Non-classical electron transport in Hall effect thrusters

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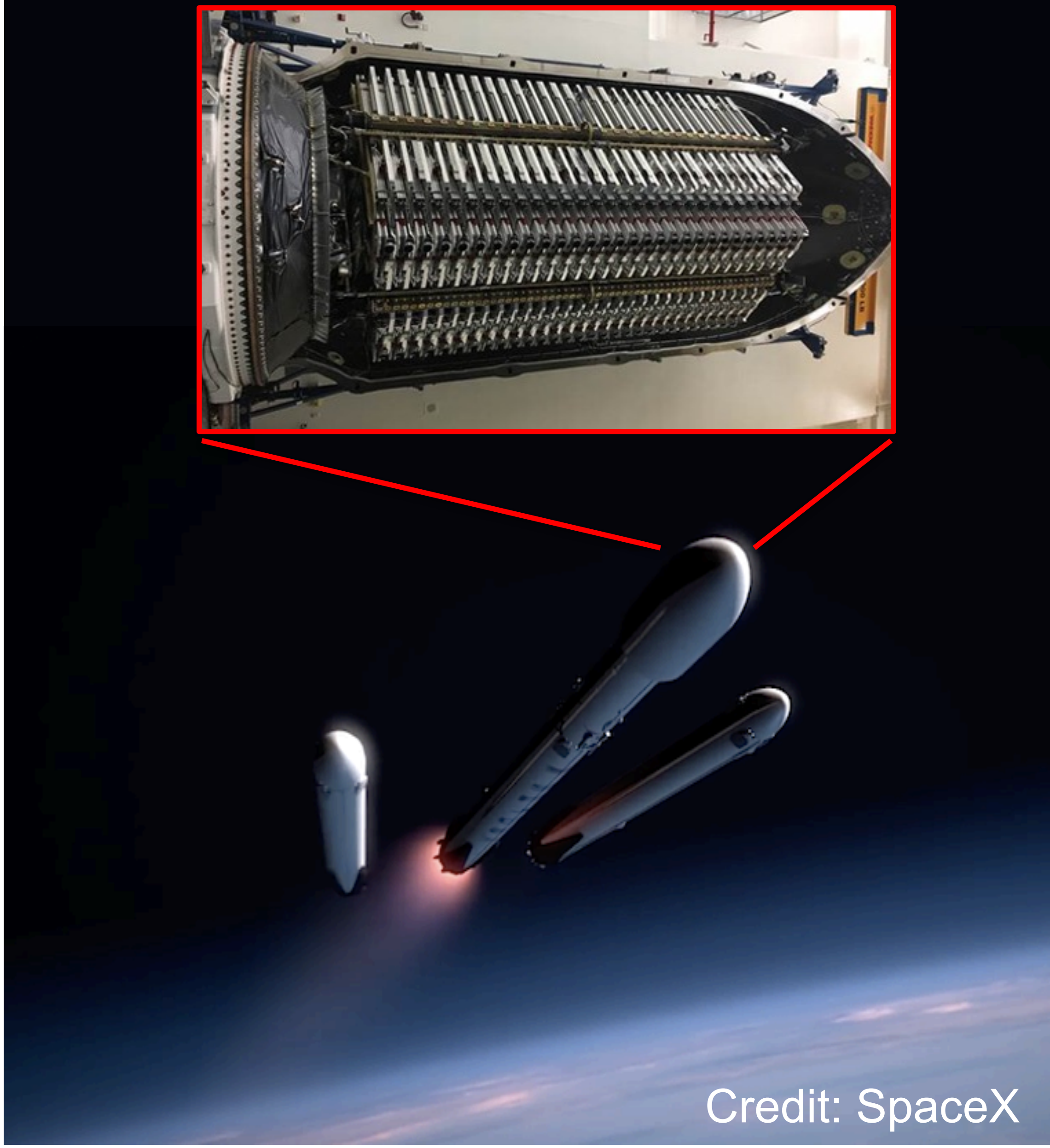
Launch cost in
$1000/kg

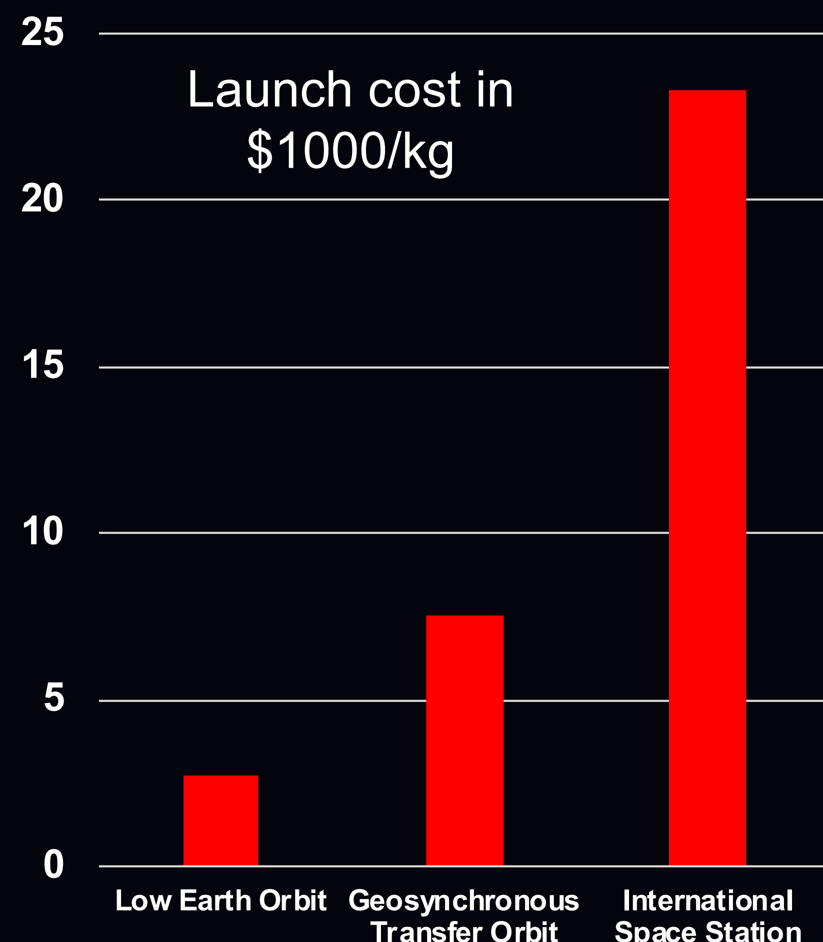
Why electric (plasma) propulsion?

Mass is at a premium for space travel

Launch vehicles have physical limits on volume and mass

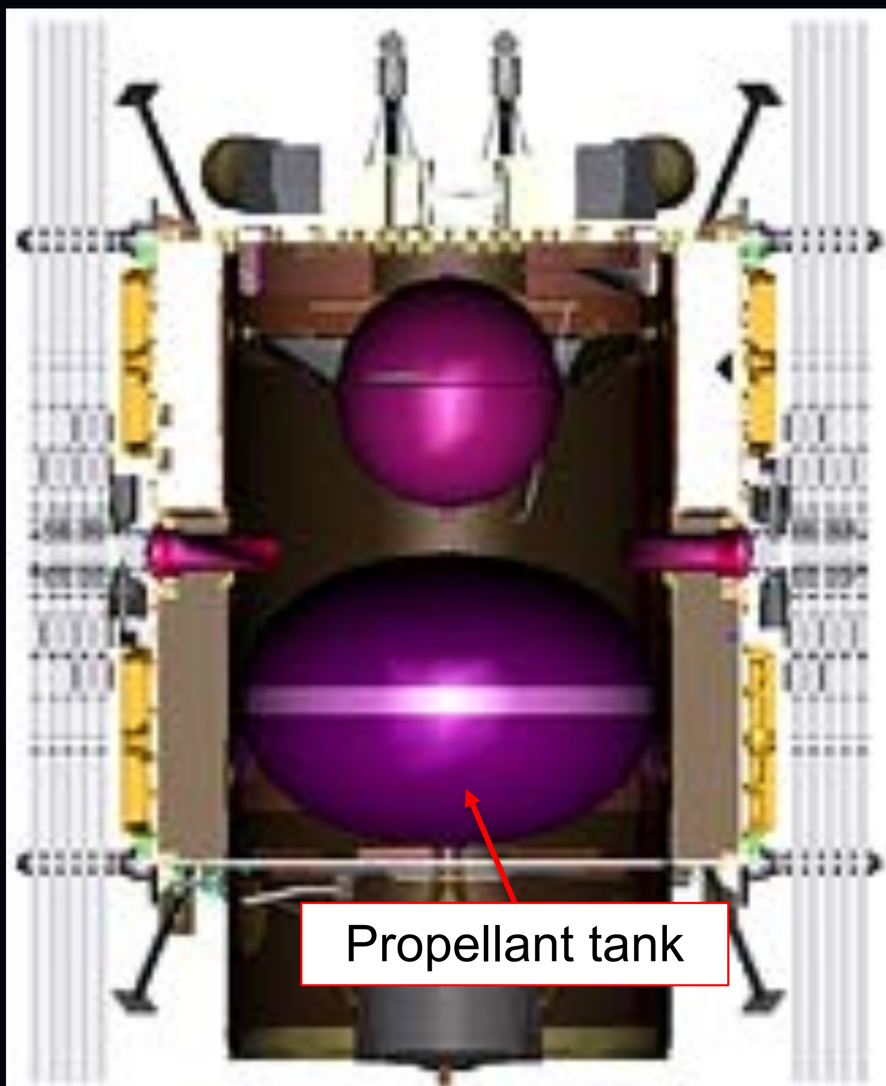


Cost scales with spacecraft mass



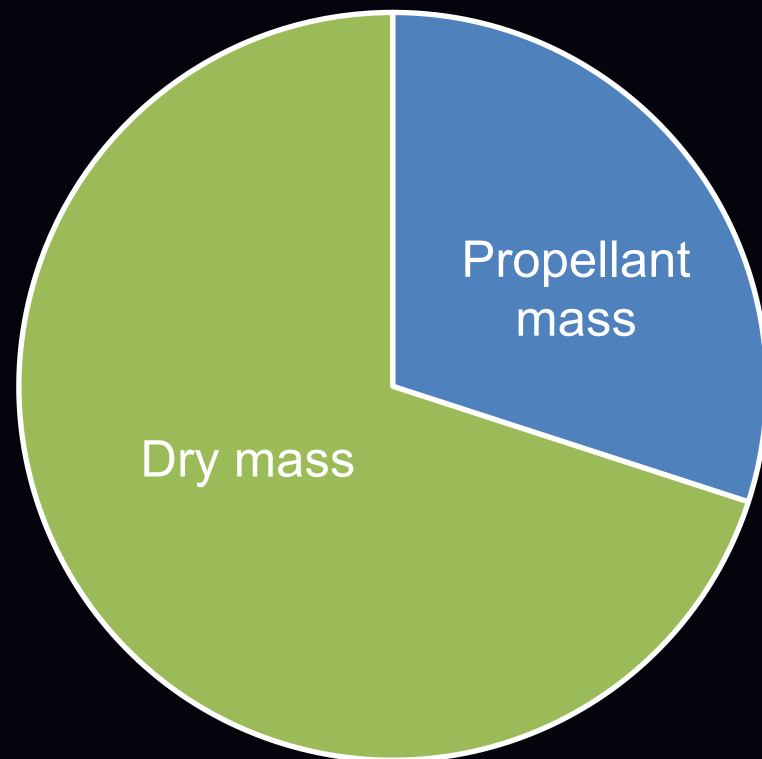
Propellant mass is a dominant fraction of overall spacecraft mass

Spacecraft cross-section



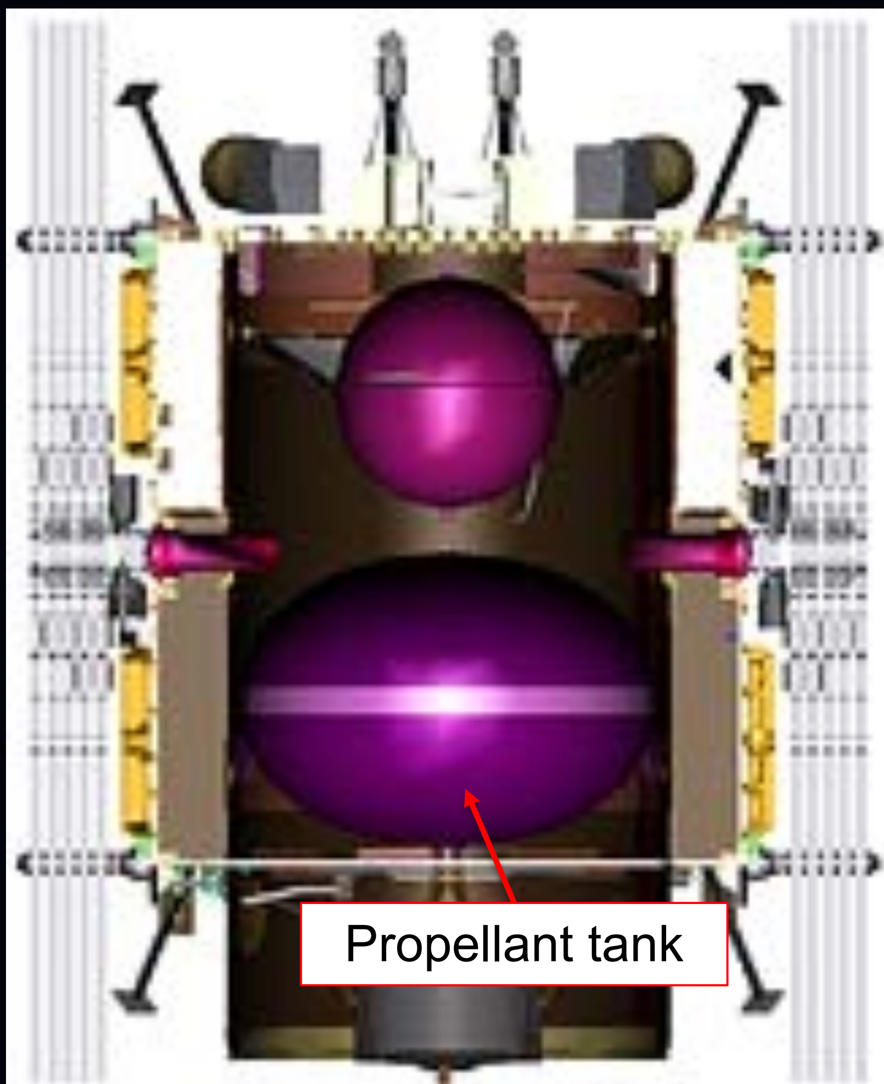
Credit: JPL

Mass budget



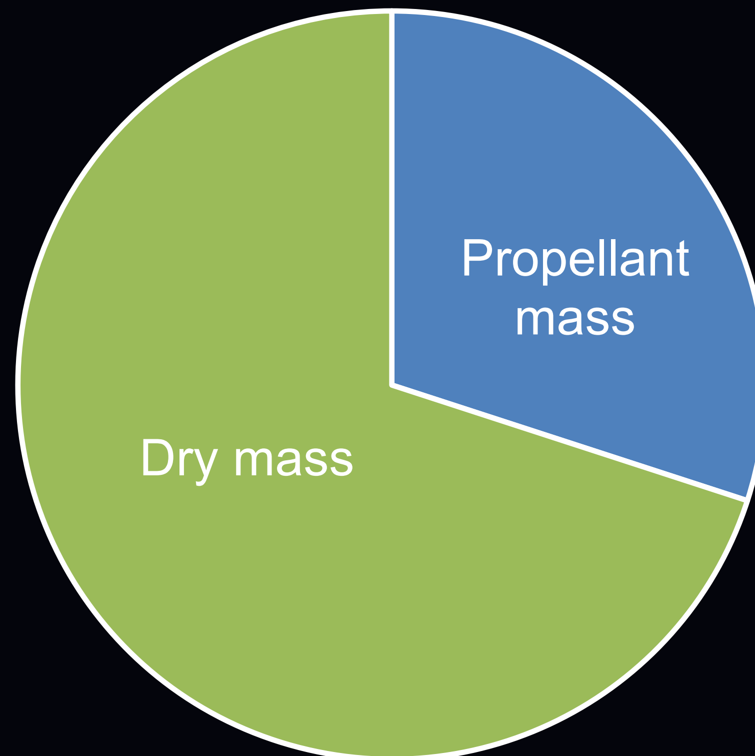
Propellant mass is a dominant fraction of overall spacecraft mass

Spacecraft cross-section



Credit: JPL

Mass budget



There is a driving need to maximize propellant efficiency of spacecraft

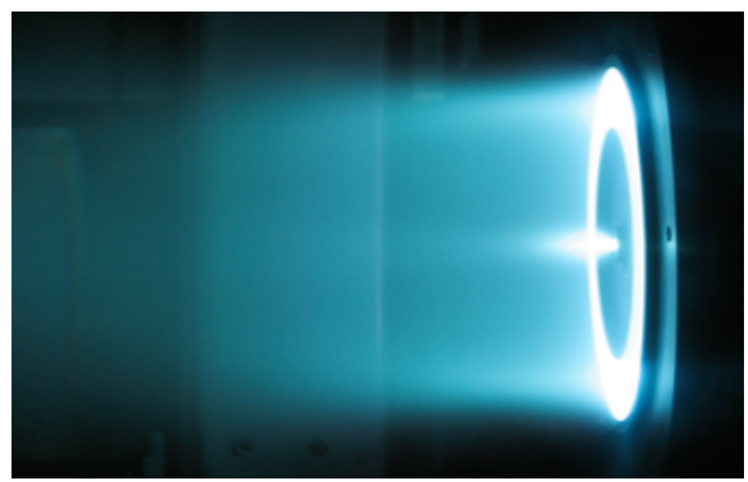
Analogy from the ground

Gas guzzling car

Electric car with great fuel economy

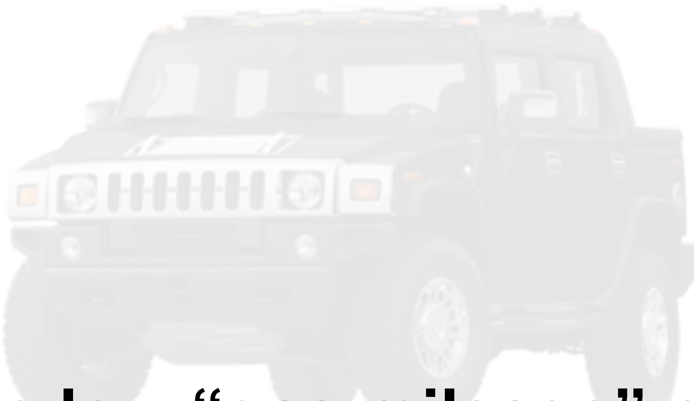
Fuel guzzling chemical rockets

Electric rockets with great propellant efficiency



Analogy from the ground

Gas guzzling car



Electric car with great fuel economy

The low "gas mileage" of "electric propulsion" makes it an enabling technology for cheaper space exploration

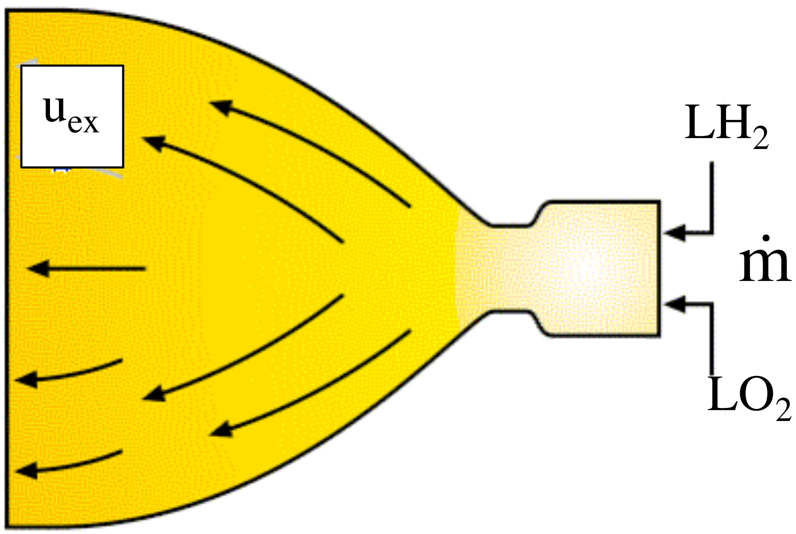
Fuel guzzling chemical rockets

Electric rockets with great propellant efficiency

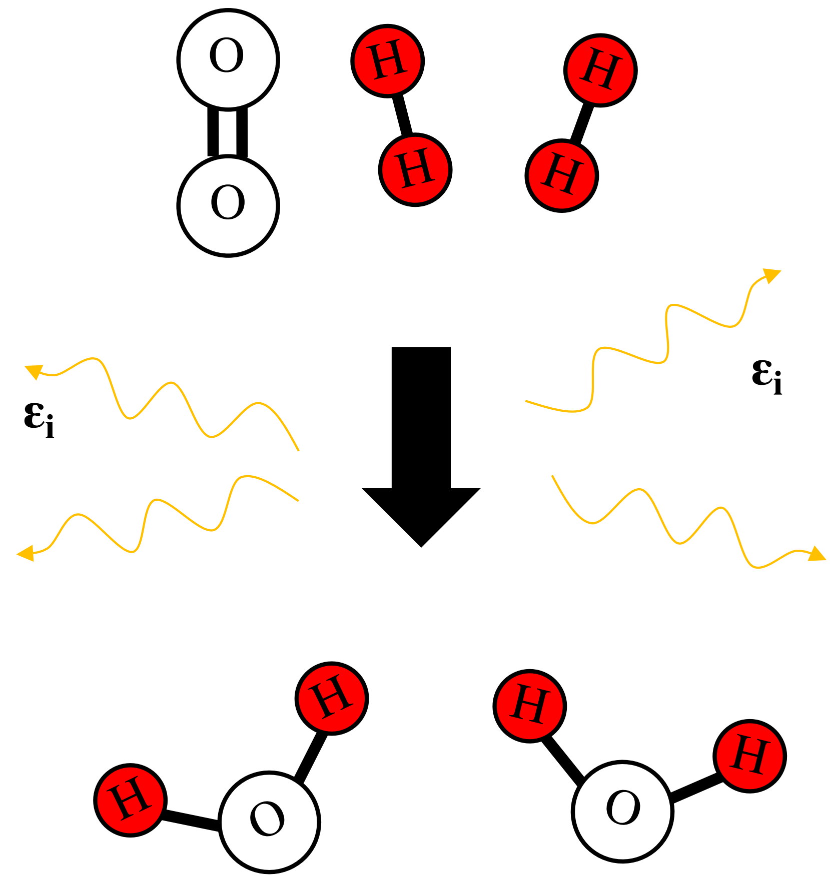
How does electric (plasma) propulsion achieve higher propellant efficiency?

Electric propulsion vs. chemical propulsion

Chemical rocket



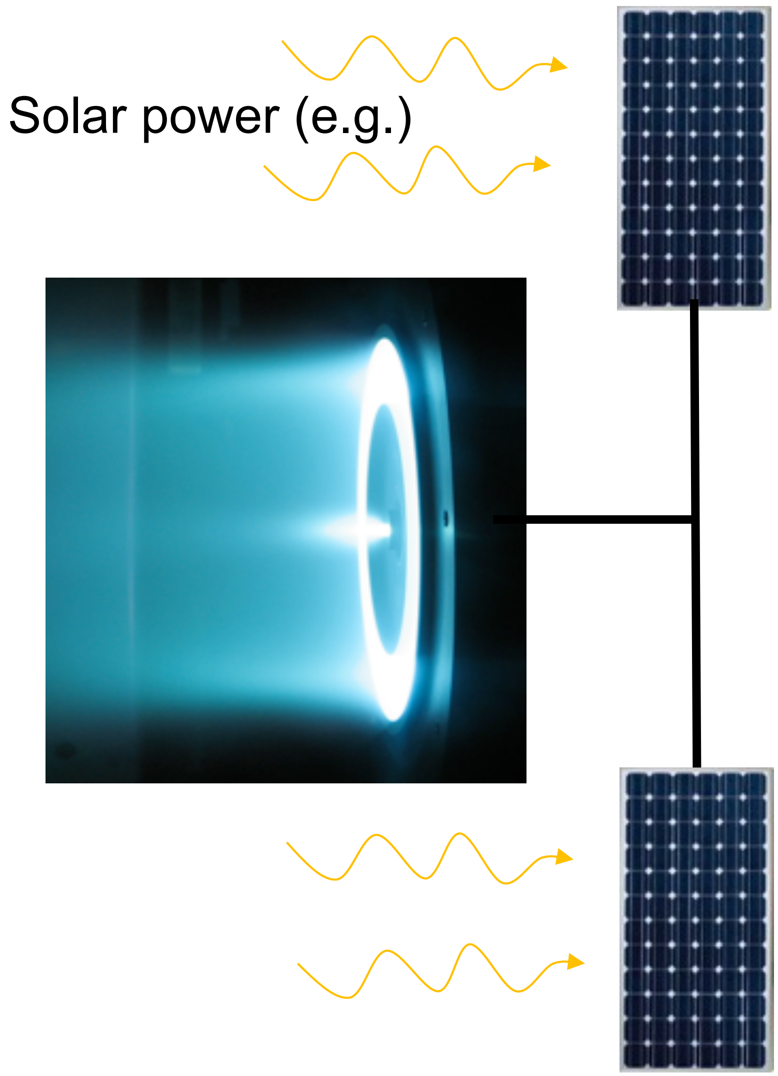
Release of internal energy in propellant



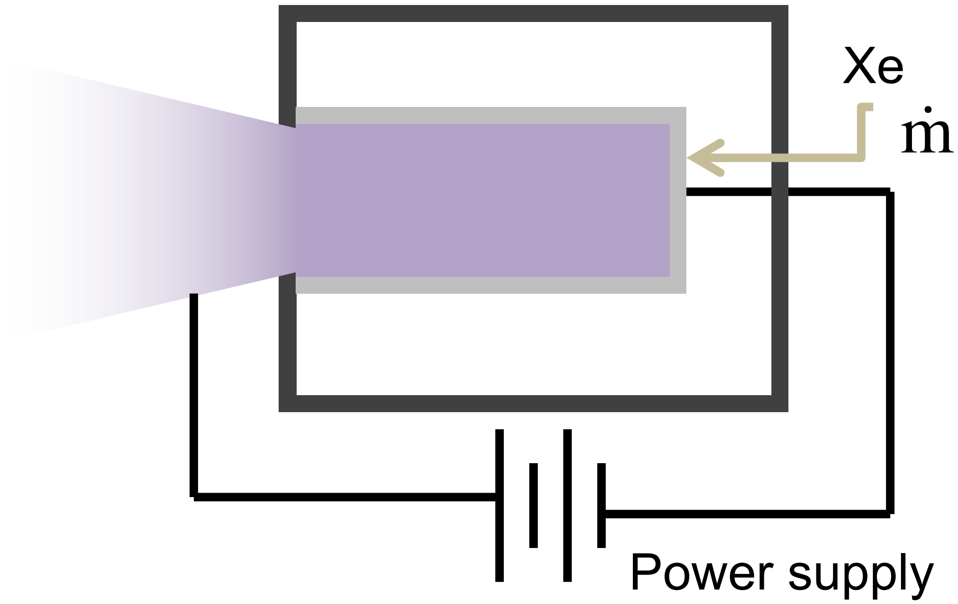
Specific internal energy $\boldsymbol{\varepsilon}_i$ in molecular bonds converted to exhaust energy

$$I_{sp} = \frac{1}{g}\sqrt{2\varepsilon_i}$$

Electric propulsion vs. chemical propulsion



Electric propulsion

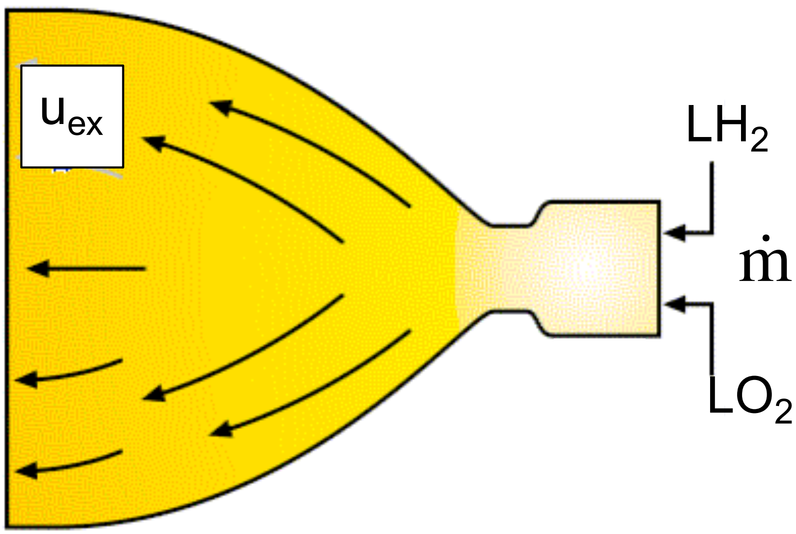


External supply with power **P** accelerates propellant

$$I_{sp} = \frac{1}{g}\sqrt{\frac{2P}{\dot{m}}}$$

Electric propulsion vs. chemical propulsion

Chemical rocket



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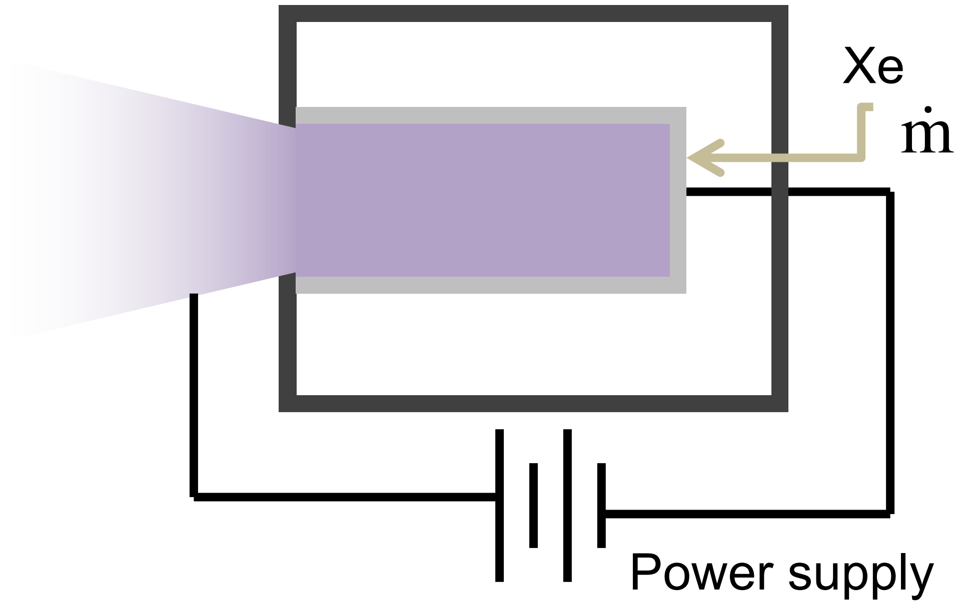
Electric propulsion



