Electron Drift Instability in a Hall Thruster: Comparisons between Standard and Sparse Grid Particle-In-Cell Approaches

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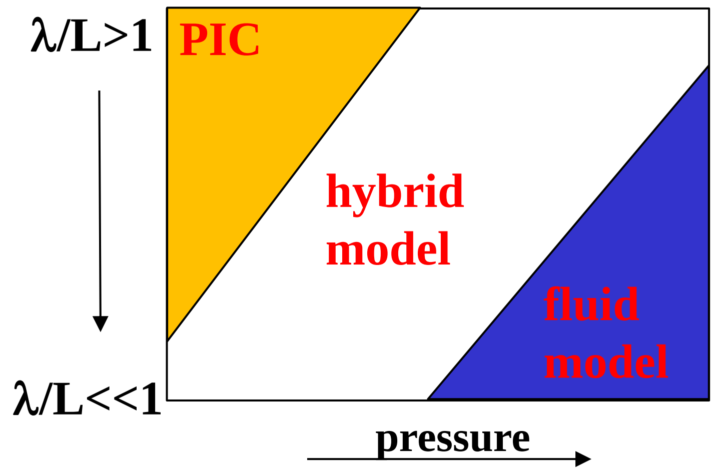
Outline

- Standard PIC schema
- Sparse grid methods
- Capacitive discharges
- Electron Drift Instability
- Conclusions and perspectives

Particle-In-Cell vs fluid approaches - LTP



$\rho(\vec{r},t)$

Boltzmann equation

$$\frac{\partial}{\partial t} f_s + \vec{v}\frac{\partial}{\partial \vec{r}} f_s + \vec{a}\frac{\partial}{\partial \vec{v}} f_s = \left\{\frac{\partial}{\partial t} f_s\right\}_{coll}$$

$$f_s \equiv f_s(\vec{r},\vec{v},t)$$

$$\vec{a} = q_s\left(\vec{E} + \vec{v}\times\vec{B}_{ext}\right)/m_s$$

Poisson equation

$$\nabla\cdot\left[\varepsilon\cdot\vec{E}(\vec{r},t)\right] = \rho(\vec{r},t)$$

$$\rho(\vec{r},t) = \sum_s q_s \int f_s(\vec{r},\vec{v},t)\, d\vec{v}$$

$\vec{a}$

Particle-In-Cell (PIC) electrostatic model

- Sampling of f_s with a finite number p of particles (**super-particules**)

$$f_s(\vec{r},\vec{v},t) = \sum_p f_{s,p}(\vec{r},\vec{v},t)$$

- Time evolution of the particles defined by position and velocity in an electric field generated and modified by themselves (with the action of an external B field), and with collisions

- Introduction of a time step (and constrains associated)

- Introduction of a space grid (and constrains associated)

Constraints of explicit PIC schema

- Constraints on time step

$$\Delta t = \left[\frac{0.2}{\omega_{p,s}}\right] \qquad \omega_{p,s} = \sqrt{\frac{q_s{}^2 n_s}{m_s \varepsilon_0}}$$

- Constraints on grid spacing

$$\Delta x < \lambda_{d,s} \qquad \lambda_{d,s} = \sqrt{\frac{\varepsilon_0 k_B T_s}{q_s{}^2 n_s}}$$

- Constraints on statistic

numerical noise $N_{s,p} > 100' N_{\text{cell}}$

Limit of standard PIC algorithm
High computational time
Exponential dependence on dimension

- Hall thruster modeling – 3D simulations

computational domain of 5 cm^3, 10^{18} m^{-3}, 500 particles/cell: 4 months on 400 000 processors to simulate 300 μs (perfect scaling)

Sparse grid methods

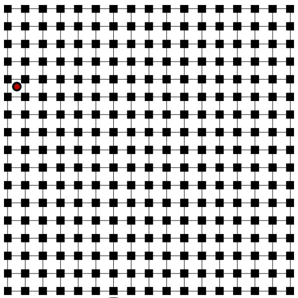
Reduction of dimension dependence for grid-based methods

- Sparse grid techniques
  - Define a hierarchy of anisotropic grids with a coarser resolution
  - Reconstruction of the solution on the initial Cartesian grid using combination techniques
  - Preserving second order approximation (for $d > 1$)

- Examples
  - Resolution of Navier-Stokes equation — *A. Rütgers and M. Griebel, Applied Math. and Comp.* ***319****, 425 (2018)*
  - Quantum mechanics: Schrödinger equation — *J. Garcke and M. Griebel, JCP* ***165****, 694 (2000)*
  - Plasma physics: gyrokinetic Vlasov equation — *D. Pflüger et al., Euro-Par 2014: Parallel Processing (2014)*
  - Financial mathematics: Black-Scholes equation — *H. J. Bungartz et al., J. Comput. Applied Math.* ***236****, 3741 (2012)*
  - **PIC method: Plasma physics** — *L. F. Ricketson and A. J. Cerfon, PPCF* ***59****, 024002 (2017)*

