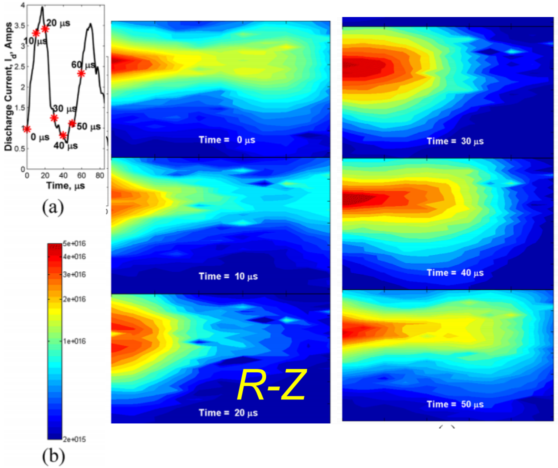
Azimuthally rotating spokes [Ellison 2011]



Gradient drift instability [Kawashima & Hara 2018]



Mode transition: ionization oscillation [Hara 2014]



Breathing mode: high-speed probe [Lobbia PhD 2010]

Physics-based modeling techniques for plasma flows

(a) Fluid (continuum) models

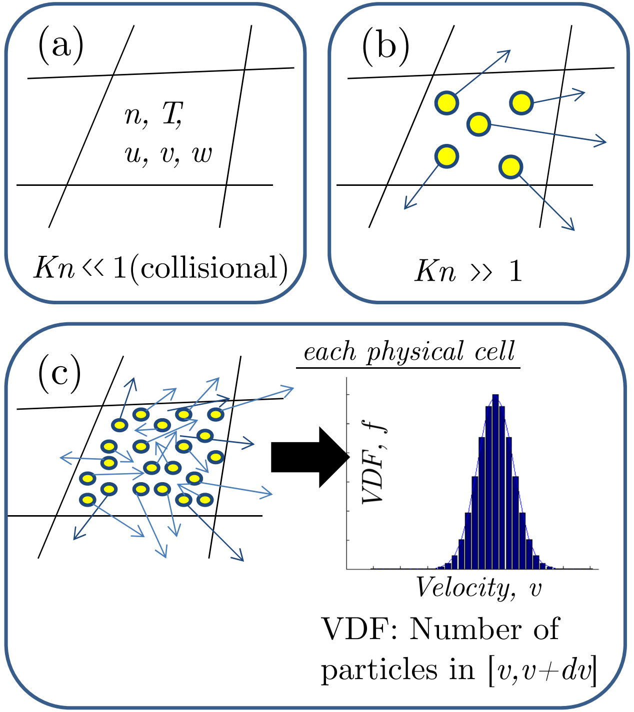
- Drift-diffusion model
- Euler/Navier-Stokes/MHD/Two-fluid
- Numerically inexpensive

(b) Particle-based kinetic methods

- 1 macroparticle ≈ 10^5-10^8 real particles
- Particle-in-cell (PIC), DSMC, MCC

(c) Grid-based direct kinetic (DK) methods

- Solve kinetic equations directly in discretized phase space
- No statistical noise vs. particle method



Cf.) Knudsen number: Kn = λ/L

Plasma fluid modeling strategies

Drift-diffusion (DD) flux models

- **Quasineutral** $(n_i \approx n_e)$ **DD model**: $\nabla \cdot (\vec{\Gamma}_i - \vec{\Gamma}_e) = 0 \Rightarrow \nabla \cdot (n_e \overline{\overline{\mu_e}} \cdot \nabla\phi) = f(p_e, \vec{\Gamma}_i)$

  [HPHall (Fife, Martinez-Sanchez, 1998), HPHall-2/3 (Ahedo 2006, Hofer 2008), Hall2De (Mikellides 2009), Detailed fluid model (Choi and Boyd 2008), other models (Boeuf and Garrigues 1998, Barral 2003, Hara 2014)]

- **Non-neutral** $(n_i \neq n_e)$ **DD model**:

  $$\frac{\partial n_e}{\partial t} + \nabla \cdot [-\overline{\overline{\mu_e}} \cdot (n_e \vec{E} + \nabla p_e)] = \dot{n}_e$$

  $$\& \ \nabla^2 \phi = -\frac{e}{\epsilon_0}(n_i - n_e)$$

  Scharfetter-Gummel scheme, Dielectric relaxation time

  Used in LTP models [Kushner 2009]