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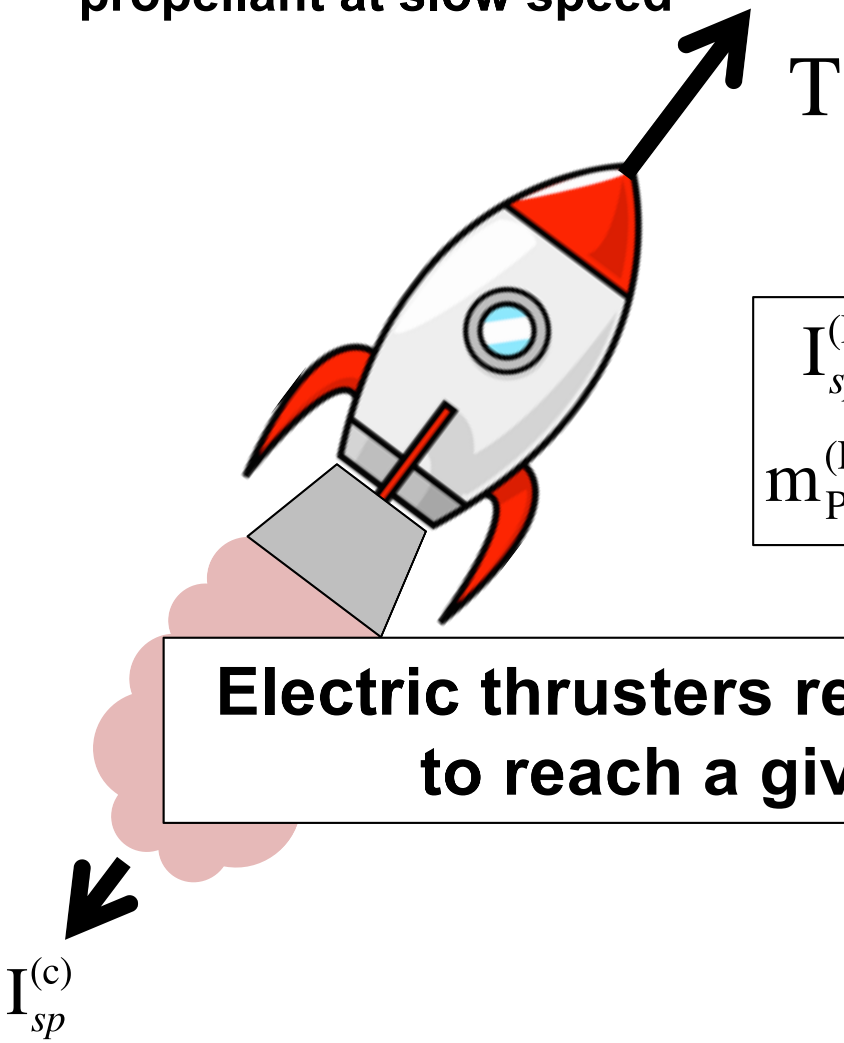
$$I_{sp} = \frac{1}{g}\sqrt{\frac{2P}{\dot{m}}}$$

Electric thrusters accelerate propellant to high velocity

Electric propulsion vs. chemical propulsion

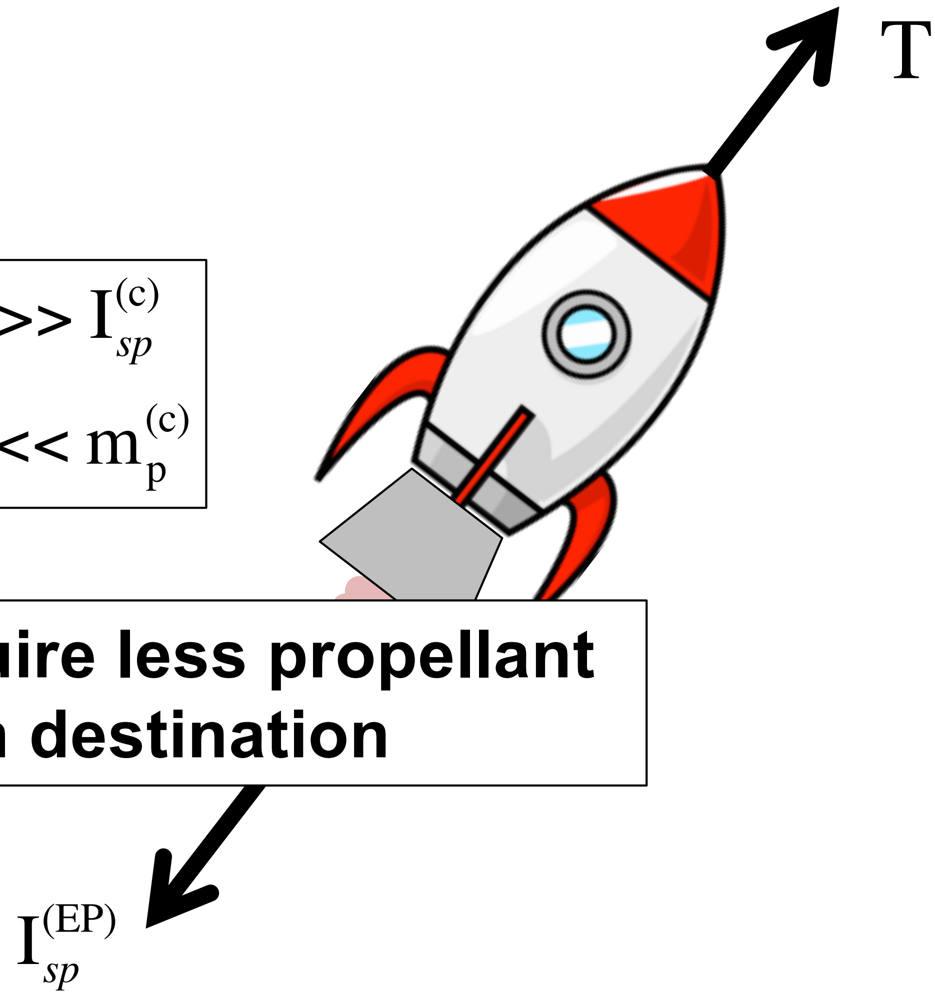
Chemical rocket

Expels large quantity of propellant at slow speed



Electric propulsion

Expels small quantity of propellant at very fast speed



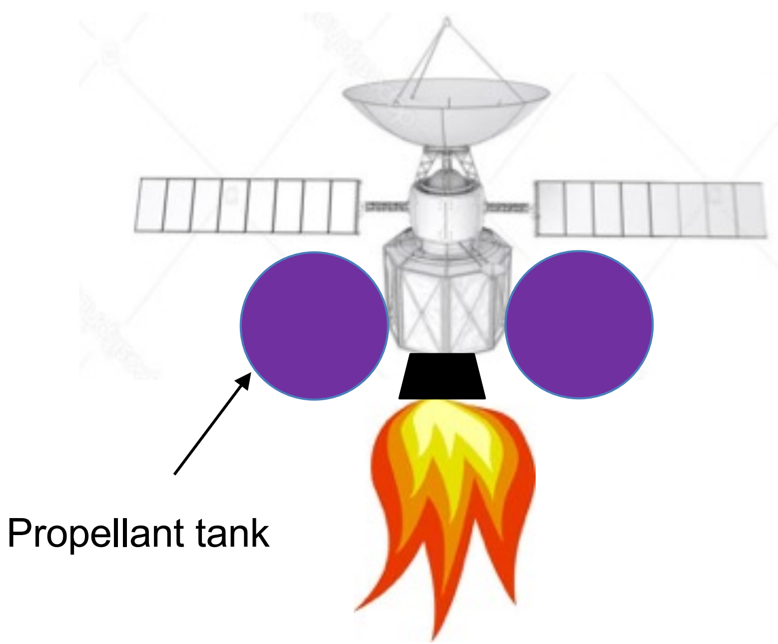
$$I_{sp}^{(EP)} >> I_{sp}^{(c)}$$

$$m_P^{(EP)} << m_p^{(c)}$$

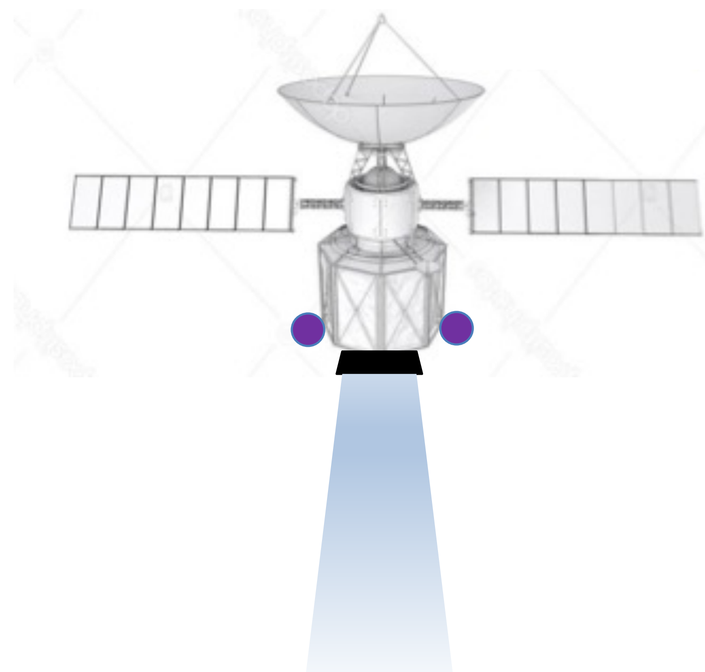
Electric thrusters require less propellant to reach a given destination

Electric propulsion systems have better “gas mileage” in space

Traditional chemical system



Electric propulsion system



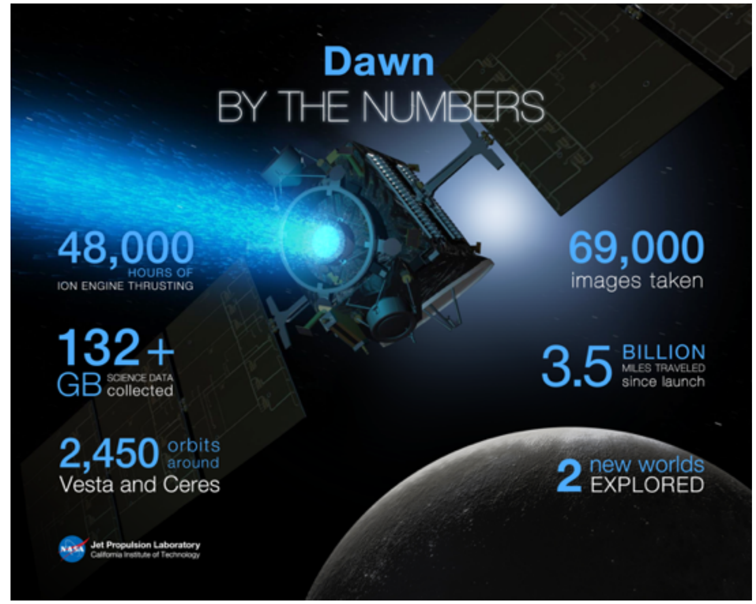
Spacecraft mass savings with EP stems from savings in propellant tank requirements

Mass savings from EP enables new and cheaper missions

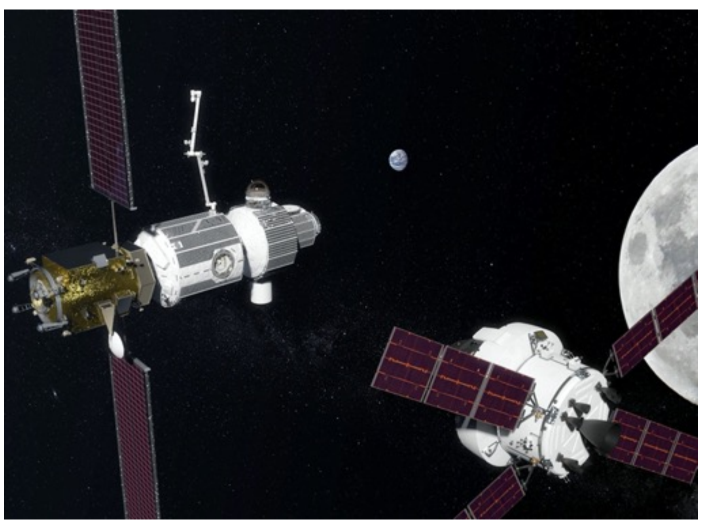
Stacked payloads to GEO

Credit: Boeing Company

Deep space exploration



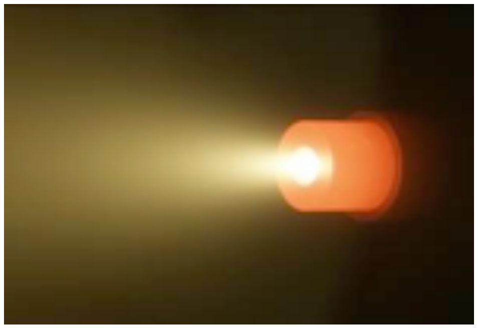
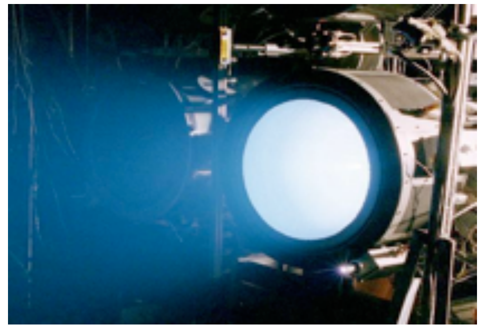
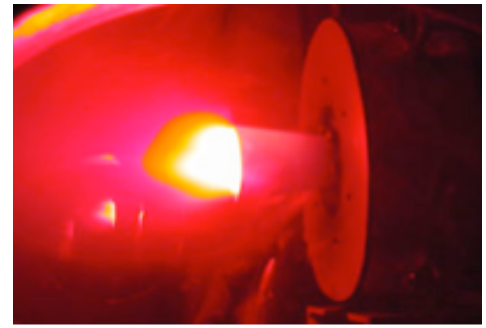
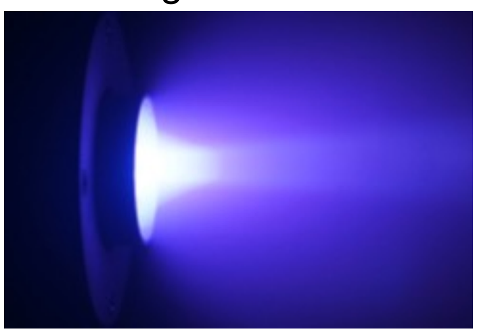
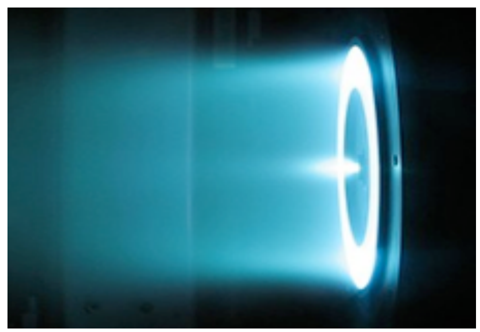
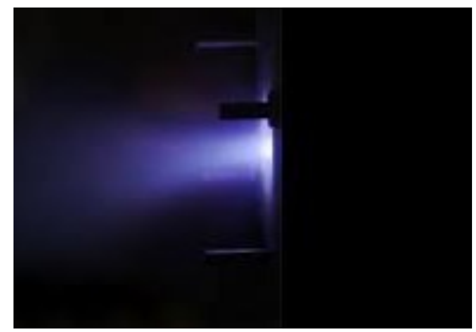
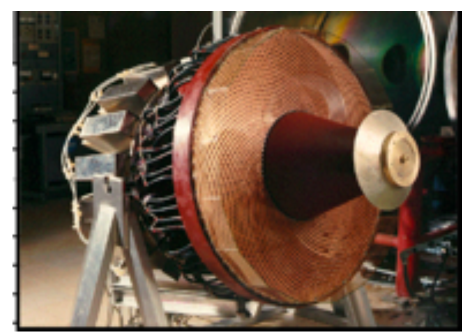
Crewed Exploration (Deep Space Gateway)



Megaconstellations (SpaceX)

What methods are employed to convert external power into thrust for electric propulsion systems?

Pantheon of Electric Propulsion Systems

Thermal expansion of plasma heated gas	Electrostatic acceleration of heavier species in a LTP	Electromagnetic acceleration through Lorentz force
Arcjets/resistojets	Gridded ion thrusters	Magnetoplasmadynamic thrusters
		
Magnetic nozzles	Hall effect thrusters	Pulsed ablative thrusters
		
	Electrospray thrusters	Pulsed inductive thrusters
		

Pantheon of Electric Propulsion Systems

Thermal expansion of plasma heated gas	Electrostatic acceleration of heavier species in a LTP	Electromagnetic acceleration through Lorentz force

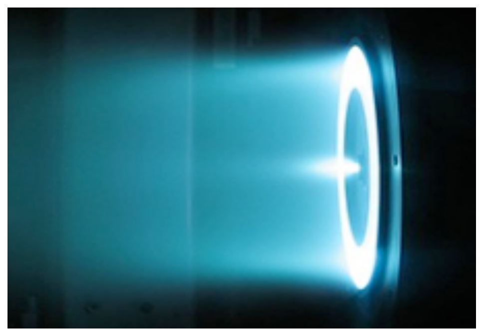
Arcjets/resistojets

Gridded ion thrusters

Magnetoplasmadynamic thrusters

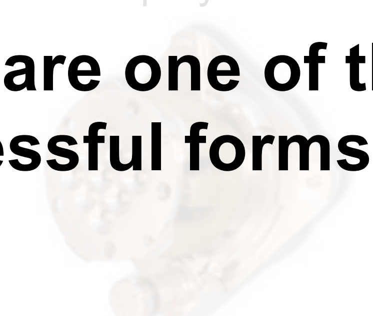
Magnetic nozzles

Hall effect thrusters



Pulsed ablative thrusters

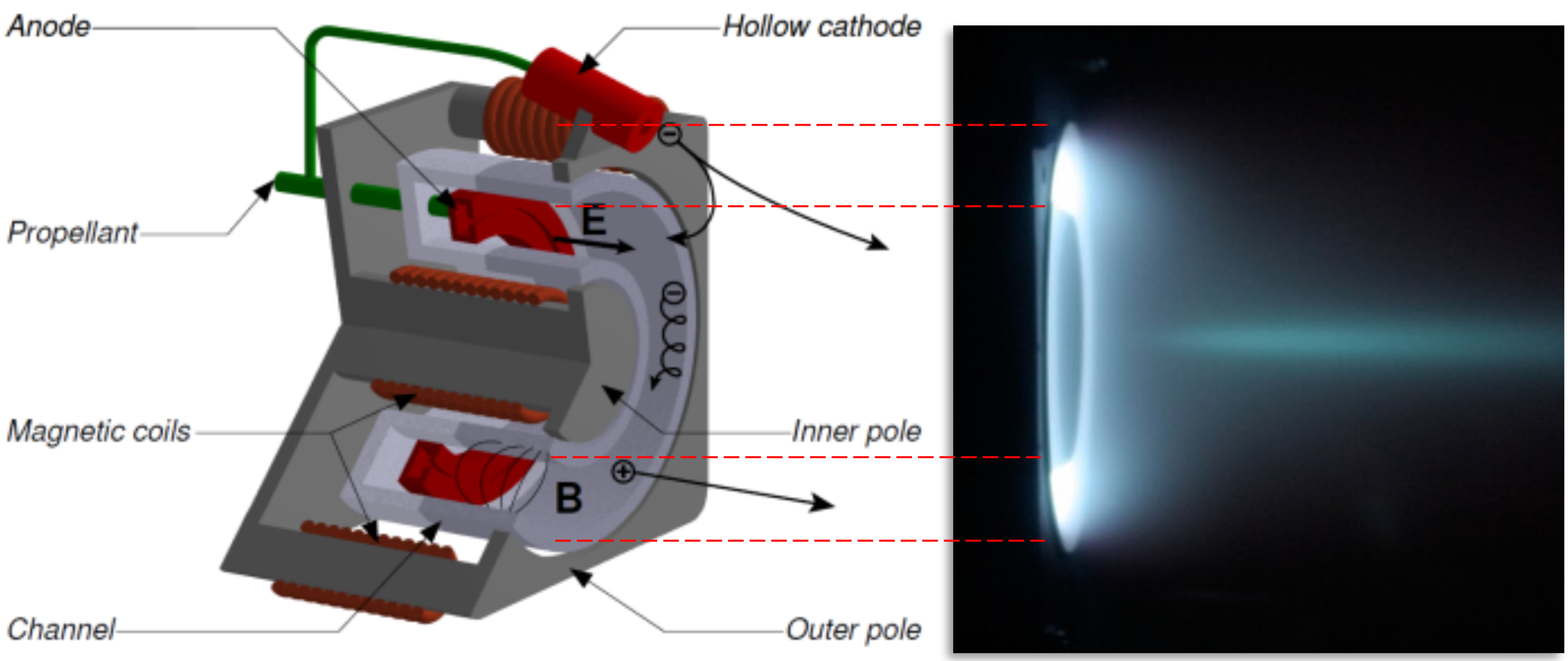
Electrospray thrusters



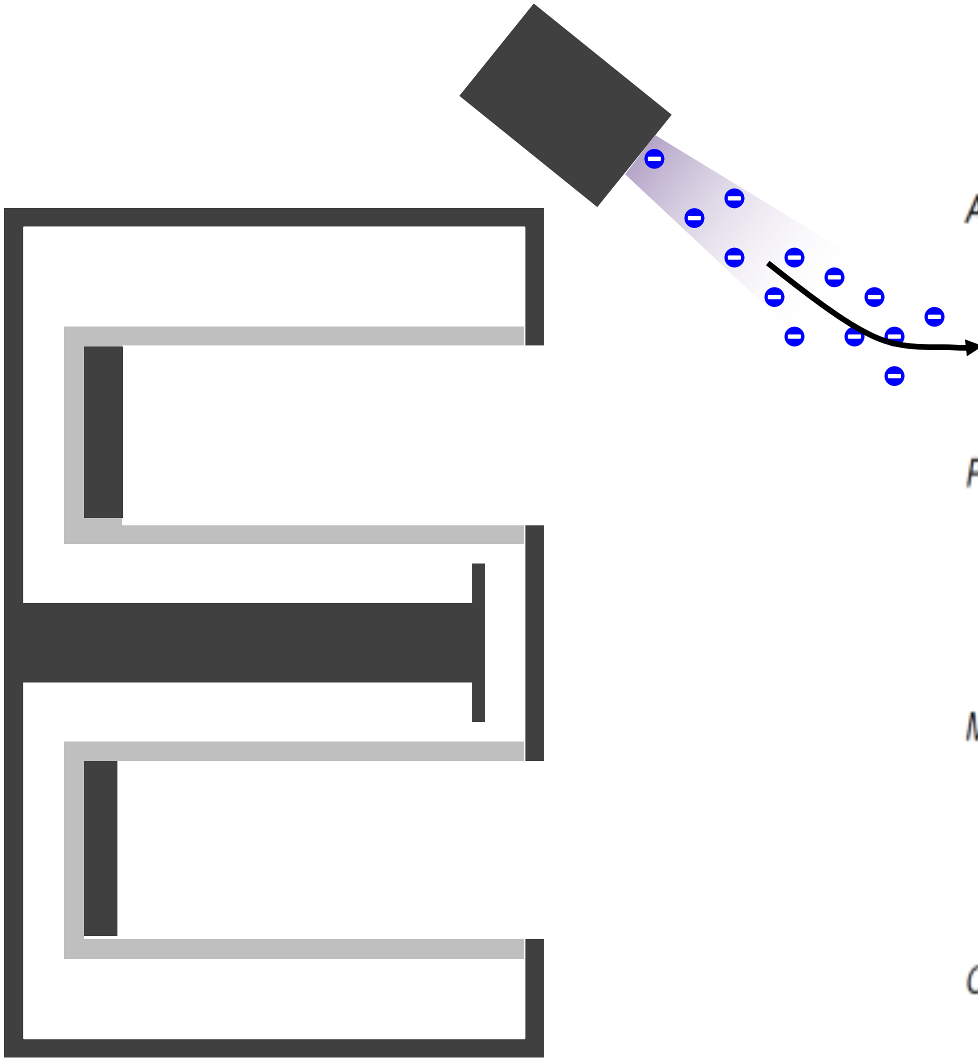
Pulsed inductive thrusters

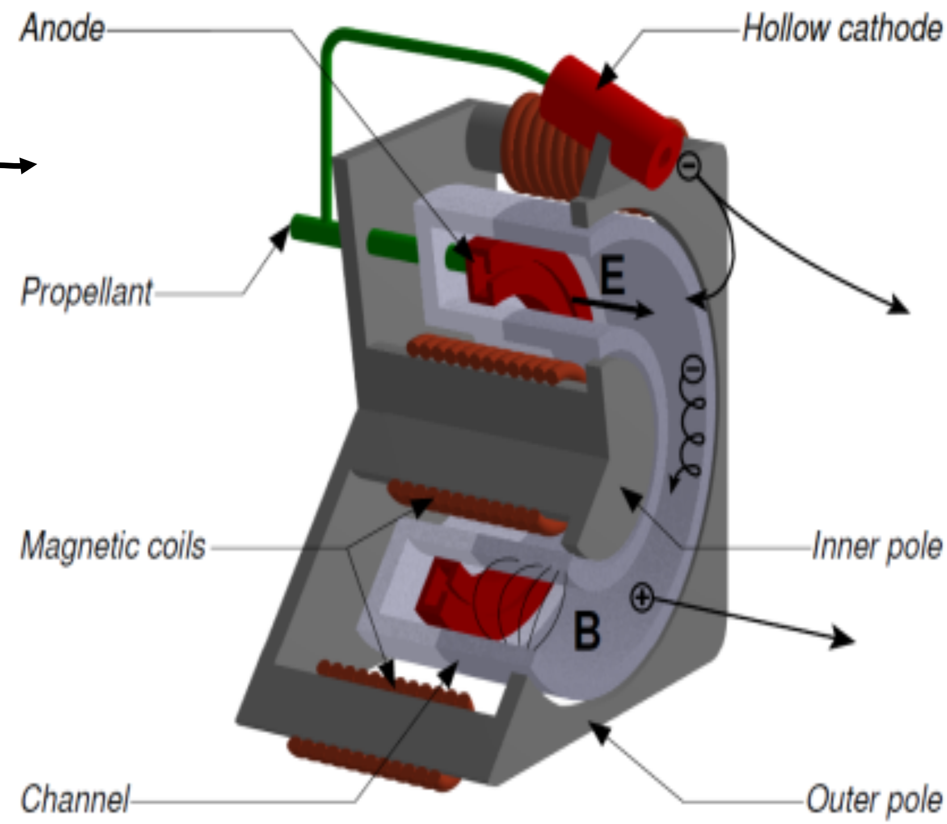
Hall effect thrusters are one of the most popular and successful forms of EP

