

Modeling transport and instabilities in low-temperature magnetized plasma sources

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Acknowledgments:

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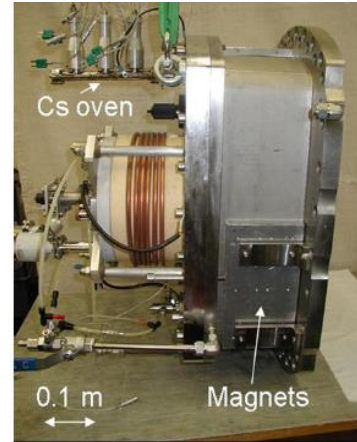
Magnetized low-temperature plasma sources



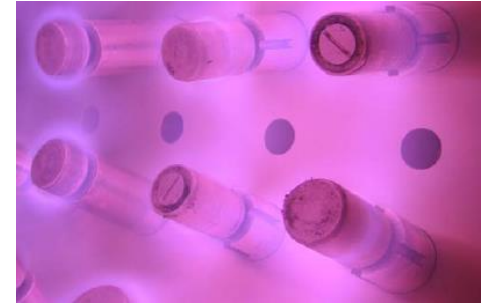
Hall thruster
(space propulsion)



End-Hall source
(materials processing)



ITER negative ion
source prototype
(neutral beam
injection)



Multidipolar ECR source

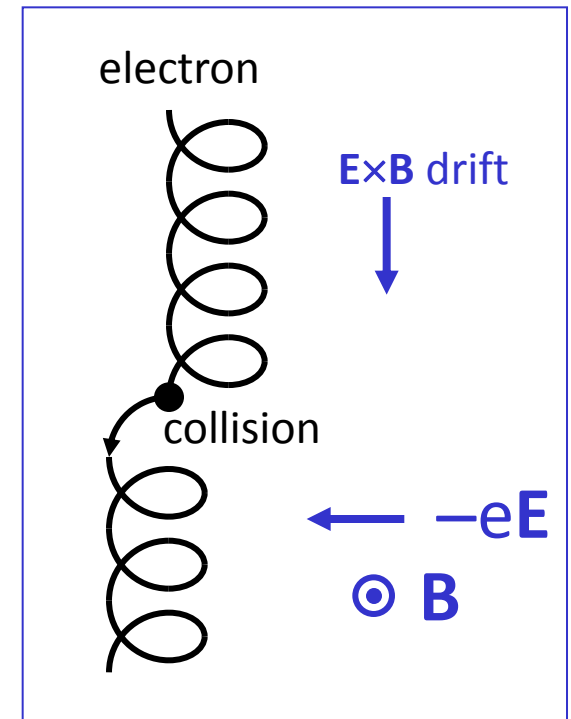
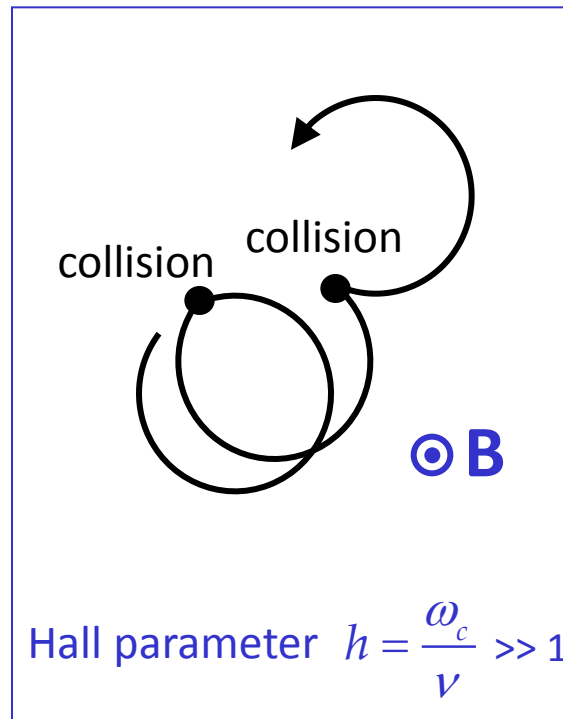
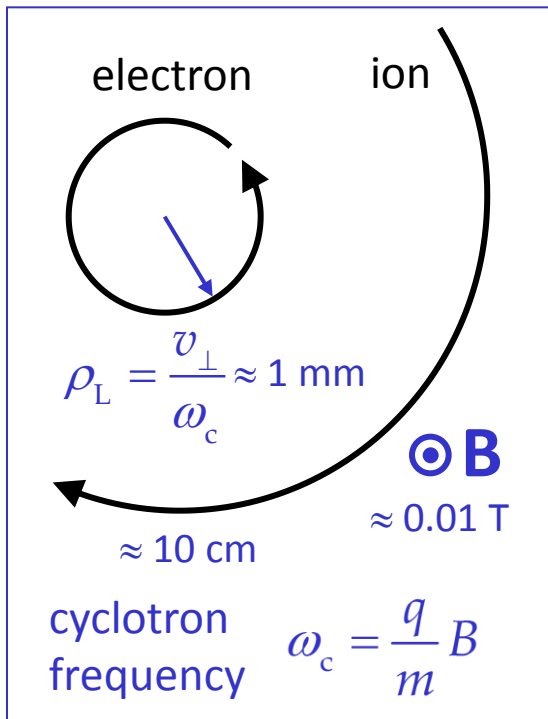


Magnetron
(sputtering
deposition)

- Many plasma sources operating at low gas pressure (< 1 Pa) use a magnetic field to limit transport losses (confinement)
- Transport in these plasma can be very complex and specific and pose major challenges for (numerical) modeling

Magnetized particle orbits

- Electrons confined by cyclotron orbits, ions only sometimes/partially
- Collisions with background gas destroy magnetic confinement, but this is effect is weak at low pressure
- Electric force causes magnetic drift in $\mathbf{E} \times \mathbf{B}$ direction



Self-consistent plasma transport models

- Separate descriptions of electrons and ions
- Coupled by Poisson equation or quasi-neutrality condition
- Boundary conditions for plasma edge
- Time stepping + 1D, 2D or 3D in space
- Different methods:
 - Particle-in-cell (PIC): trajectories of "super" particles tracked on grid, accurate but heavy and cumbersome
 - Fluid models: equations for continuity, momentum & energy, fast and instructive, but inaccurate and missing some kinetic effects
 - Hybrid models: combination of previous, e.g. fluid magnetized electrons + PIC non-magnetized ions

Electron fluid equations

■ Continuity equation: $\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{w}_e) = S = n_e \nu_{iz}(T_e)$ ionization frequency

■ Electron momentum equation:

$$m_e \frac{\partial n_e \mathbf{w}_e}{\partial t} + m_e \nabla \cdot (n_e \mathbf{w}_e \otimes \mathbf{w}_e) + m_e \nu_e n_e \mathbf{w}_e = \overbrace{-en_e \mathbf{E} - \nabla(n_e T_e)}^{\text{net driving force}} - en_e \mathbf{w} \times \mathbf{B}$$

inertia
collisions
electric force
pressure force
magnetic force

Electron fluid equations

■ Continuity equation: $\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{w}_e) = S = n_e \nu_{iz}(T_e)$ ionization frequency

■ Electron momentum equation:

$$\begin{aligned}
 m_e \frac{\partial n_e \mathbf{w}_e}{\partial t} + m_e \nabla \cdot (n_e \mathbf{w}_e \otimes \mathbf{w}_e) + m_e \nu_e n_e \mathbf{w}_e &= \overbrace{-en_e \mathbf{E} - \nabla(n_e T_e)}^{\text{net driving force}} - en_e \mathbf{w} \times \mathbf{B} \\
 &\quad \text{inertia} \qquad \text{collisions} \qquad \text{electric force} \quad \text{pressure force} \qquad \text{magnetic force}
 \end{aligned}$$

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■ Electron momentum equation:

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inertia collisions net driving force

■ Mobility tensor

electron flux = $n_e \mathbf{w}_e = -\boldsymbol{\mu} \cdot (n_e \mathbf{E} + \nabla(n_e T_e)) \equiv$

$$\equiv \underbrace{-\mu_{//} (n_e \mathbf{E}_{//} + \nabla_{//}(n_e T_e))}_{\text{parallel}} - \underbrace{\mu_{\perp} (n_e \mathbf{E}_{\perp} + \nabla_{\perp}(n_e T_e))}_{\text{perpendicular}} + \underbrace{\mu_{\times} (n_e \mathbf{E}_{\perp} + \nabla_{\perp}(n_e T_e)) \times (\mathbf{B} / B)}_{\text{perpendicular to both } \mathbf{B} \text{ and forces}}$$

$$\mu_{//} = \frac{e}{m_e \nu_e}$$

parallel to $\mathbf{B}$
very large, no confinement

$$\mu_{\perp} = \frac{\mu_{//}}{1 + h^2} \approx \frac{m_e \nu_e}{e B^2}$$

perpendicular to $\mathbf{B}$
very small

$$\mu_{\times} = \frac{h \mu_{//}}{1 + h^2} \approx \frac{1}{B}$$

perpendicular to both $\mathbf{B}$ and forces
= magnetic drift

typically $\mu_{//} : \mu_{\perp} : \mu_{\times} = 1000000 : 1 : 1000$

$$\mathbf{E}_{//} = \mathbf{E} \cdot (\mathbf{B} / B)$$

$$\mathbf{E}_{\perp} = \mathbf{E} - \mathbf{E}_{//}$$

Electron fluid equations (2)

- Drift-diffusion-continuity & current conservation equations:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (-n_e \boldsymbol{\mu} \cdot \mathbf{E} - \boldsymbol{\mu} \cdot \nabla (n_e T_e)) = S(T_e)$$

$$\nabla \cdot \mathbf{j} = \nabla \cdot (-en_e \boldsymbol{\mu} \cdot \nabla \Phi + e\boldsymbol{\mu} \cdot \nabla (n_e T_e) + en_i \mathbf{w}_i) = 0 \quad (n_e = n_i)$$

Electron fluid equations (2)

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- Electron energy equation:

$$\frac{3}{2} \frac{\partial n_e T_e}{\partial t} + \frac{5}{2} \nabla \cdot (n_e \mathbf{w}_e T_e) + \nabla \cdot \mathbf{Q} = -en_e \mathbf{w}_e \cdot \mathbf{E} + P_{\text{ext}} - C(T_e)$$

heat flux
electric
external
collisional

power
heating
losses

- Heat flux equation:

~~$$\frac{\partial \mathbf{Q}}{\partial t} + \nu_e \mathbf{Q} - \frac{e}{m_e} \mathbf{B} \times \mathbf{Q} = -\frac{5}{2} n_e T_e \nabla T_e$$~~

$$\longrightarrow \mathbf{Q} = -\underset{\substack{\uparrow \\ \text{thermal conductivity tensor}}}{\boldsymbol{\kappa}} \cdot \nabla T_e \equiv -\frac{5}{2} n_e T_e \boldsymbol{\mu} \cdot \nabla T_e$$

Electron fluid equations (2)

- Drift-diffusion-continuity & current conservation equations:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (-n_e \boldsymbol{\mu} \cdot \mathbf{E} - \boldsymbol{\mu} \cdot \nabla (n_e T_e)) = S(T_e)$$

$$\nabla \cdot \mathbf{j} = \nabla \cdot (-en_e \boldsymbol{\mu} \cdot \nabla \Phi + e \boldsymbol{\mu} \cdot \nabla (n_e T_e) + en_i \mathbf{w}_i) = 0 \quad (n_e = n_i)$$

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heat flux
electric
external
collisional

power
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losses

- Heat flux equation:

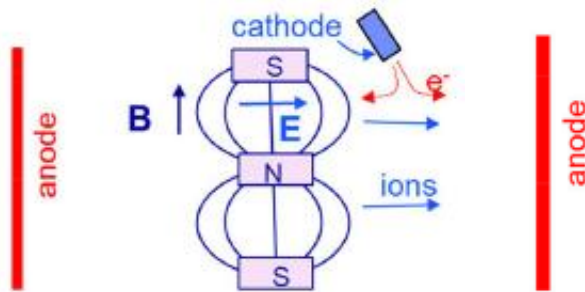
$$\cancel{\frac{\partial \mathbf{Q}}{\partial t}} + \nu_e \mathbf{Q} - \frac{e}{m_e} \mathbf{B} \times \mathbf{Q} = -\frac{5}{2} n_e T_e \nabla T_e \quad \longrightarrow \quad \mathbf{Q} = -\underset{\substack{\uparrow \\ \text{thermal conductivity tensor}}}{\boldsymbol{\kappa}} \cdot \nabla T_e \equiv -\frac{5}{2} n_e T_e \boldsymbol{\mu} \cdot \nabla T_e$$

- Boundary conditions for fluxes at walls
- Coupled with ion transport model via quasineutrality or Poisson equation

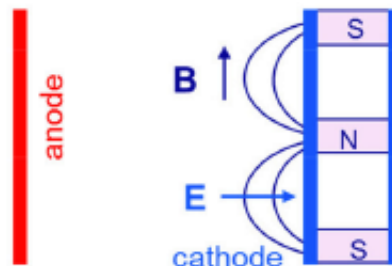
Axisymmetric E×B discharge plasmas

- Axisymmetric geometry with $\mathbf{B}$ in axial-radial plane and voltage applied across magnetic field lines
- 2D fluid equations are elliptic with strong anisotropy due to $\mu_{\perp} \ll \mu_{\parallel}$ (μ_x plays no role)

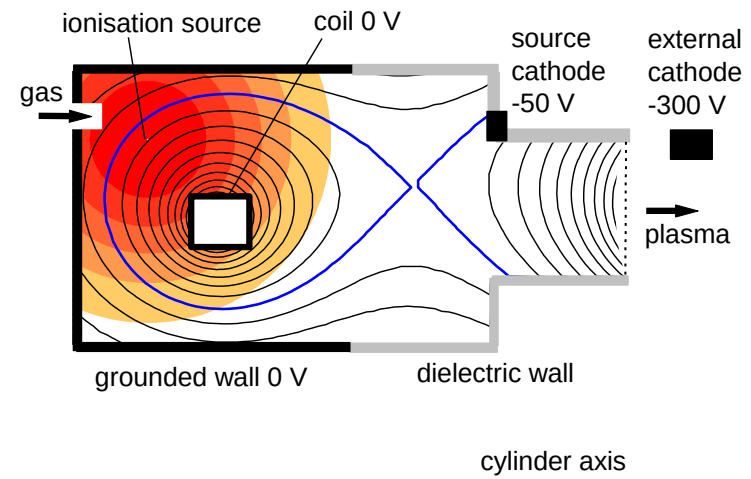
$$\nabla \cdot \mathbf{j} = \nabla \cdot \left(-e\mu_{\perp} (n_e \nabla_{\perp} \Phi - \nabla_{\perp} (n_e T_e)) - e\mu_{\parallel} (n_e \nabla_{\parallel} \Phi - \nabla_{\parallel} (n_e T_e)) + en_i \mathbf{w}_i \right) = 0$$