Full fluid moment (FFM) model

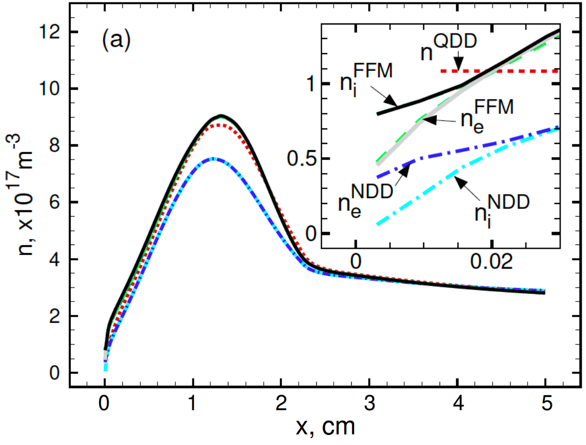
Need non-oscillatory schemes for the nonlinear inertia term (cf. hyperbolic PDE)

Used in other fields, e.g. CFD and HTP [Hakim, Hammett, Srinivasan, Shumlak]

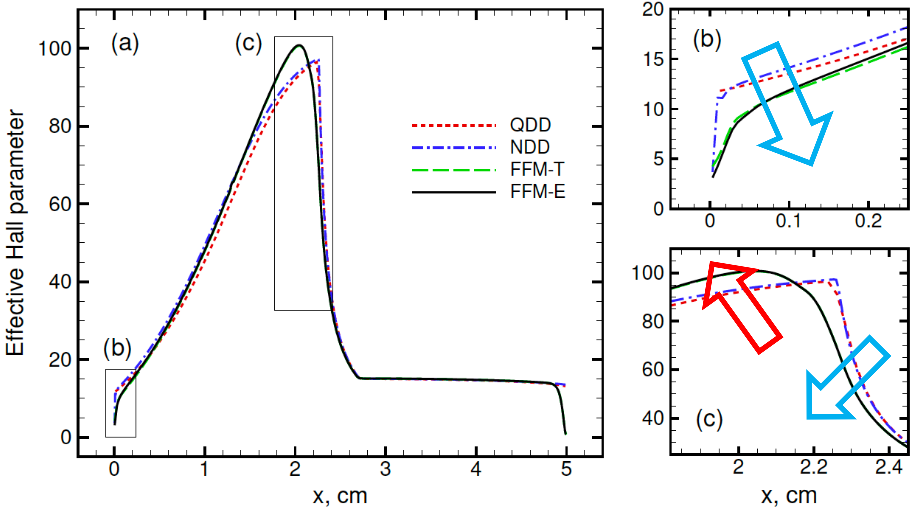
$$\frac{\partial U}{\partial t} + \nabla \cdot \vec{F} = S, \text{ where } U = [n, n\vec{u}, n\varepsilon] \ \ \& \ \ \nabla^2 \phi = -\frac{e}{\epsilon_0}(n_i - n_e)$$

Fluid model: Comparison of DD vs FFM models

- Non-oscillatory results are obtained from all solvers (Quasineutral drift-diffusion [QDD], Non-neutral drift-diffusion [NDD], and full fluid moment models [FFM-T] [FFM-E]).
- Effects of the nonlinear inertia term (e.g. shear) on electron transport are observed: **shear diamagnetic drift**.



Number density



Effective Hall parameter

$$\frac{\partial u_x}{\partial x}$$

$$\frac{\partial u_y}{\partial x}$$

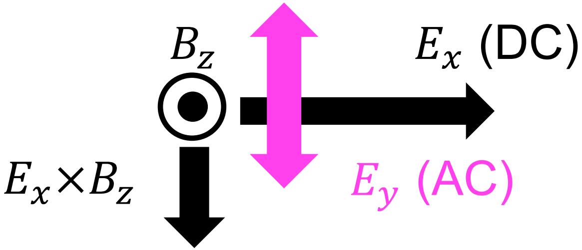
$$\Omega_{eff} = \frac{\overline{nu_y}}{\overline{nu_x}}$$

[R. Sahu, A. R. Mansour, K. Hara, Phys. Plasmas, 2020.]