Hall thruster principle of operation



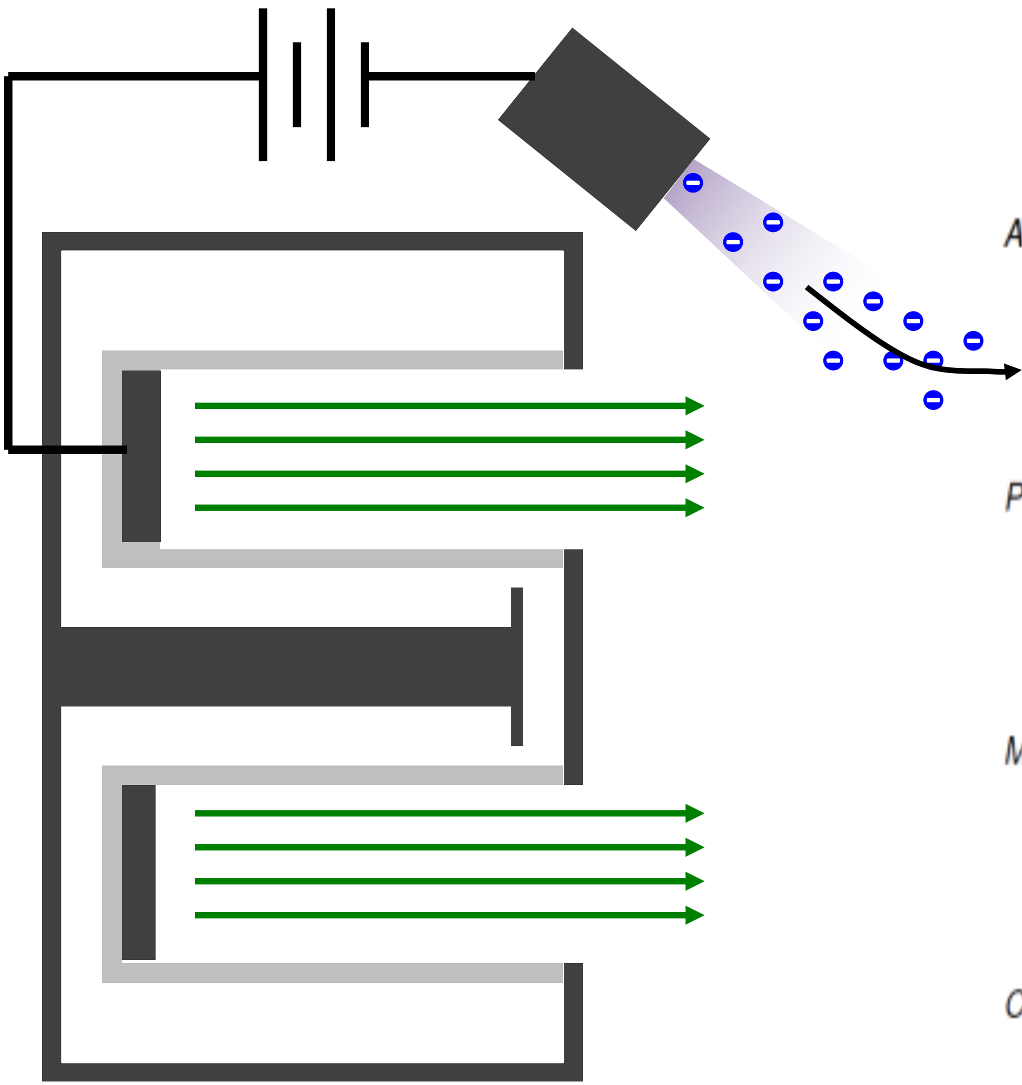
Hall thruster principle of operation

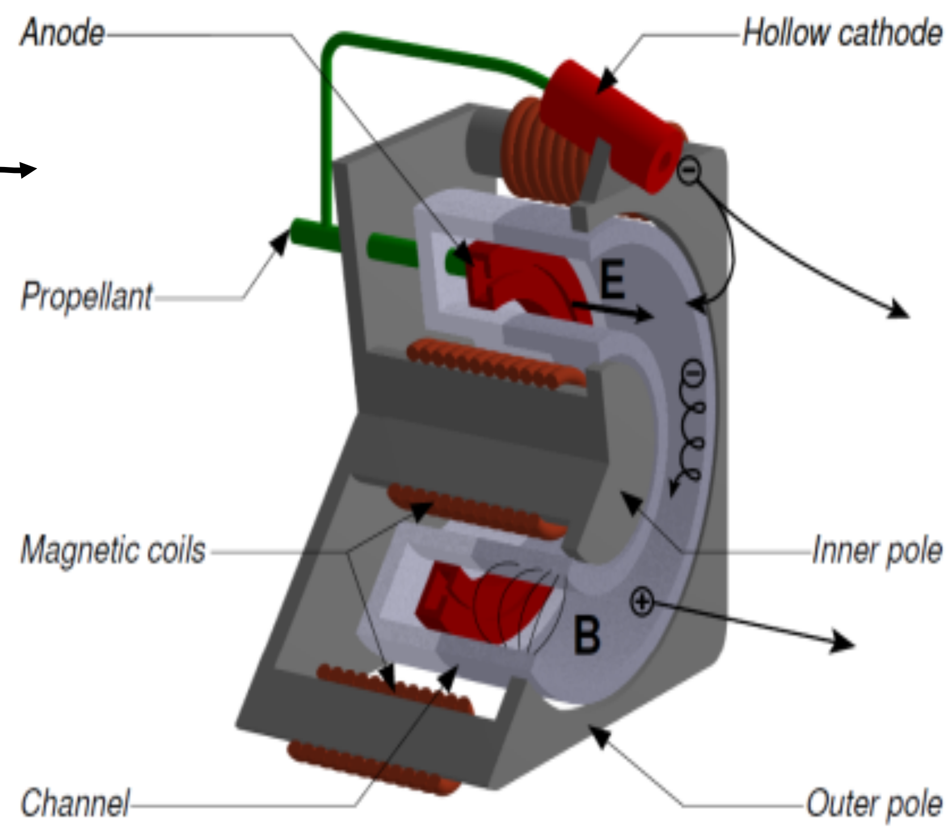




Hollow cathode heated until thermionically emitting electrons

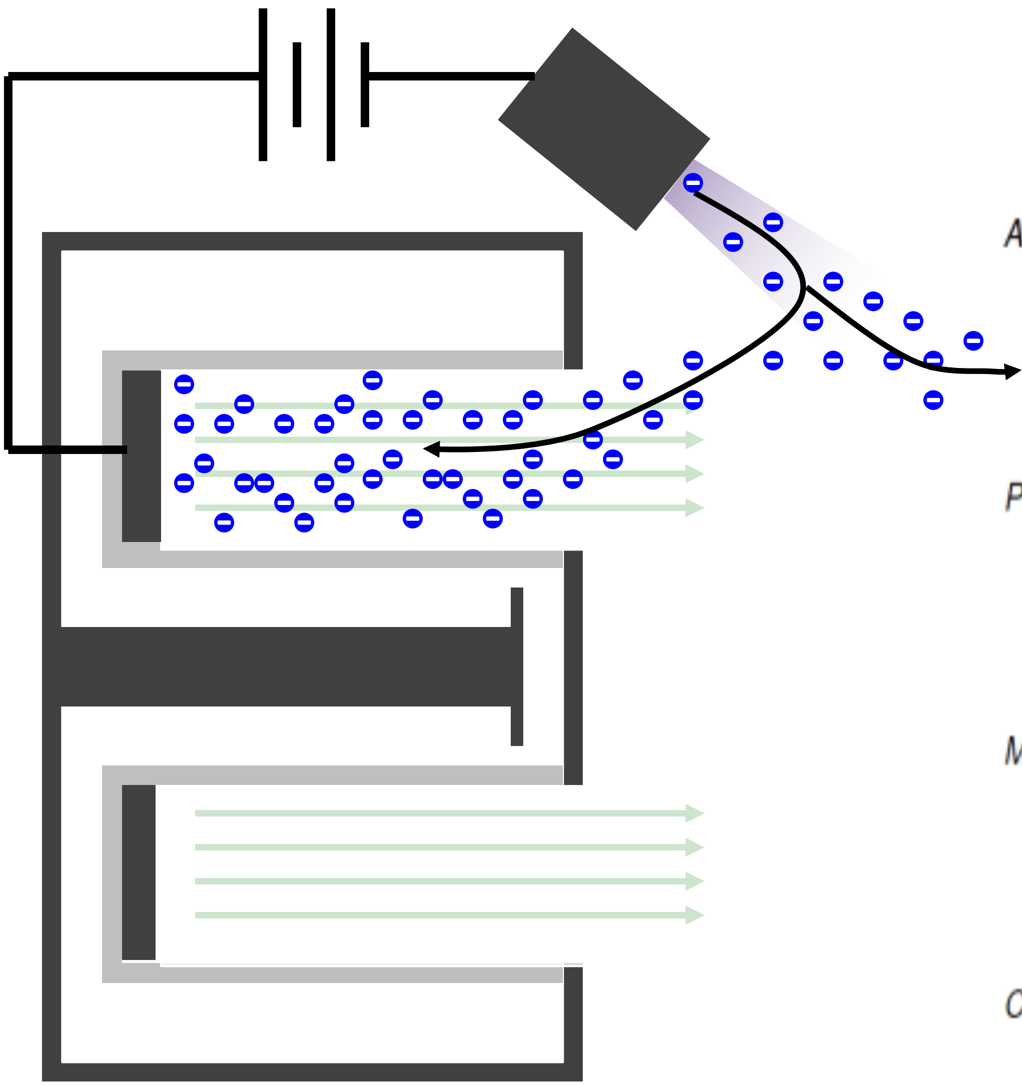
Hall thruster principle of operation

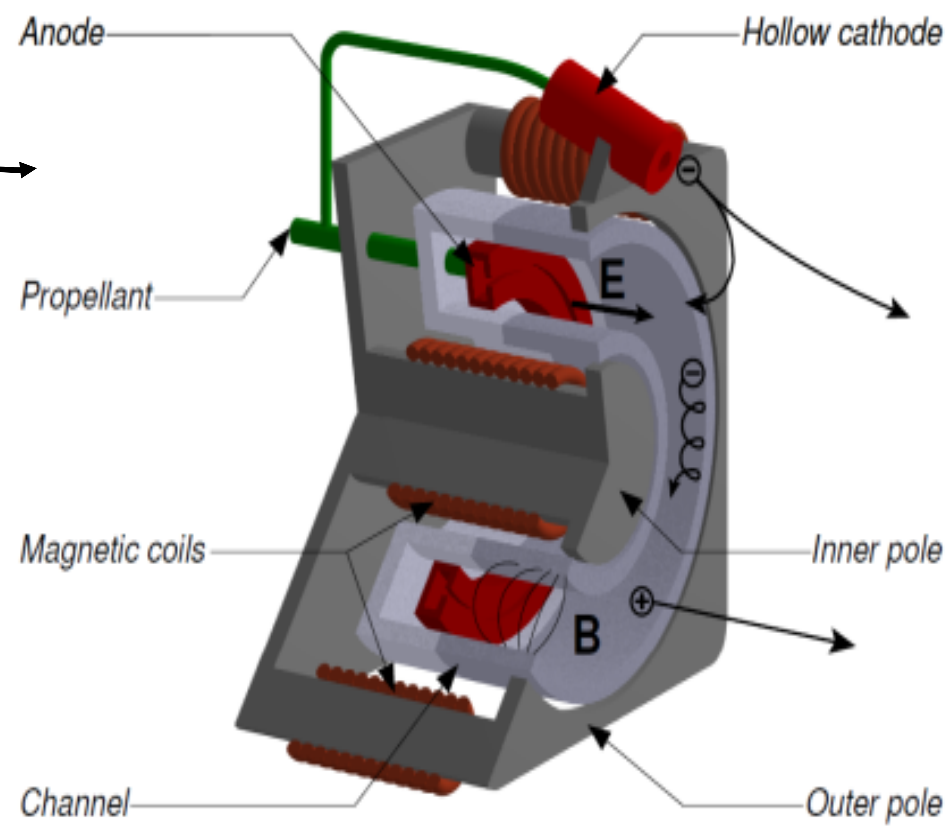




Electric field applied between anode and cathode

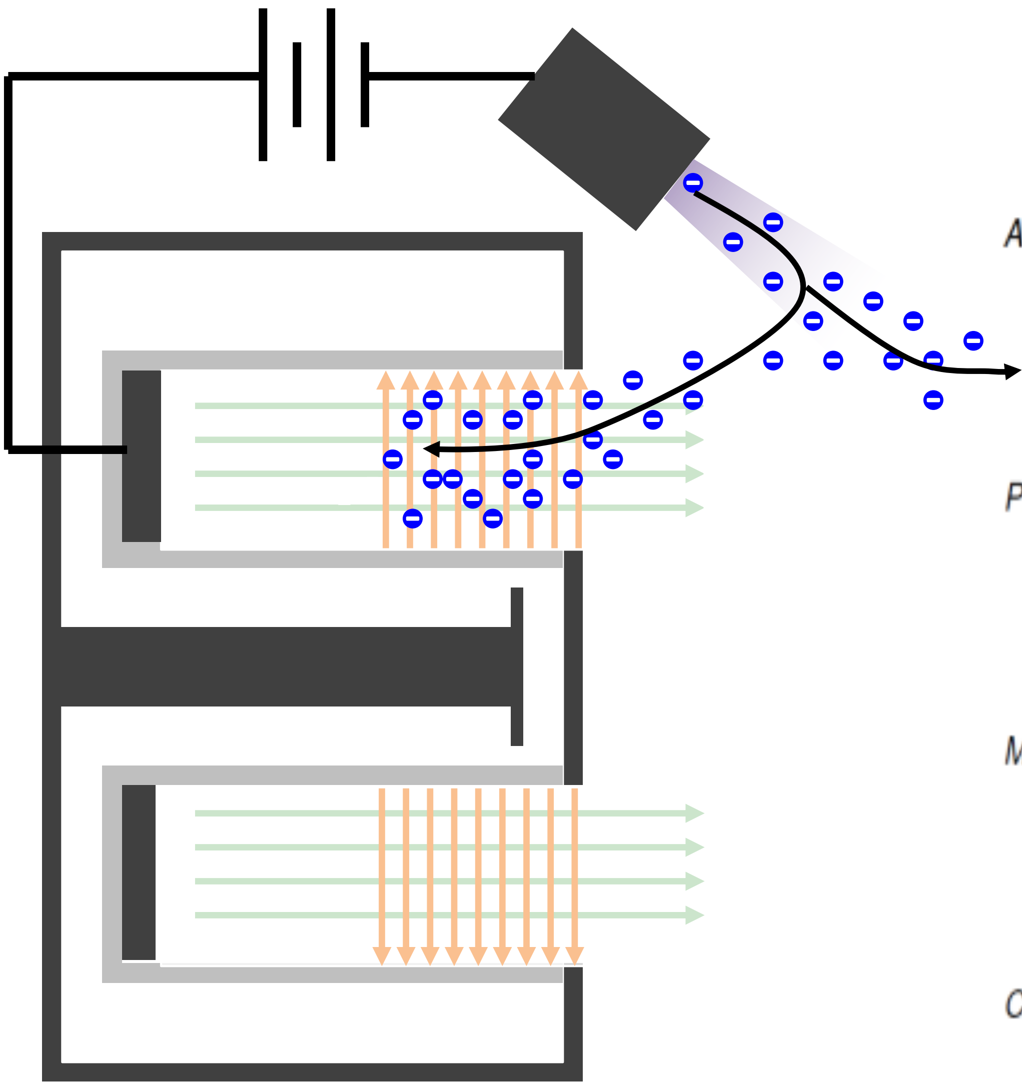
Hall thruster principle of operation

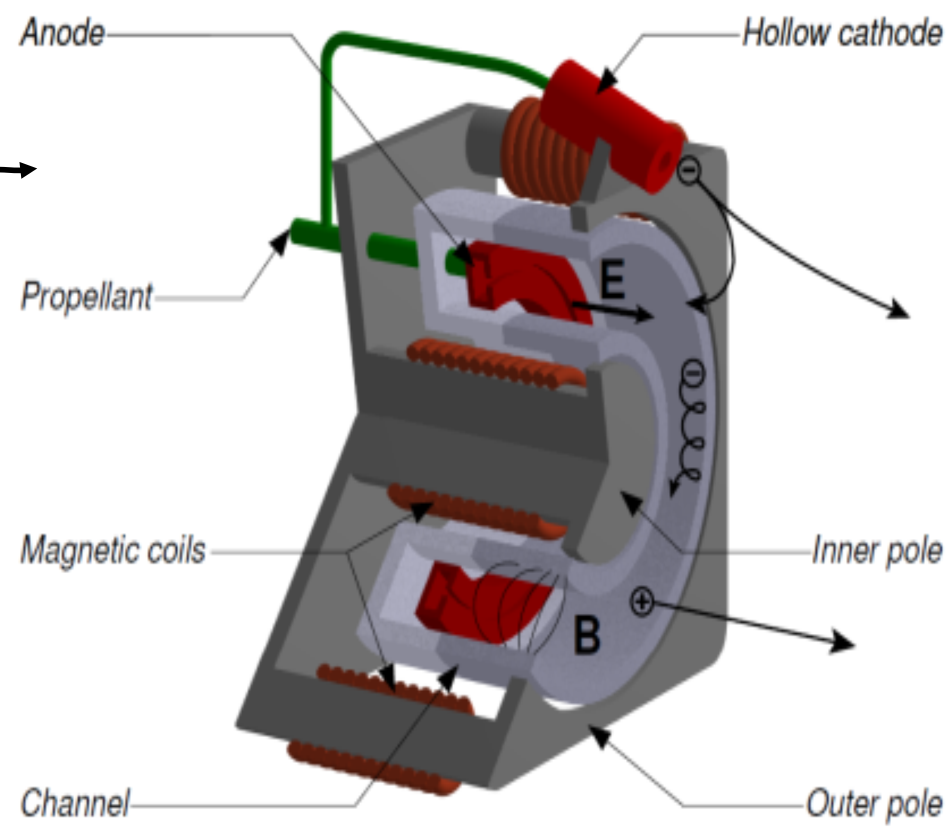




Electrons follow electric field and current flows between cathode and anode

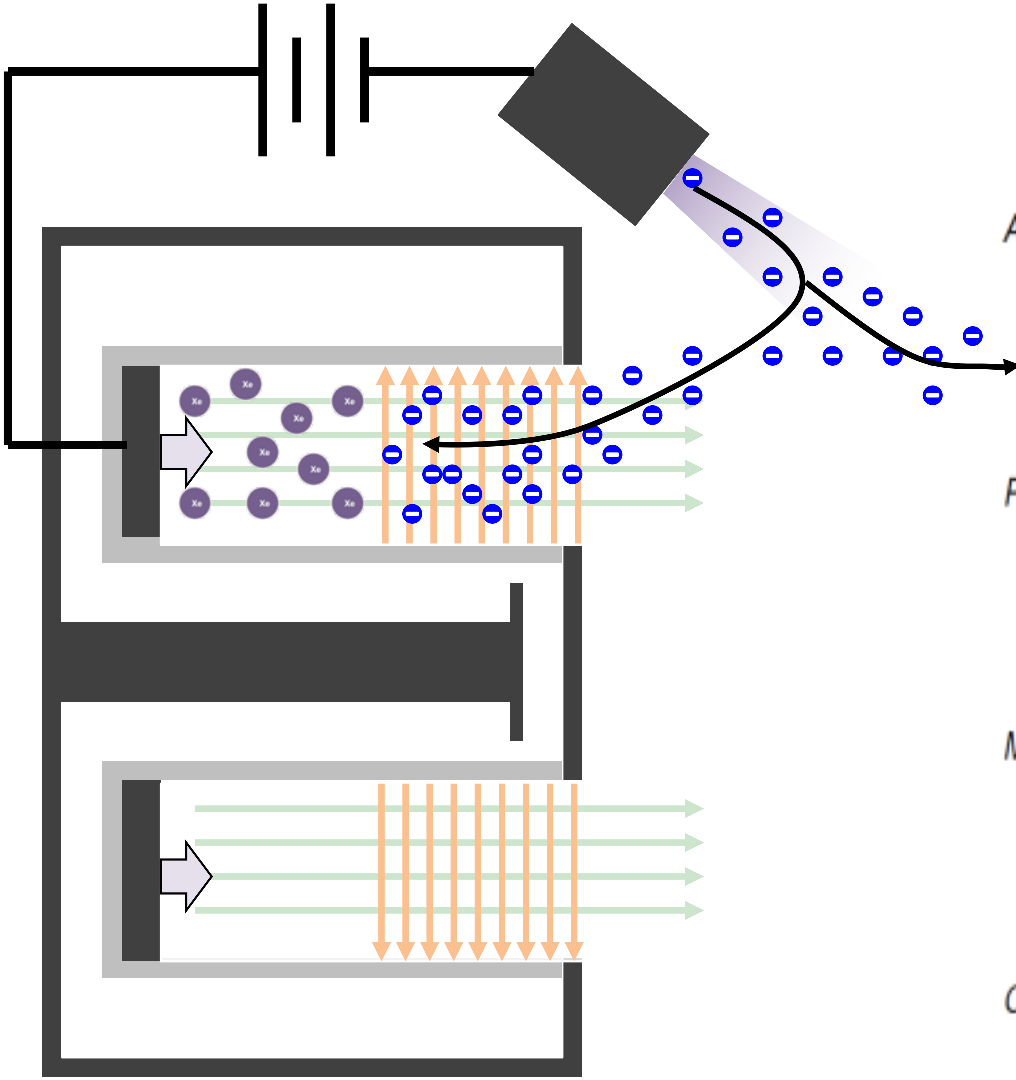
Hall thruster principle of operation

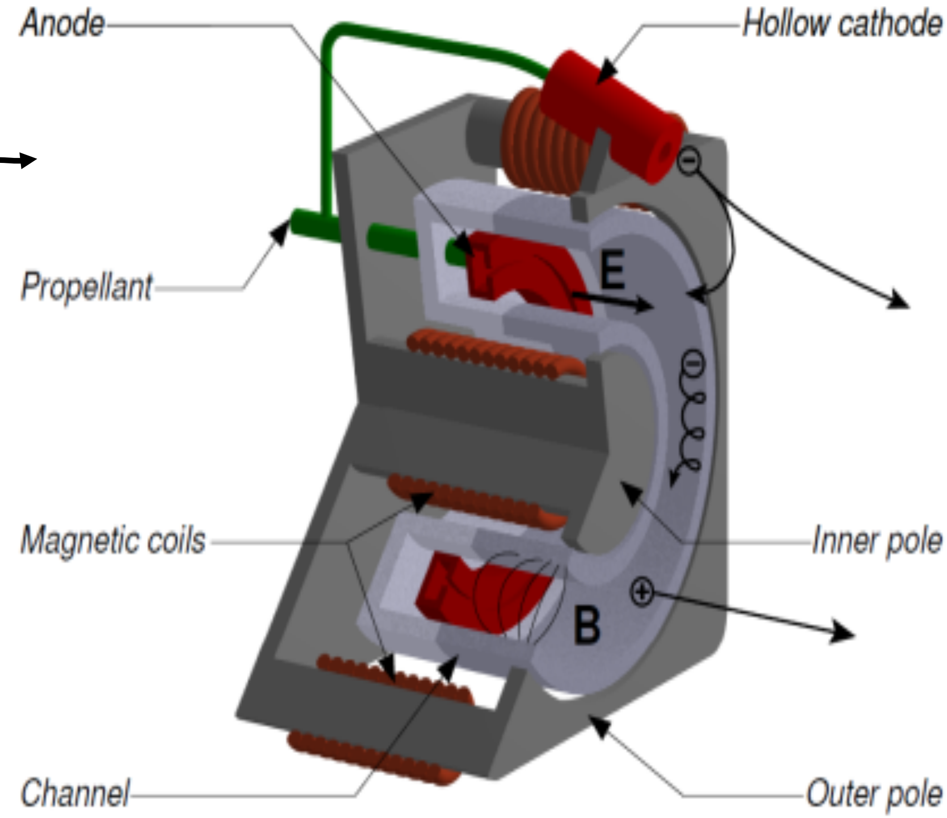




Magnetic field is applied in the radial direction. Electrons are trapped in **E × B** azimuthal drift: *Hall effect*

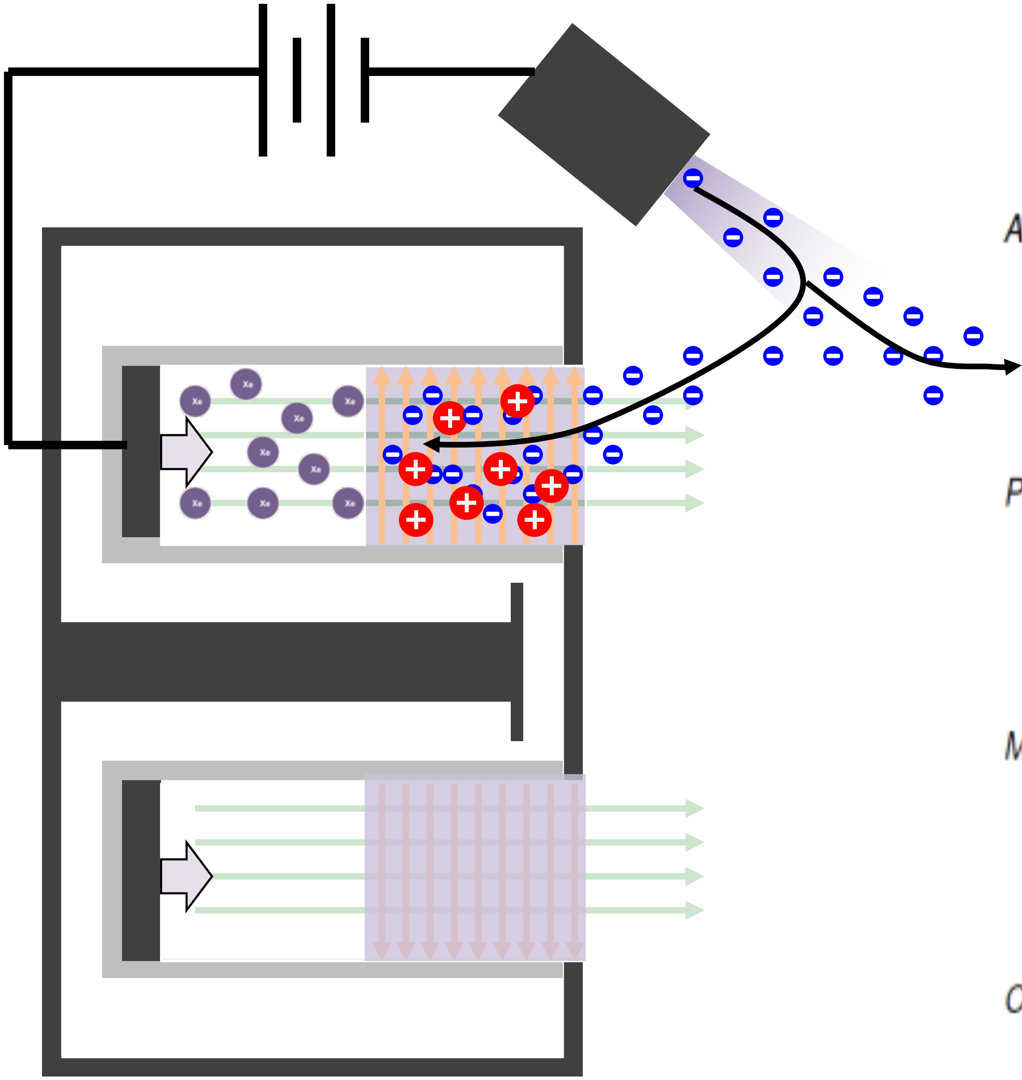
Hall thruster principle of operation

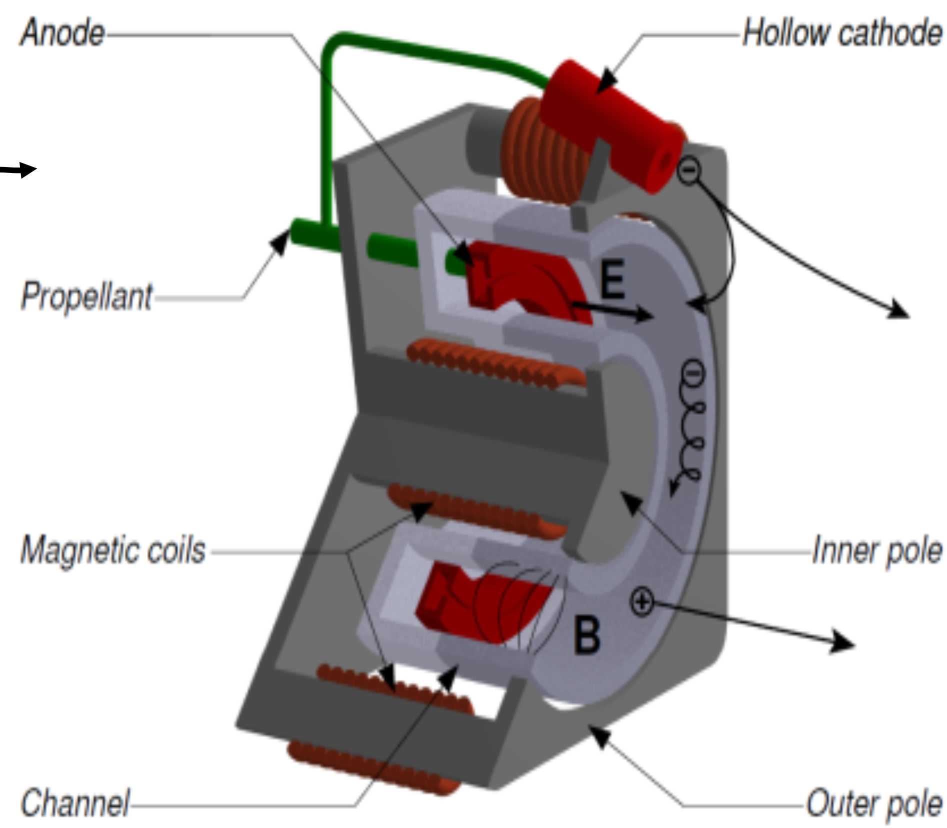




Neutral gas (xenon) flows through thruster anode.