Application of sparse PIC approach to low temperature plasma simulations

Construction of sparse grid sub-domains



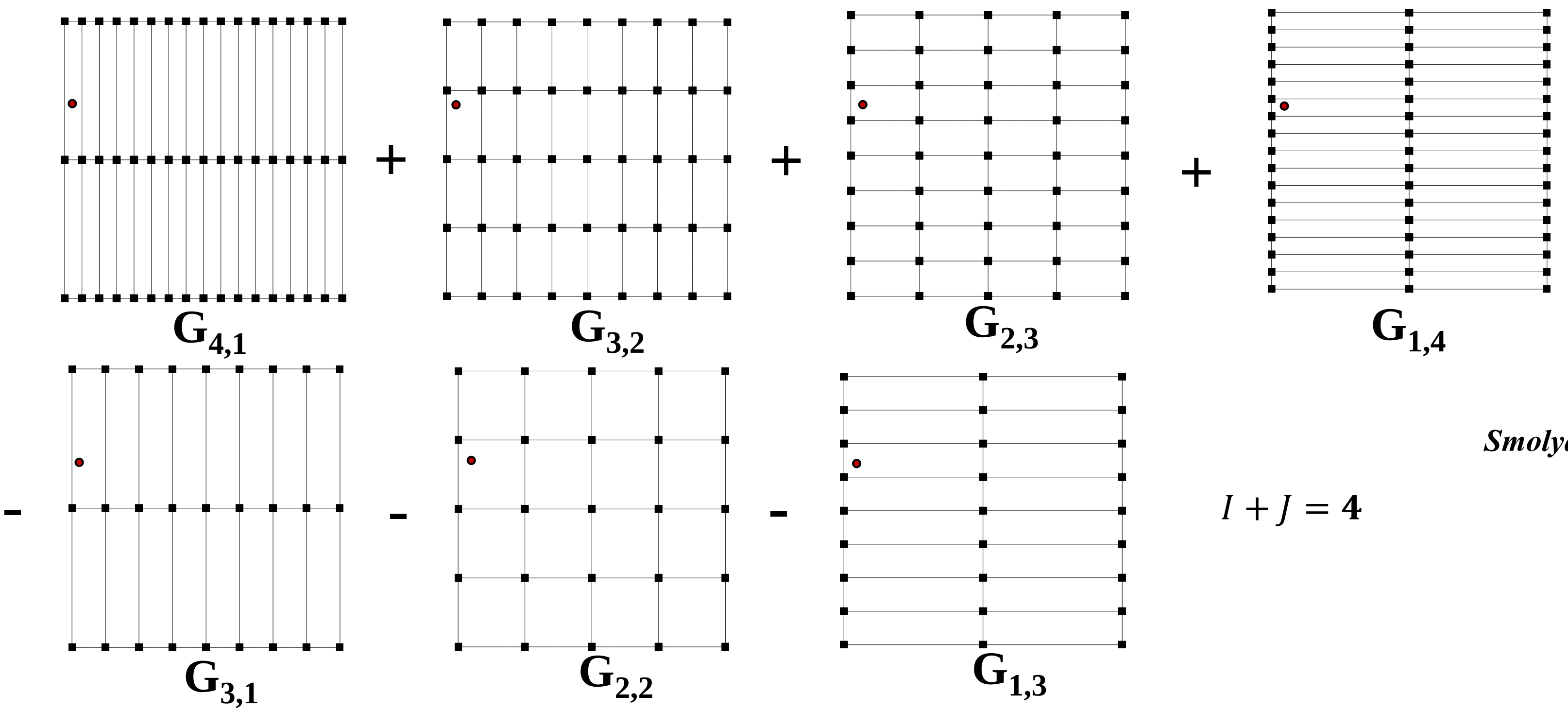
$G_{regular}$

16x16

2^nx2^n , n=4

Recombination technique

$$U_N = \sum_{I+J=N+1} U_{I,J} - \sum_{I+J=N} U_{I,J}$$

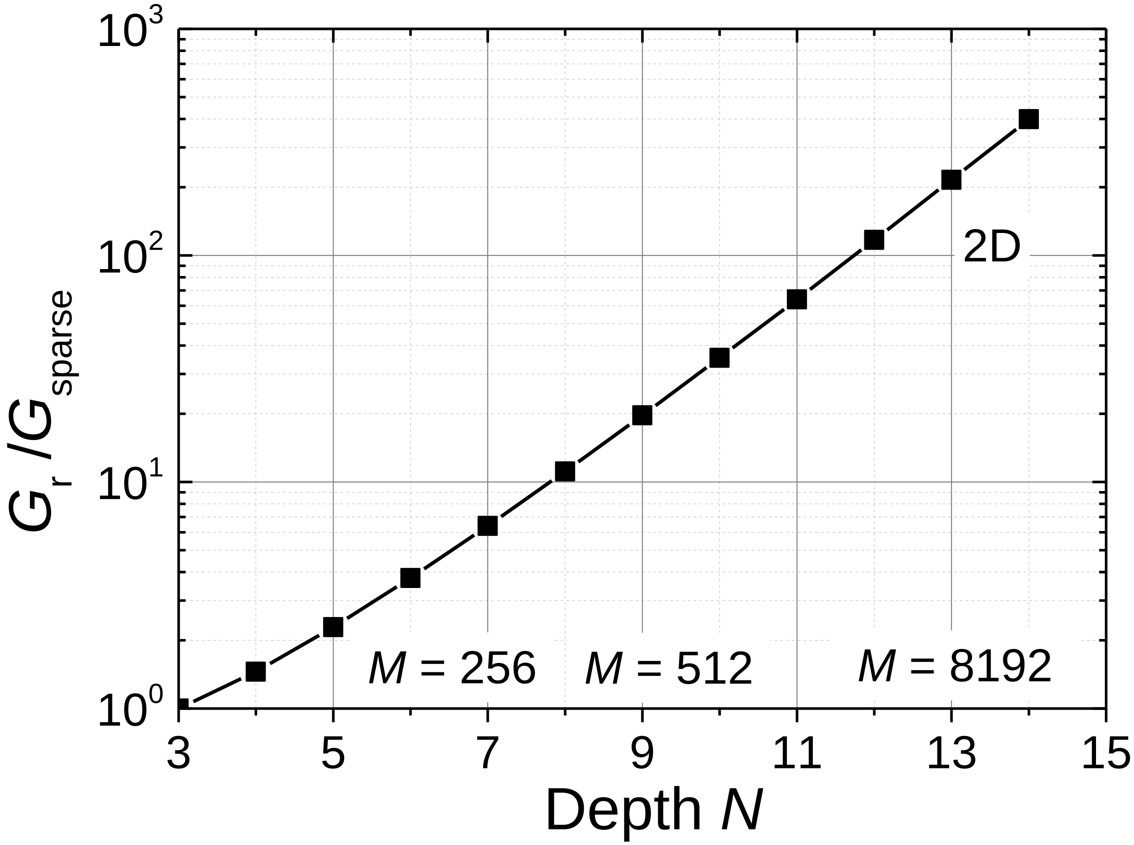


$I + J = 5$

Smolyak's grid

$I + J = 4$

Reduction of the number cells in 2D



$$\{cells(512^2)\}_{sparse}$$
$$=$$
$$\{cells(512^2)\}_{standard}/20$$

Gain in number of cells, gain in number of particles
Reduction of computational time

PIC cycle revisited with sparse approach

Integration of equations of motion

$$\mathbf{F}\left(\mathbf{x}_{\text{old}}^{l,\alpha}\right) \rightarrow \mathbf{v}_{\text{new}}^{l,\alpha} \rightarrow \mathbf{x}_{\text{new}}^{l,\alpha}$$