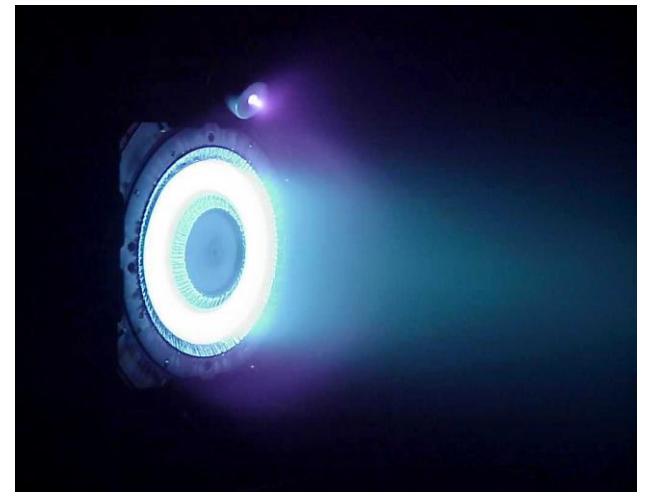
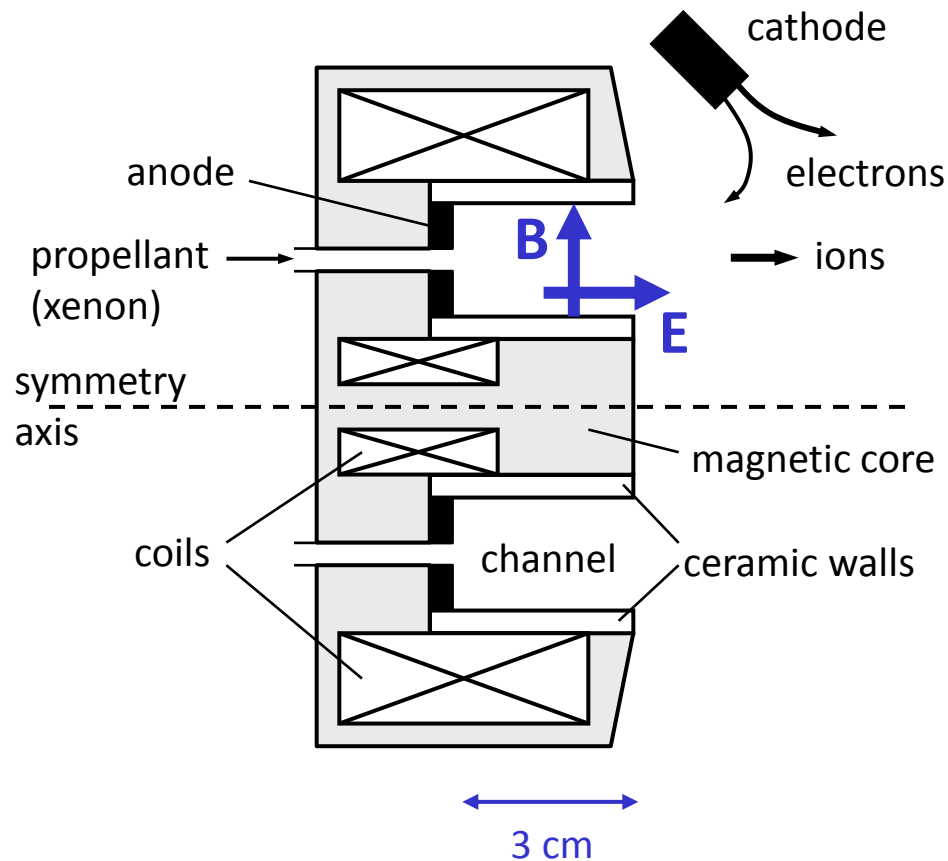
Hall ion source



Planar magnetron



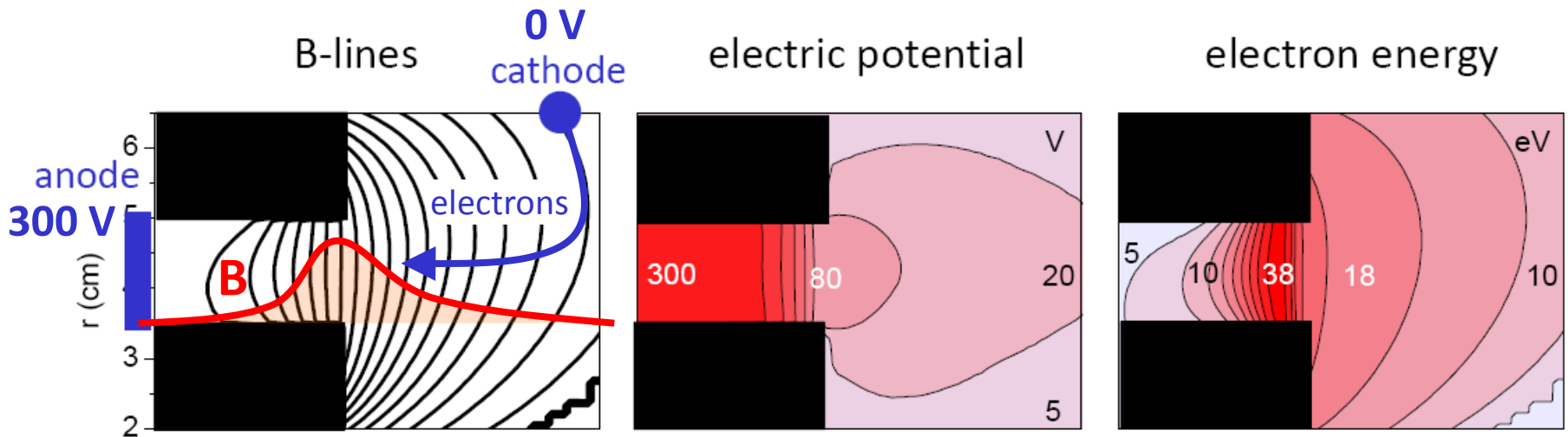
Example: Hall thruster for satellites



V. V. Zhurin et al, Plasma Sources Sci. Technol. **8** R1-R20 (1999).

A. I. Morozov and V. V. Savelyev, Reviews of Plasmas **21**, 203-391 (2000)

Electron transport in Hall thruster



- Along B:** large mobility $\rightarrow$ Boltzmann equilibrium:

$$\mu_{||} n_e \nabla_{||} \Phi \approx \mu_{||} \nabla_{||} (n_e T_e)$$

$$\rightarrow \Phi \approx \text{const} + T_e \ln(n / n_0) \quad T_e \approx \text{const}$$

Field lines are equipotential + diffusion term
- Across B:** transport by small perpendicular mobility $\rightarrow$

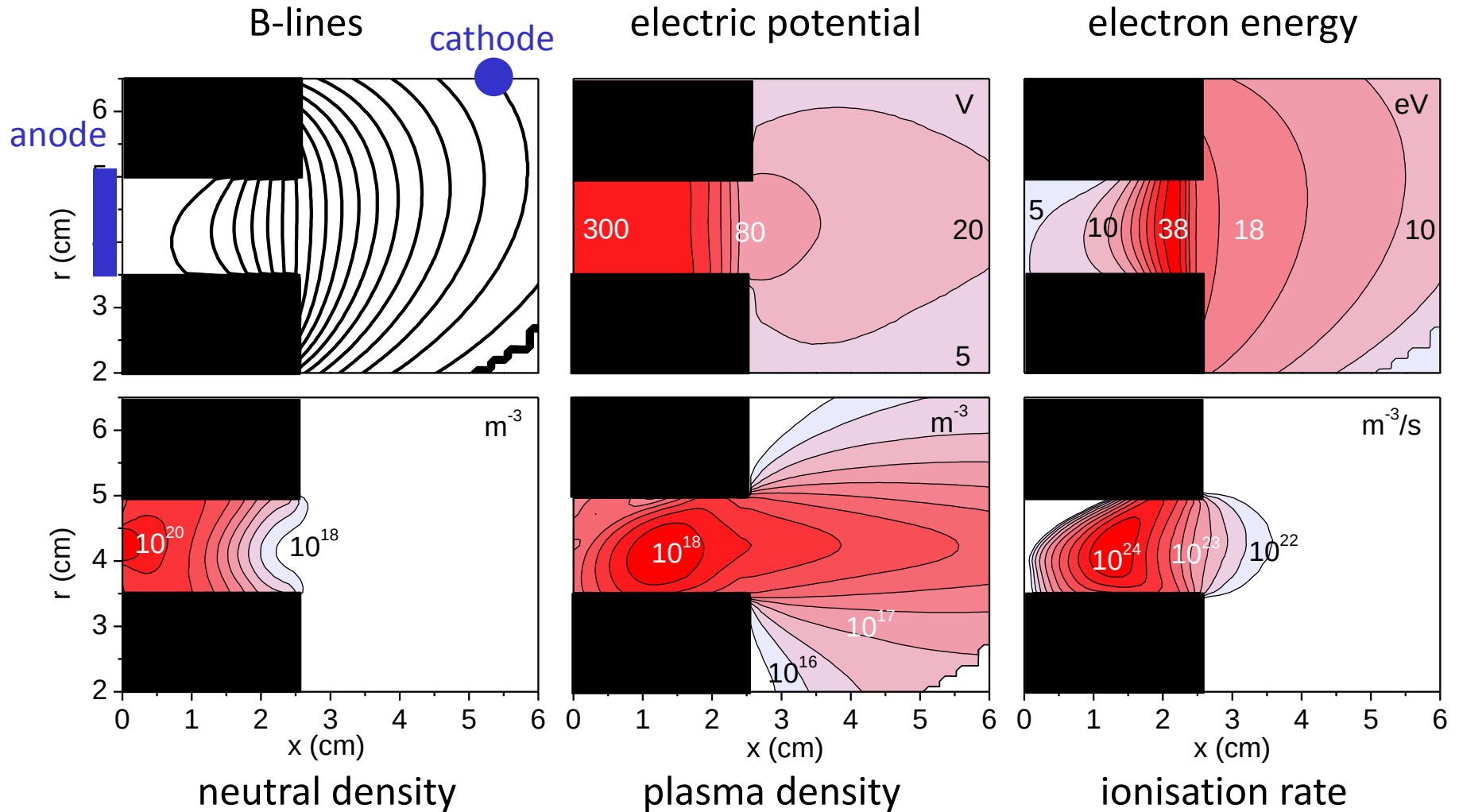
$$\mu_{\perp} \approx \frac{mv}{eB^2} + \dots$$

Electric potential drop in magnetic field barrier to satisfy current continuity:

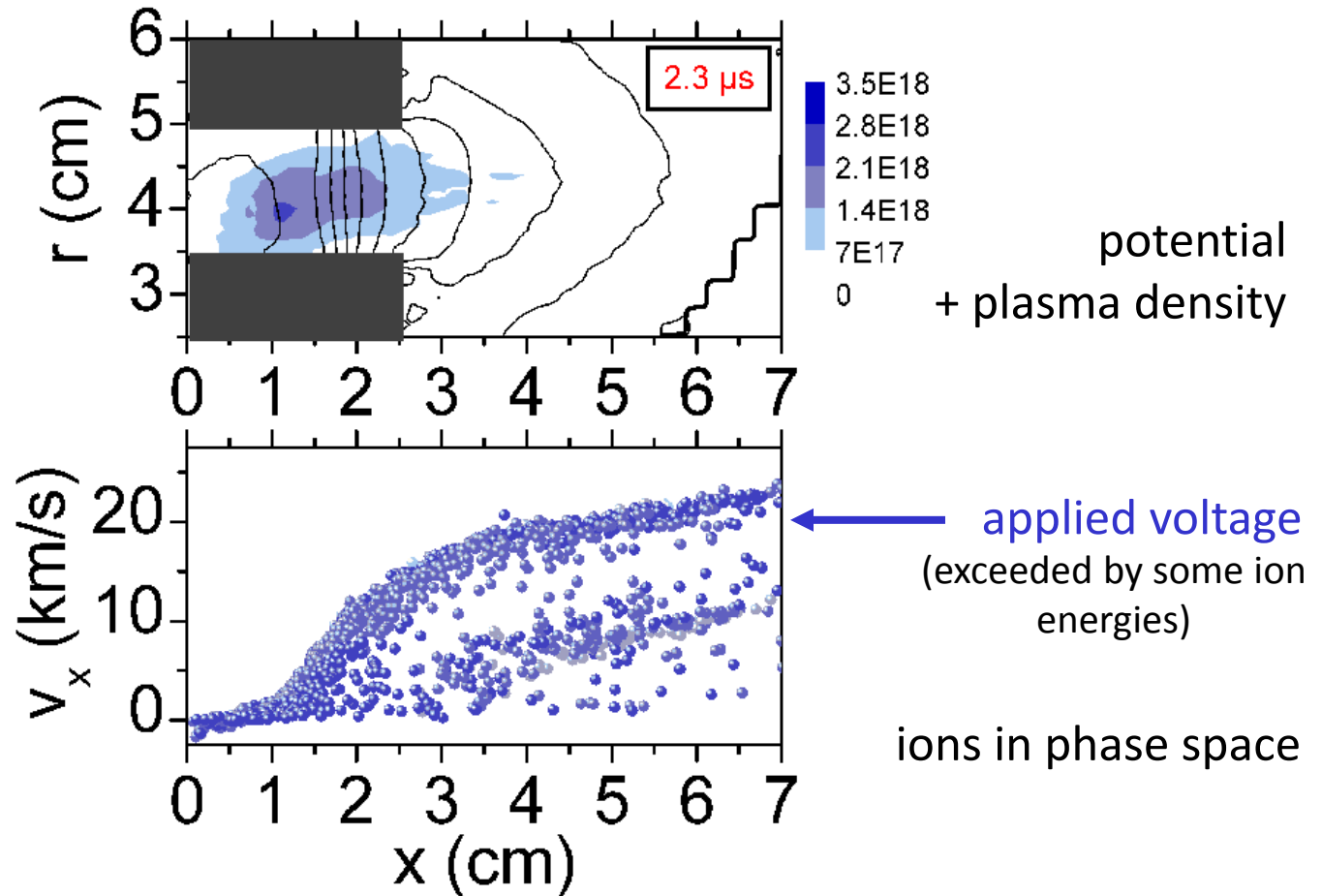
$$-e \int \mu_{\perp} n \nabla_{\perp} \Phi ds = I - e \int \Gamma_{i\perp} ds - e \int \mu_{\perp} \nabla_{\perp} (nT) ds$$

Conductance $\times$ Voltage = Current (Ohm's law)

Structure of Hall thruster discharge (time averaged)