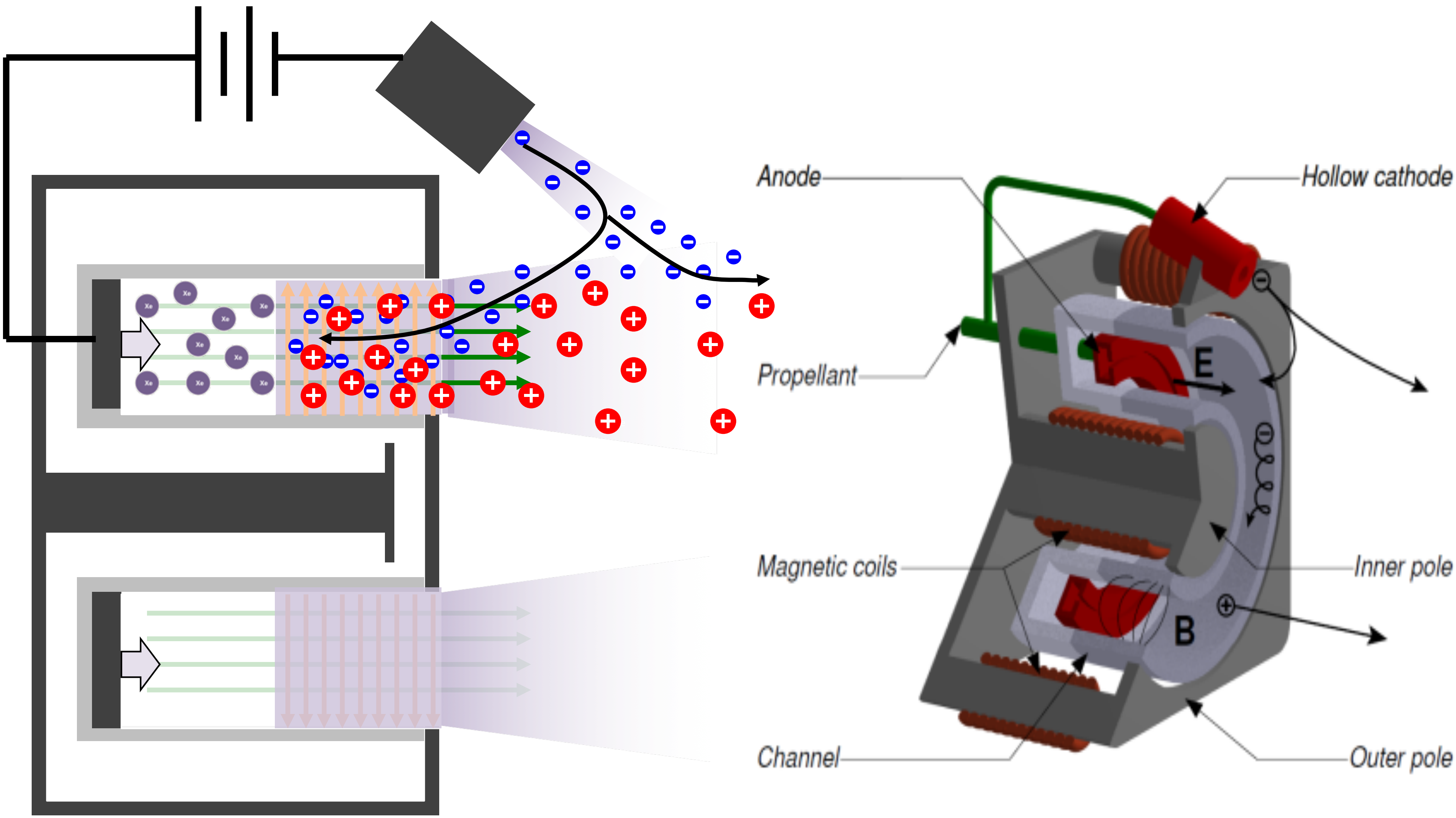
Hall thruster principle of operation





Electron azimuthal drift “buzz saw” ionizes neutral xenon. Plasma is created.

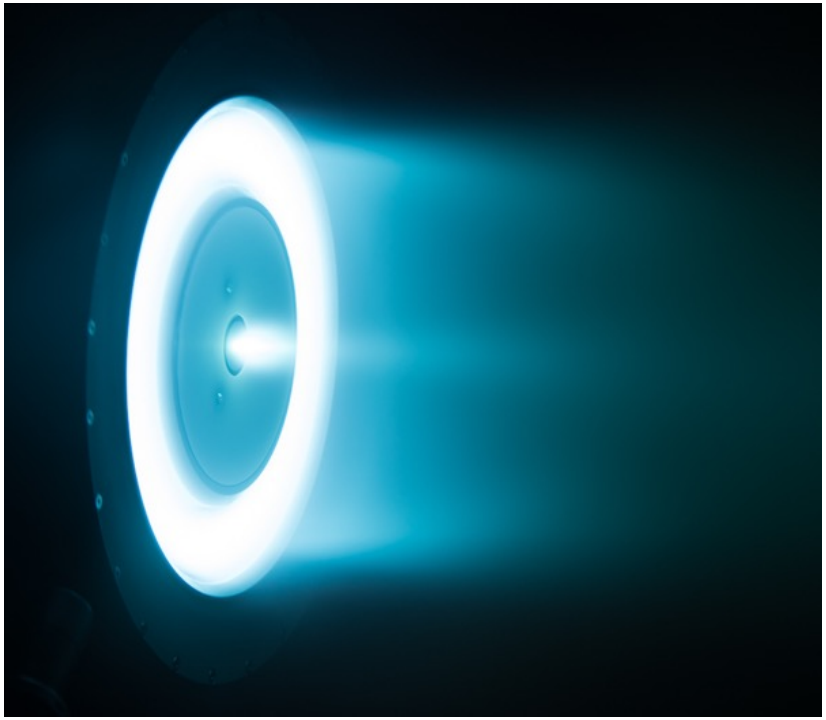
Hall thruster principle of operation



Ions are unmagnetized and therefore follow electric field lines directly.
Accelerated ions are neutralized by electrons in plume

Hall Effect Thruster

6-kW thruster at JPL



- **1970s**: > 240 flown since first launch by Soviet Union
- **2004:** First Western use of Hall thrusters (Space Systems Loral)
- **2011:** American manufactured Hall thrusters flown (Aerojet BPT-4000)
- **Present:** 12.5 kW thruster baselined for NASA return to moon

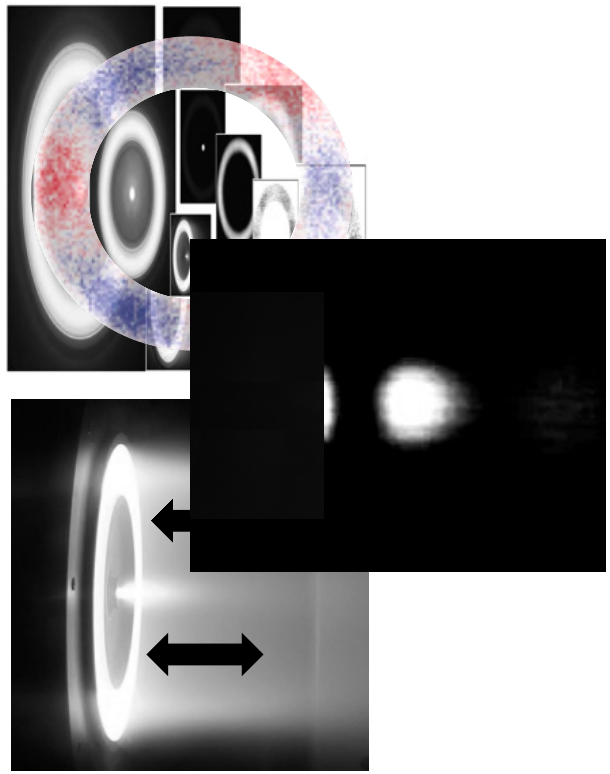
- Moderate thrust density (smaller mass for thrust)
- High specific impulse (1000-3000 s)
- High electrical efficiency (> 60%)
- **100s currently flown commercially and DoD spacecraft**

Challenges with Hall thrusters

Facility effects

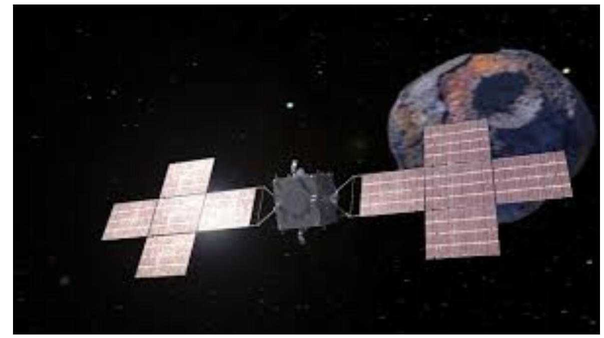
- Ground based test facilities are not representative of on orbit environment.
- This poses a challenge for qualifying new thruster designs for operational use

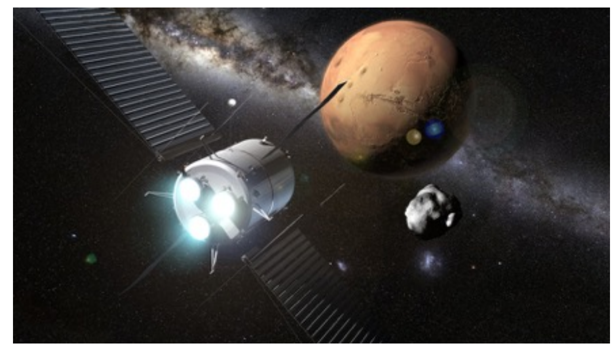
Stability



- Hall thrusters are dynamical systems that can transition into instability. Transition not well understood
- Transition in stability can adversely impact performance/lifetime

Lifetime validation



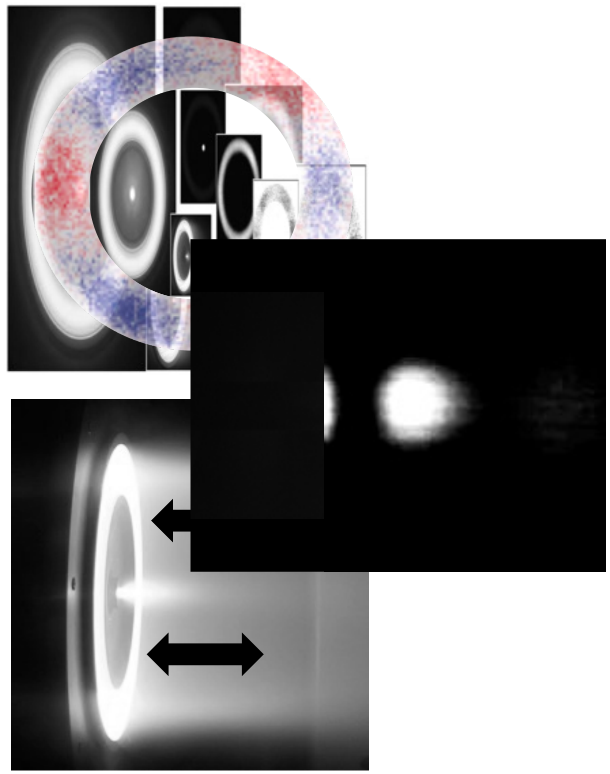


- Hall thrusters are low thrust and require years of continuous operation
- Demonstrating lifetime through testing for next generation missions is impractical

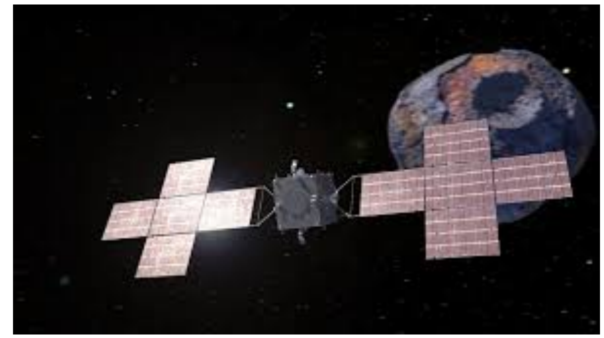
Challenges with Hall thrusters

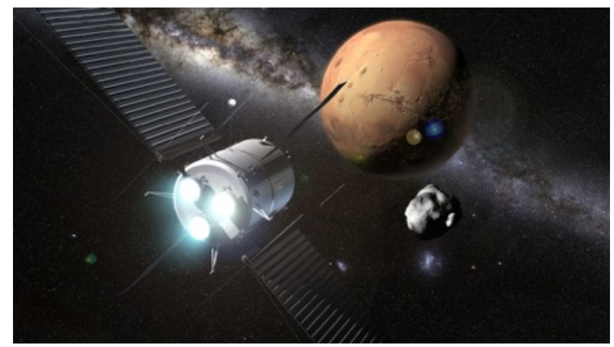
Facility effects

Stability



Lifetime validation

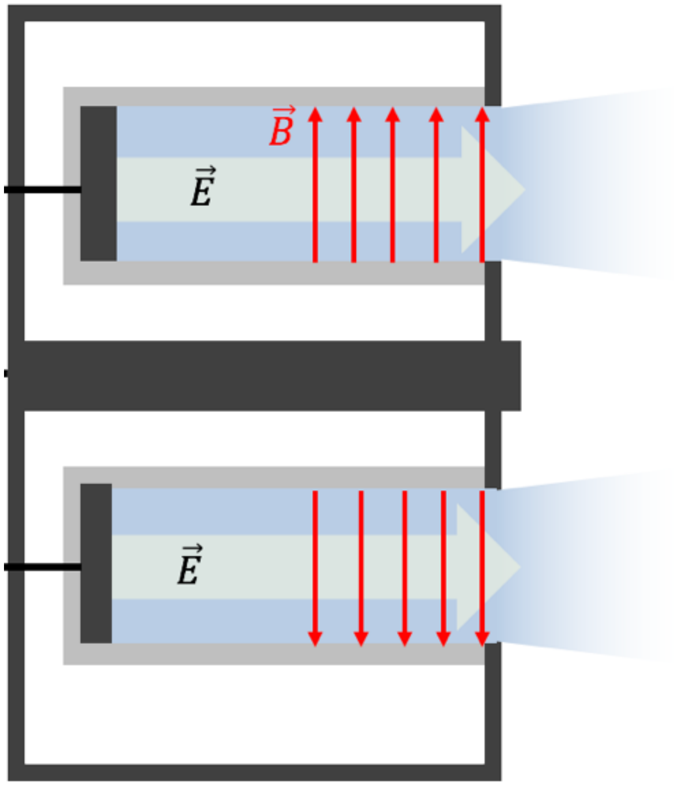


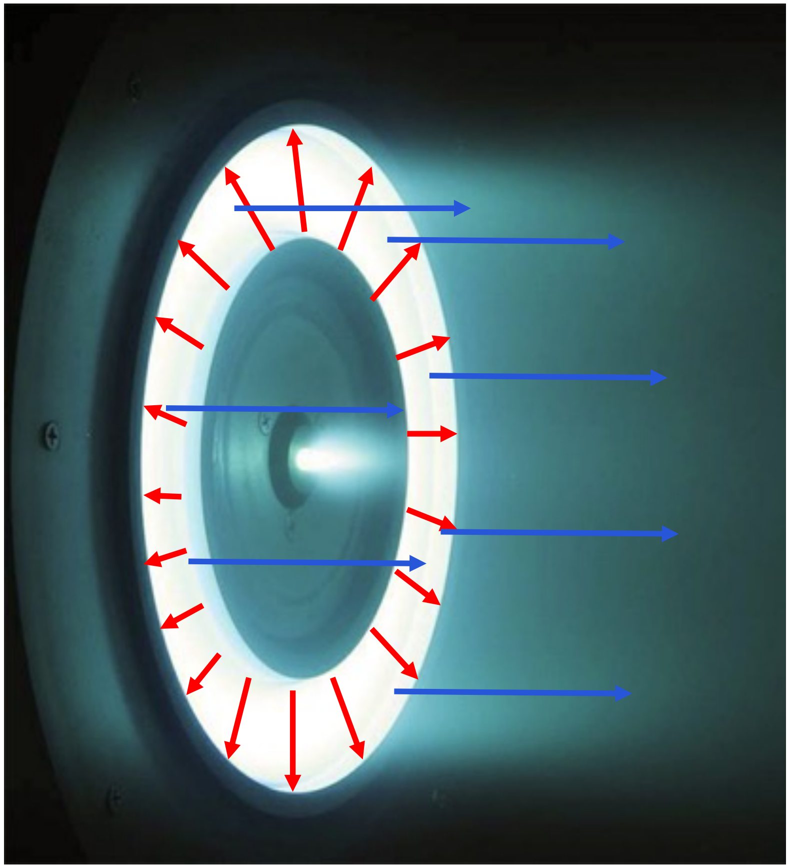


- Many of these challenges *could* be addressed with predictive modeling, but there are currently no fully predictive models for Hall thruster operation
- Lack of predictive models is largely related to problem of non-classical electron transport

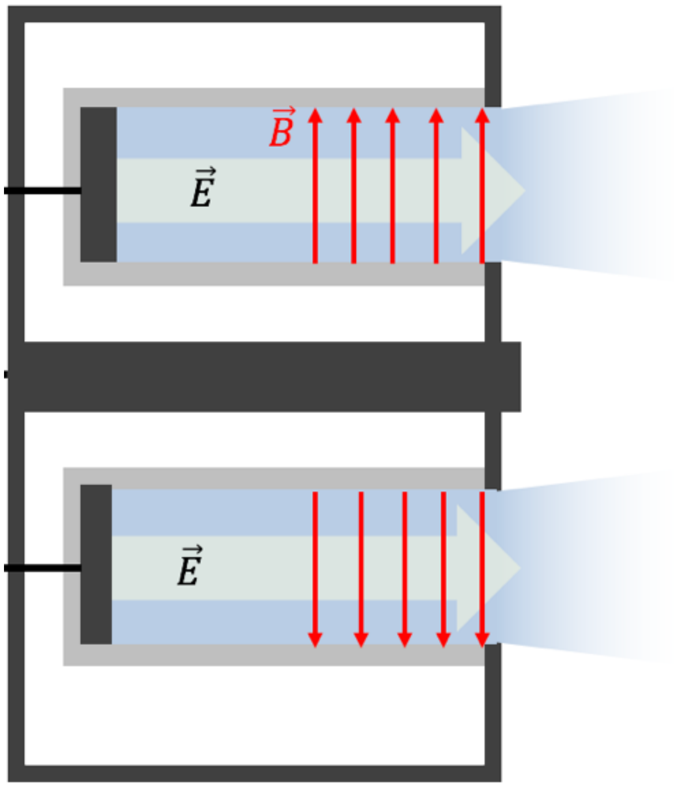
The problem of non-classical (anomalous) electron transport in Hall thruster modeling

Problem of non-classical (anomalous) electron transport in Hall thruster modeling





Problem of non-classical (anomalous) electron transport in Hall thruster modeling



Closed set of classical equations that can be evaluated with standard techniques

Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B}$$

Electron Energy

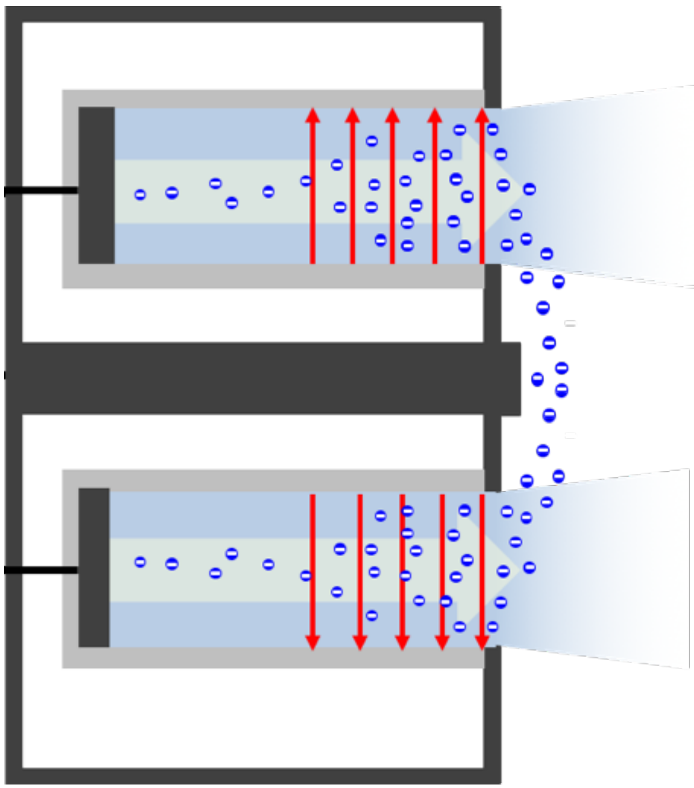
$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q n_e \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e \quad + \boldsymbol{Q_e} \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Problem of non-classical electron transport in Hall thruster modeling

$I_e / I_d \sim 0.1\%$



Electron cross-field current from evaluating classical equations

Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B}$$

Electron Energy

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q n_e \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e \quad + \boldsymbol{Q_e} \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

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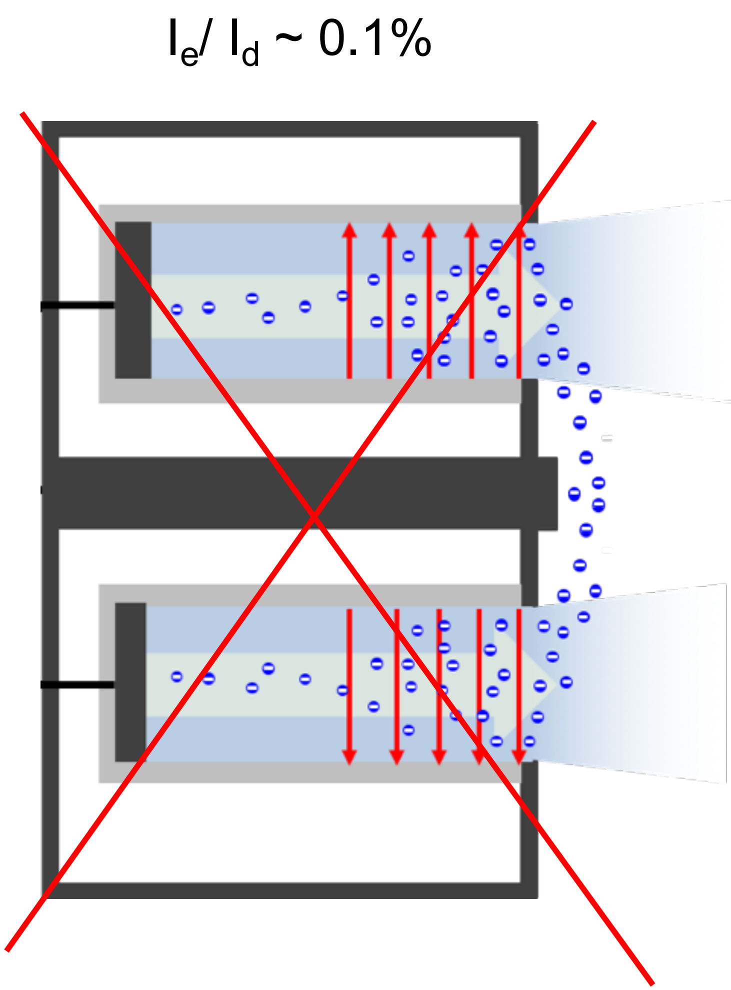
Problem of non-classical electron transport in Hall thruster modeling



Actual cross-field current from evaluating equations <u>1000 x higher</u>

Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B}$$

Electron Energy

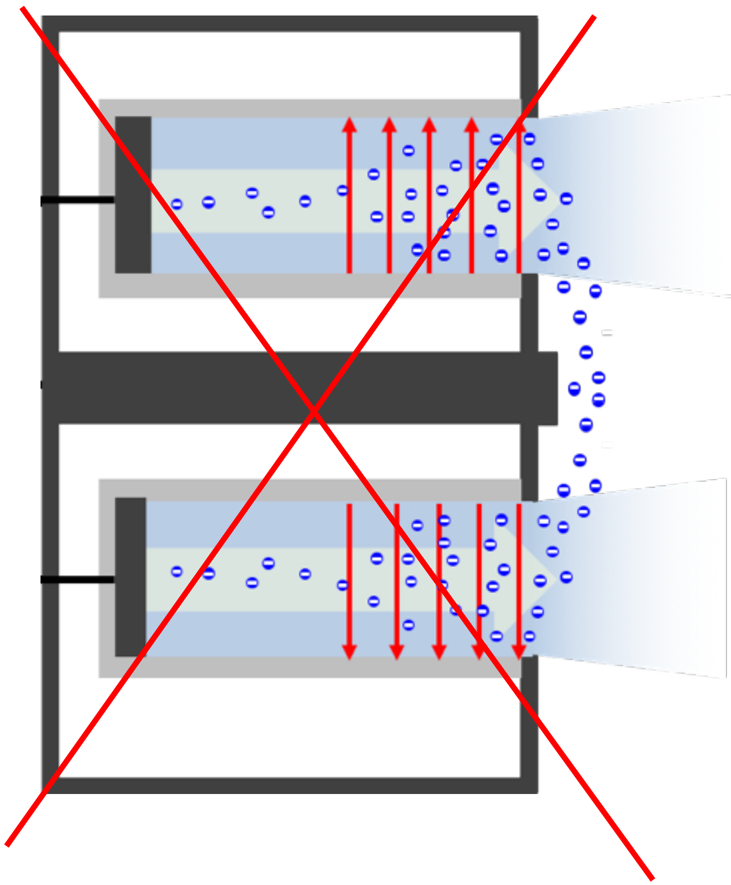
$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q n_e \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q_e} \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Problem of non-classical electron transport in Hall thruster modeling

$I_e / I_d \sim 0.1\%$



Actual cross-field current from evaluating equations <u>1000 x higher</u>

Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B} - n_e m_e\ \nu_{AN}\, \boldsymbol{u_e},$$

Need to introduce ad hoc factor

"Anomalous collision frequency"