Particle losses/gain at boundaries (absorption, emission)

Collisions

$$\mathbf{v}_{\text{old}}^{l,\alpha} \rightarrow \mathbf{v}_{\text{new}}^{l,\alpha}$$

Interpolation of particle sources to grid for the $2N-1$ sub-grids

$$\sum_{\alpha}\sum_{l} \mathbf{x}_{\text{old}}^{l,\alpha} \rightarrow \rho_{I,J}$$

Integration of the $2N-1$ Poisson's equations

$$\rho_{I,J} \rightarrow \mathbf{E}_{I,J}$$

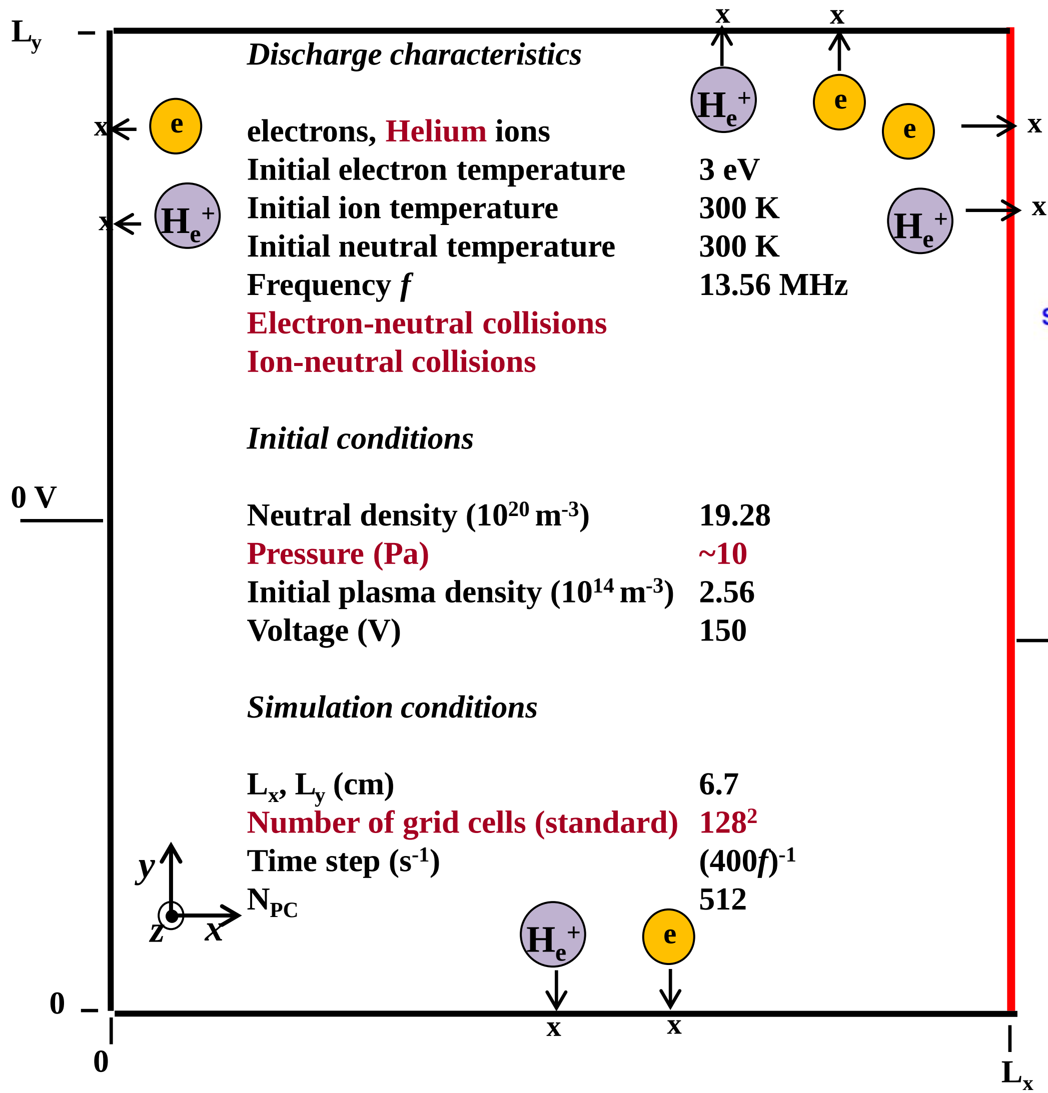
Assignation of E field to particles

$$\mathbf{E}\left(\mathbf{x}_{\text{old}}^{l,\alpha}\right) = \sum_{I+J=N+1} \mathbf{E}_{I,J}\left(\mathbf{x}_{\text{old}}^{l,\alpha}\right) - \sum_{I+J=N} \mathbf{E}_{I,J}\left(\mathbf{x}_{\text{old}}^{l,\alpha}\right)$$