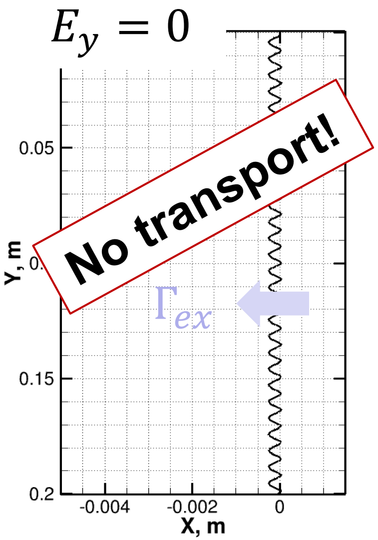
"Anomalous" cross-field electron transport

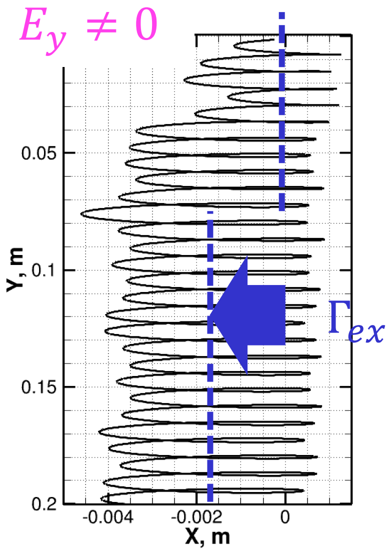
Classical theory: electrons are trapped by the magnetic field

$$\frac{d\vec{x}}{dt} = \vec{v}; \quad \frac{d\vec{v}}{dt} = \frac{q}{m}\left(\vec{E} + \vec{v}\times\vec{B}\right)$$



What if there is a plasma wave (i.e. fluctuation in the electric field)?



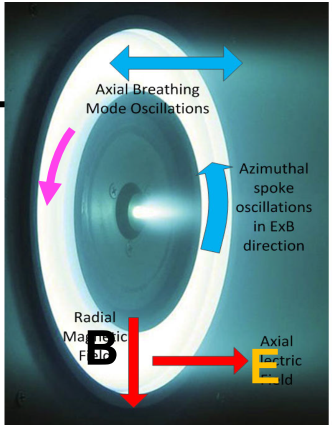


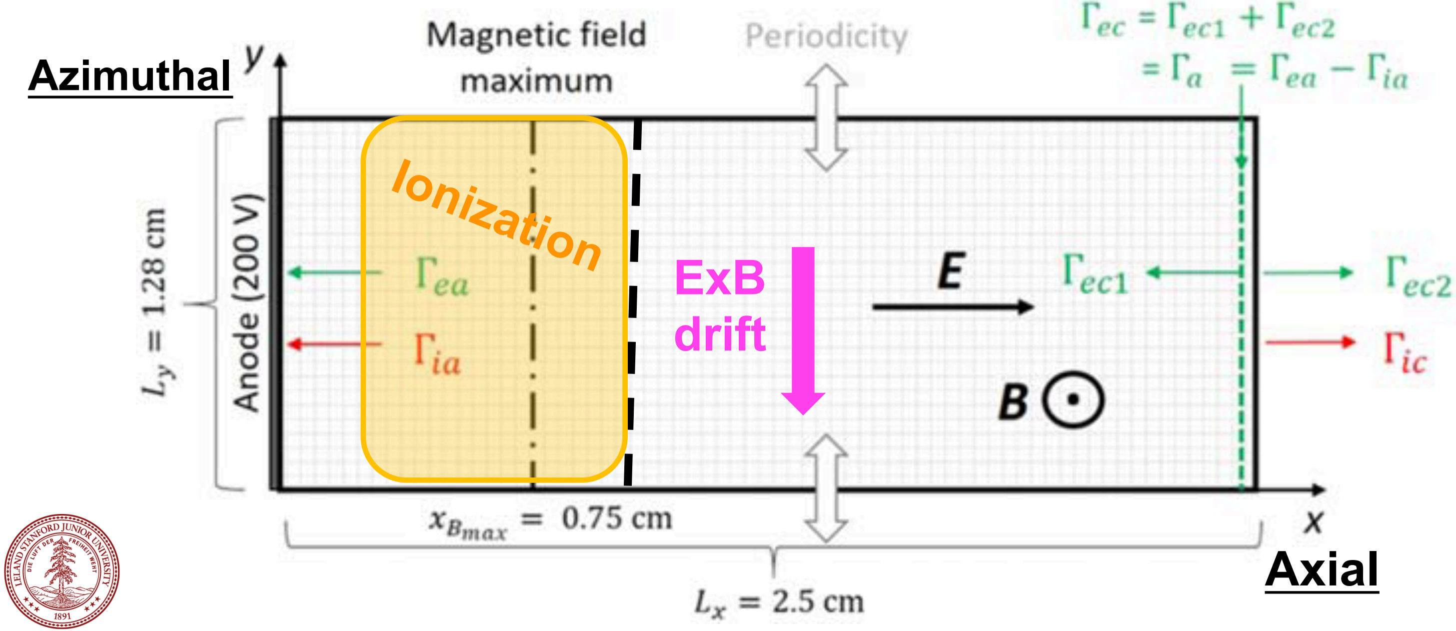
Turbulence: *Electron transport enhanced by plasma wave (<u>1D, 2D, 3D?</u>)*