Electron Energy

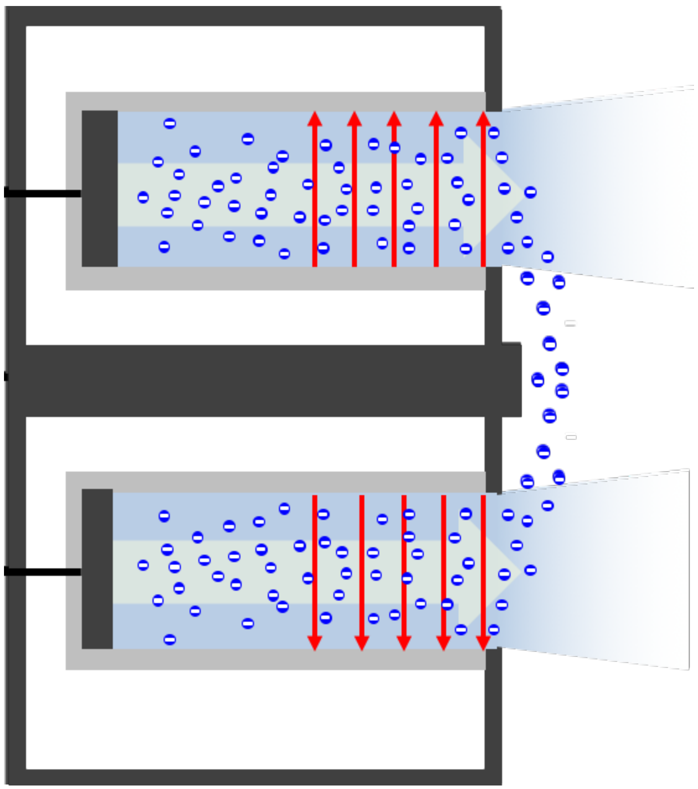
$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q \frac{\rho_i}{m} \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q}_e\right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Problem of non-classical electron transport in Hall thruster modeling

$I_e / I_d \sim 10\%$



Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B} - n_e m_e\ \nu_{AN}\, \boldsymbol{u_e},$$

Electron Energy

Need to introduce ad hoc factor

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q \frac{\rho_i}{m} \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q}_e \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

"Anomalous collision frequency "

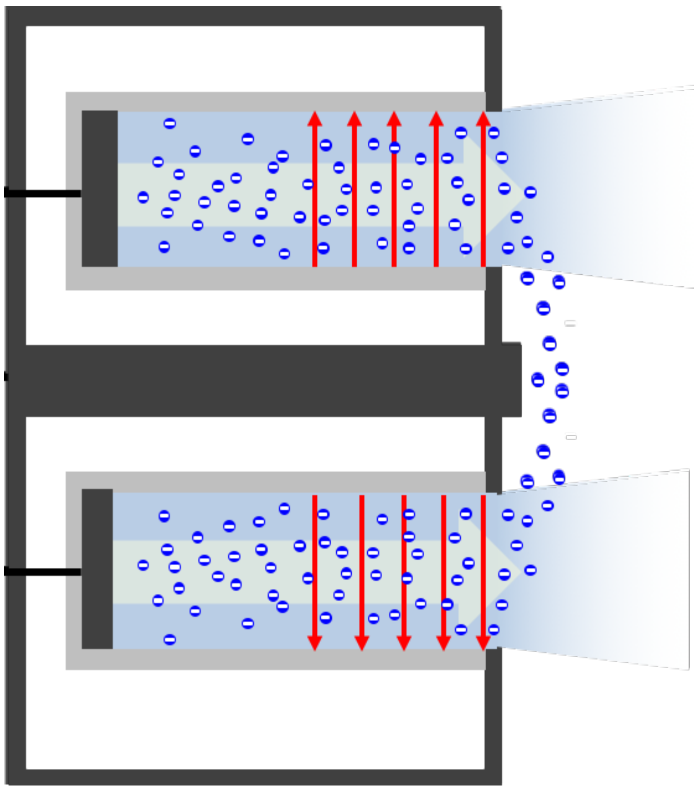
Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Anomalous friction term promotes additional cross-field current

Problem of non-classical electron transport in Hall thruster modeling

$I_e / I_d \sim 10\%$



Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B} - n_e m_e\ \nu_{AN}\, \boldsymbol{u_e},$$

Electron Energy

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q \frac{\rho_i}{m} \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q}_e\right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

Current conservation

Need to introduce ad hoc factor

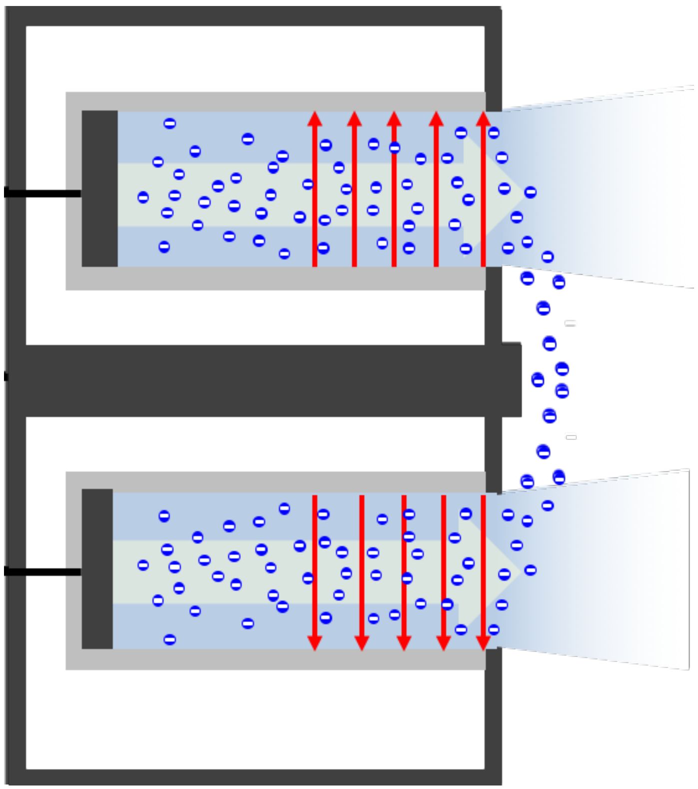
"Anomalous collision frequency "

Anomalous friction term promotes additional cross-field current

***Problem of closure*: introducing ad-hoc term opens set of equations (too many unknowns).**

Problem of non-classical electron transport in Hall thruster modeling

$I_e / I_d \sim 10\%$



We need a functional form for $\nu_{AN}(T_e, n_e, ..)$ that depends on classical fluid parameters

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B} - n_e m_e \, \nu_{AN} \, \boldsymbol{u_e},$$

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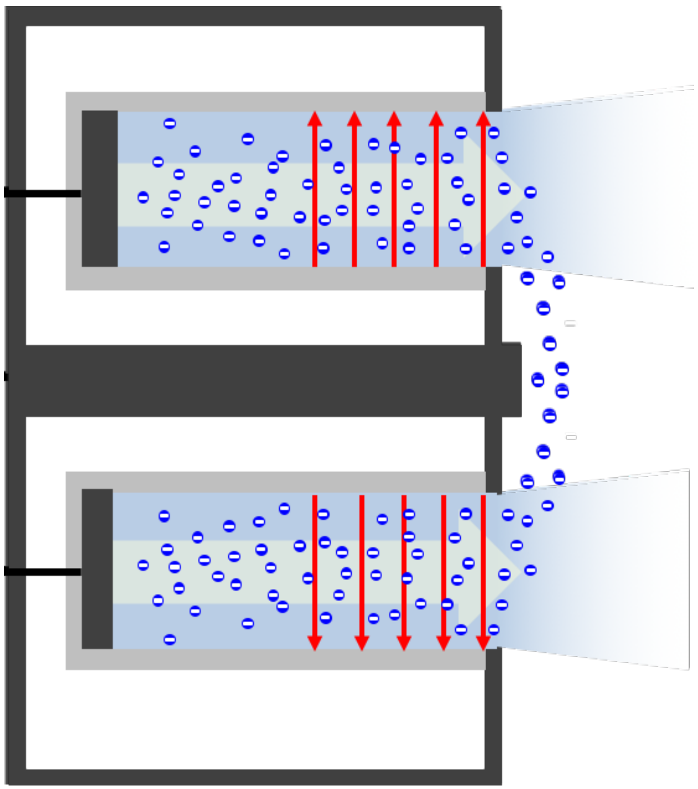


Aside: why not use a fully kinetic model?

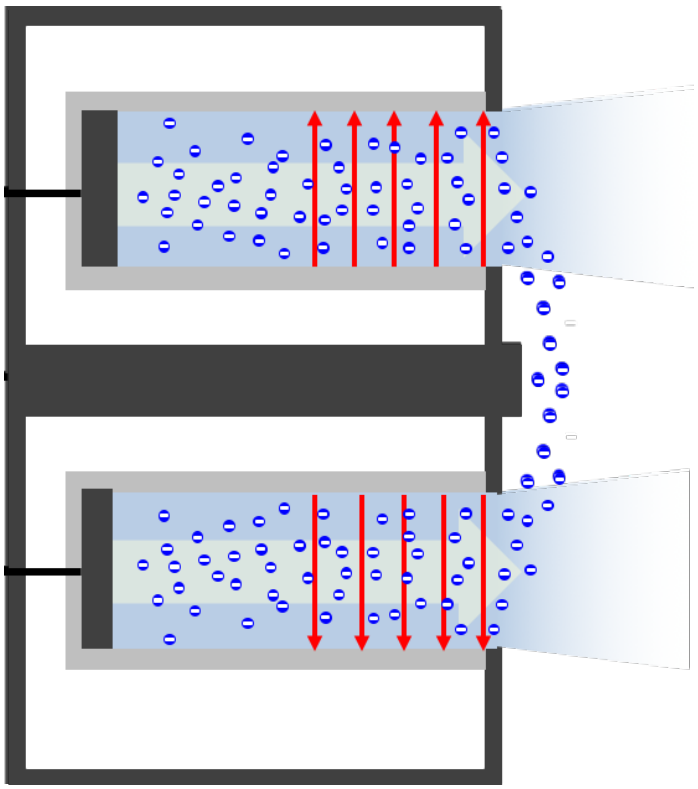
In principle, this should capture all processes including non-classical one

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q \frac{\rho_i}{m_i} \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q}_e \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

Anomalous friction term promotes additional cross-field current

Problem of non-classical electron transport in Hall thruster modeling

$I_e / I_d \sim 10\%$



Ion continuity

Assuming we use a fluid model, we need a functional form for $\nu_{AN}(T_e, n_e, ..)$ that depends on classical fluid parameters

$$\frac{\partial(m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B} - n_e m_e\, \nu_{AN}\, \boldsymbol{u_e},$$

Electron Energy

Need to introduce ad hoc factor

"Anomalous collision frequency"

Current conservation

Anomalous friction term promotes additional cross-field current

***Problem of closure*: introducing ad-hoc term opens set of equations (too many unknowns).**

Closing the Hall thruster fluid model

We need a functional form for $\nu_{AN}(T_e, n_e, ..)$ that depends on classical fluid parameters

There have been many attempts. . .

Mikellides, I. G. and Katz, I., “Numerical Simulations of Hall-effect Plasma Accelerators on a Magnetic-Field-Aligned Mesh,” Physical Review E, Vol. 86, No. 4, 2012, pp. 1–17.

Barral, S., Makowski, K., Peradzy´nski, Z., Gascon, N., and Dudeck, M., “Wall material effects in stationary plasma thrusters. II. Near-wall and in-wall conductivity, Physics of Plasmas, Vol. 10, No. 10, 2003, pp. 4137–4152.

Cappelli, M. A., Young, C. V., Cha, E., and Fernandez, E., “A zero-equation turbulence model for two-dimensional hybrid Hall thruster simulations,” Physics of Plasmas, Vol. 22, No. 11, 2015.

Garrigues, L., Hagelaar, G. J., Boniface, C., and Boeuf, J. P., “Anomalous conductivity and secondary electron emission in Hall effect thrusters,”Journal of Applied Physics, Vol. 100, No. 12, 2006.

Hofer, R., Katz, I., Mikellides, I., Goebel, D. M., Jameson, K. K., Sullivan, R. M., and Johnson, L. K., “Efficacy of Electron Mobility Models in Hybrid-PIC Hall Thruster Simulations,” Proceedings of the 44th AIAA Joint Propulsion Conference, No.AIAA-2008-4924, 2008.

Katz, I., Mikellides, I. G., Jorns, B. a., and Ortega, A. L., “Hall2De Simulations with an Anomalous Transport Model,”Proceedings of the 34th International Electric Propulsion Conference, No. IEPC-2015-402, 2015.

Koo, J. W. and Boyd, I. D., “Modeling of anomalous electron mobility in Hall thrusters,”Physics of Plasmas, Vol. 13,No. 3, 2006, pp. 1–7.

Lafleur, T., Baalrud, S. D., and Chabert, P., “Theory for the anomalous electron transport in Hall effect thrusters. I.Insights from particle-in-cell simulations,”Physics of Plasmas, Vol. 23, No. 5, 2016.

Mikellides, I., Ortega, A., Katz, I., and Jorns, B., “Hall2De simulations with a first-principles electron transport model based on the electron cyclotron drift instability,” , No. AIAA-2016-4618.

Morozov, A. I. and Savelyev, V. V., Fundamentals of Stationary Plasma Thruster Theory, Springer US, Boston, MA,2000, pp. 203–391.

Reza, M., Faraji, F., Andreussi, T., and Andrenucci, M., “A Model for Turbulence-Induced Electron Transport in Hall Thrusters,” , No. IEPC-2017-367.

14Scharfe, M. K., Thomas, C. A., Scharfe, D. B., Gascon, N., Cappelli, M. A., and Fernandez, E., “Shear-Based Model for Electron Transport in 2D Hybrid Hall Thruster Simulations,” IEEE Transactions on Plasma Science, Vol. 36, No. July, 2007,pp. 1–10.

15Katz, I., Lopez Ortega, A., Jorns, B., and Mikellides, I., “Growth and saturation of ion acoustic waves in hall thrusters,”, No. AIAA-2016-4534.

16Jorns, B., “Predictive , Data-Driven Model for the Anomalous Electron Collision Frequency in a Hall Effect Thruster,” Plasma Sources Science and Technology, Vol. 27, No. 104007, 2018.

17Lopez Ortega, A., Katz, I., and Chaplin, V., “A First-Principles Model Based on Saturation of the Electron Cyclotron Drift Instability for Electron Transport in Hydrodynamics Simulations of Hall Thruster Plasmas,”35th International Electric Propulsion ConferenceGeorgia Institute of Technology, Atlanta, Georgia * USA, IEPC-2017-178

Closing the Hall thruster fluid model

We need a functional form for $\nu_{AN}(T_e, n_e, ..)$ that depends on classical fluid parameters

Physics-based approach

Propose mechanism for what drives transport

↓

Verify experimentally if mechanism dominates transport

↓

Formulate expression for $\nu_{AN}(T_e, n_e, ..)$ based on validated process

Data-driven approach

Use experimental data to measure $\nu_{AN}(T_e, n_e, ..)$ over several operating conditions

↓

Regress data to build empirical closure model

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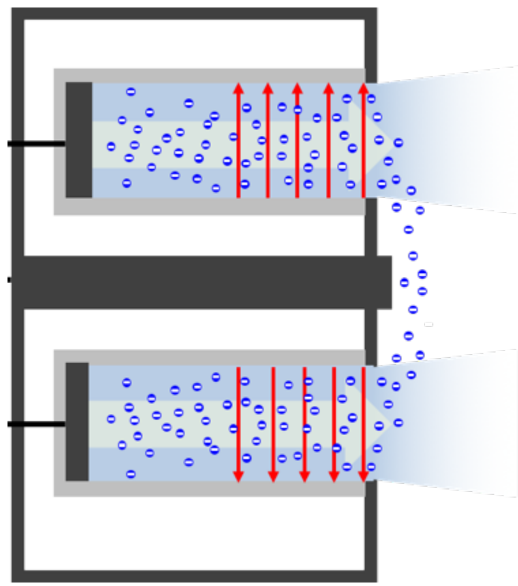
Proposed mechanisms for enhanced cross-field transport in Hall thrusters

Low-frequency, coherent modes

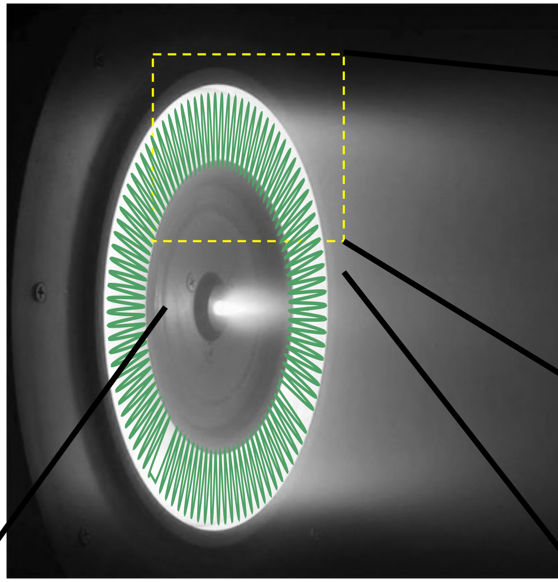
Wall Interactions



High-frequency, low amplitude Instabilities

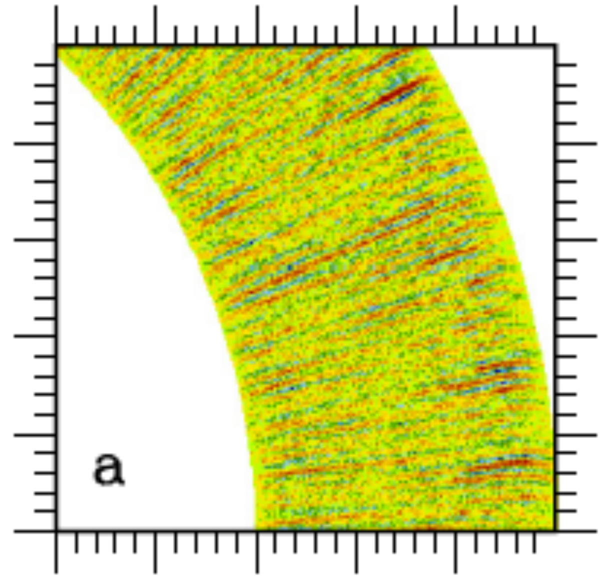
Bohm Diffusion

Numerical and experimental evidence of turbulent fluctuations



Simulation

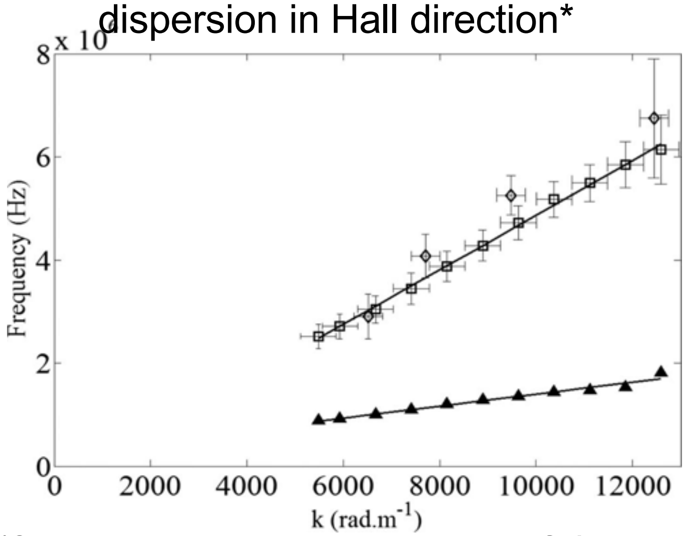
Density fluctuations from PIC, 2D model



* A. Héron and J. C. Adam. Physics of Plasmas 20 , 082313 (2013);

Experiment

Coherent Thomson Scattering measurements of dispersion in Hall direction*



*S. Tsikata, N. Lemoine, V. Pisarev, and D. Grésillon, *Physics of Plasmas*. Vol. 16., No. 3. 2009.

- 2D kinetic simulations predict high frequency (>1 MHz) propagating mode in Hall direction
- Experiments confirm there are broadband instabilities propagating in Hall direction
- Previously proposed that the simulated/measured waves are the electron cyclotron drift instability
- Azimuthal drift waves are known to drive cross-field transport (e.g. space weather)

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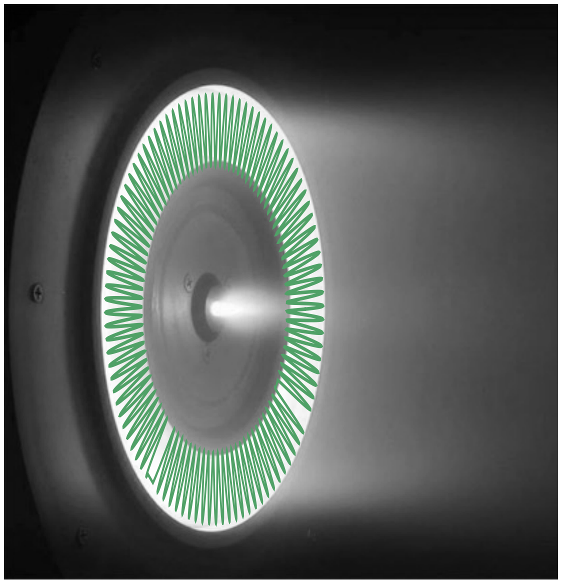
Data-driven approach

Use experimental data to measure $\nu_{AN}(T_e, n_e, ..)$ over several operating conditions

↓

Regress data to build empirical closure model

Relating experimentally measured wave properties to transport



The first moment of the second order Boltzmann equation yields

$$\nu_{AN} \approx \frac{\langle q\ \delta n_e \delta E_{(E\times B)} \rangle}{\left| m_e n_e u_{e(E\times B)} \right|}$$

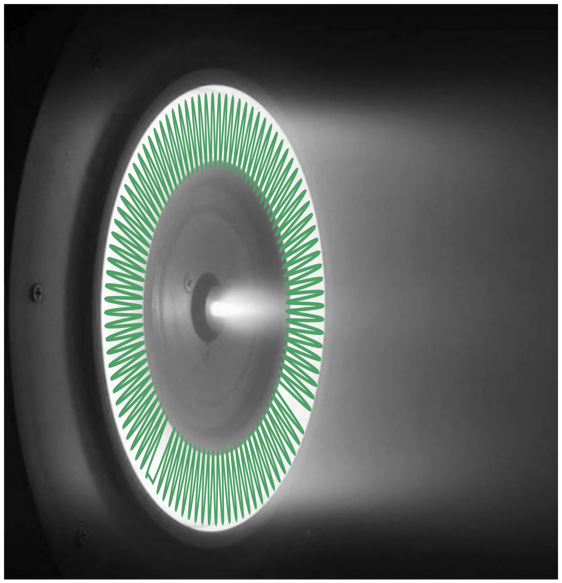
δn_e	Wave density fluctuation amplitude
$\delta E_{(E\times B)}$	Wave electric field amplitude e
$u_{e(E\times B)}$	Electron drift in Hall direction
$\langle\ \ \rangle$	Average over wave spectrum

Practical problem: While it straightforward to measure density fluctuations, it has not been possible to perform simultaneous electric field measurements

Solution: Employ quasilinear theory to relate density and electric field amplitudes

*Z. Brown, E. Dale, and B. Jorns, IEPC-2019-843, 2019.

Relating experimentally measured wave properties to transport



The first moment of the second order Boltzmann equation yields

$$\nu_{AN} \approx \frac{T_e}{\left|m_e u_{e(E\times B)}\right|} \left\langle \left(\frac{\delta n_e}{n_e}\right)^2 k_i \right\rangle$$

δn_e	Wave density fluctuation amplitude
k_i	Linear wave growth growth rate
$u_{e(E\times B)}$	Electron drift in Hall direction
$\langle\ \rangle$	Average over wave spectrum

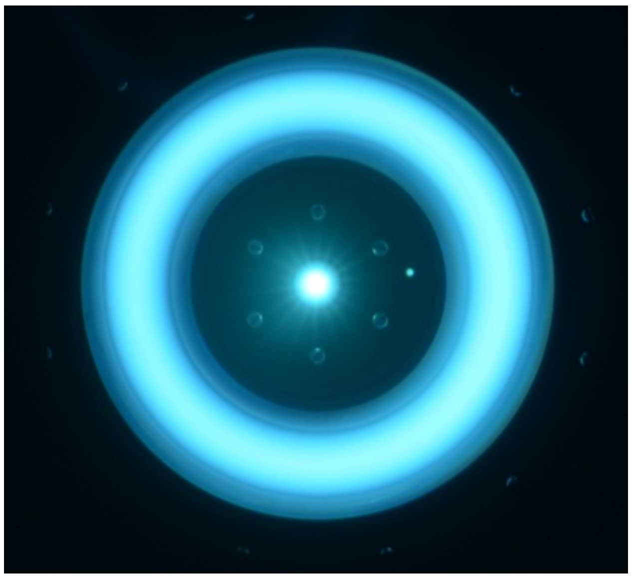
Practical problem: While it straightforward to measure density fluctuations, it has not been possible to perform simultaneous electric field measurements

Solution: Employ quasilinear theory to relate density and electric field amplitudes

*Z. Brown, E. Dale, and B. Jorns, IEPC-2019-843, 2019.