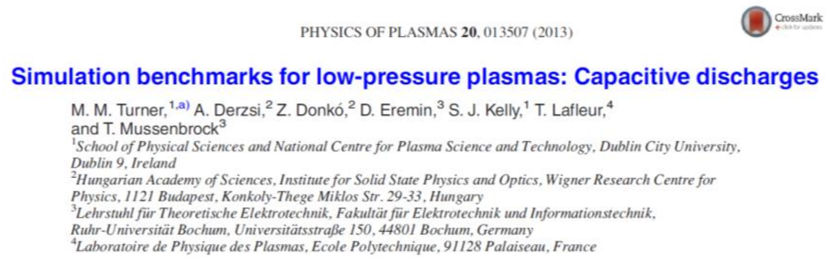
recombination technique

Δt

RF discharge - 2D benchmark conditions



Discharge characteristics	
electrons, Helium ions	
Initial electron temperature	3 eV
Initial ion temperature	300 K
Initial neutral temperature	300 K
Frequency *f*	13.56 MHz
Electron-neutral collisions	
Ion-neutral collisions	
Initial conditions	
Neutral density (10^{20} m^{-3})	19.28
Pressure (Pa)	~10
Initial plasma density (10^{14} m^{-3})	2.56
Voltage (V)	150
Simulation conditions	
L_x, L_y (cm)	6.7
Number of grid cells (standard)	128^2
Time step (s^{-1})	$(400f)^{-1}$
N_{PC}	512



PHYSICS OF PLASMAS **20**, 013507 (2013)

Simulation benchmarks for low-pressure plasmas: Capacitive discharges

M. M. Turner,[1,a)] A. Derzsi,[2] Z. Donkó,[2] D. Eremin,[3] S. J. Kelly,[1] T. Lafleur,[4] and T. Mussenbrock[3]
[1]*School of Physical Sciences and National Centre for Plasma Science and Technology, Dublin City University, Dublin 9, Ireland*
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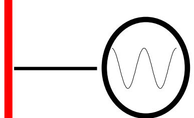
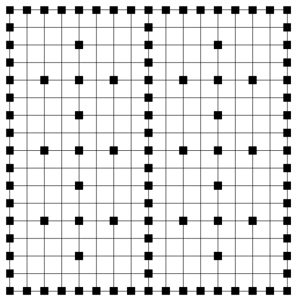
[4]*Laboratoire de Physique des Plasmas, Ecole Polytechnique, 91128 Palaiseau, France*