2D z-theta benchmark testcase

- *Test case proposed by Boeuf and Garrigues 2018*
- BC: anode + electron injection
- **Fixed ionization rate**: run time ~30 μs (If neutrals are resolved, 1 ms is needed)

ExB drift





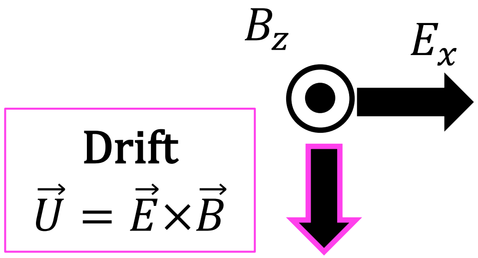
- 500x256 cells
- ~250 ppc
- 200 V
- 0.01 T peak
- j_i =400 A/m²
- **Xe^+**
- 200 CPUs
- 2 weeks

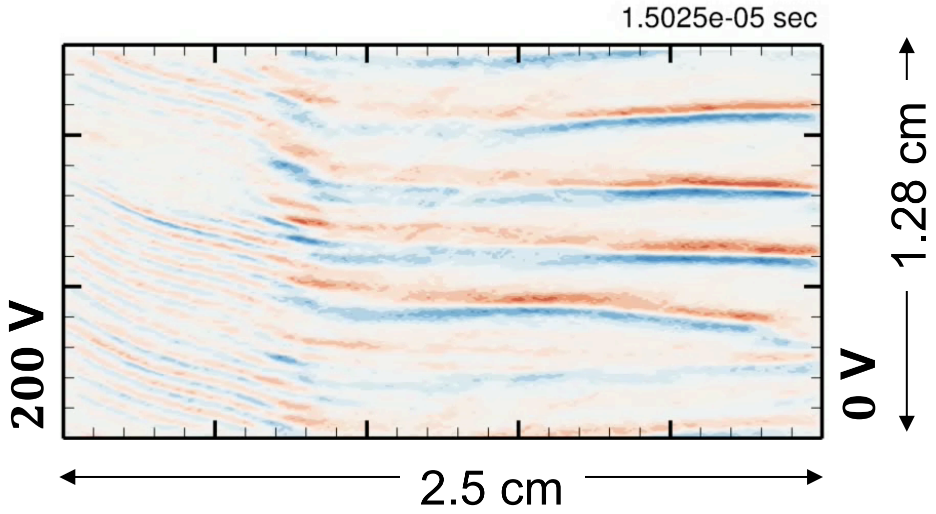
Azimuthal plasma wave initiated by electron cyclotron drift instability (ECDI)

Movie

- ExB drift in the y-direction causes the plasma wave formation in the y-direction.
- Upstream: small scale fluctuation (dominant mode: ~1 mm)
- Fluctuating E_y perturbs and detraps the electrons from the magnetic field lines

- **Singly charge ions: Xe^+**
- **Plasma wave (1D in y)**



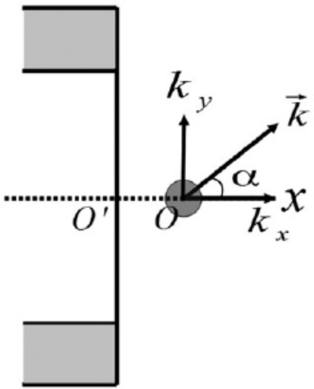


Azimuthal electric field, E_y

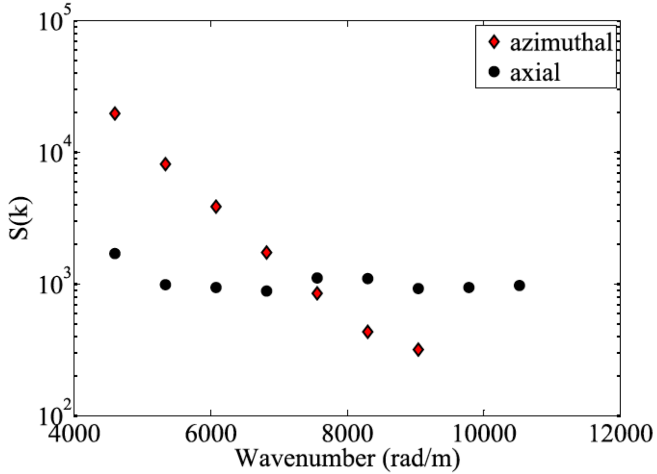
[Charoy T et al. PSST 2019]