Transit-time instability in Hall thruster



Problem: anomalous mobility

- Measured perpendicular mobility $\gg$ classical expression

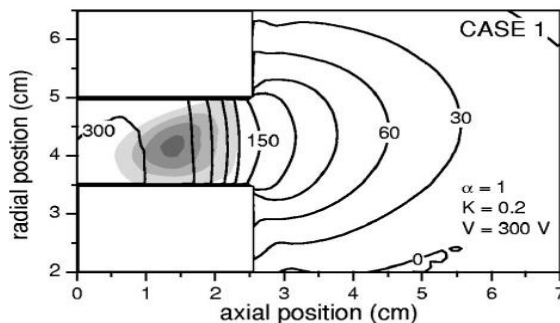
- Add anomalous mobility with fit parameters:

$$\mu_{\perp} = \frac{mv}{eB^2} + \alpha \frac{mv_0}{eB^2} + K \frac{1}{16B}$$

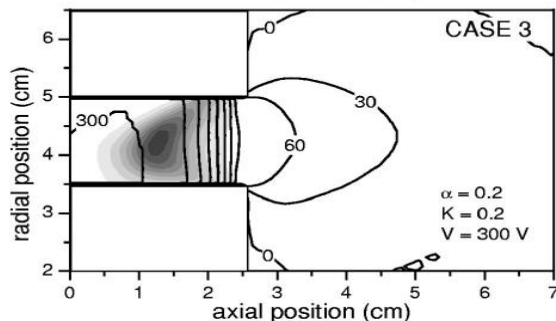
Bohm limit
(1949)

- Due to wall collisions and turbulence

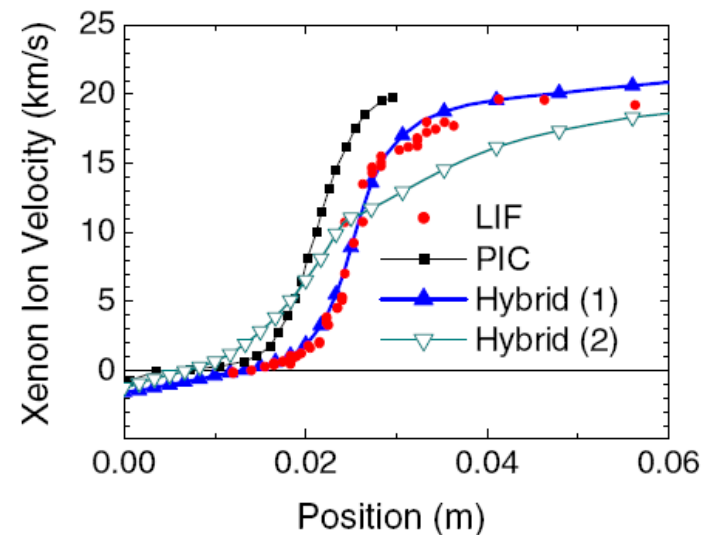
Potential + ionization rate



$\alpha = 1$
 $K = 0.2$



$\alpha = 0.2$
 $K = 0.2$



J. Bareilles et al, Phys. Plasmas **11**, 3035-3046 (2004)

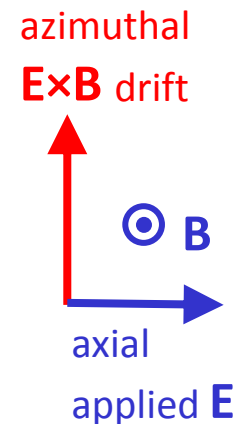
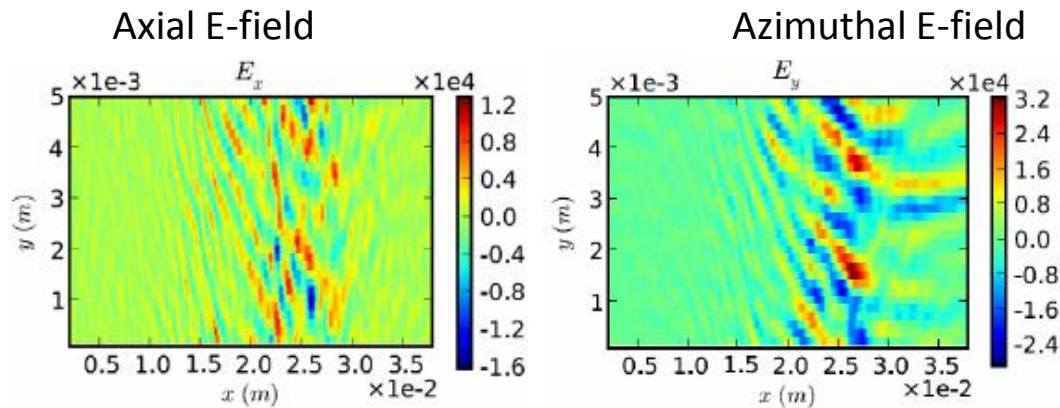
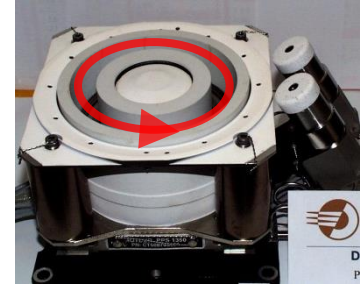
Adam et al, Plasma Phys. Contr. Fusion **50**, 124041 (2008)

E×B drift instabilities

- Closed E×B drift along azimuthal direction tends to develop instabilities/turbulence
- Instable/turbulent E×B drift can "leak" across barrier, contributing to "anomalous mobility"

$$n_e w_{e,z} = -\mu_{\perp} n_e E_z - \mu_{\perp} \frac{\partial n_e T_e}{\partial z} - \mu_{\times} \langle \tilde{n}_e \tilde{E}_{\phi} \rangle$$

- 2D PIC simulations and collective diffusion experiments: electron-cyclotron drift instabilities (ECDI)

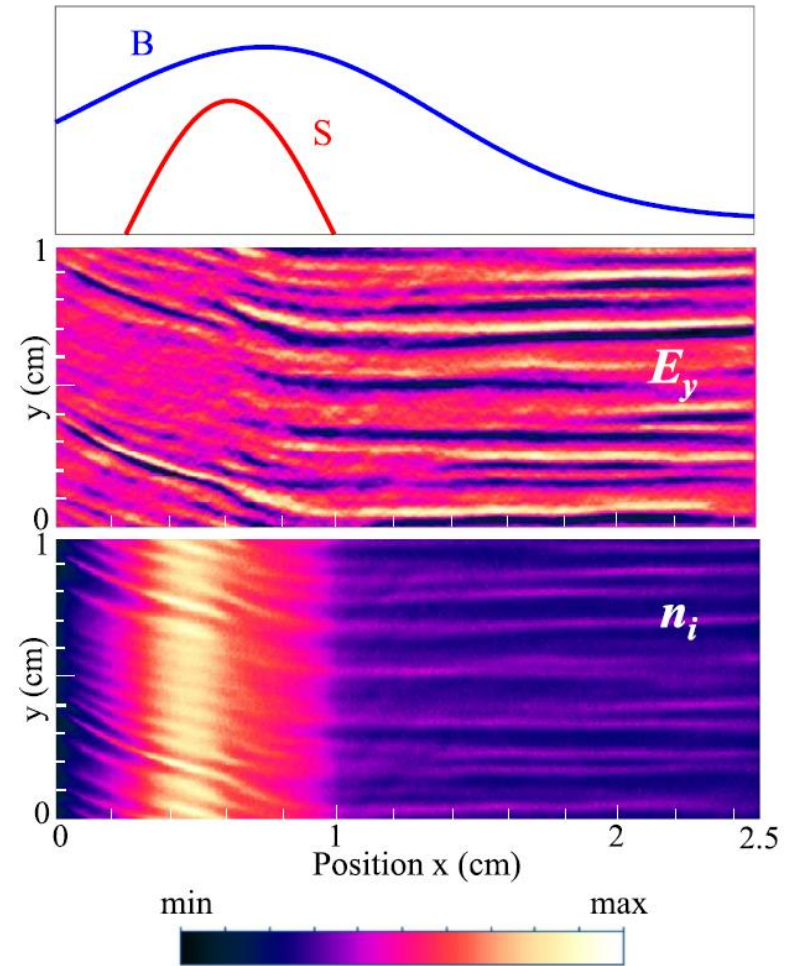
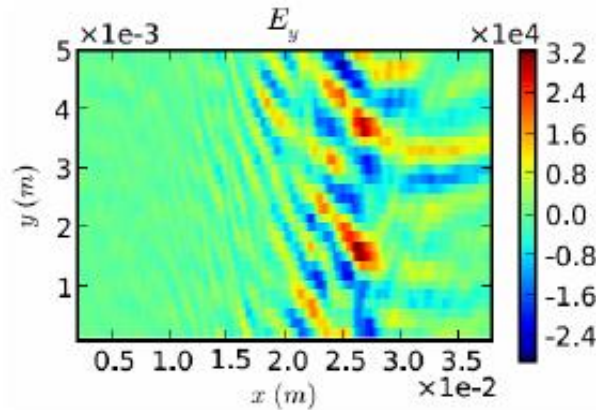


J.C. Adam et al, Plasma Phys. Control. Fusion **50**, 124041 (2008)

Tsikata et al, Phys. Plasmas **16**, 033506 (2009)

PIC simulations of ECDI in Hall thruster

- Electron Cyclotron Drift Instabilities: resonance of Bernstein and ion acoustic modes – kinetic!



Adam et al, Plasma Phys. Contr. Fusion **50**, 124041 (2008)

Tsikata et al, Phys. Plasmas **16**, 033506 (2009)

Lafleur et al, Phys. Plasmas **23**, 053503 (2016)

Boeuf et al, Phys. Plasmas **25**, 061204 (2018)

LANDMARK project – www.landmark-plasma.com

re | <https://www.landmark-plasma.com>

LANDMARK

Low temperature magnetized plasma benchmarks

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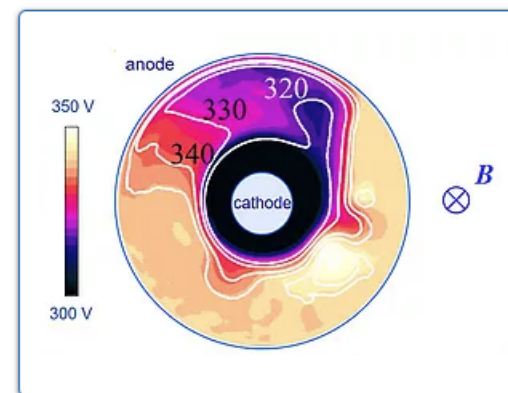
Aim and scope

The physics of low temperature magnetized plasmas or ExB plasma devices is very specific and different from that of fusion plasmas. This distinct area of basic plasma physics has been poorly explored and is not well understood but is within the reach of current analytical and numerical capabilities as well as powerful modern diagnostics tools.

The physics of magnetized plasmas and ExB devices is however complex and very non-linear and the scientific community must organize itself to better understand and quantify the properties of these plasmas.

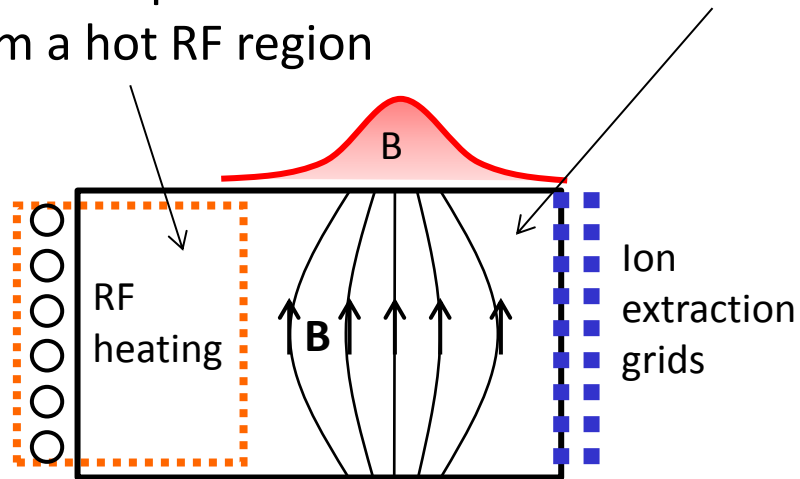
The LANDMARK project aims at

- providing an open forum for evaluating methods of description of plasma transport in non-fusion magnetized plasmas (eg ion sources, HALL thrusters, magnetrons, cusped-field thrusters ...)
- defining benchmark test cases for PIC, fluid and hybrid methods



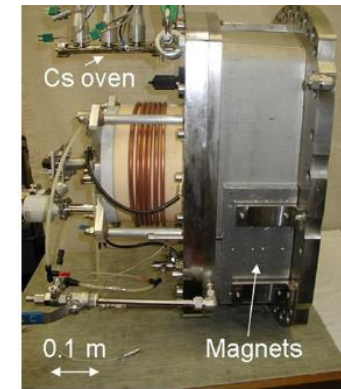
Magnetic filtering

Some (negative) ion sources use a magnetic filter to separate a cold ion extraction region from a hot RF region

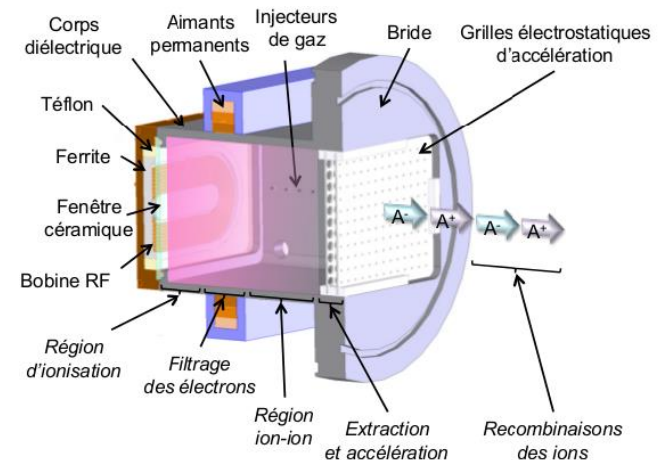


Difference with $\mathbf{E} \times \mathbf{B}$ discharge devices:

- Transport driven by pressure rather than applied voltage
- Rectangular geometry!