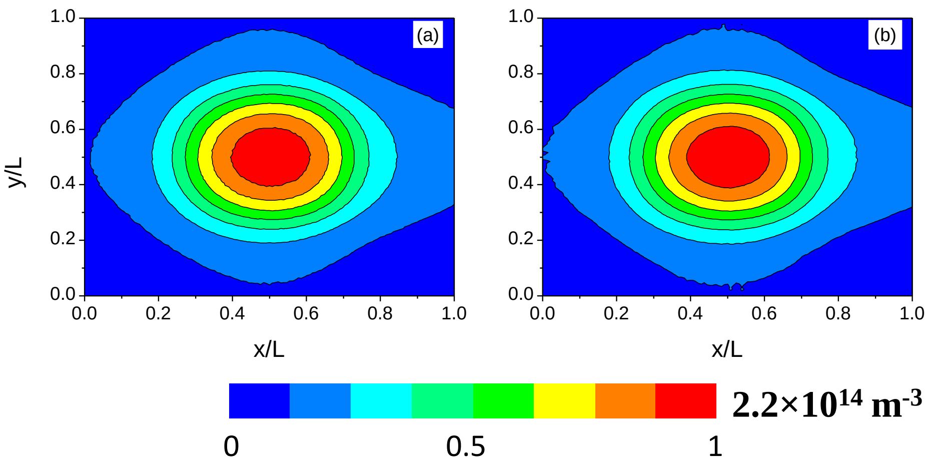




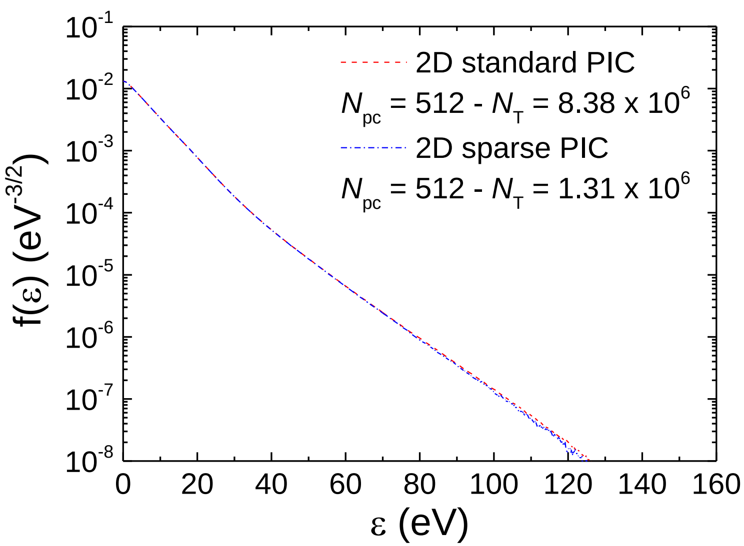
G_{sparse}

Standard vs sparse grid PIC approaches

Standard (N_{pc} = 512) **Sparse (N_{pc} = 512)**



Ion density

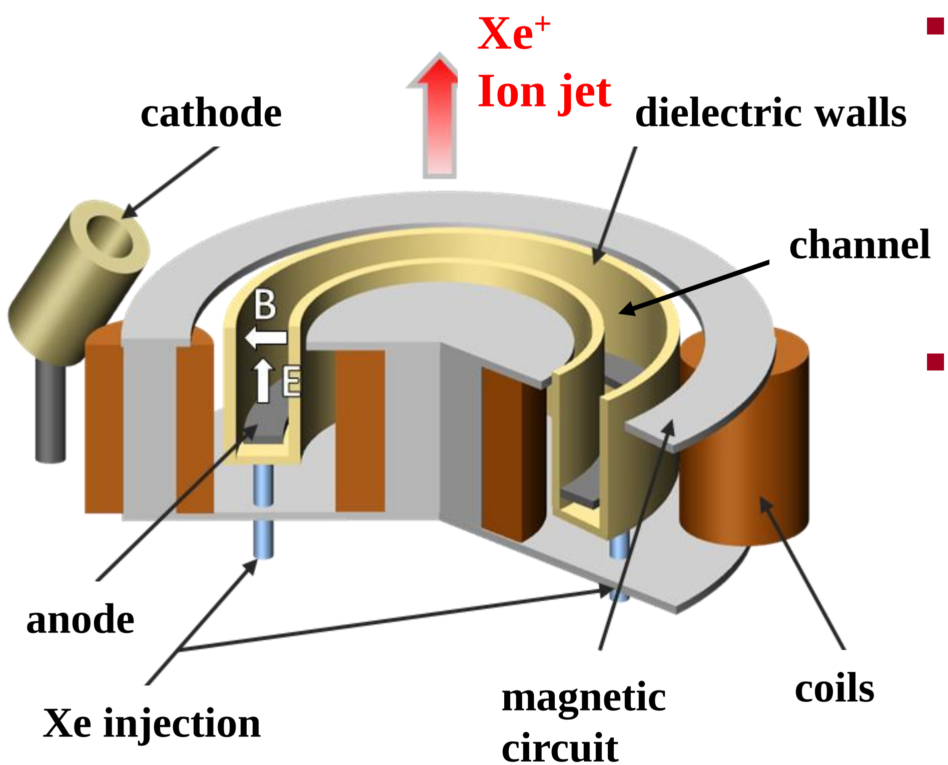


L. Garrigues et al., JAP ***129****, 153303 (2021)*

$$\epsilon(\varphi) = \frac{\|\varphi - \varphi_{\text{ref}}\|_{L^2}}{\|\varphi_{\text{ref}}\|_{L^2}} = \sqrt{\frac{\int |\varphi - \varphi_{\text{ref}}|^2 \mathrm{du}}{\int |\varphi_{\text{ref}}|^2 \mathrm{du}}}$$

Error (%)	
	Standard Vs Sparse
n_i	1.8
$EEDF$	0.1

Hall thruster configuration

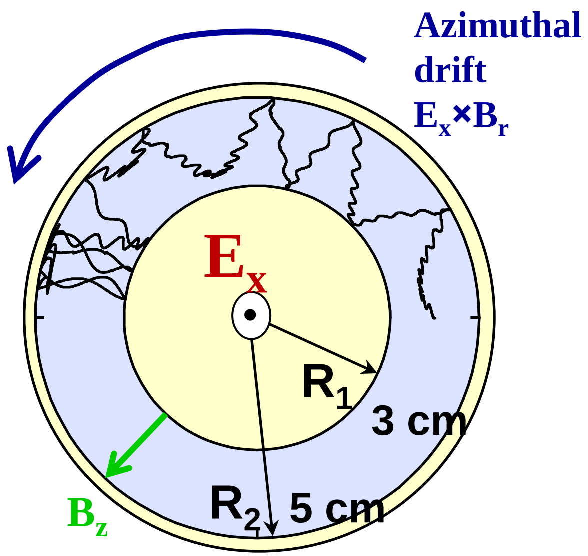


- **Principle**
  - Mfp << L
  - Magnetic field strength: trap only electrons
  - Ions are collisionless

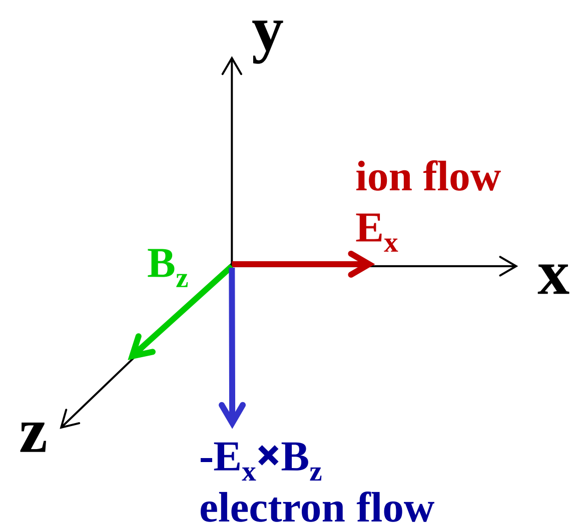
- **Typical numbers**
  - Voltage: 300 V, current ~ 4 A
  - Diameter: 10 cm, L = 2.5 cm
  - Xe mass flow: 5 mg/s (density ~ 10^{20} m^{-3})
  - Plasma density: 10^{18} m^{-3}
  - Electron temperature: 50 eV

- **Missions**
  - Very high ion velocity
  - Well adapted for satellite station keeping

Hall thruster: Electron drift instability (EDI)