Experimental evidence of axial (cross-field) plasma wave due to ion-ion two-stream instability (IITSI)

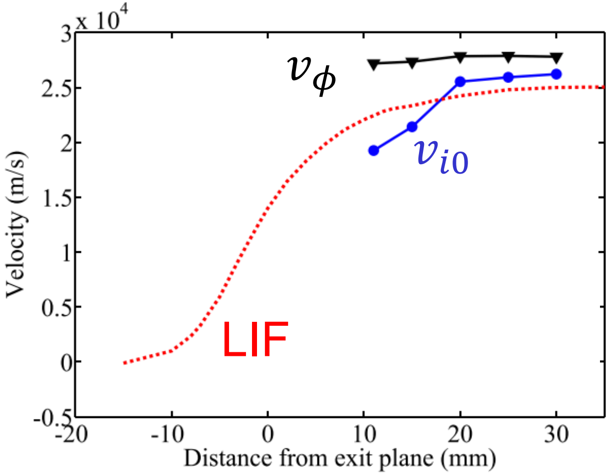
- Coherent Thomson scattering (CTS) detected signature of unambiguous **axial** plasma wave in the plume of a cross-field discharge (with **azimuthal** wave).
- Phase velocity of axial wave (v_ϕ) is *faster* than Xe⁺ ion velocity (v_{i0})



$x \in [11, 30]$ mm



Density fluctuation amplitude



Axial plasma wave

[Tsikata S, Cavalier J, Héron A, et al. Phys. Plasmas **21**, 072116 (2014)]

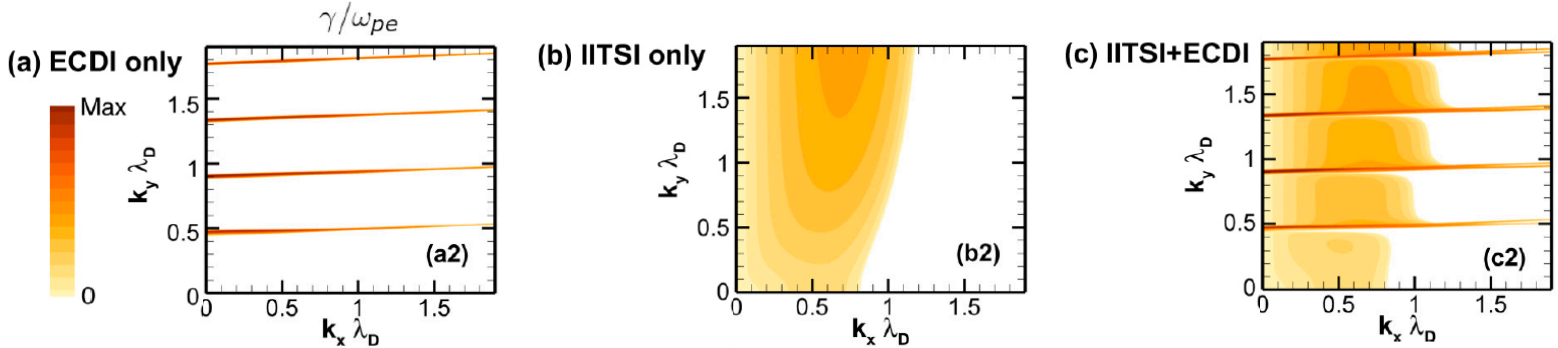
IITSI + ECDI dispersion theory

From the CTS measurement, it was observed that the stream of singly-charged ions (Xe^{+}) and doubly-charged ions (Xe^{2+}) causes ion-ion two-stream instability (IITSI)

Doubly charged ion fraction: $\alpha = \frac{2n_i^{2+}}{n_e}$

$$(k_\perp \lambda_D)^2 \left[1 - \frac{(1-\alpha)\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^{+})^2} - \frac{\alpha\omega_{pi}^2}{\left(\omega - \mathbf{k}\cdot\mathbf{U}_i^{2+}\right)^2} \right] + 1 - I_0(b)\exp(-b) + \sum_{n=1}^{\infty} \frac{2\omega^2 I_n(b)\exp(-b)}{(n\omega_B)^2 - \omega^2} = 0$$