BATMAN prototype negative ion source for ITER NBI @ IPP Garching



PEGASES thruster @ Ecole Polytechnique

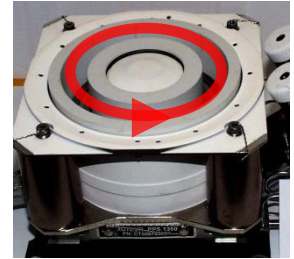
E. Speth et al., Nucl. Fusion 46, S220 (2006)

A. Aanesland et al, PSST 23, 044003 (2014)

Closed drift versus bounded drift

Cylindrical symmetry:

magnetic drift forms closed loop →
optimal v/B^2 confinement
(if no instabilities)



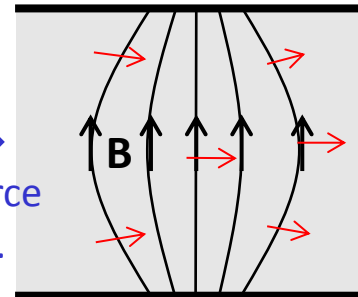
No cylindrical symmetry:

drift bounded by walls →

Hall effect: plasma asymmetry & increased
 $1/B$ transport across magnetic field barrier

Side view:

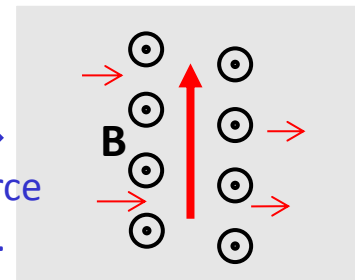
driving force
 $E, \nabla p, \dots$



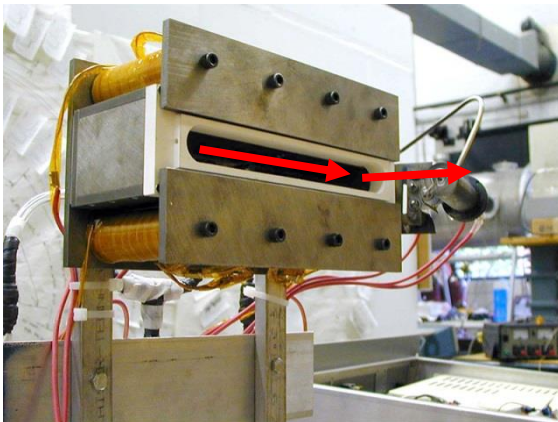
$$\mu_{\perp} \approx \frac{mv}{eB^2}$$

Top view:

driving force
 $E, \nabla p, \dots$



$$\mu_x \approx \frac{1}{B}$$



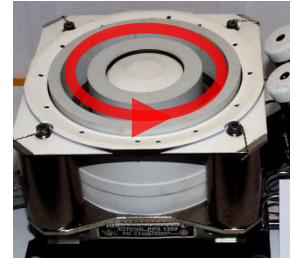
Linear plasma thruster
@ University of Michigan

Beal and Gallimore AIAA-2001-3649

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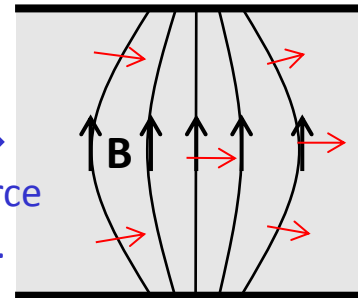
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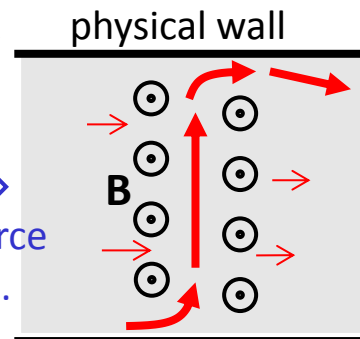
driving force
 $E, \nabla p, \dots$



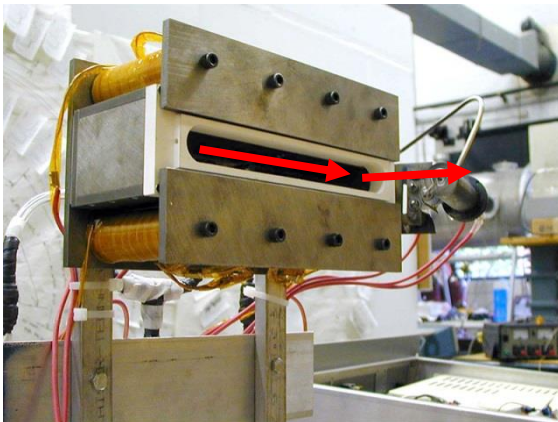
$$\mu_{\perp} \approx \frac{mv}{eB^2}$$

Top view:

driving force
 $E, \nabla p, \dots$



$$\mu_x \approx \frac{1}{B}$$



Linear plasma thruster
@ University of Michigan

Beal and Gallimore AIAA-2001-3649

MAGNIS fluid model @ LAPLACE Toulouse

- Multi-fluid: electrons, ions, neutrals
 - Continuity equations with chemistry source terms
 - Full momentum equations including inertia terms
 - Electron energy equation with magnetized heat flux
 - Quasi-neutrality: plasma potential deduced from current continuity
 - Boundary conditions from sheath theory, allowing for different wall materials: grounded, biased, dielectric
- Evolution in time and 2D plane perpendicular to magnetic field lines
 - Parallel losses (along field lines) included via effective source terms (2D+½D) allowing for dielectric, grounded, or biased walls
 - Magnetized fluxes handled through original numerical scheme using prediction/correction on shifted numerical grids

Main fluid model equations

- Continuity & momentum equations for each species α :

$$\frac{\partial n_\alpha}{\partial t} + \nabla_\perp \cdot (n_\alpha \mathbf{w}_\alpha) = S_\alpha - L_{\parallel\alpha}$$

ionization

$$L_{\parallel\alpha} = \overline{\nabla_\parallel \cdot (n_\alpha \mathbf{w}_\alpha)} \propto n_\alpha$$

losses along magnetic field lines
included in source term

Main fluid model equations

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$$\frac{\partial n_\alpha}{\partial t} + \nabla_\perp \cdot (n_\alpha \mathbf{w}_\alpha) = S_\alpha - L_{\parallel\alpha} \qquad L_{\parallel\alpha} = \overline{\nabla_\parallel \cdot (n_\alpha \mathbf{w}_\alpha)} \propto n_\alpha$$

$$m_\alpha \underbrace{\frac{\partial \mathbf{w}_\alpha}{\partial t}}_{\text{inertia}} + m_\alpha \mathbf{w}_\alpha \cdot \nabla \mathbf{w}_\alpha + m_\alpha \underbrace{\left(\nu_\alpha + \frac{S_\alpha}{n_\alpha} \right)}_{\text{collisions}} \mathbf{w}_\alpha + q_\alpha \underbrace{\mathbf{B} \times \mathbf{w}_\alpha}_{\text{magnetic force}} = -q_\alpha \underbrace{\nabla \Phi}_{\text{electric force}} - \underbrace{\frac{\nabla(n_\alpha T_\alpha)}{n_\alpha}}_{\text{pressure force}}$$

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$$m_\alpha \frac{\partial \mathbf{w}_\alpha}{\partial t} + m_\alpha \mathbf{w}_\alpha \cdot \nabla \mathbf{w}_\alpha + m_\alpha \left(v_\alpha + \frac{S_\alpha}{n_\alpha} \right) \mathbf{w}_\alpha + q_\alpha \mathbf{B} \times \mathbf{w}_\alpha = -q_\alpha \nabla \Phi - \frac{\nabla(n_\alpha T_\alpha)}{n_\alpha}$$

- Quasi-neutrality & current conservation:

$$n_e = \frac{1}{e} \sum_i q_i n_i \qquad \nabla_\perp \cdot (en_e \mathbf{w}_e(\Phi)) = \nabla_\perp \cdot \left(\sum_i q_i n_i \mathbf{w}_i \right) + \sum_\alpha q_\alpha L_{\parallel\alpha}$$

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$$\frac{\partial n_\alpha}{\partial t} + \nabla_\perp \cdot (n_\alpha \mathbf{w}_\alpha) = S_\alpha - L_{//\alpha} \quad L_{//\alpha} = \overline{\nabla_{//} \cdot (n_\alpha \mathbf{w}_\alpha)} \propto n_\alpha$$

$$m_\alpha \frac{\partial \mathbf{w}_\alpha}{\partial t} + m_\alpha \mathbf{w}_\alpha \cdot \nabla \mathbf{w}_\alpha + m_\alpha \left(\nu_\alpha + \frac{S_\alpha}{n_\alpha} \right) \mathbf{w}_\alpha + q_\alpha \mathbf{B} \times \mathbf{w}_\alpha = -q_\alpha \nabla \Phi - \frac{\nabla(n_\alpha T_\alpha)}{n_\alpha}$$

- Quasi-neutrality & current conservation:

$$n_e = \frac{1}{e} \sum_i q_i n_i \quad \nabla_\perp \cdot (en_e \mathbf{w}_e(\Phi)) = \nabla_\perp \cdot \left(\sum_i q_i n_i \mathbf{w}_i \right) + \sum_\alpha q_\alpha L_{//\alpha}$$

- Electron energy: $\frac{3}{2} \frac{\partial n_e T_e}{\partial t} + \frac{5}{2} \nabla_\perp \cdot (n_e \mathbf{w}_e T_e) + \nabla_\perp \cdot \mathbf{Q}_e = P_{\text{ext}} + en_e \mathbf{w}_e \cdot \nabla_\perp \Phi - L_{\text{col}} - L_{//T}$

$$\frac{\partial \mathbf{Q}_e}{\partial t} + \nu \mathbf{Q}_e - \frac{e}{m_e} \mathbf{B} \times \mathbf{Q}_e = -\frac{5n_e T_e}{2m_e} \nabla T_e$$

Main fluid model equations

- Continuity & momentum equations for each species α :

$$\frac{\partial n_\alpha}{\partial t} + \nabla_\perp \cdot (n_\alpha \mathbf{w}_\alpha) = S_\alpha - L_{//\alpha} \quad L_{//\alpha} = \overline{\nabla_{//} \cdot (n_\alpha \mathbf{w}_\alpha)} \propto n_\alpha$$

$$m_\alpha \frac{\partial \mathbf{w}_\alpha}{\partial t} + m_\alpha \mathbf{w}_\alpha \cdot \nabla \mathbf{w}_\alpha + m_\alpha \left(\nu_\alpha + \frac{S_\alpha}{n_\alpha} \right) \mathbf{w}_\alpha + q_\alpha \mathbf{B} \times \mathbf{w}_\alpha = -q_\alpha \nabla \Phi - \frac{\nabla(n_\alpha T_\alpha)}{n_\alpha}$$

- Quasi-neutrality & current conservation:

$$n_e = \frac{1}{e} \sum_i q_i n_i \quad \nabla_\perp \cdot (en_e \mathbf{w}_e(\Phi)) = \nabla_\perp \cdot \left(\sum_i q_i n_i \mathbf{w}_i \right) + \sum_\alpha q_\alpha L_{//\alpha}$$

- Electron energy: $\frac{3}{2} \frac{\partial n_e T_e}{\partial t} + \frac{5}{2} \nabla_\perp \cdot (n_e \mathbf{w}_e T_e) + \nabla_\perp \cdot \mathbf{Q}_e = P_{\text{ext}} + en_e \mathbf{w}_e \cdot \nabla_\perp \Phi - L_{\text{col}} - L_{//T}$

$$\frac{\partial \mathbf{Q}_e}{\partial t} + \nu \mathbf{Q}_e - \frac{e}{m_e} \mathbf{B} \times \mathbf{Q}_e = -\frac{5n_e T_e}{2m_e} \nabla T_e$$

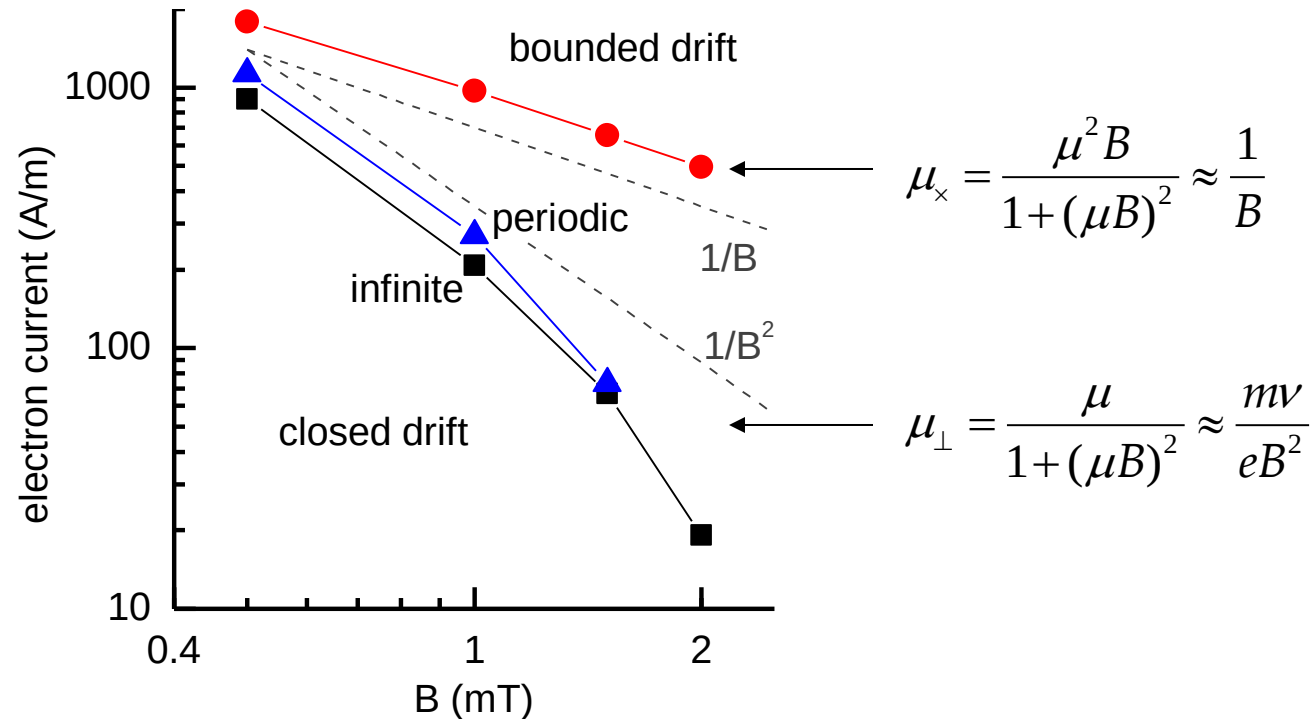
- Boundary conditions from sheath theory: $\mathbf{w}_i \cdot \mathbf{n} \geq (T_e / m_i)^{1/2}$

$$\mathbf{w}_e \cdot \mathbf{n} = (T_e / 2\pi m_e)^{1/2} \exp(-e(\Phi - \Phi_w) / T_e) \quad \mathbf{Q}_e \cdot \mathbf{n} = n_e \mathbf{w}_e \cdot \mathbf{n} (e(\Phi - \Phi_w) - \frac{1}{2} T_e)$$