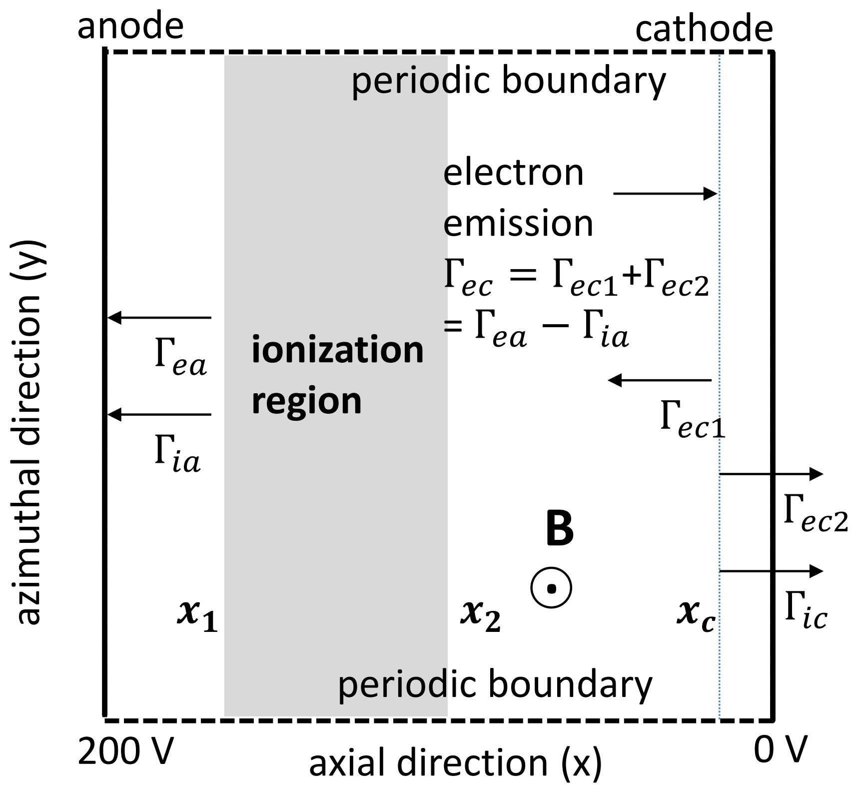
$$V_E = \frac{ExB}{B^2} = \frac{E}{B}$$

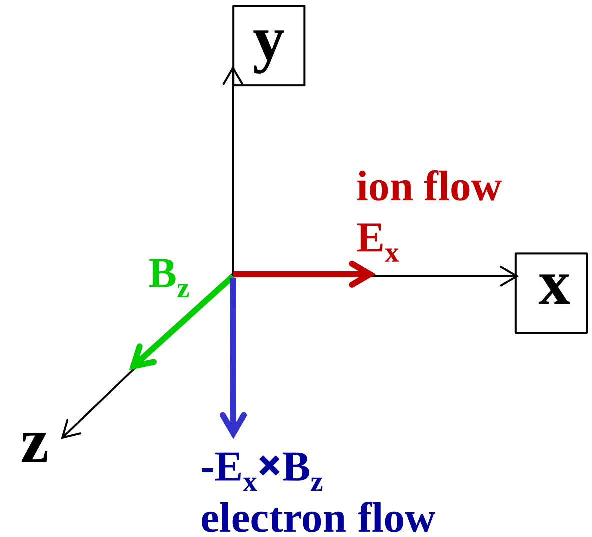


- **ExB discharge – partially magnetized LTP**
  - Electrons are magnetized
  - Ions are not magnetized
  - Closed electron drift in the azimuthal direction
  - EDI induced by the different velocity between electrons and ions in the azimuthal direction
  - EDI tends to be a modified ion acoustic instability

T. Lafleur et al., PoP ***23****, 053503 (2016)*

2D PIC simplified model – (x,y) plane





*J. P. Boeuf and L. Garrigues, PoP **25**, 061204 (2018)*
*T. Charoy et al., PSST **28**, 105010 (2019)*
T. Charoy, International Online Plasma Seminar, 2021

- **Cartesian simulation domain (x,y) directions**
- **B ⊥ to simulation plane (with fixed profile)**
- **Periodic boundary conditions along y (angular sector, w = 2.5 cm)**
- **Fixed source term, no collisions**
- **W=d=2.5 cm, applied voltage 200 V**

2D Benchmark conditions

Discharge characteristics

electrons, xenon ions	
Initial electron temperature	**10 eV**
Initial ion temperature	**0.5 eV**

Initial conditions

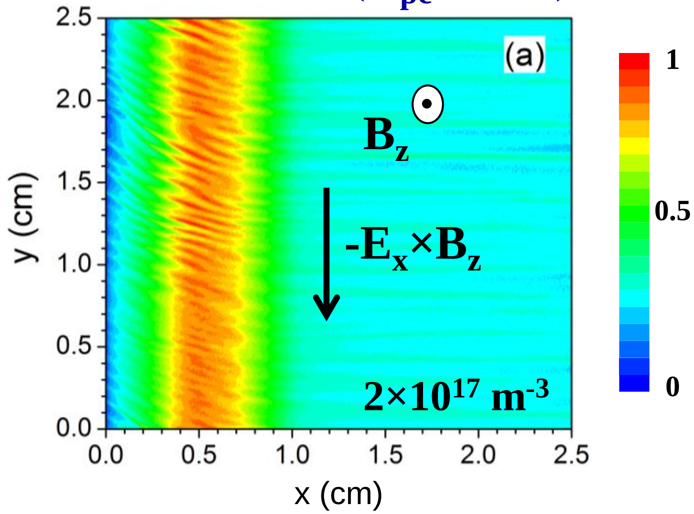
Initial plasma density (10^{16} m^{-3})	**8**
Voltage (V)	**200**
Current density J_M (A/m^2)	**150**
Maximum of B field (G)	**75**