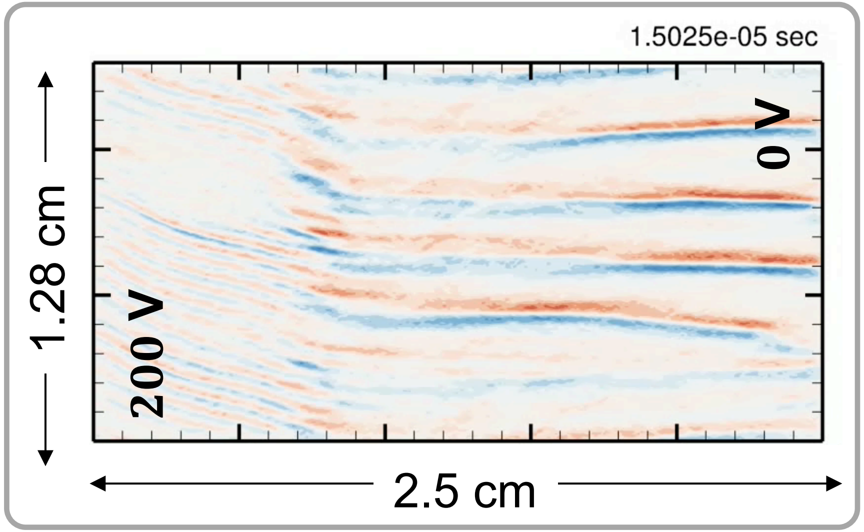
Xe^{+} $\quad$ Xe^{2+} $\quad$ *Magnetized electrons*



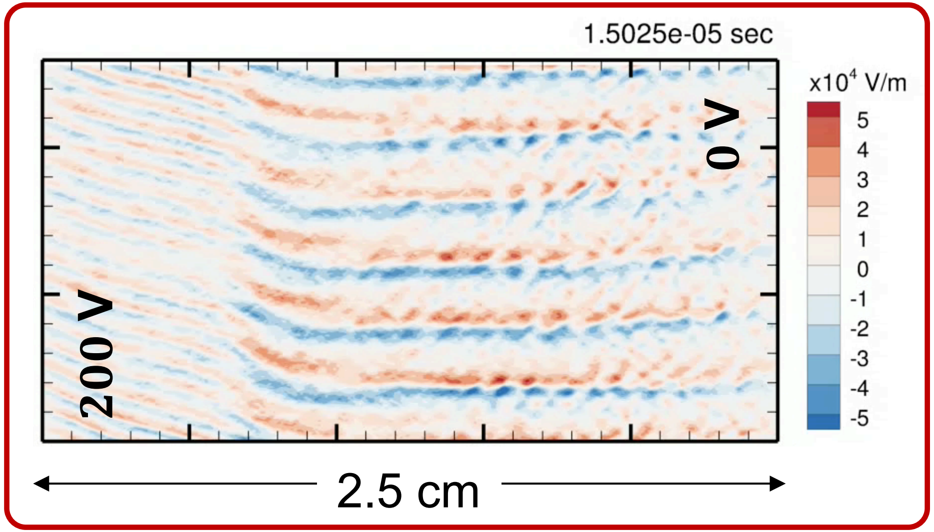
Multidimensional plasma microturbulence

Movie

Multidimensional (2D) plasma wave is observed due to the coupling of (i) **axial** oscillation via IITSI and (ii) **azimuthal** oscillation via ECDI



100% Xe^{+}
(ECDI only)