Experimental Setup at UM

H9: 9-kW magnetically shielded Hall effect thruster at 300 V and 4.5 kW

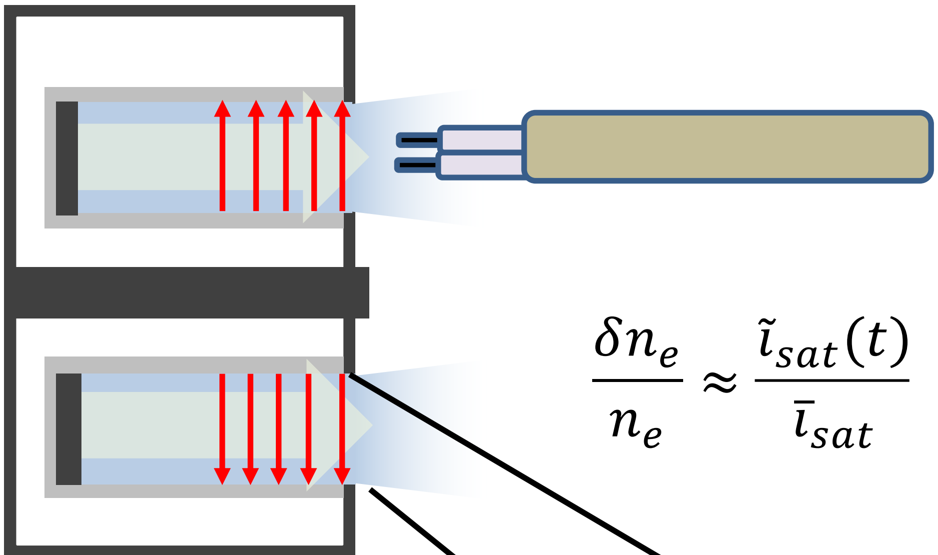


Large Vacuum Test Facility at UM

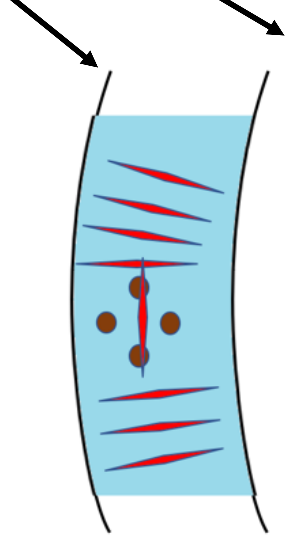
550 kl/s pumping speed on xenon

Experimental Setup at UM

Ion saturation probes



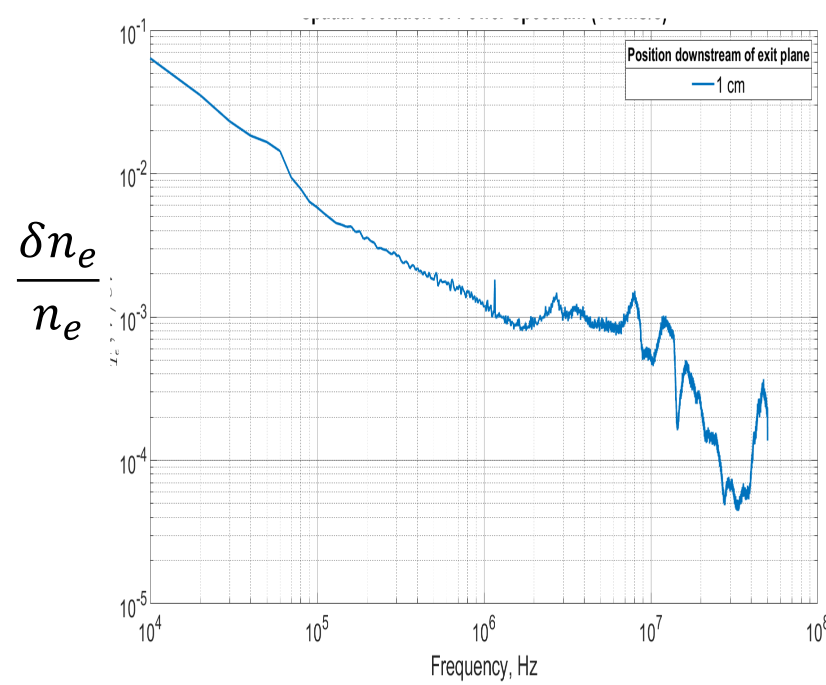
$$\frac{\delta n_e}{n_e} \approx \frac{\tilde{\iota}_{sat}(t)}{\bar{\iota}_{sat}}$$



Measure fluctuations and phase delay in Hall direction

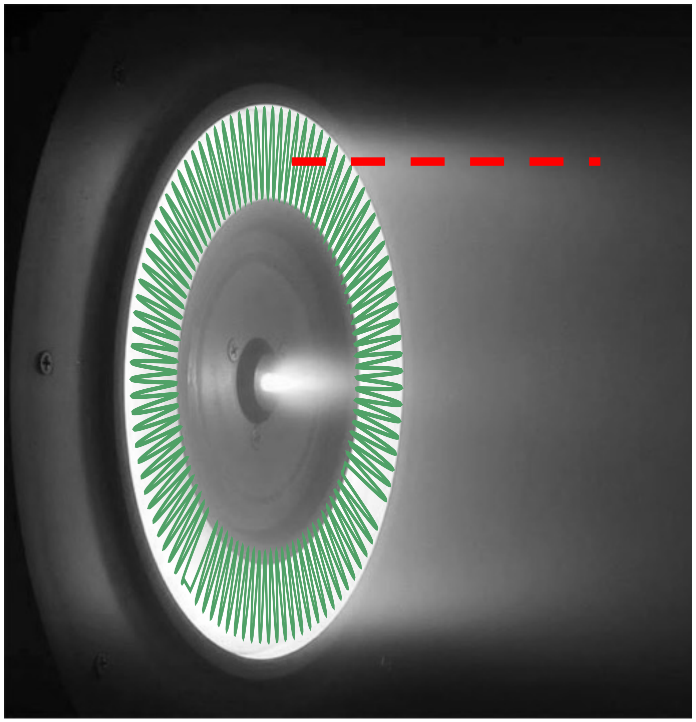
Analysis

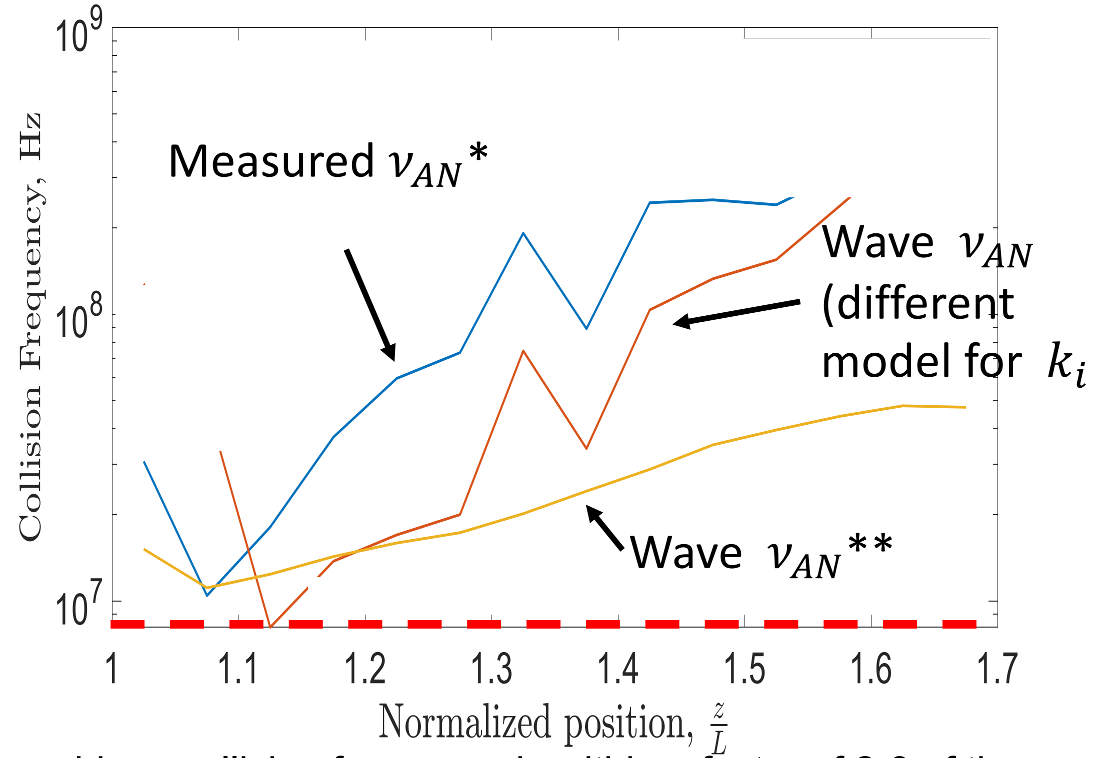
- Fourier analysis to find
  - Power spectrum $\phi(\omega)$
  - Cross-correlation $\omega(k)$
- Averaging yields statistical dispersion relation $\phi(\omega, k)$



Measured collision frequency from instabilities compared to actual non-classical collision frequency

$$\nu_{AN} \approx \frac{T_e}{|m_e u_{e(E\times B)}|} \left\langle \left(\frac{\delta n_e}{n_e}\right)^2 k_i \right\rangle$$





- Depending on model for k_i, the wave-driven collision frequency is within a factor of 2-3 of the measured collision frequency. Captures the right shape
- There are also multiple potential reasons for this discrepancy (measurement techniques, imperfect model for k_i , etc.)
- **Continuing to refine these results, but they at least show waves <u>might</u>** be dominant factor

Closing the Hall thruster fluid model

We need a functional form for $\nu_{AN}(T_e, n_e, ..)$ that depends on classical fluid parameters

Physics-based approach

Propose mechanism for what drives transport

↓

Verify experimentally if mechanism dominates transport

↓

Formulate expression for $\nu_{AN}(T_e, n_e, ..)$ based on validated process

Data-driven approach

Use experimental data to measure $\nu_{AN}(T_e, n_e, ..)$ over several operating conditions

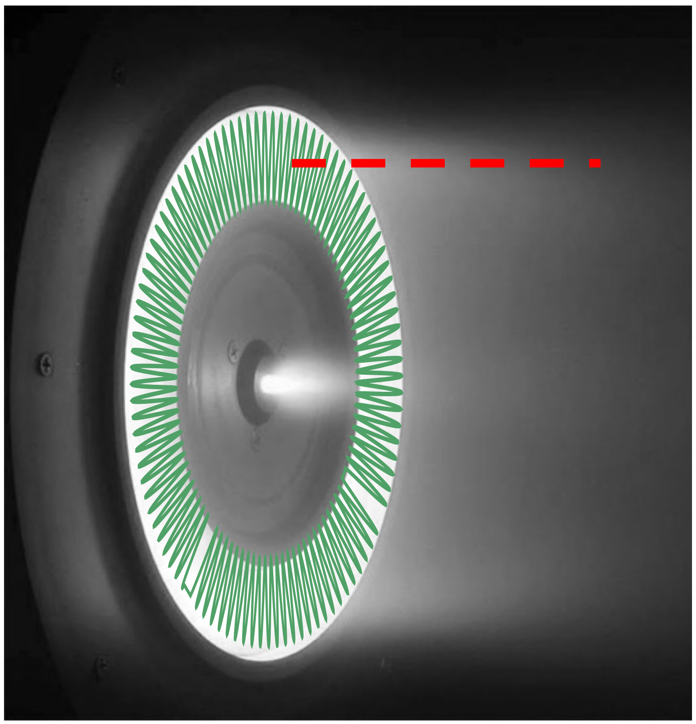
↓

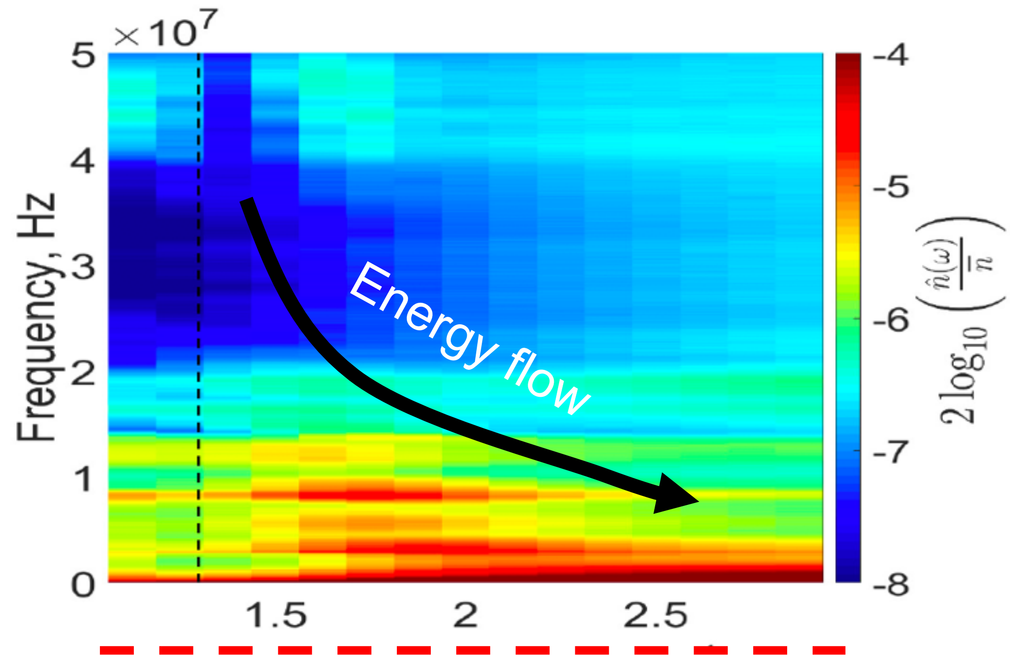
Regress data to build empirical closure model

Formulating closure model for collision frequency

Governing hypothesis: anomalous collision frequency in Hall thruster is result of wave-particle interaction with low-level electrostatic waves

Challenge: how do we model the evolution of the wave energy and the rate at which these waves exchange energy with the electrons?





There is experimental evidence of nonlinear energy cascade as waves propagate. This suggests evolution of wave properties is nonlinear. This complicates the formulation for how the energy convects in plasma

*Z. Brown and B. Jorns, Physics of Plasmas, 26 2019.

Formulating closure model for collision frequency

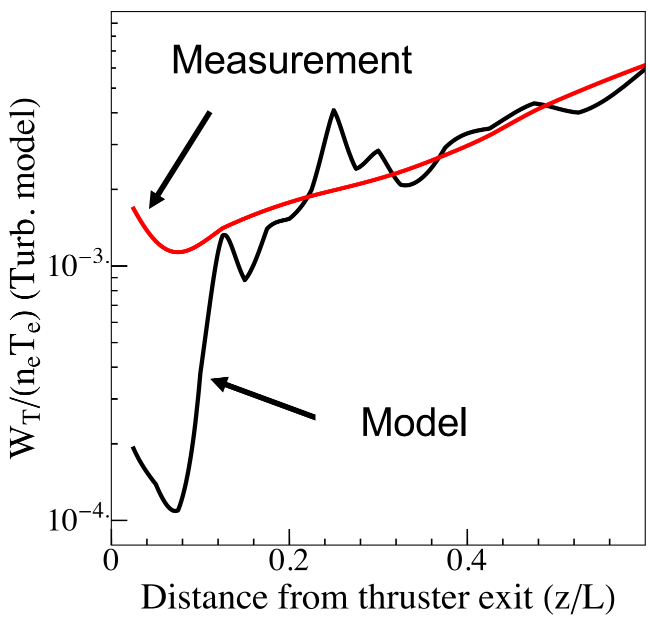
Approach: Two equation closure model based on moments of nonlinear plasma wave kinetic equation

$$\nu_{AN} \propto \langle \omega_i \rangle W_T$$

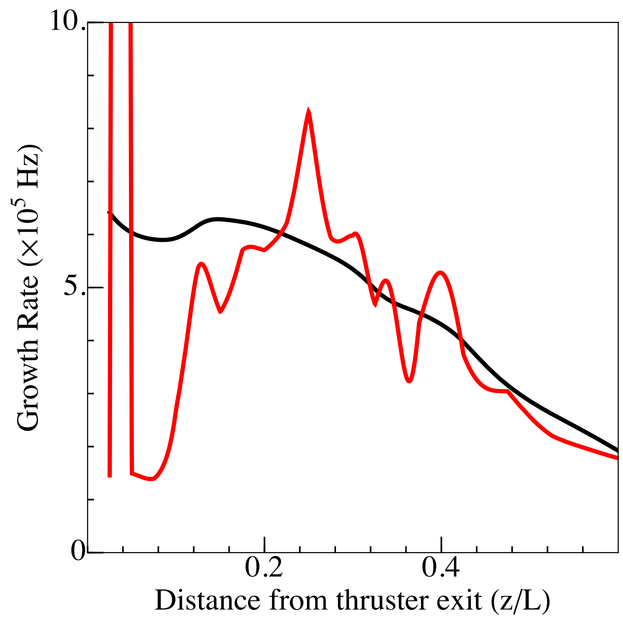
Average rate at which wave extracts energy from electrons → $\langle \omega_i \rangle$

Total energy density of waves → W_T

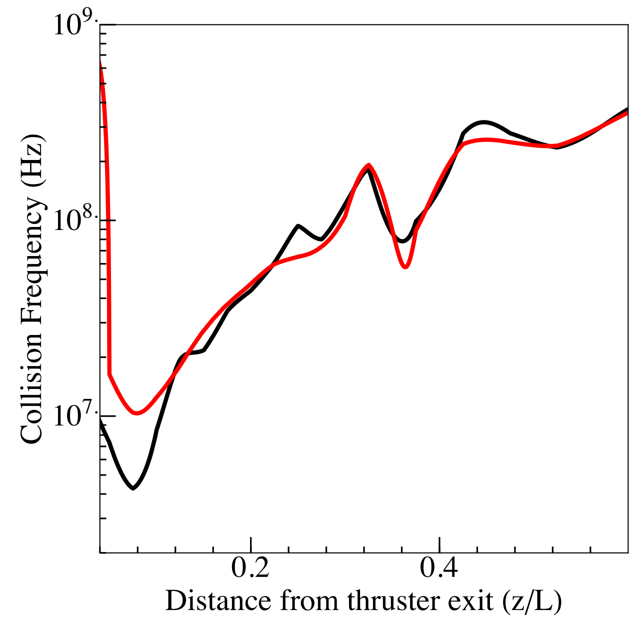
W_T



$\langle \omega_i \rangle$



ν_{AN}



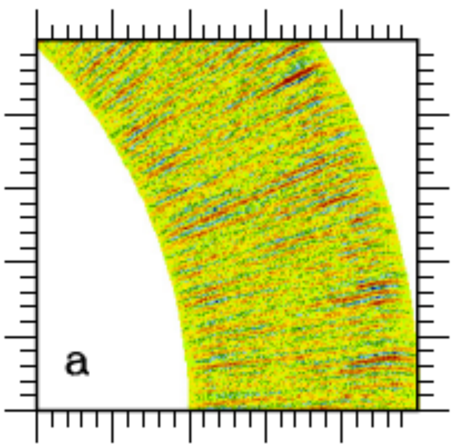
Preliminary formulation has yielded promising results, though model is still not predictive (needs to be calibrated against data)

Summary of physics-based approach to closure for Hall thruster models

Propose mechanism for what drives transport

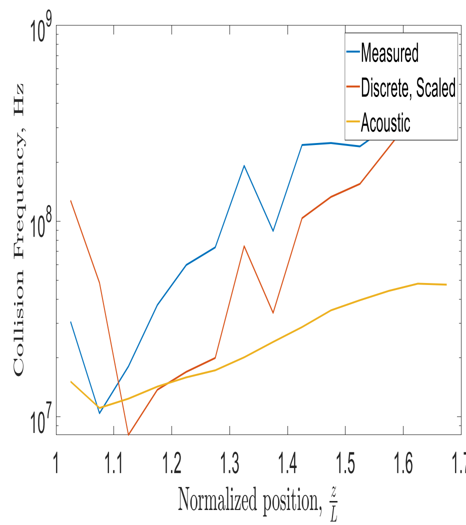
Verify experimentally if mechanism dominates transport

Formulate expression for $\nu_{AN}(T_e, n_e, ..)$ based on validated process

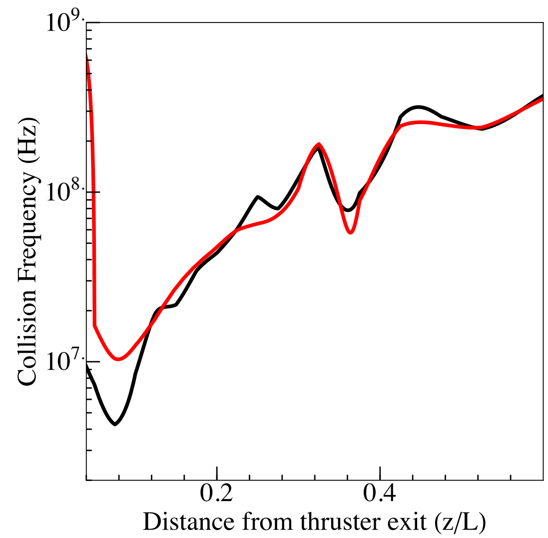


* A. Héron and J. C. Adam. Physics of Plasmas 20 , 082313 (2013);

ν_{AN}



ν_{AN}



- There may be other mechanisms impacting transport (boundary effects or lower frequency modes)
- The formulation relating wave properties to transport is oversimplified
- We have derived one closure model, though there are other approaches* based on zero and single equation formulations. **Predictive models remain elusive**

Closing the Hall thruster fluid model

We need a functional form for $\nu_{AN}(T_e, n_e, ..)$ that depends on classical fluid parameters

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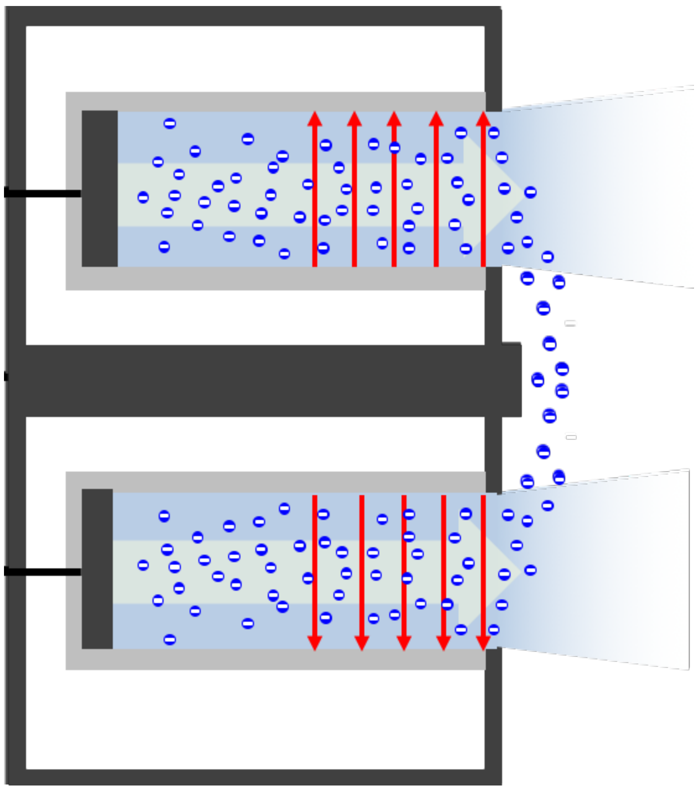
Regress data to build empirical closure model

Current practice for engineering models: data calibration



Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B} - n_e m_e\ \nu_{AN}\, \boldsymbol{u_e},$$

Electron Energy

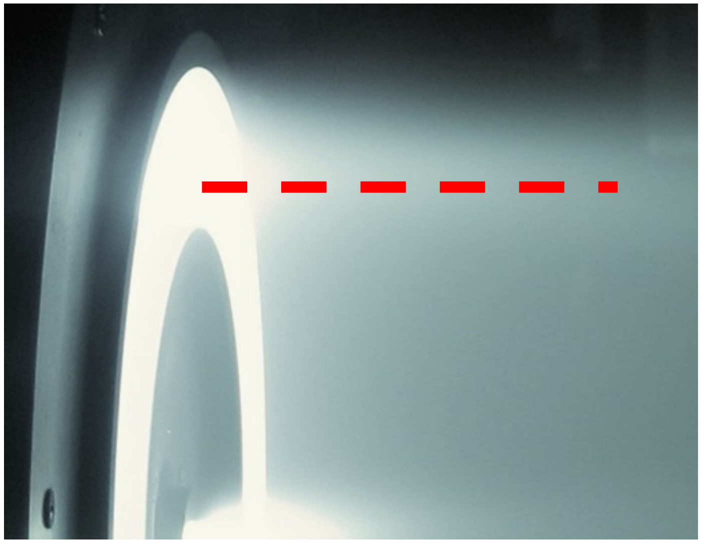
$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q \frac{\rho_i}{m_i} \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e \quad + \boldsymbol{Q_e} \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

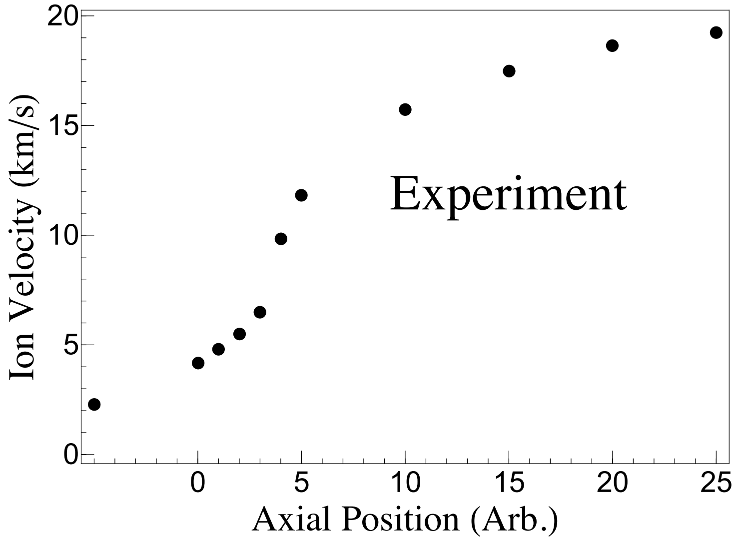
Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

SOA Hall thruster fluid models use data to calibrate collision frequency term

Closures for anomalous collision frequency: empirical estimate





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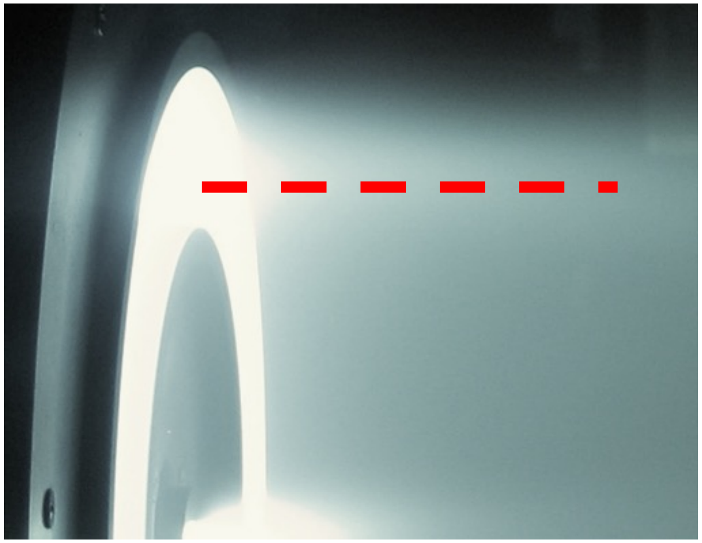

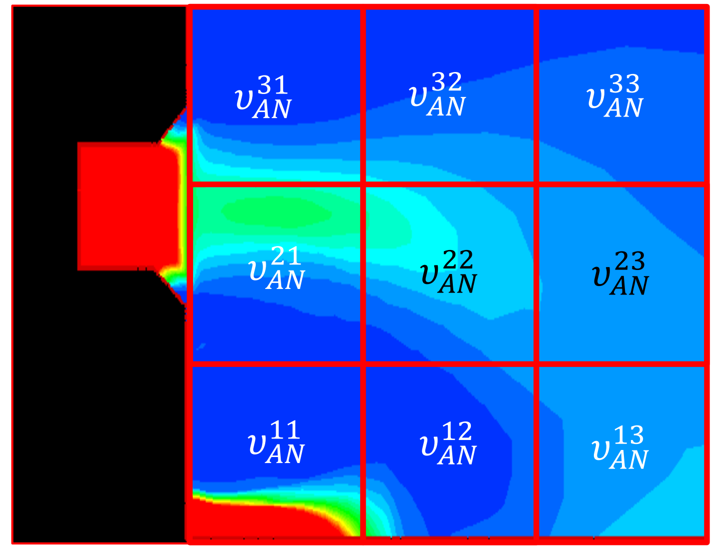

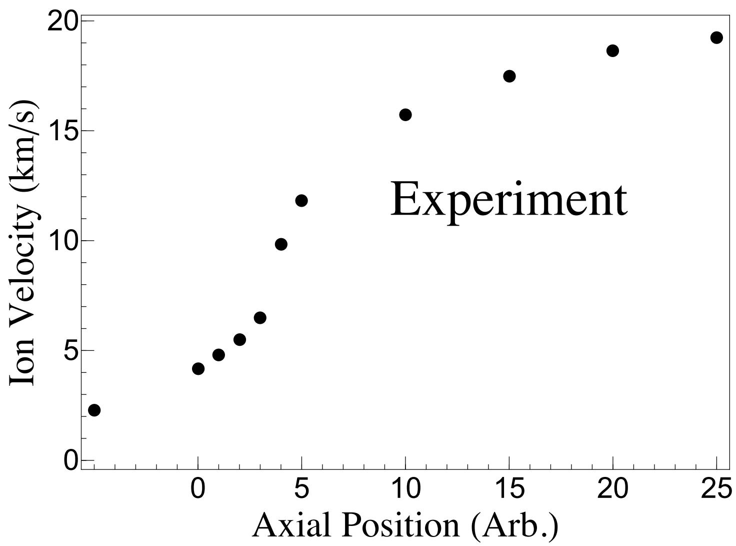

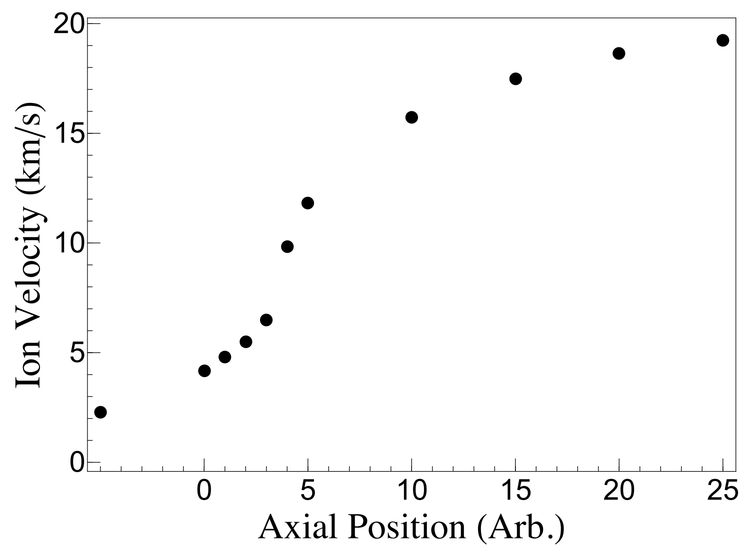

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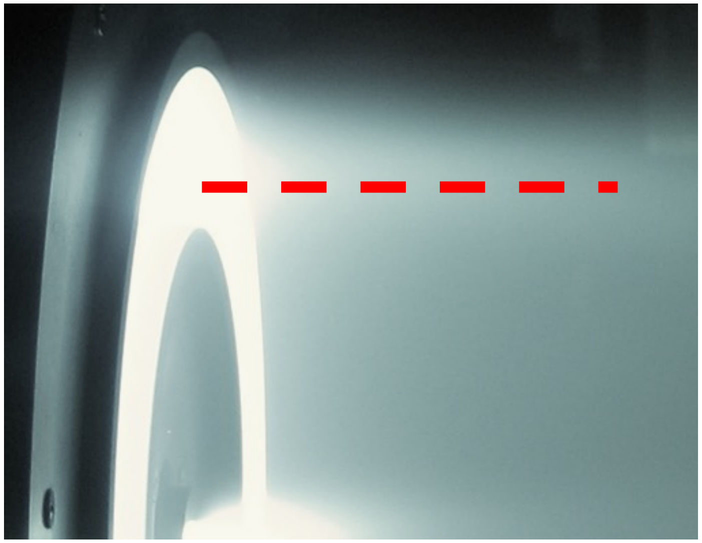
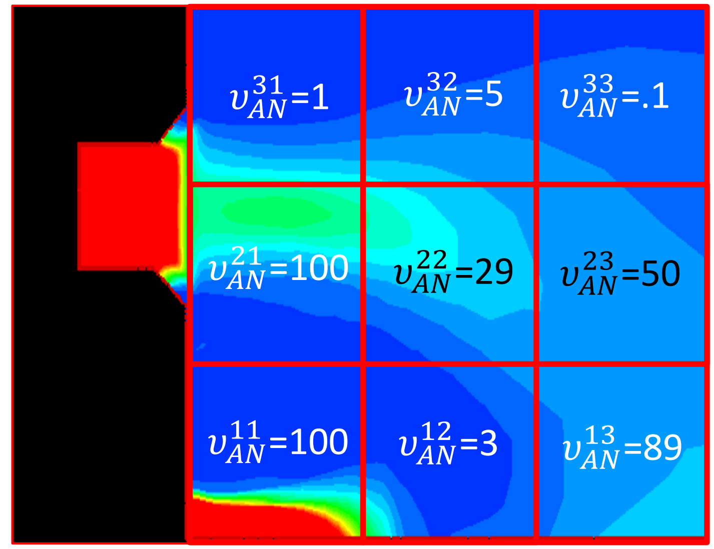
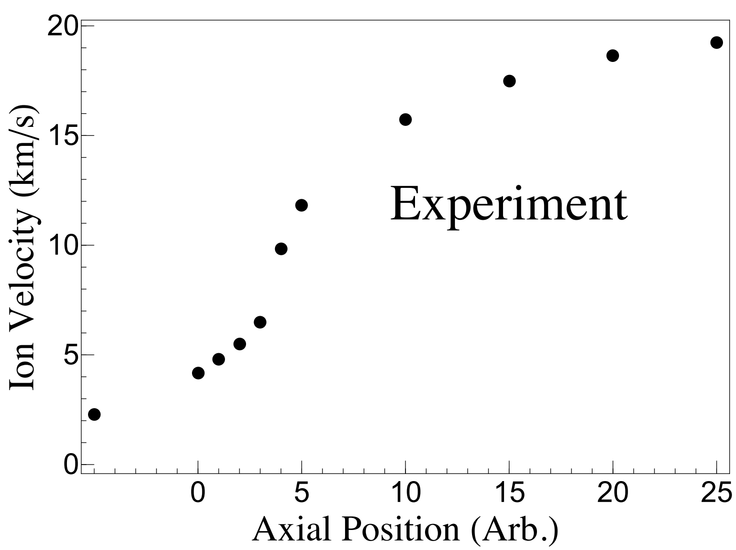
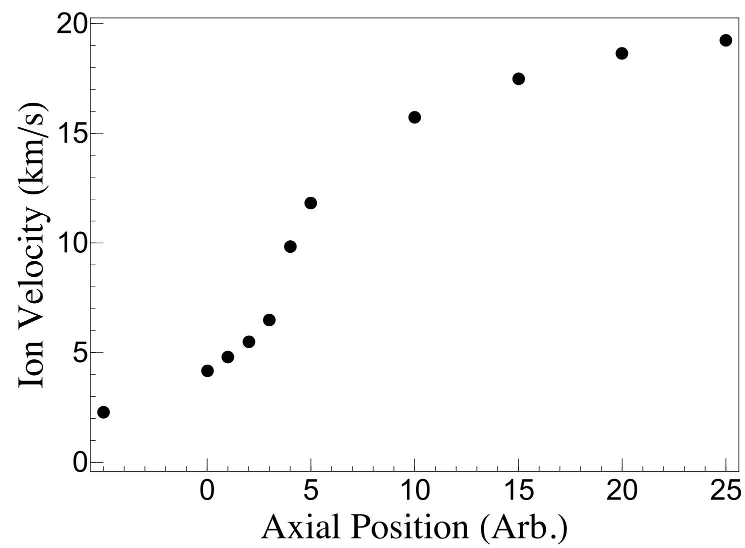