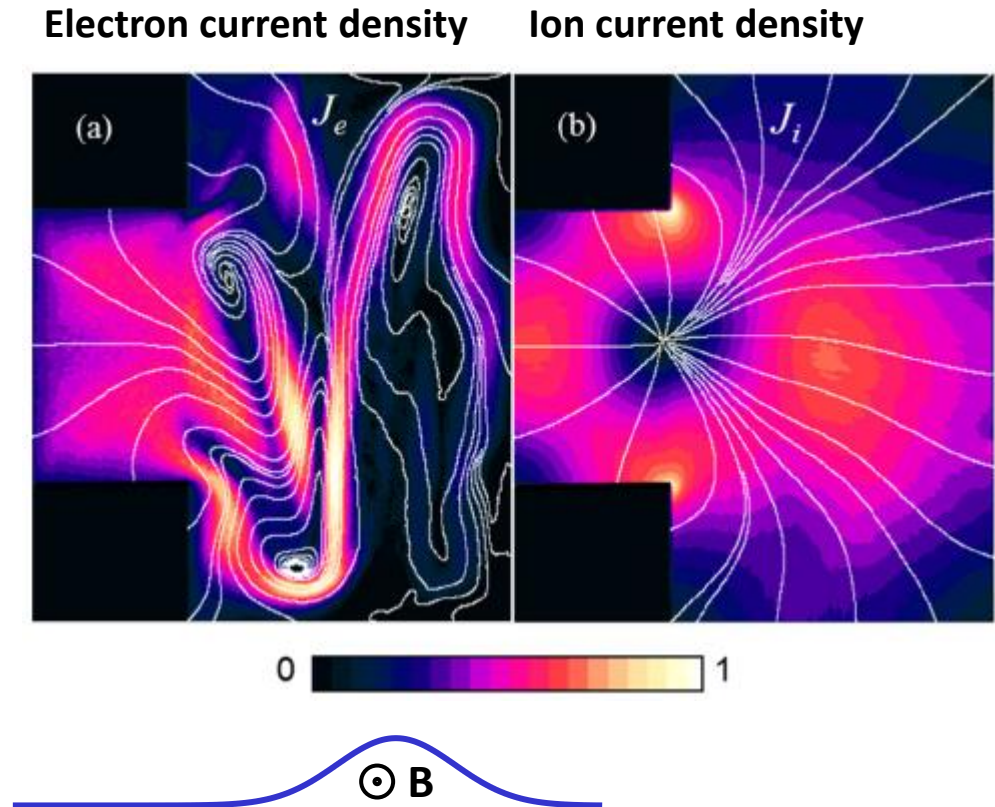
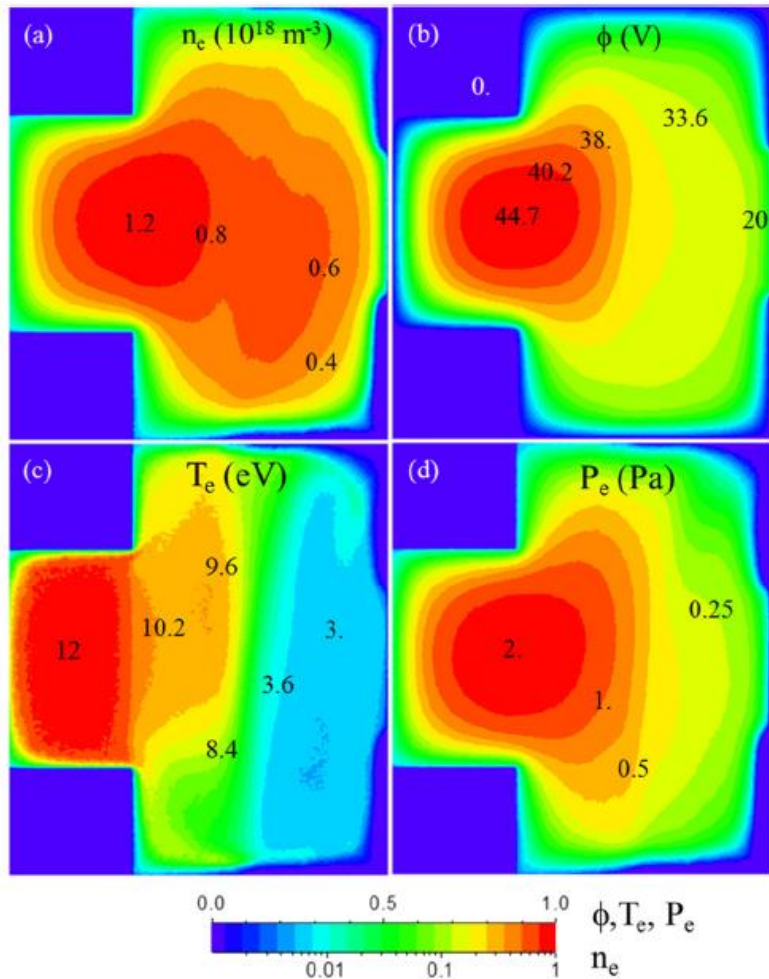
-
- A schematic diagram of a tokamak cross-section. The central region is labeled "RF heating". To the left, a vertical column of six white circles represents the electron population. To the right, a vertical column of six white circles represents the ion population. A red arrow points upwards between these columns, labeled "B", indicating the magnetic field. A red curved arrow at the top indicates the direction of the magnetic field lines. A red curved arrow at the bottom indicates the direction of the magnetic field lines. A red curved arrow at the top indicates the direction of the magnetic field lines.

Electron current across magnetic filter



- Closed/infinite drift → transport governed by $\mu_{\perp} \sim 1/B^2$
- Bounded drift → transport governed by $\mu_x \sim 1/B$
- Bounded drift effect scales as “anomalous” transport $\mu = 1/16B$ (Bohm)

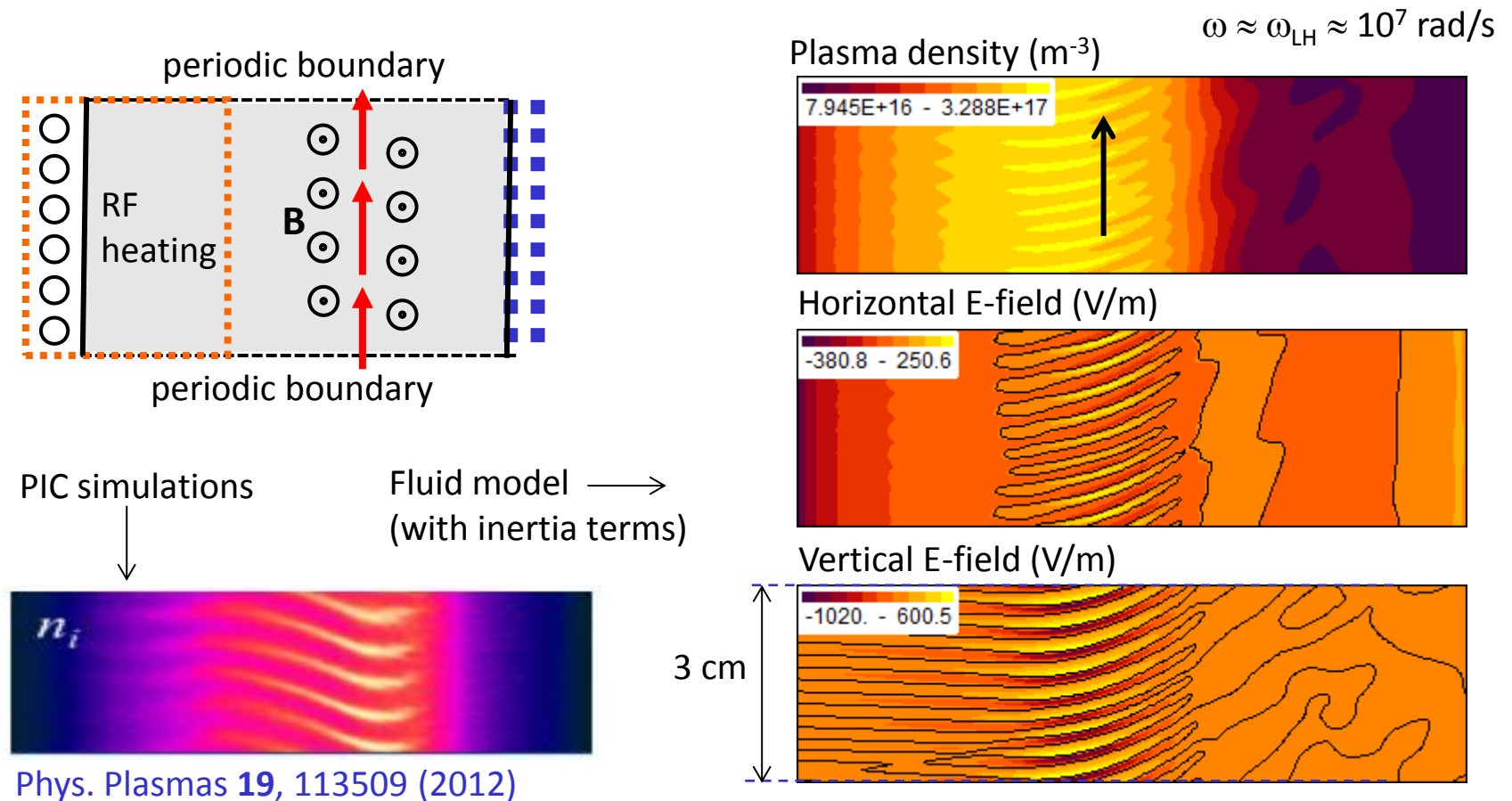
PIC simulations of prototype ITER negative ion source



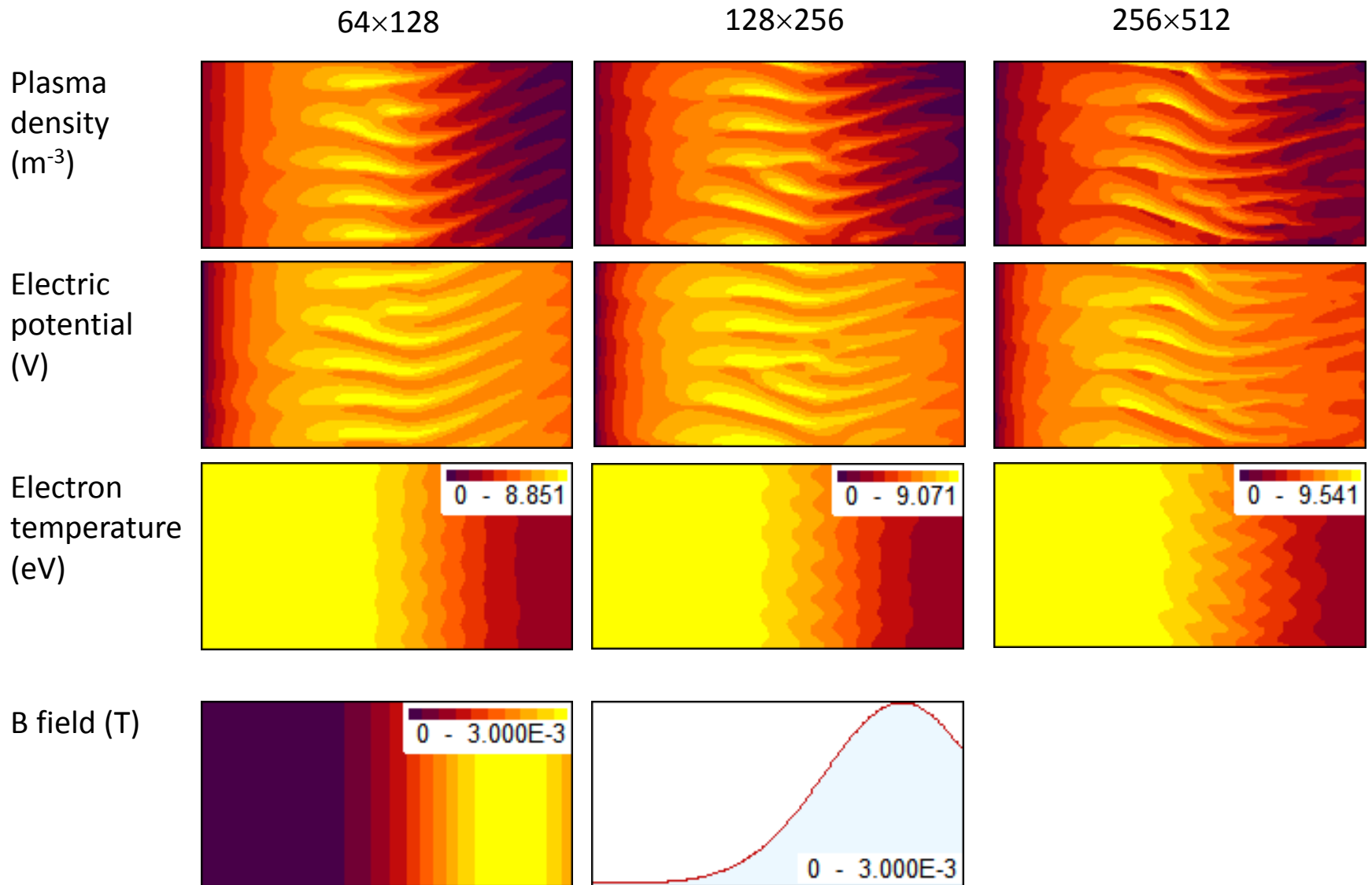
Fully kinetic PIC simulations

Drift instabilities in magnetic filter

- Replacing walls by periodic BCs $\rightarrow$ no Hall effect $\rightarrow$ small-scale drift instabilities, observed in both PIC and fluid simulations