Simulation conditions

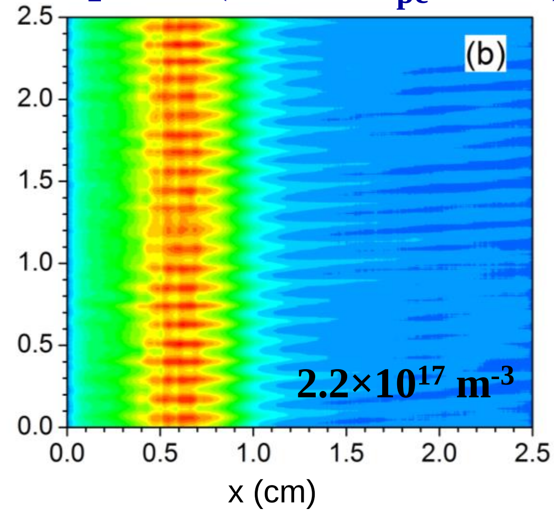
L_x, L_y (cm)	**2.5**
Axial zone of injection (cm)	**0.25-1.0**
Number of grid cells (standard)	**512^2**
Time step (s^{-1})	**10^{-11}**
N_{PC}	**20-400**
No collisions	

2D ion density profile

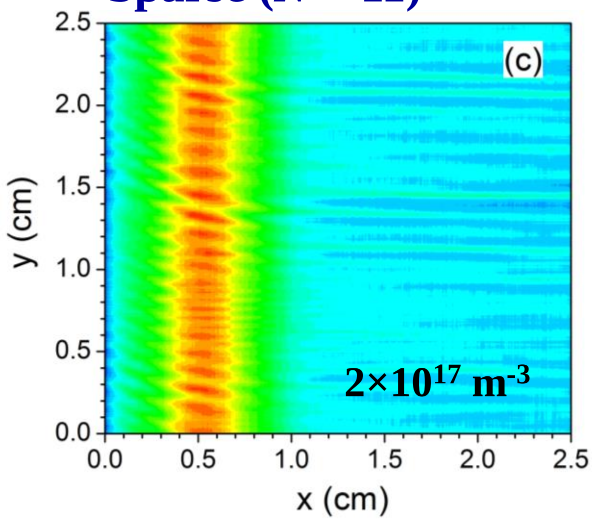
Standard (N_{pc} = 400)



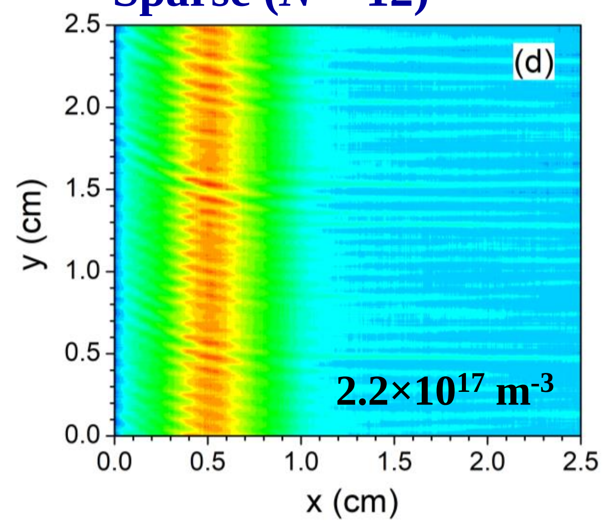
Sparse (*N* = 9, N_{pc} = 400)



Sparse (*N* = 11)

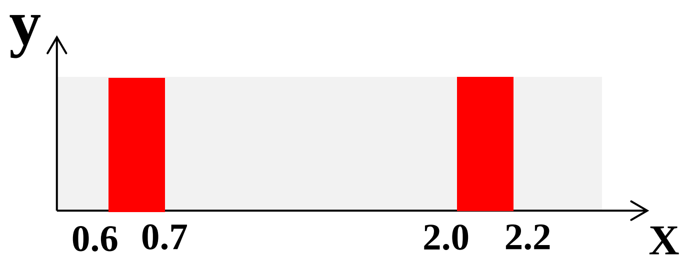


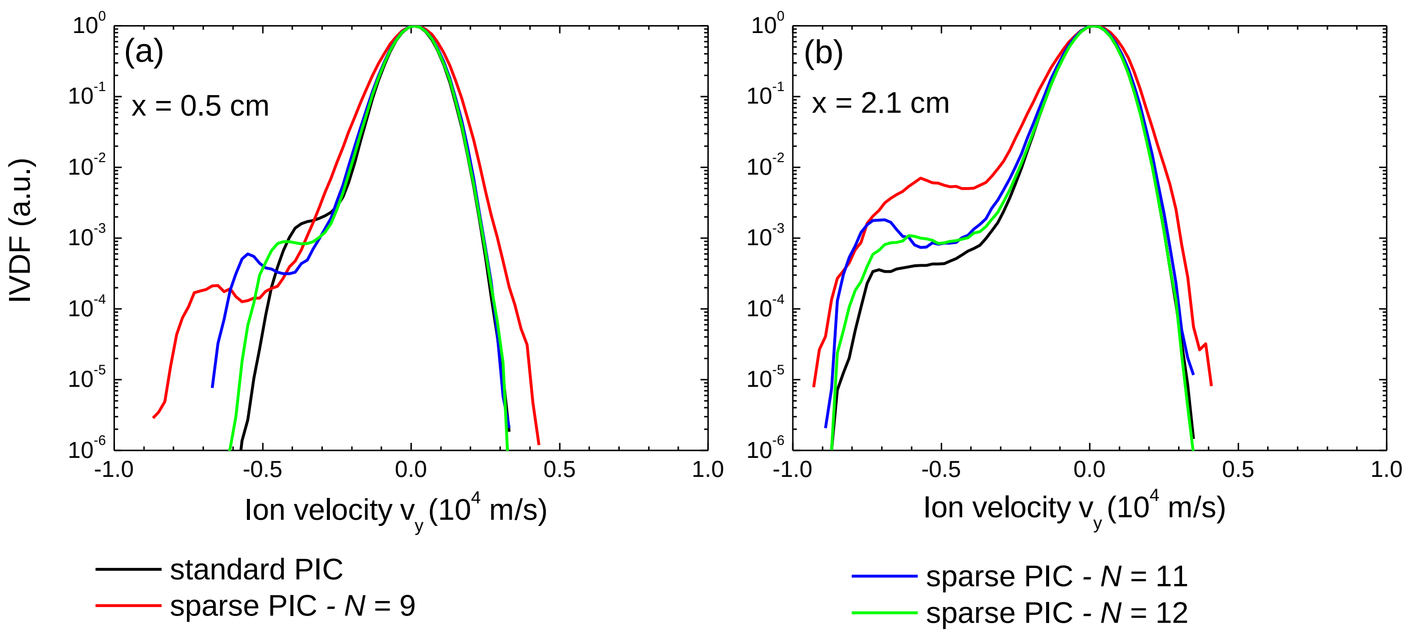
Sparse (*N* = 12)