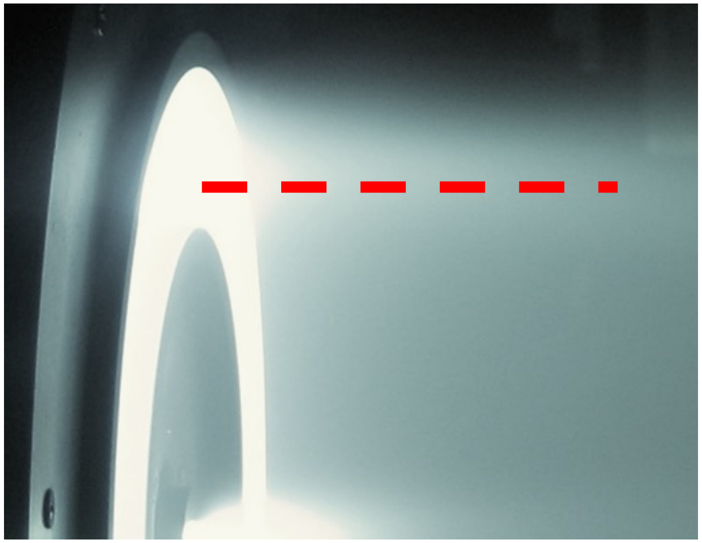


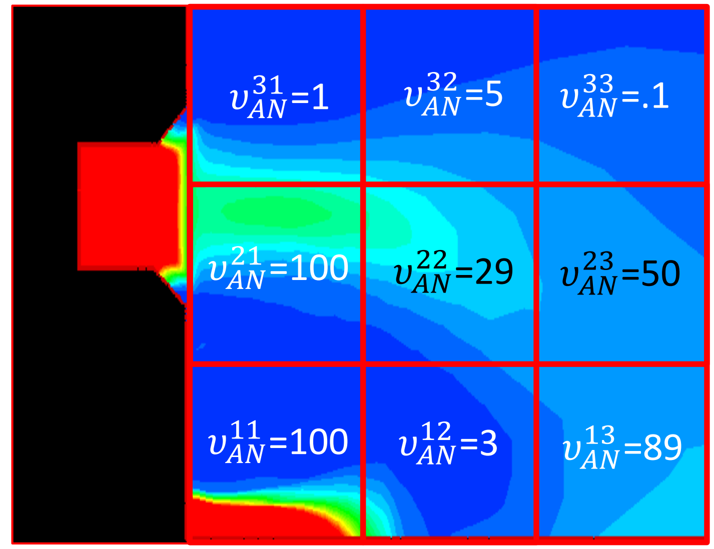


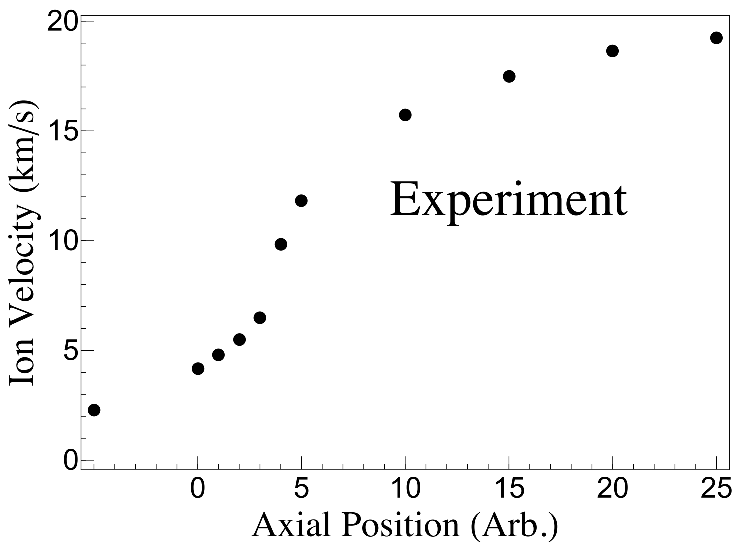
Iteration #1



Closures for anomalous collision frequency: empirical estimate







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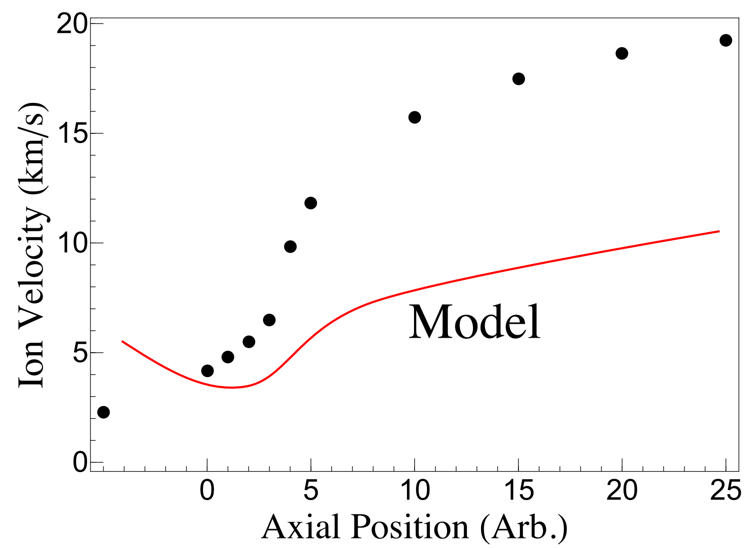
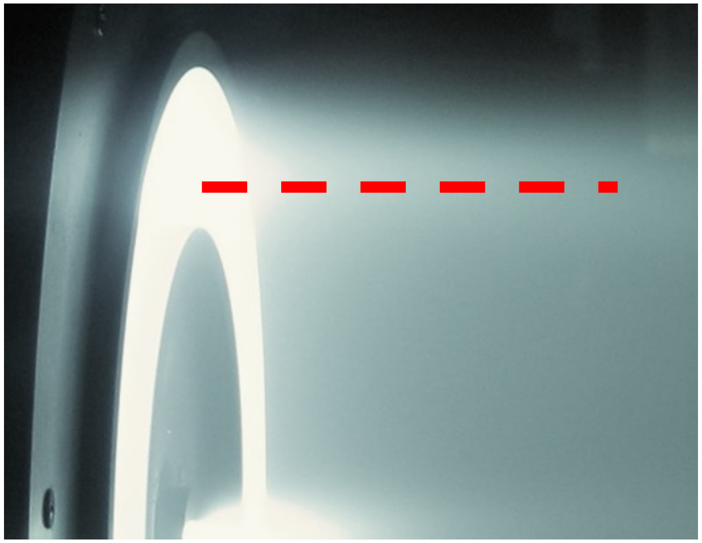
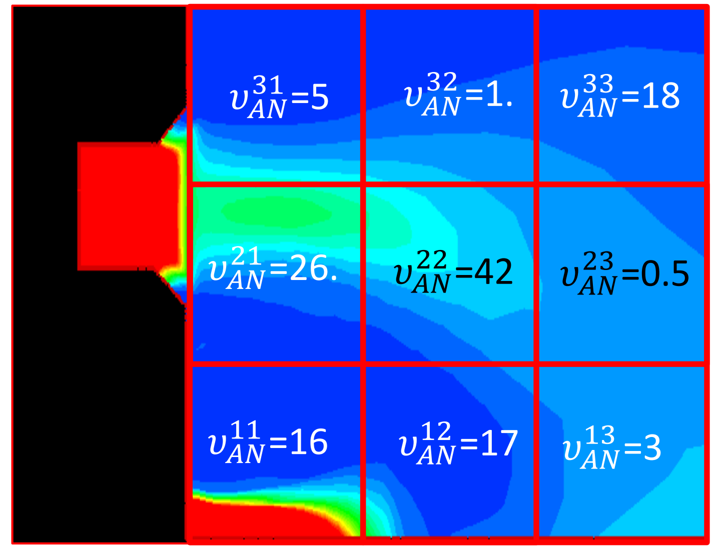
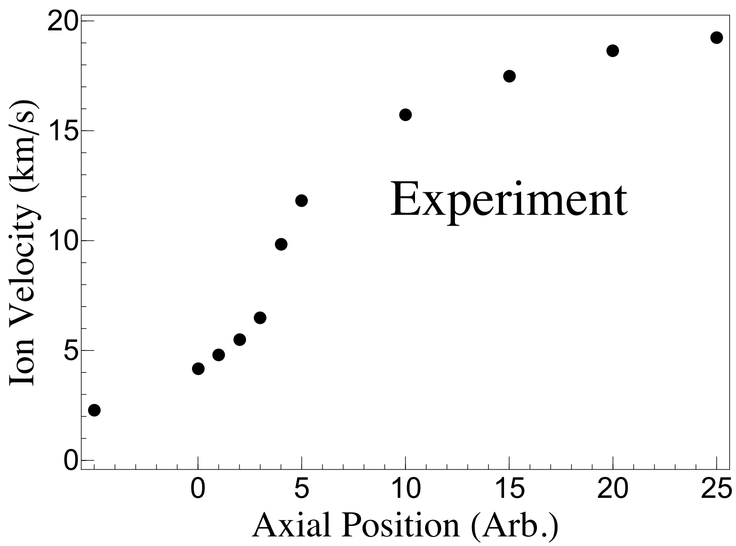


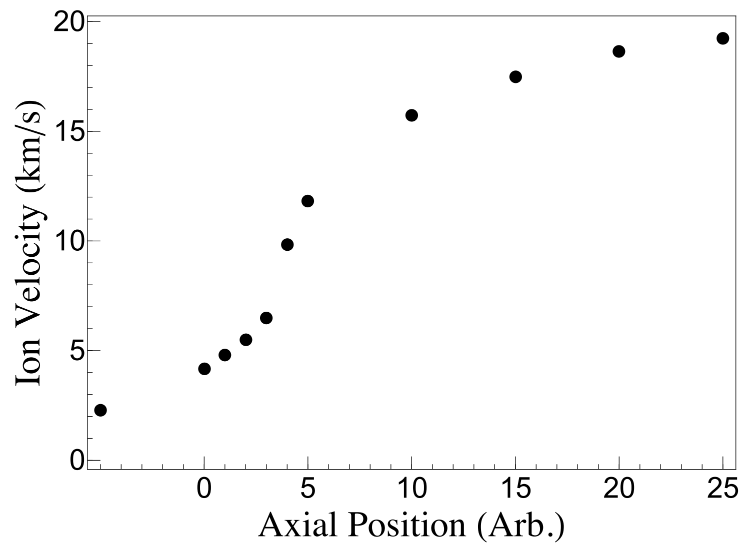
Closures for anomalous collision frequency: empirical estimate



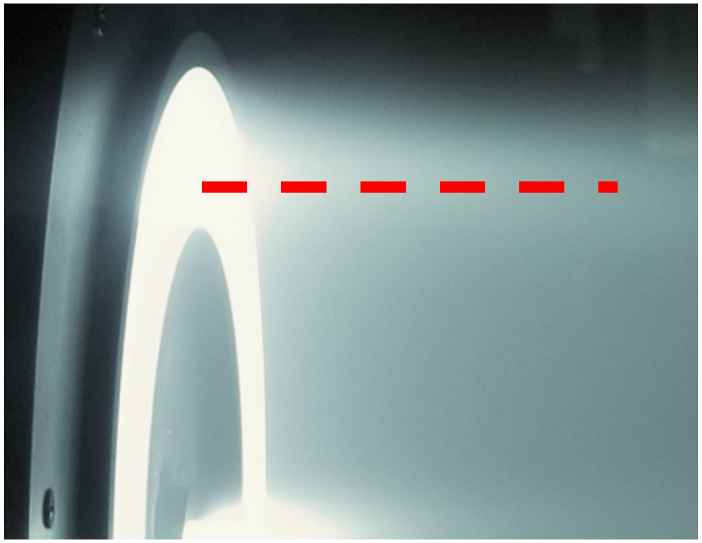


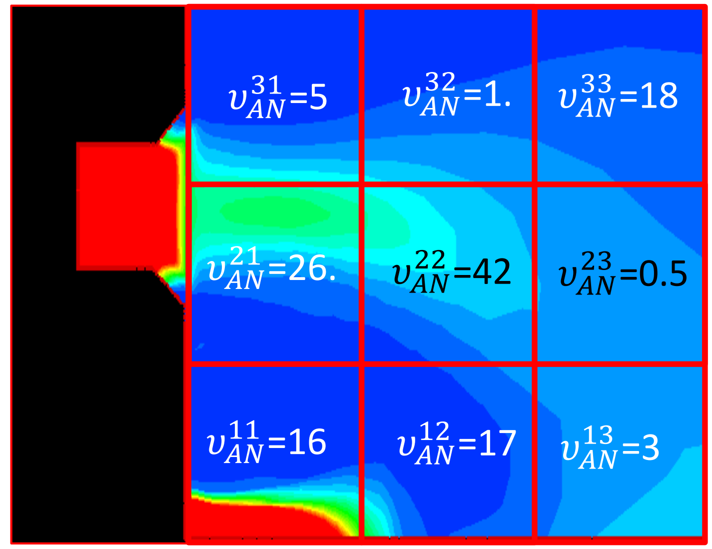


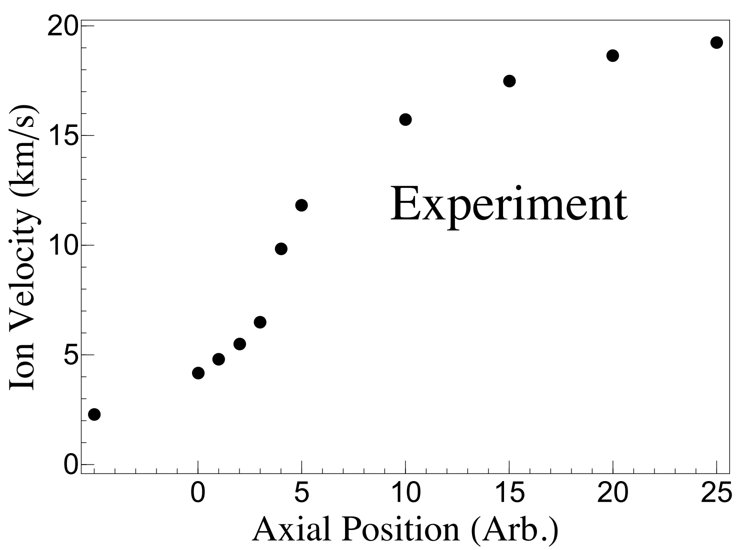
Iteration #2



Closures for anomalous collision frequency: empirical estimate







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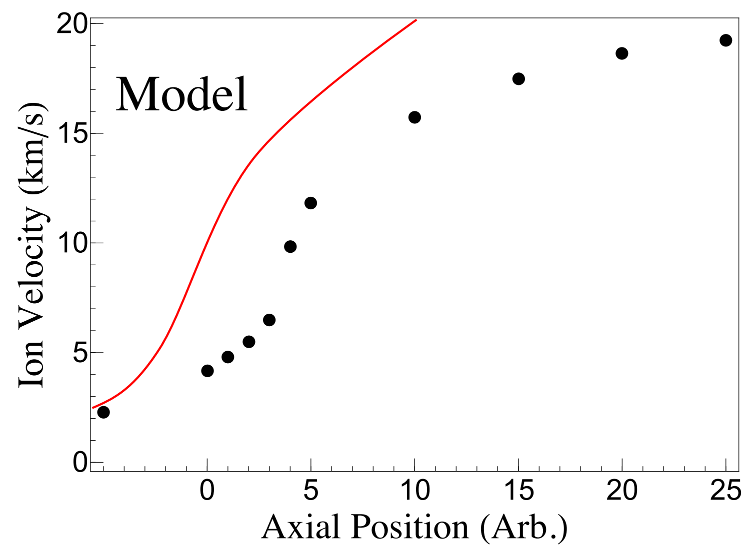
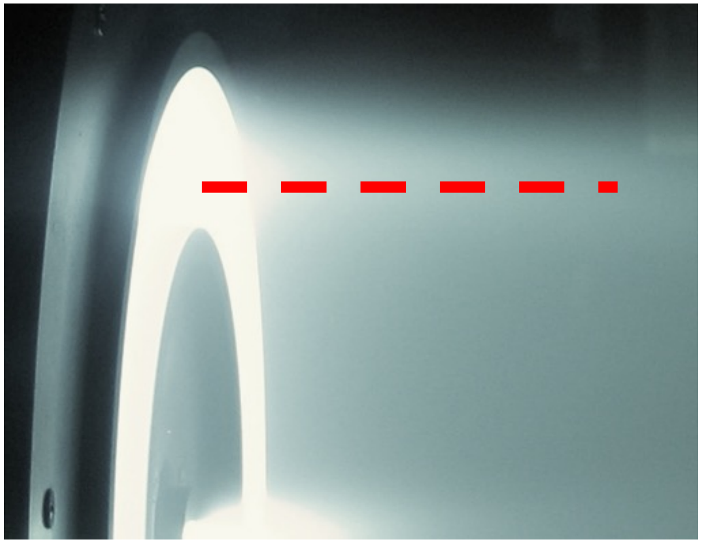
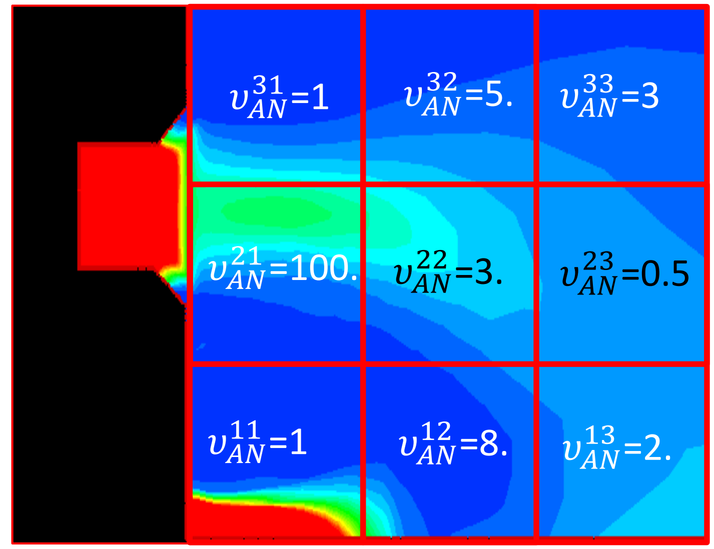
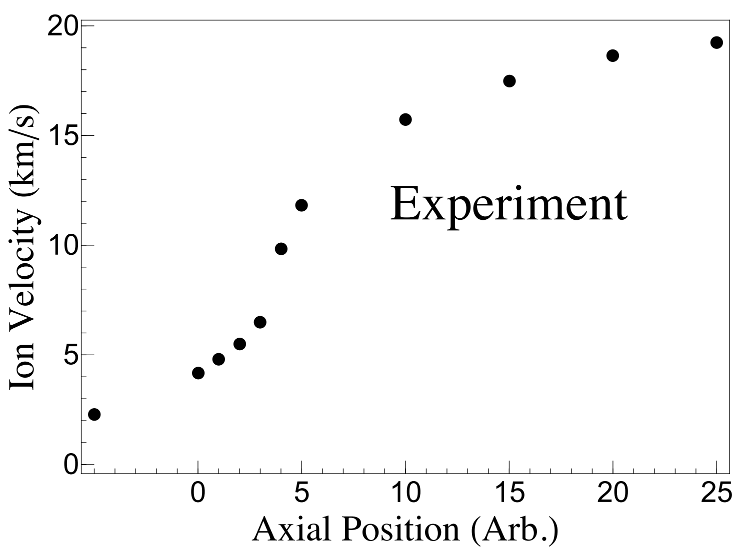


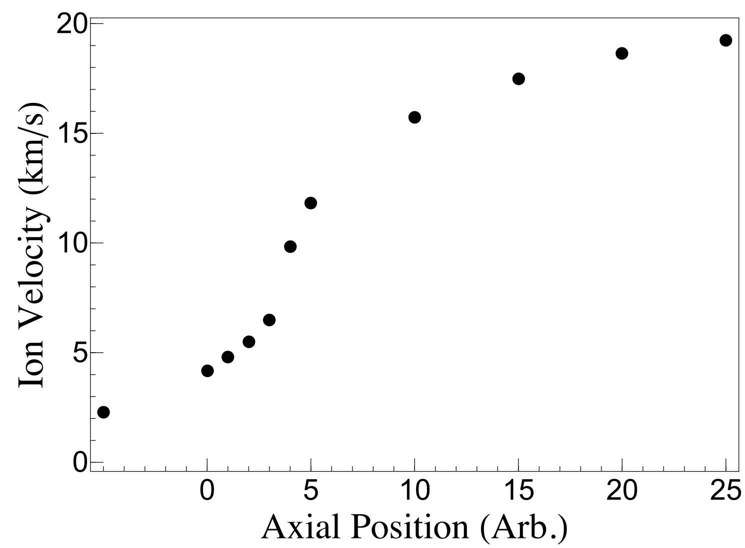
Closures for anomalous collision frequency: empirical estimate