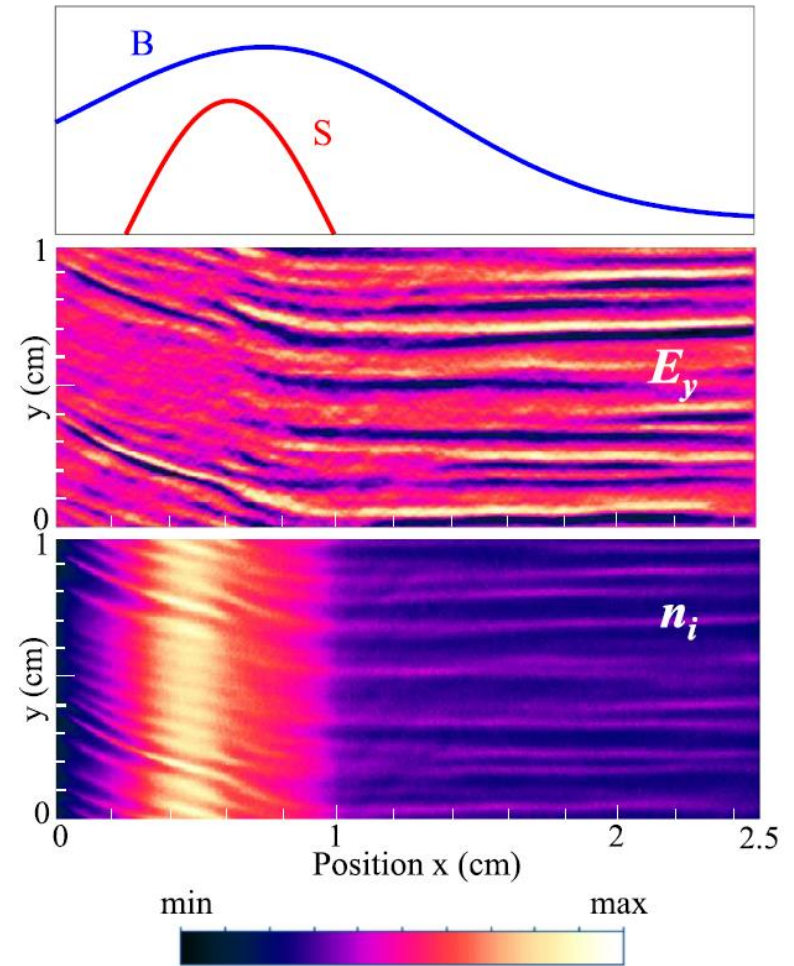
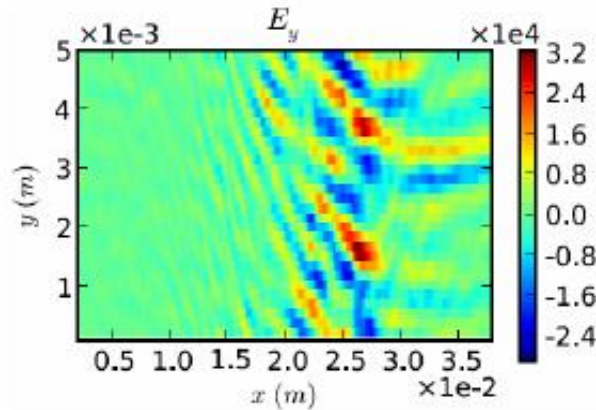


Grid convergence



PIC simulations of ECDI in Hall thruster

- Electron Cyclotron Drift Instabilities: resonance of Bernstein and ion acoustic modes – kinetic!



Adam et al, Plasma Phys. Contr. Fusion **50**, 124041 (2008)

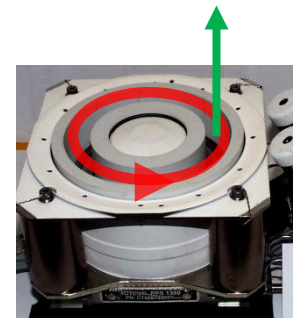
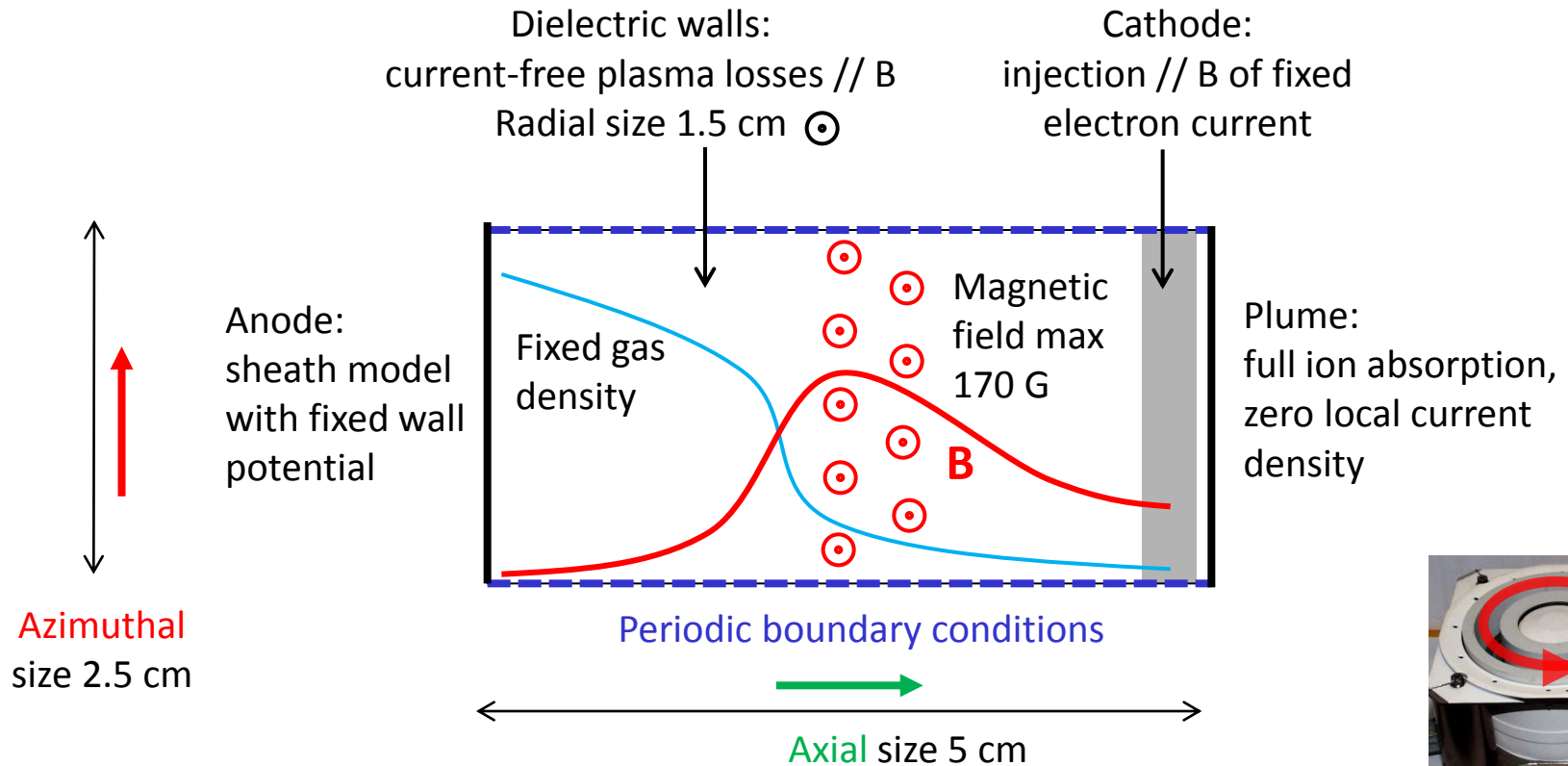
Tsikata et al, Phys. Plasmas **16**, 033506 (2009)

Lafleur et al, Phys. Plasmas **23**, 053503 (2016)

Boeuf et al, Phys. Plasmas **25**, 061204 (2018)

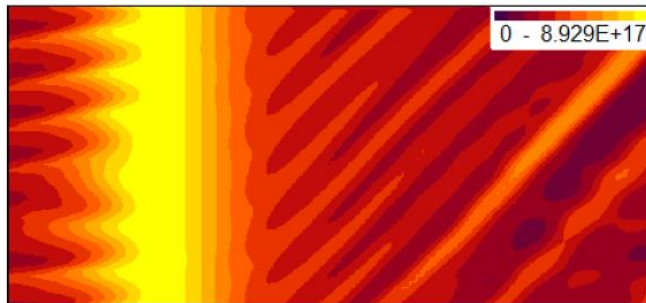
$E \times B$ discharge in Hall thruster

- Transport due to applied voltage rather than pressure
- Electrons injected at cathode
- Neutral gas depletion

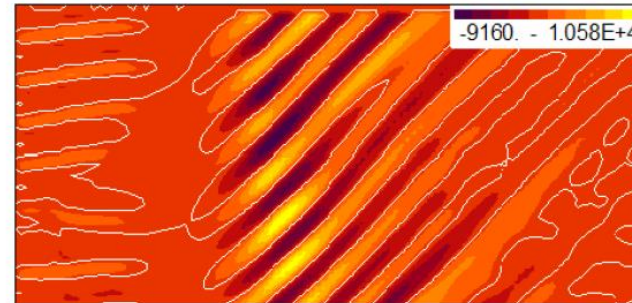


Problem with grid convergence

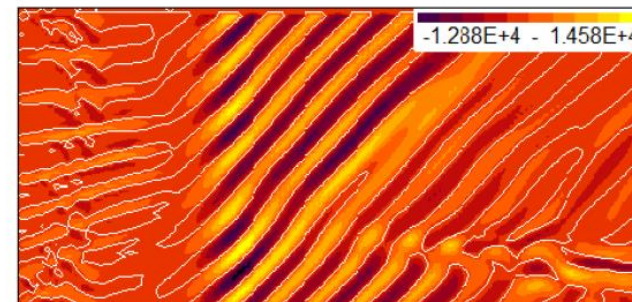
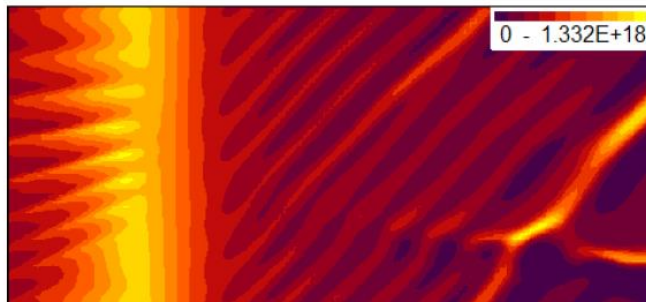
Plasma density (m^{-3})



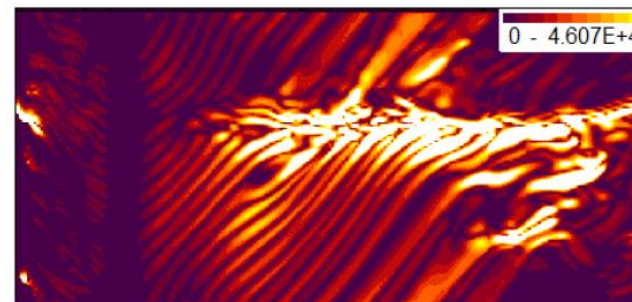
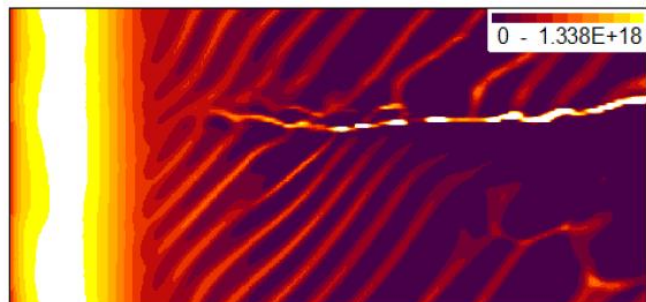
Azimuthal electric field (V/m)



Maillage : 96x48



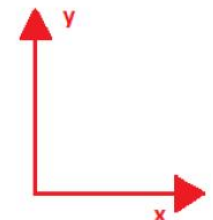
Maillage : 128x64



Maillage : 192x96

Densité de plasma (m^{-3})

Champ électrique azimuthal (V/m)

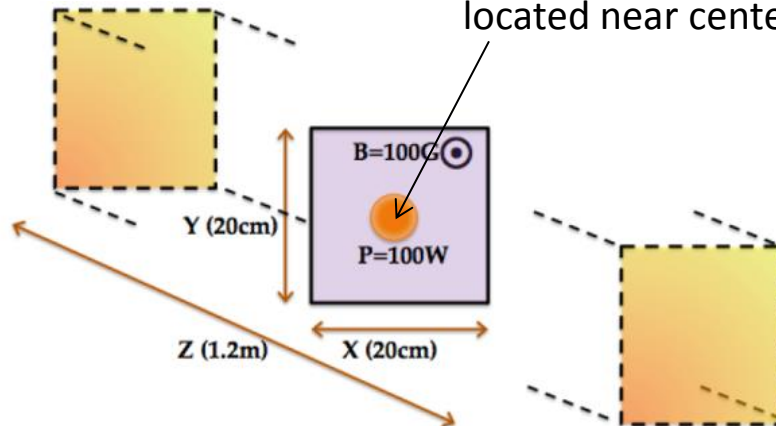


Magnetized plasma column

Classical problem: long (cylindrical) plasma column with axial magnetic field, relevant to some low temperature plasma devices (CYBELE, Mistral, Mirabelle, ...)

Boundary conditions at ends of column play important role

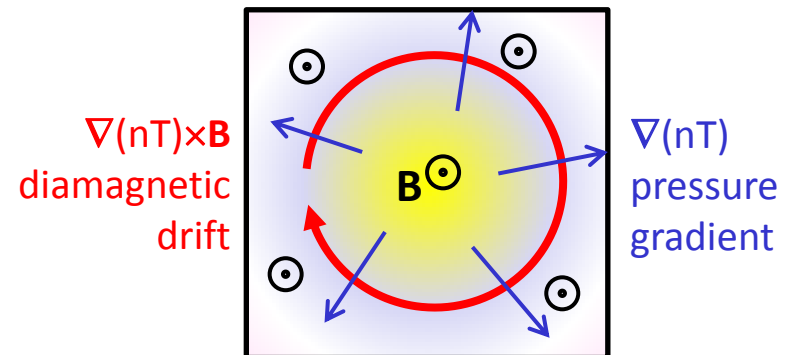
Plasma sustained by RF heating or filaments located near center



1.7 mTorr hydrogen



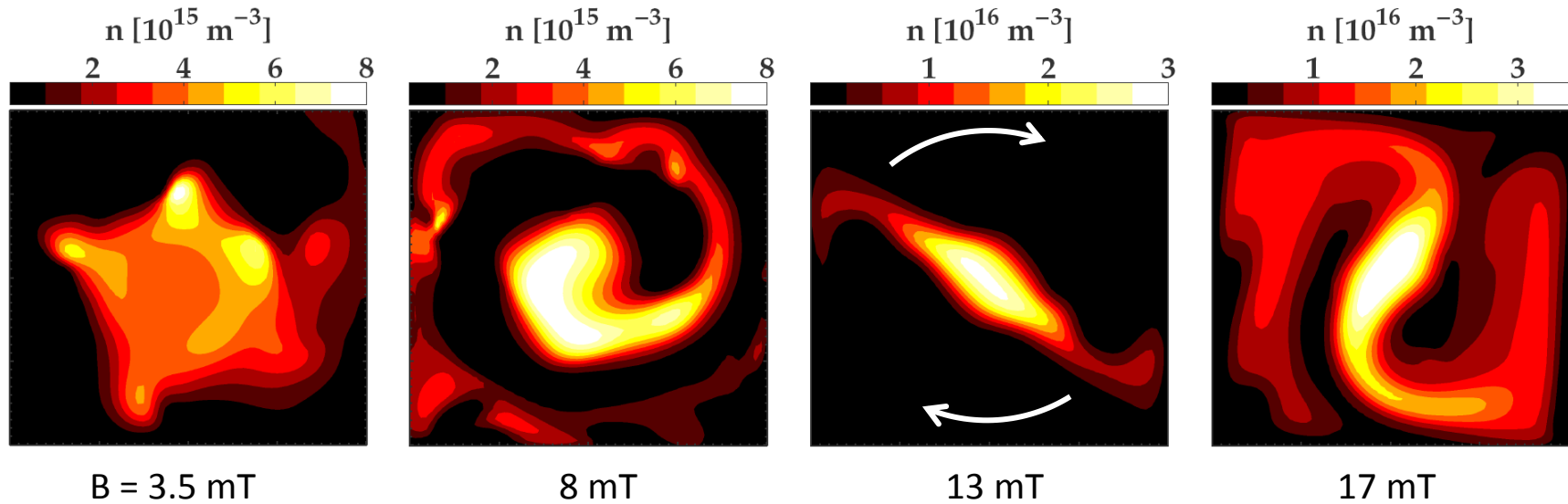
CYBELE negative ion source for NBI under development @ CEA Cadarache