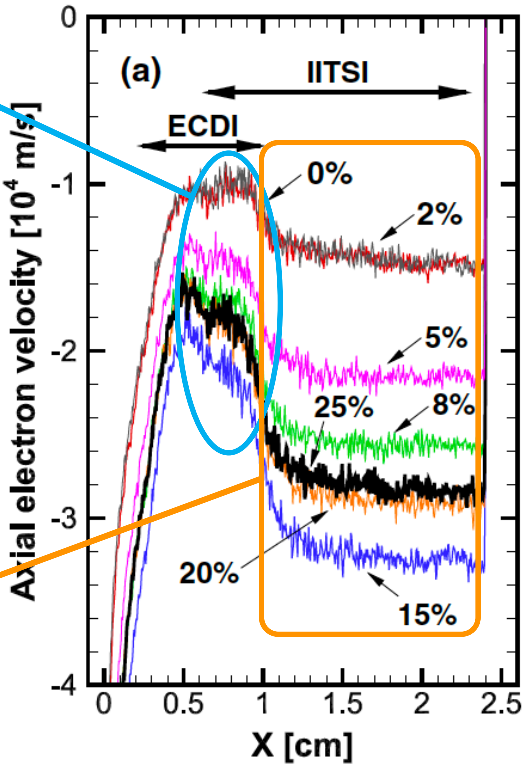


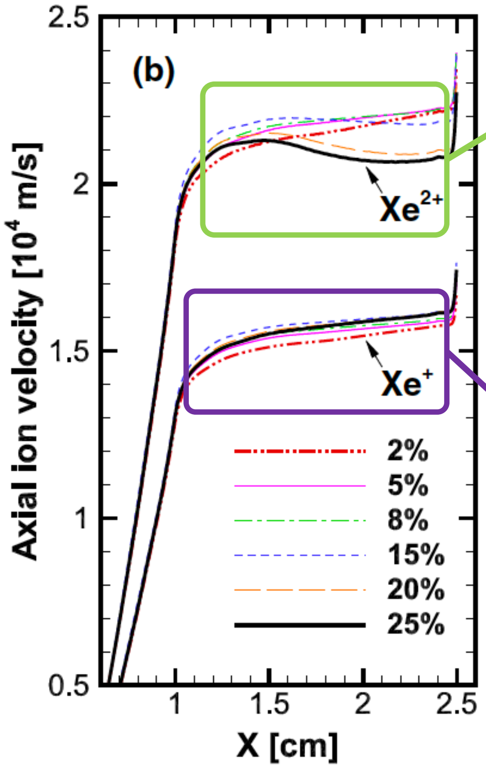
80% Xe^{+}, 20% Xe^{2+}
(ECDI+IITSI)

Cross-field transport: electrons and ions

Nonlinear coupling of ECDI and IITSI

$|u_{xe}| \approx \mu_{\perp} E_x$ increases with more Xe^{2+}





Deceleration of doubly charged ions

Acceleration of singly charged ions

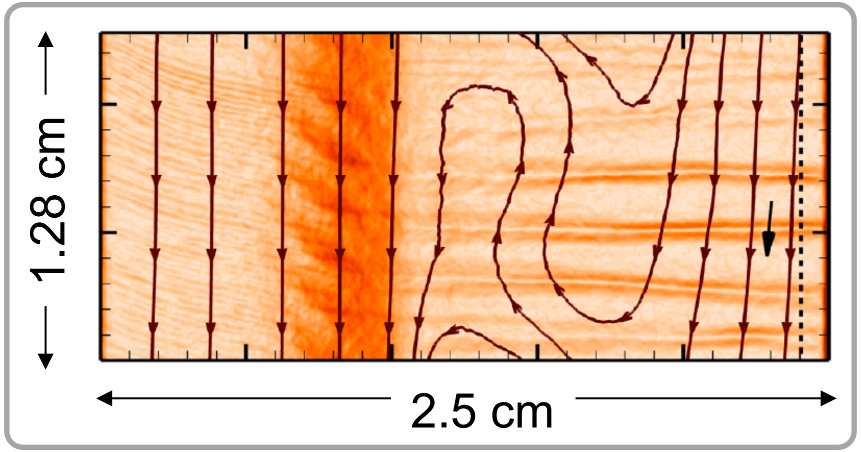
Time-averaged electron streamline

Fluctuation-based electron transport
[Waltz PoF 1982, Liewer Nucl Fusion 1985]

$$\langle \Gamma_{ex} \rangle = \frac{\langle n'_e E'_y \rangle}{B_z} \qquad \langle \Gamma_{ey} \rangle = -\frac{n_{e0} E_{x0}}{B_z} - \frac{\langle n'_e E'_x \rangle}{B_z}$$

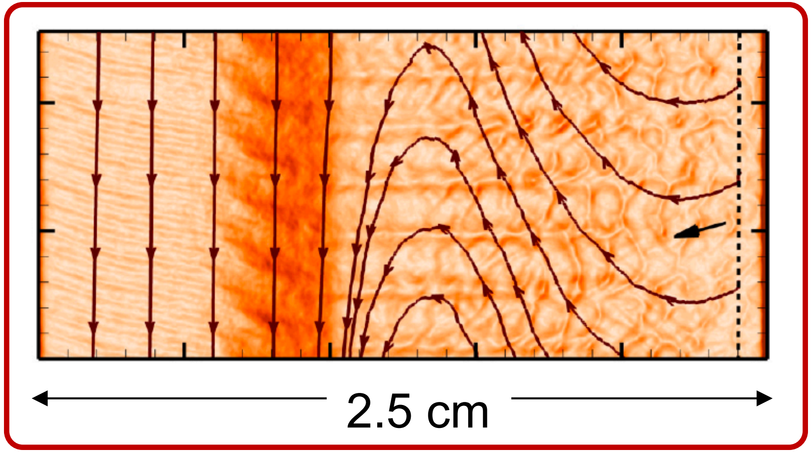
100% Xe^{+}
(ECDI only)

$|\langle \Gamma_{ex} \rangle| < |\langle \Gamma_{ey} \rangle|$



80% Xe^{+}, 20% Xe^{2+}
(ECDI+IITSI)