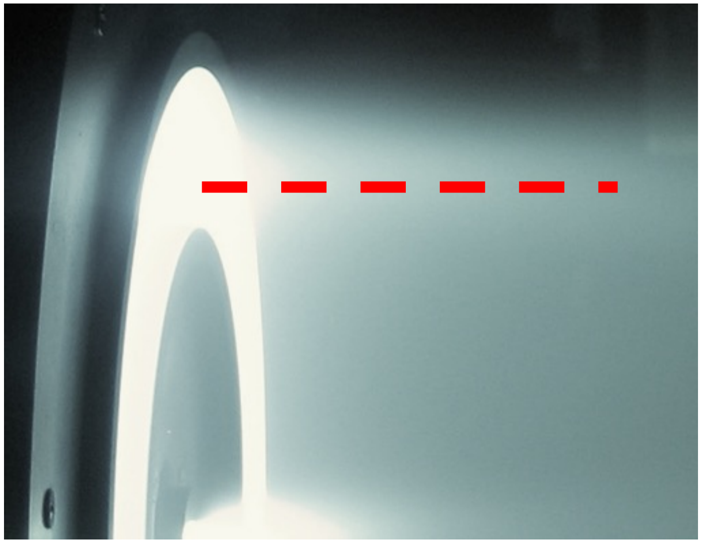


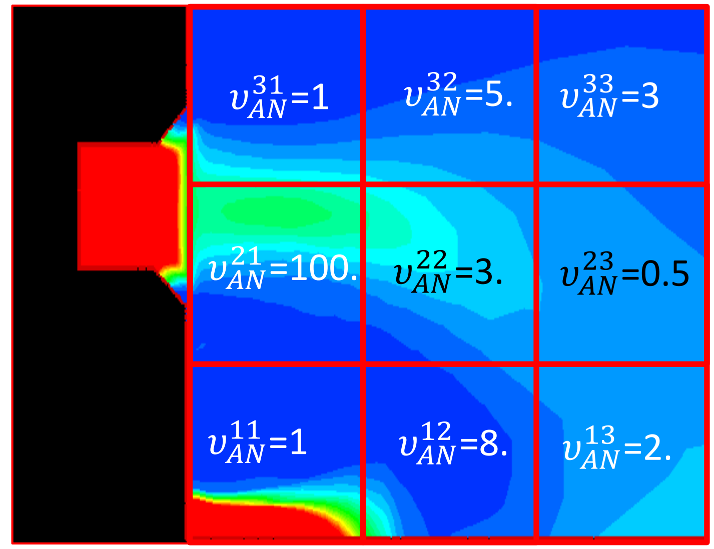


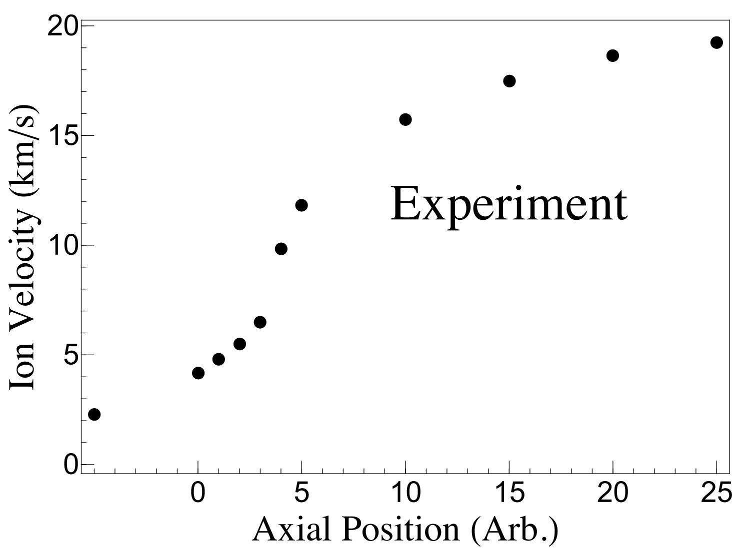
Iteration #3

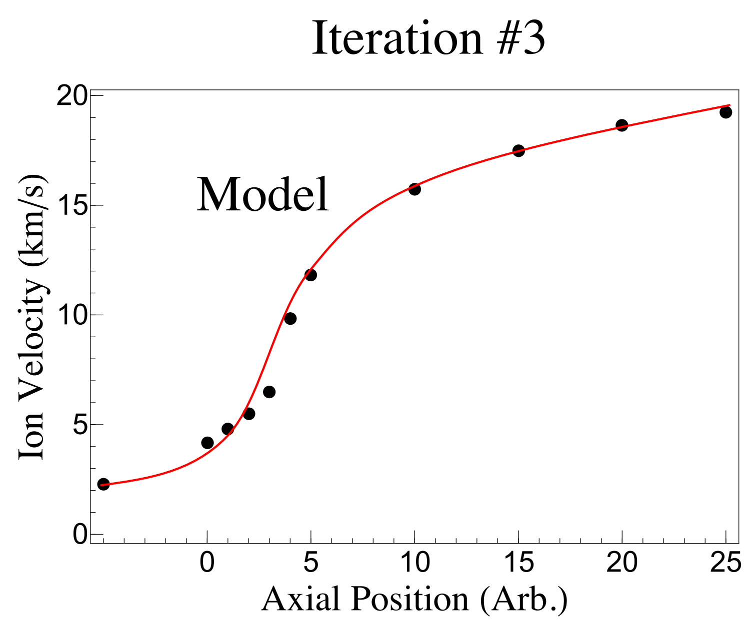


Closures for anomalous collision frequency: empirical estimate





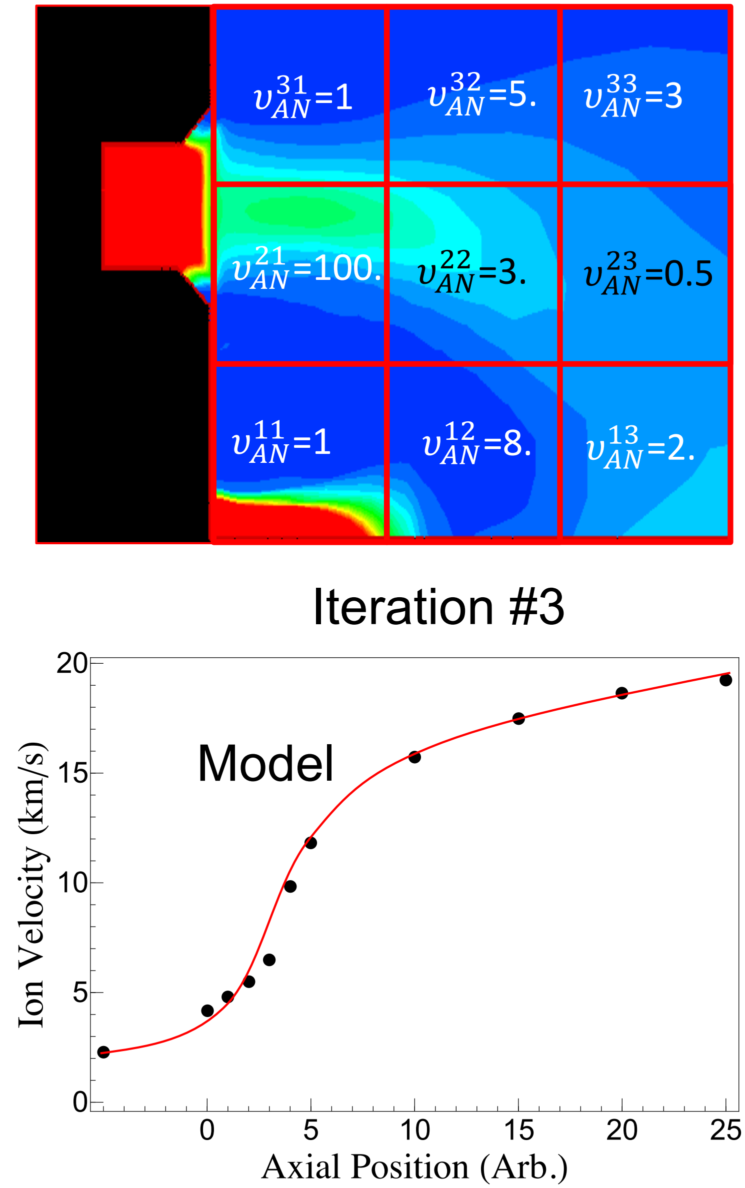




Closures for anomalous collision frequency: empirical estimate

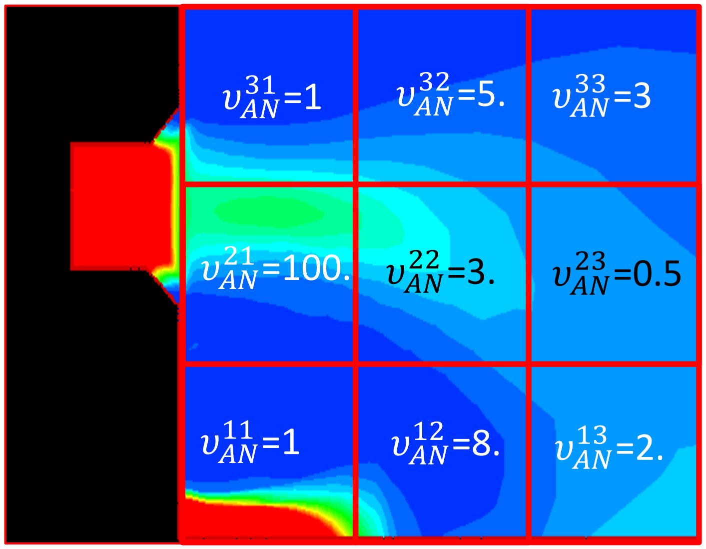
- Yields excellent agreement with experimental results for a given operating condition

- Collision frequency is specified empirically. Only applicable for data set used for validation

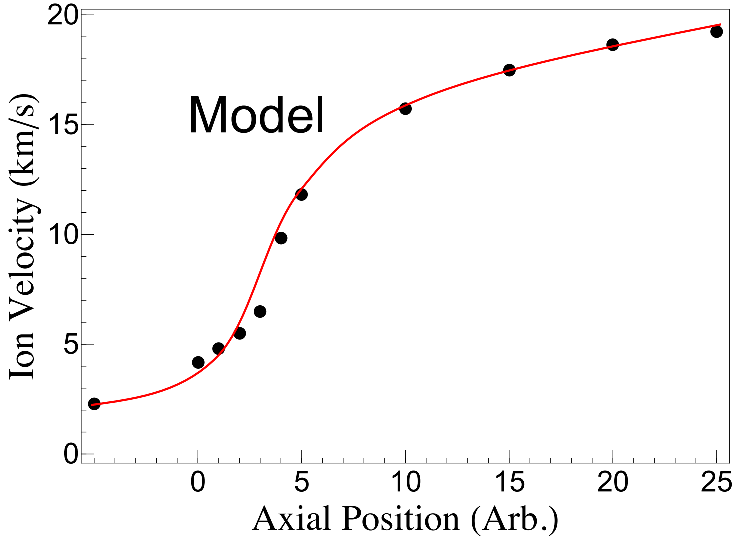


Closures for anomalous collision frequency: empirical estimate

- Yields excellent agreement with experimental results for a given operating condition
- Collision frequency is specified empirically. Only applicable for data set used for validation
- **To date, empirical models have not been predictive**



Iteration #3



Closures for anomalous collision frequency: empirical estimate

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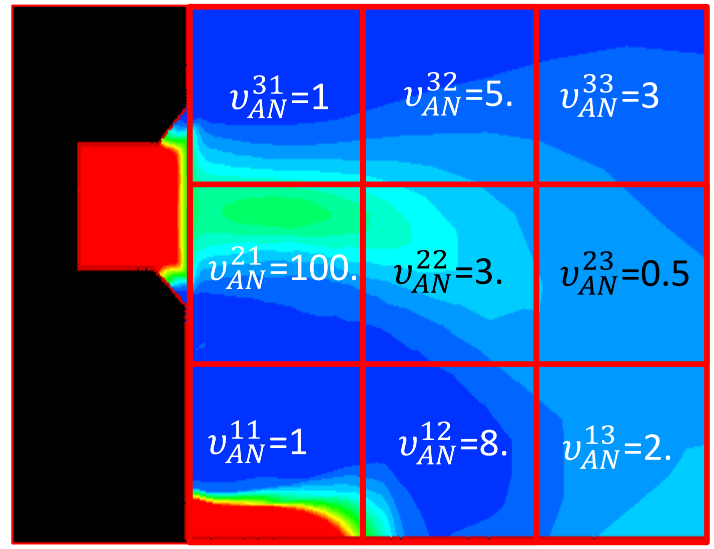
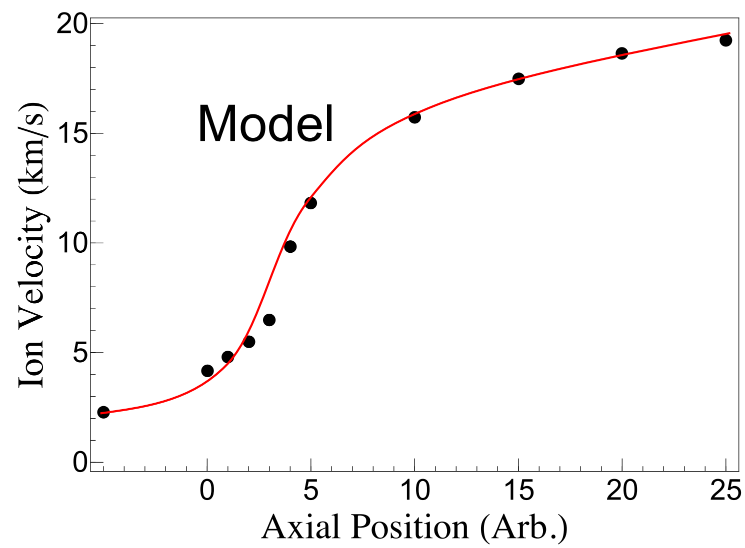
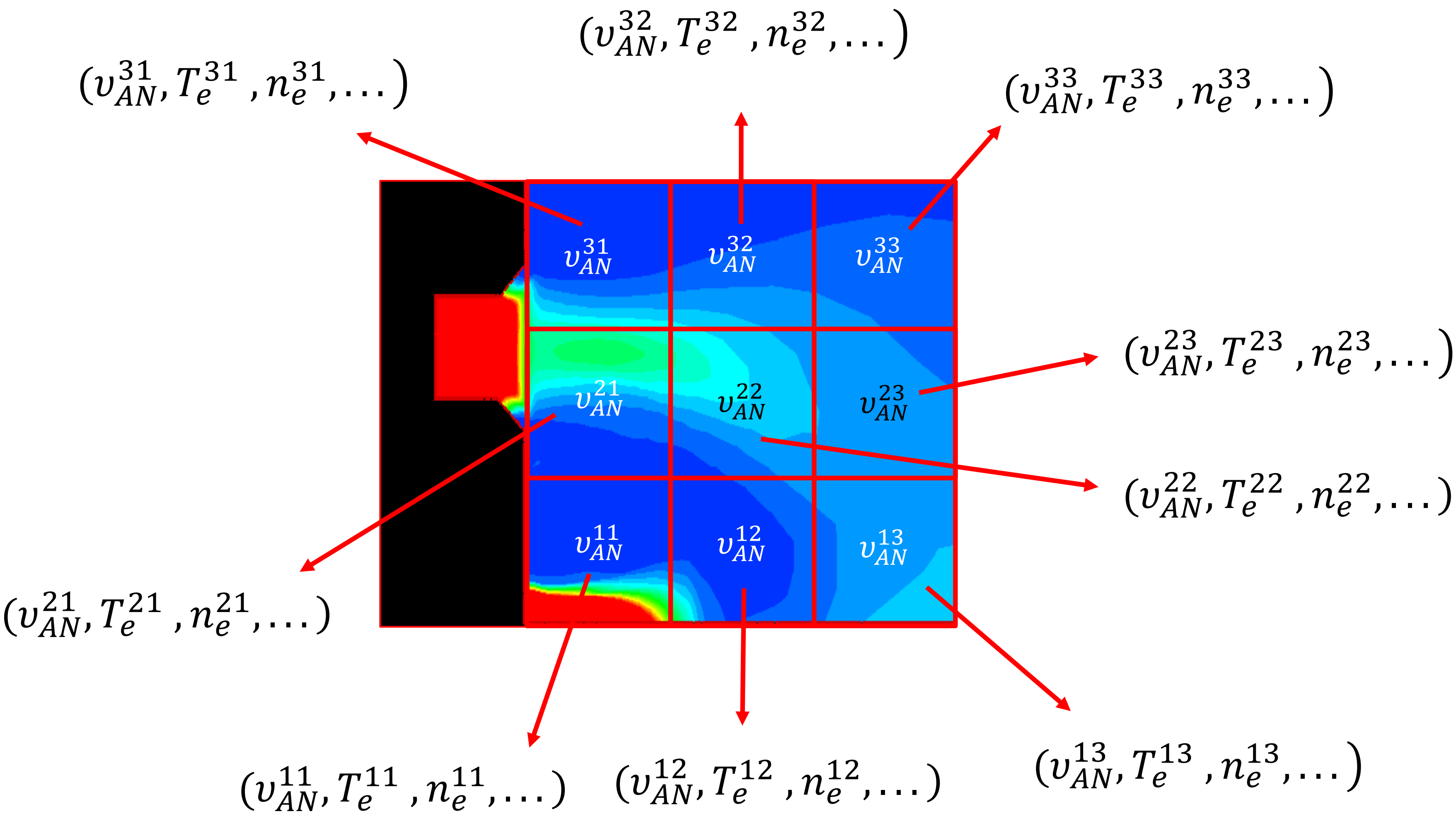
Hypothesis: we can use empirical data to generate a functional form, $\nu_{AN}(T_e, n_e, ..)$



Iteration #3



Model from regression

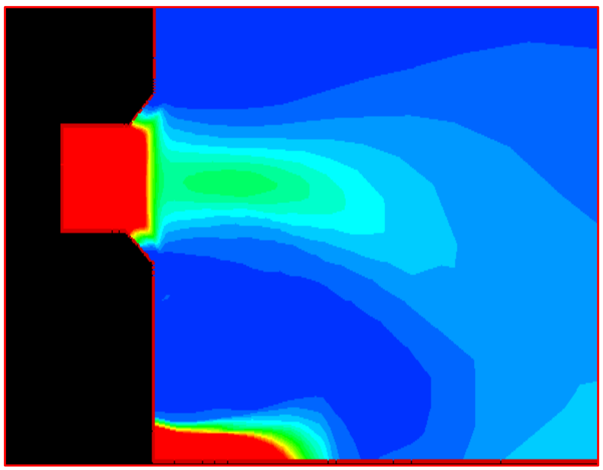


Each point from empirical model yields data point

Maybe there is a function, $\nu_{AN}(T_e, n_e, ...)$, that fits the data

Regression with machine learning

Generate datasets from empirically validated codes



7 operating conditions from 4 different thrusters from Hall2De*: 700 data points

Prepare datasets for regression

Frequencies normalized by electron cyclotron frequency, ω_{ce}	
Ion plasma frequency	ω_{pi}
Classical electron collision frequency	f_e
Classical ion collision frequency	f_i
Velocities normalized by ion sound speed, c_s	
Ion axial velocity	u_i
Electron Hall velocity	v_{de}
Length scales normalized by electron Larmor radius, r_{ce}	
Debye length	λ_{de}
Pressure gradient length-scale	$L_P = P_e/\nabla P_e$
Ion drift velocity length-scale	$L_{ui} = u_i/\nabla u_i$

8 normalized lengthscales, velocities, and frequencies

Apply ML regression algorithm

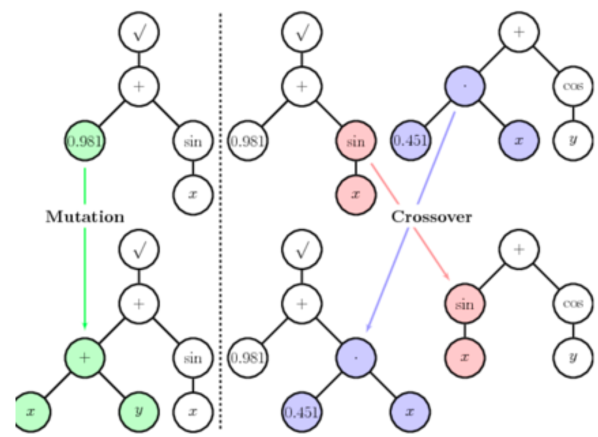
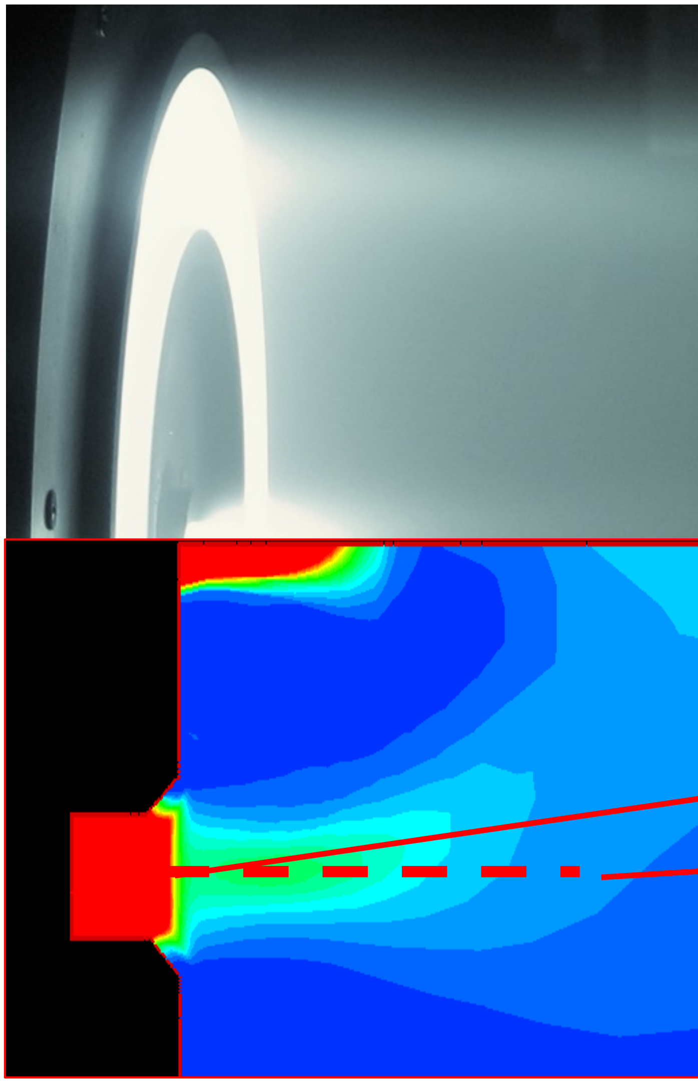


Image credit: M. Quade, Phys Rev. E. no 1. 2016

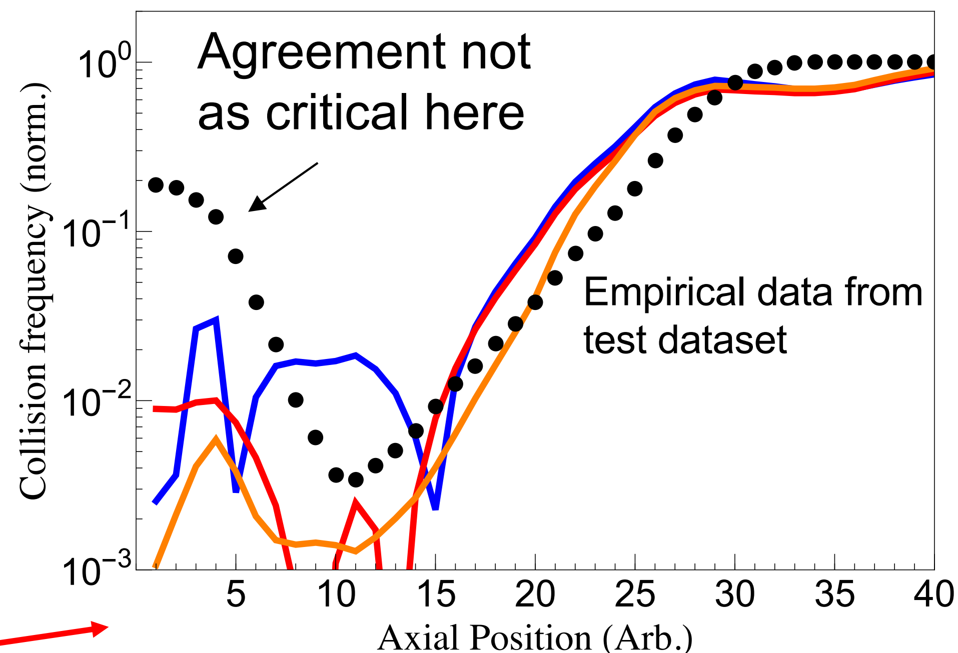
DataModeler symbolic regression from *Evolved Analytics*

*I. G. Mikellides and I. Katz, *Phys. Rev. E* vol. 86, no. 4, pp. 1–17, 2012.

Data-driven model predictions



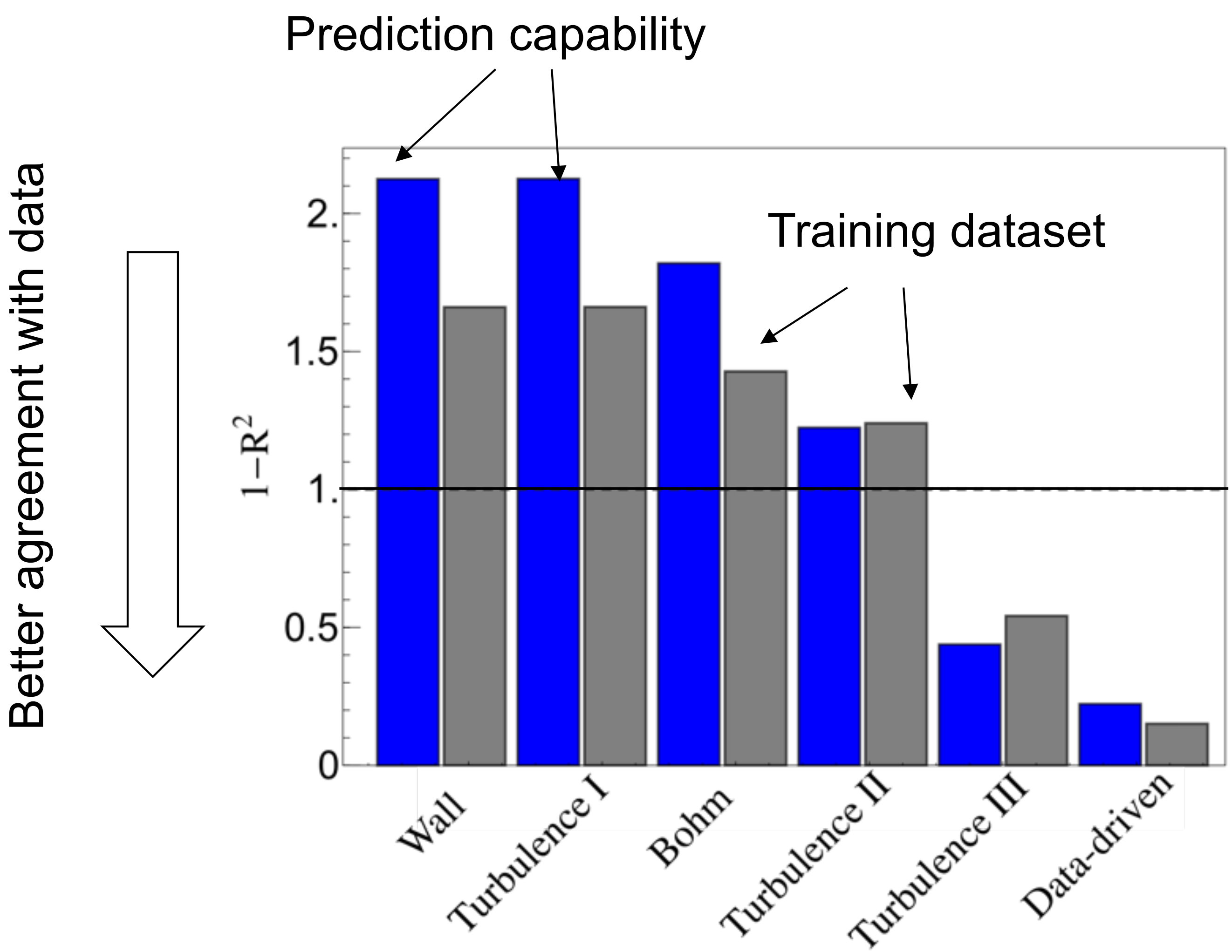
Model predictions for collision frequency



Note: model collision frequency independent of position

B. Jorns, Plasma Sources Science and Technology, Vol. 27, No. 10, 2018

Comparison of ML to first-principles models

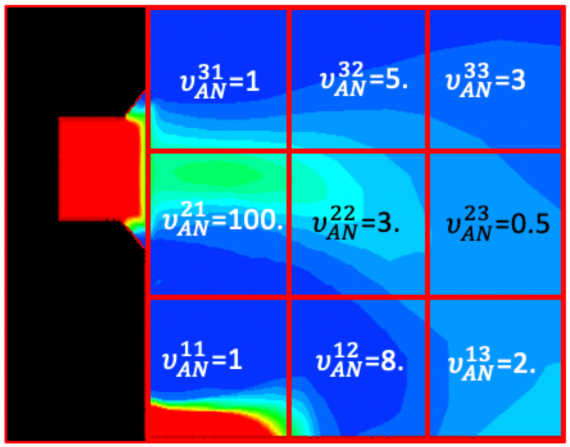


ML model has best correspondence and predictive capability of proposed closures

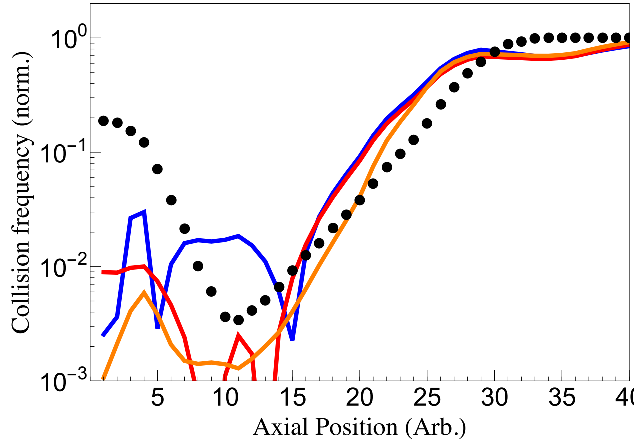
<u>Practically</u>, can this be used for predictive models?

Summary of data-driven approach to closure

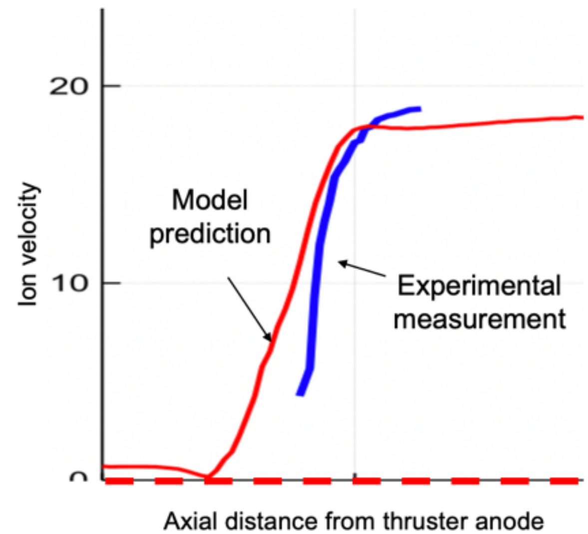
Use experimental data to measure $\nu_{AN}(T_e, n_e, ..)$ over several operating conditions



Regress data to build empirical closure model



Implement self-consistently into model



- Data-driven approach yields closures that better match experimental measurements of collision frequency
- The formulations are symbolic and can be incorporated into self-consistent models
- Not clear about the quality of predictions or extensibility outside datasets.

Summary

- Electric propulsion is a highly enabling technology for space exploration
- The Hall effect thruster is one of the most popular and successful forms of electric propulsion
- There are still outstanding and operationally critical open questions about Hall thruster operation. These mostly relate to the lack of fully predictive models
- The problem of non-classical electron transport must be solved (or bounded) to develop predictive models.
- We are exploring both physics-based and data-driven approaches.
- Results will have direct implications for future qualification and development efforts

Acknowledgments

- Zach Brown (PhD Candidate, NSF Fellow)
- Dr. Ethan Dale
- Thomas Marks (PhD Candidate)