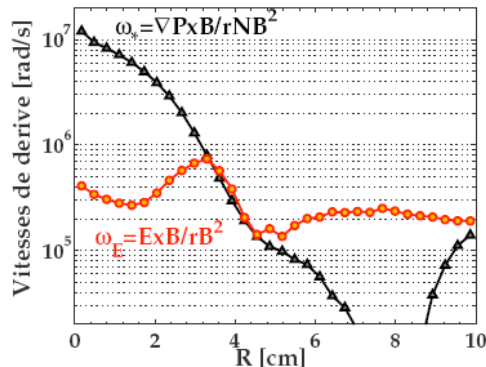
A. Simonin et al, Nucl. Fusion **55**, 123020 (2015)

R. Futtersack, PhD thesis (University of Toulouse, 2014)

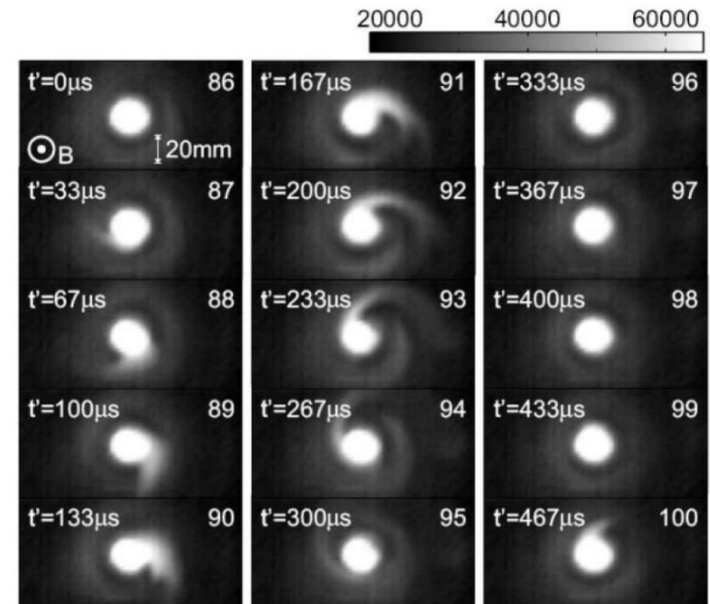
Rotating instability in plasma column with central heating



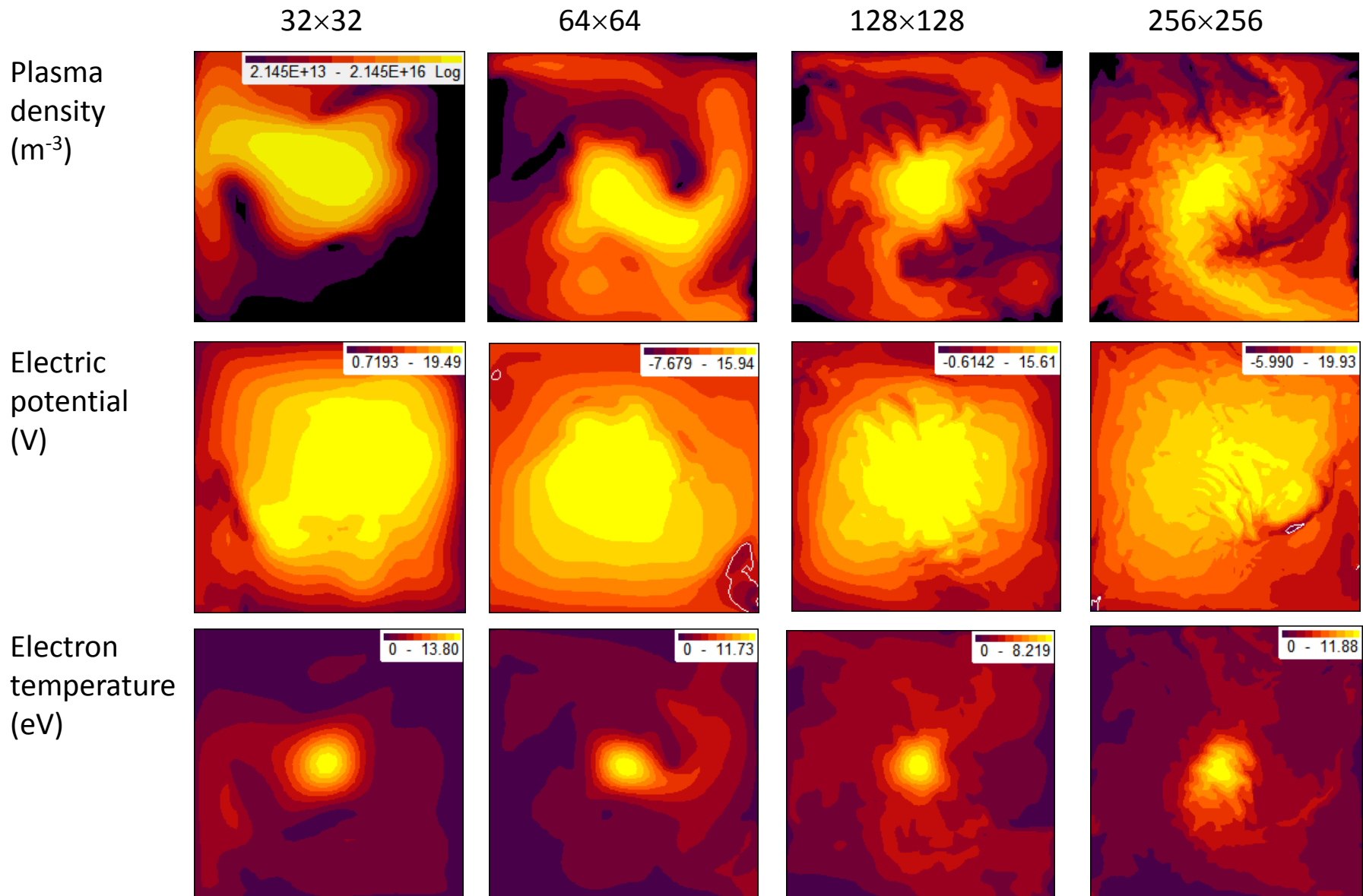
Plasma tends to develop one or two "arms" rotating at $2\text{-}4 \times 10^5 \text{ rad/s}$, more or less as rigid objects



Successive CCD camera images taken in linear plasma device NAGDIS. From: H. Tanaka et al, Contrib. Plasma Phys. 52 (2012)

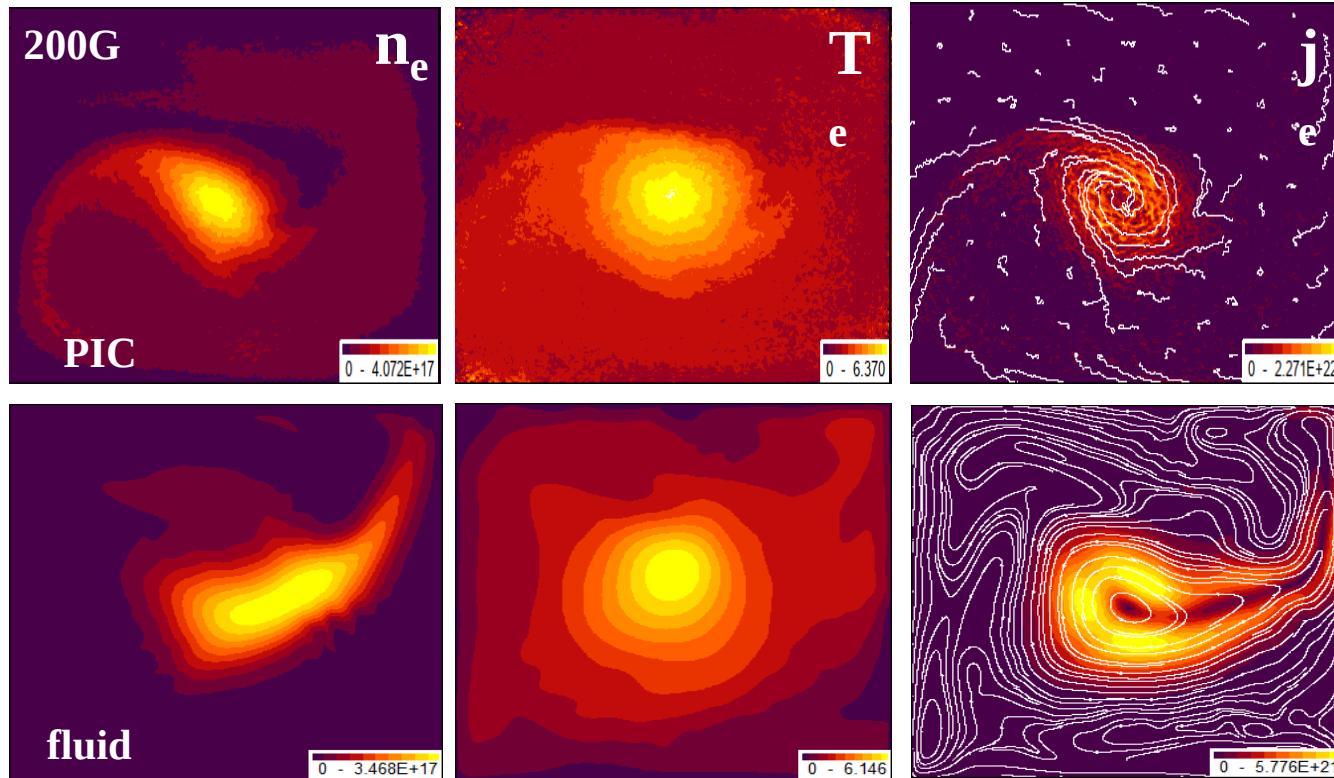


Grid convergence



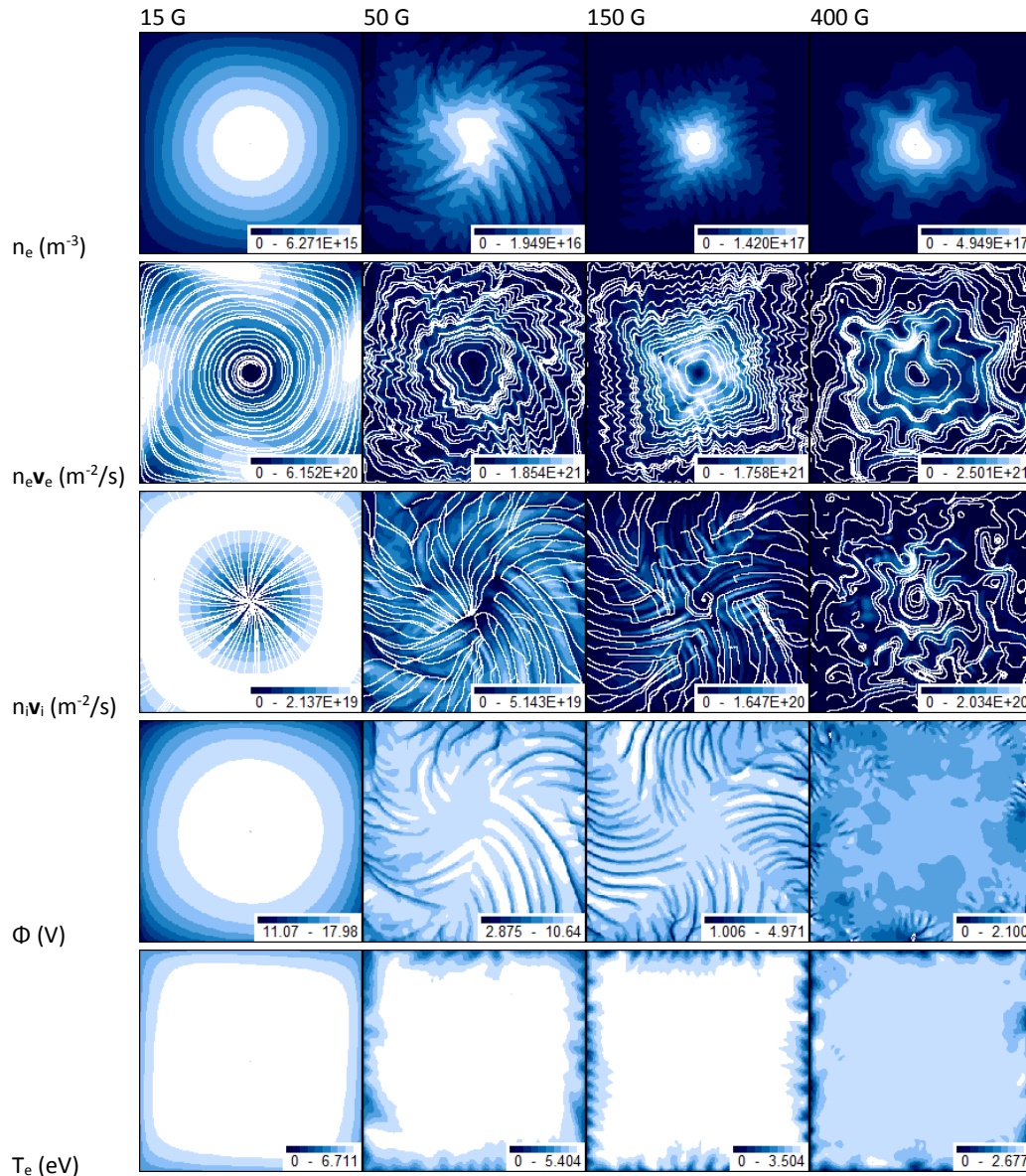
Comparison with 2.5D PIC simulations

PIC simulations of RAID plasma source (G. Fubiani)