L. Garrigues et al.,
JAP ***129****, 503304 (2021)*

Ion energy distribution functions





*L. Garrigues et al., JAP **129**, 503304 (2021)*

Conclusions

- **Demonstration of applicability of sparse approach for PIC simulations in the context of LTP**
  - Self-consistent description of sheaths
  - Collisions
  - Different types of boundary conditions

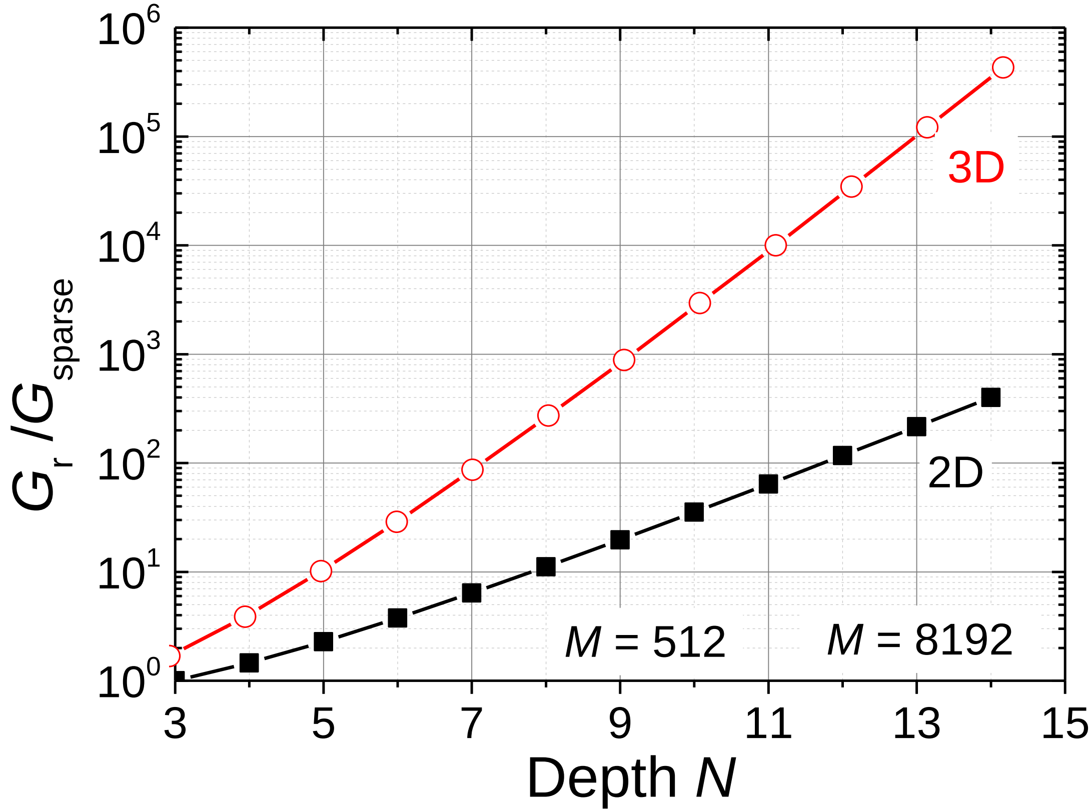
- **High precision for RF discharges modeling**
  - **128^2, 44 cores, computational speed up of 2**
  - **512^2, 180 cores, computational speed up of 5**

*L. Garrigues et al., JAP **129**, 153303 (2021)*

- **Electron drift instability, 512^2, 180 cores**
  - **25 % of error, computational speed up: 5**
  - **< 5 %, need for a finer grid size, no speed up for 2D simulations**

*L. Garrigues et al., JAP **129**, 503304 (2021)*

Number of cells in 3D



$$\{cells(512^3)\}_{\mathbf{sparse}} < \{cells(512^2)\}_{\mathbf{standard}}$$

large speed up expected for 3D simulations

Perspectives

- **Numerical analysis about sparse method – ASPIC3D**

PhD grant Université de Toulouse/Région Occitanie – ADI call 2020

Clément Guillet (IMT/LAPLACE)

- **3D models of a Hall thruster using sparse PIC approach**

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Collaborators

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- **Fabrice Deluzet, Institut de Mathématiques de Toulouse**

- **Jacek Narski, Institut de Mathématiques de Toulouse**