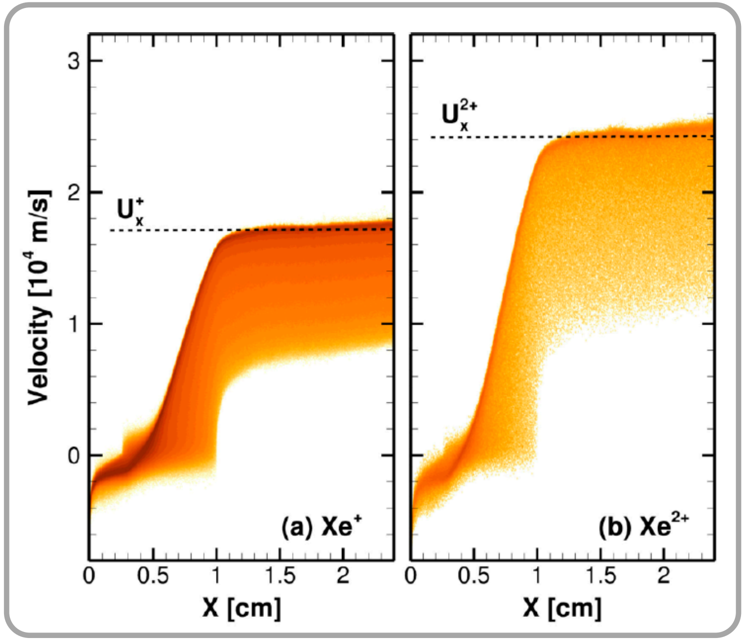
$|\langle \Gamma_{ex} \rangle| > |\langle \Gamma_{ey} \rangle|$



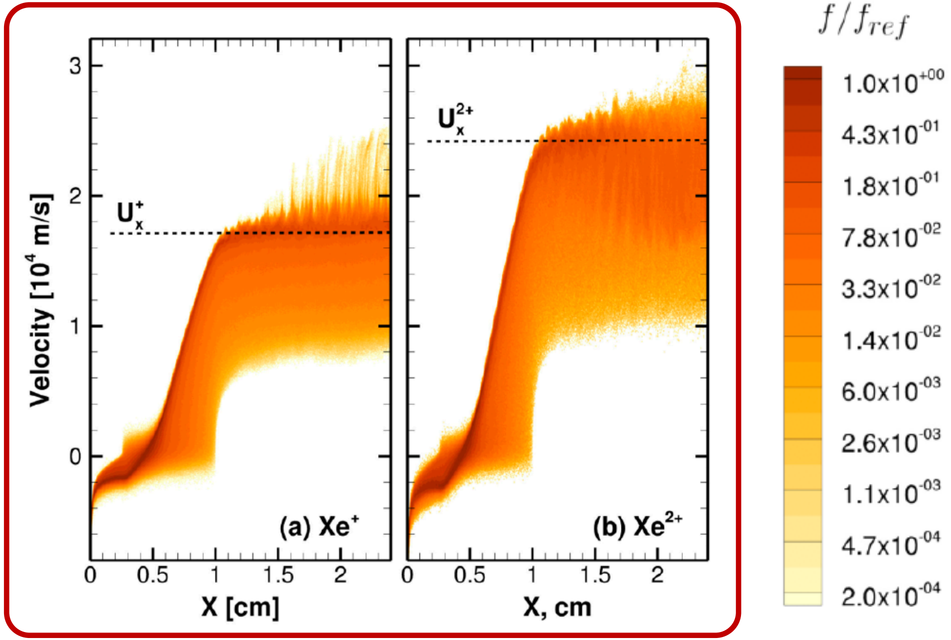
Color: amplitude of electric field. Line: electron streamlines.

Ion phase space

Broadening (heating) of the ion VDFs is observed: saturation of IITSI



100% Xe⁺
(ECDI only)



80% Xe⁺, 20% Xe²⁺
(ECDI+IITSI)

Summary: anomalous transport

- We investigated the coupling of ECDI (**azimuthal**) and IITSI (**axial**) modes using a 2D parallel PIC code. The numerical results are consistent with the theoretical analysis and experimental observation (cf. coherent Thomson scattering).

- The enhancement of cross-field electron transport is observed due to the excitation of multidimensional (**2D**) plasma turbulence via the nonlinear saturation of the linear instabilities.

- Reference: K. Hara and S. Tsikata, “Cross-field electron diffusion due to the coupling of drift-driven microinstabilities”, Physical Review E **102**, 023202 (2020)

Thank you for your attention

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