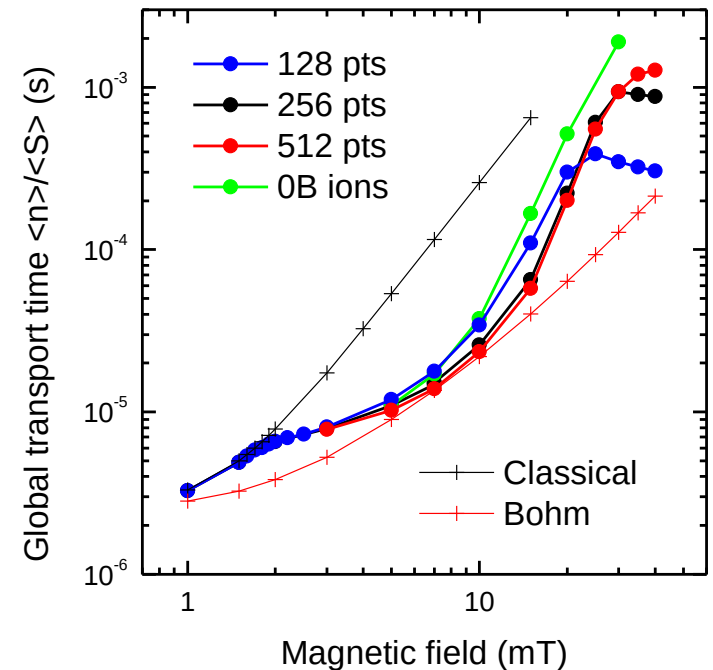
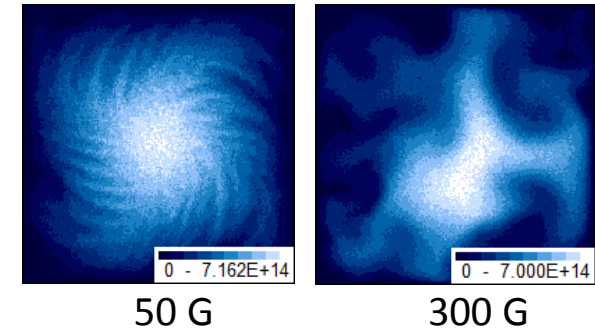


Magnetized plasma column with uniform heating

Fluid simulation



PIC simulation



Linear stability analysis

- Consider equilibrium solution and small perturbation

$$\mathbf{u}_0(\mathbf{r}) = [n_0, \mathbf{E}_0, \mathbf{w}_{e0}, \mathbf{w}_{i0}, S_0, \dots]$$

$$\tilde{\mathbf{u}}(\mathbf{r}, t) = [\tilde{n}, \tilde{\mathbf{E}}, \tilde{\mathbf{w}}_e, \tilde{\mathbf{w}}_i, \tilde{S}, \dots]$$

- Solve perturbation from linearized model equations, e.g.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{w}) = S \quad \longrightarrow \quad \frac{\partial \tilde{n}}{\partial t} + \mathbf{w}_0 \cdot \nabla \tilde{n} + (\nabla \cdot \mathbf{w}_0) \tilde{n} + n_0 \nabla \cdot \tilde{\mathbf{w}} + (\nabla n_0) \cdot \tilde{\mathbf{w}} = \tilde{S} \quad \dots \text{etcetera}$$

- Assume quasi-constant equilibrium: $|\nabla \mathbf{u}_0 / \mathbf{u}_0| \ll |\nabla \tilde{\mathbf{u}} / \tilde{\mathbf{u}}| \quad k^2 \gg |\mathbf{k} \cdot \nabla \mathbf{u}_0| / |\mathbf{u}_0|$

- Then: $\tilde{\mathbf{u}}(\mathbf{r}, t) \propto \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ $\tilde{\mathbf{u}}(\mathbf{r}, t) = \text{Re} \left(\sum_n A_n \exp(i\mathbf{k}_n \cdot \mathbf{r} - i\omega_n t) \right)$

Wave number $\nearrow$
 $\nwarrow$ Complex angular frequency: $\omega = \omega_r + i\omega_i$

propagation + growth

- Linearized model yields dispersion relation $\omega \leftrightarrow \mathbf{k}$
- Compare with Fourier transform of numerical results

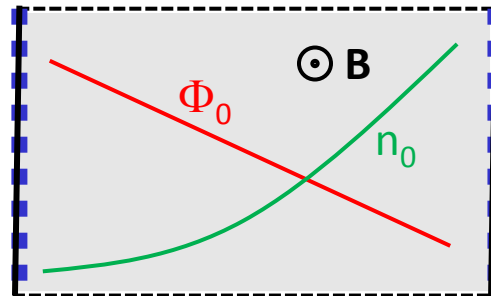
Test case with exact dispersion relation

- Equilibrium solution of simple model system

$$\begin{aligned} n_0(x) &= n_0(0) \exp(gx) & \mathbf{w}_{0e} &= -\frac{e(E + T_e g)}{m_e(\nu_e^2 + \Omega_e^2)} (\nu_e \hat{\mathbf{x}} + \Omega_e \hat{\mathbf{y}}) & \mathbf{w}_{0i} &= \frac{e(E - T_i g)}{m_i(\nu_i^2 + \Omega_i^2)} (\nu_i \hat{\mathbf{x}} - \Omega_i \hat{\mathbf{y}}) \\ \Phi_0(x) &= -Ex \end{aligned}$$

- Such that perturbation of the form $[\ln \tilde{n}, \tilde{\Phi}, \tilde{\mathbf{w}}_e, \tilde{\mathbf{w}}_i] \propto \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$
- Dispersion relation (4th order complex equation for ω)

$$\frac{m_i(\omega - \mathbf{k} \cdot \mathbf{w}_{0i})((i\alpha_i\omega - i\beta_i\mathbf{k} \cdot \mathbf{w}_{0i} - \nu_i)^2 + \Omega_i^2)}{(i\alpha_i\omega - i\beta_i\mathbf{k} \cdot \mathbf{w}_{0i} - \nu_i)(ik^2 + \mathbf{g} \cdot \mathbf{k}) + \Omega_i\mathbf{k} \times \mathbf{b} \cdot \mathbf{g}} + \frac{m_e(\omega - \mathbf{k} \cdot \mathbf{w}_{0e})((i\alpha_e\omega - i\beta_e\mathbf{k} \cdot \mathbf{w}_{0e} - \nu_e)^2 + \Omega_e^2)}{(i\alpha_e\omega - i\beta_e\mathbf{k} \cdot \mathbf{w}_{0e} - \nu_e)(ik^2 + \mathbf{g} \cdot \mathbf{k}) - \Omega_e\mathbf{k} \times \mathbf{b} \cdot \mathbf{g}} = e(T_e + T_i)$$



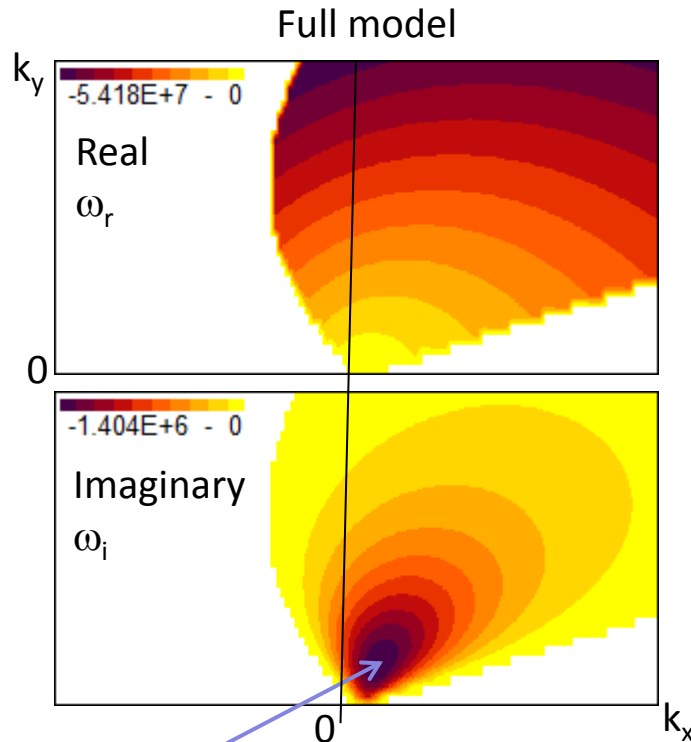
periodic boundary

$$\Omega = \frac{q}{m} B \quad \mathbf{b} = \frac{\mathbf{B}}{B} = \hat{\mathbf{x}} \times \hat{\mathbf{y}}$$

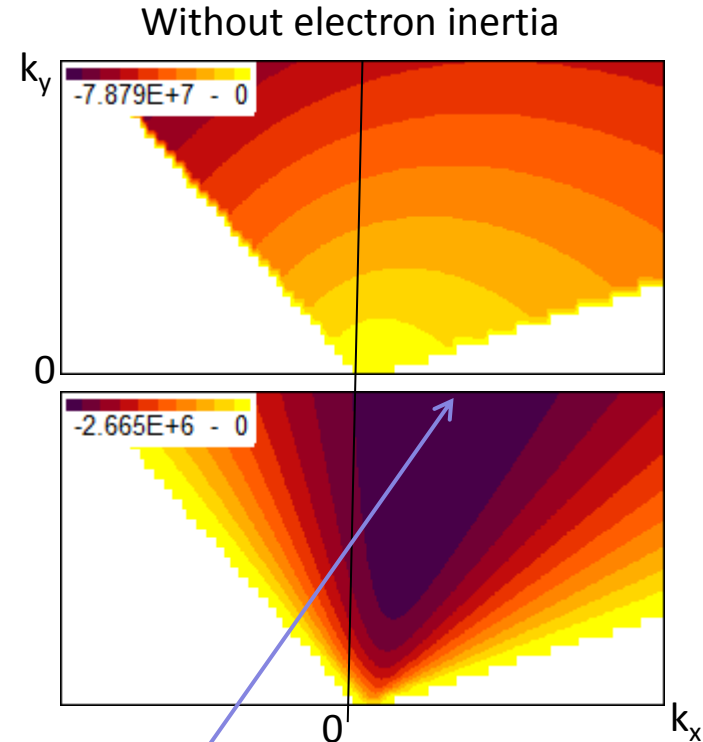
imposed voltage and density drop

Example of result from linear instability analysis

Find most unstable root (ω_r , ω_i) of dispersion relation versus (k_x , k_y)



Growth rate $-\omega_i = 1.40 \times 10^6 \text{ s}^{-1}$
 Frequency $\omega_r/2\pi = 0.80 \text{ MHz}$
 Wave length $2\pi/k_y = 1.00 \text{ cm}$
 $2\pi/k_x = 1.60 \text{ cm}$

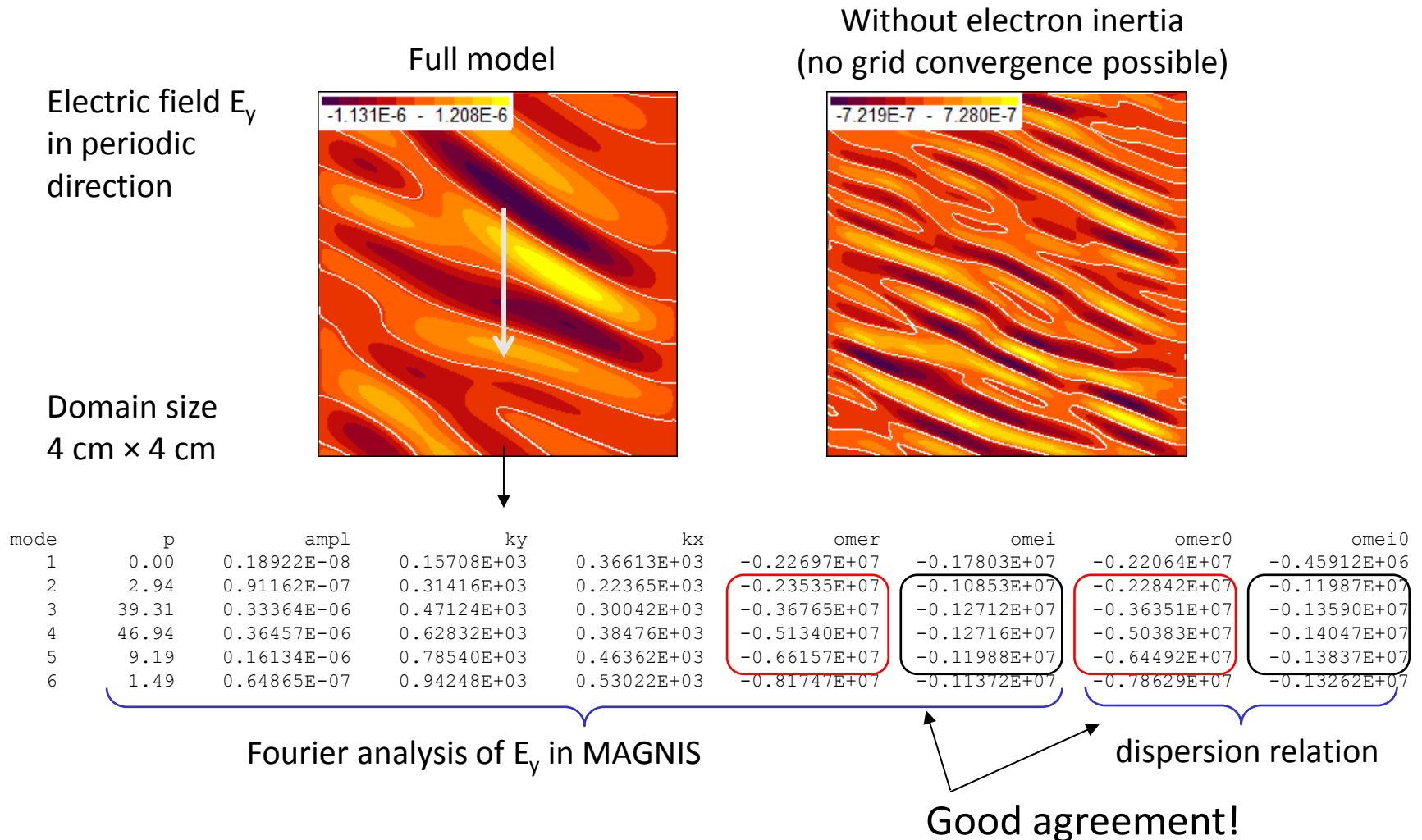


Growth rate keeps increasing with increasing k
 $\rightarrow$ impossible to simulate, always instabilities on scale of grid spacing!

Conditions: H_2 gas, $N = 10^{20} \text{ m}^{-3}$, $E = 25 \text{ V/m}$, $g = 25 \text{ m}^{-1}$, $T_e = 1 \text{ eV}$, $T_i = 0.2 \text{ eV}$

Verification of fluid code against dispersion relation

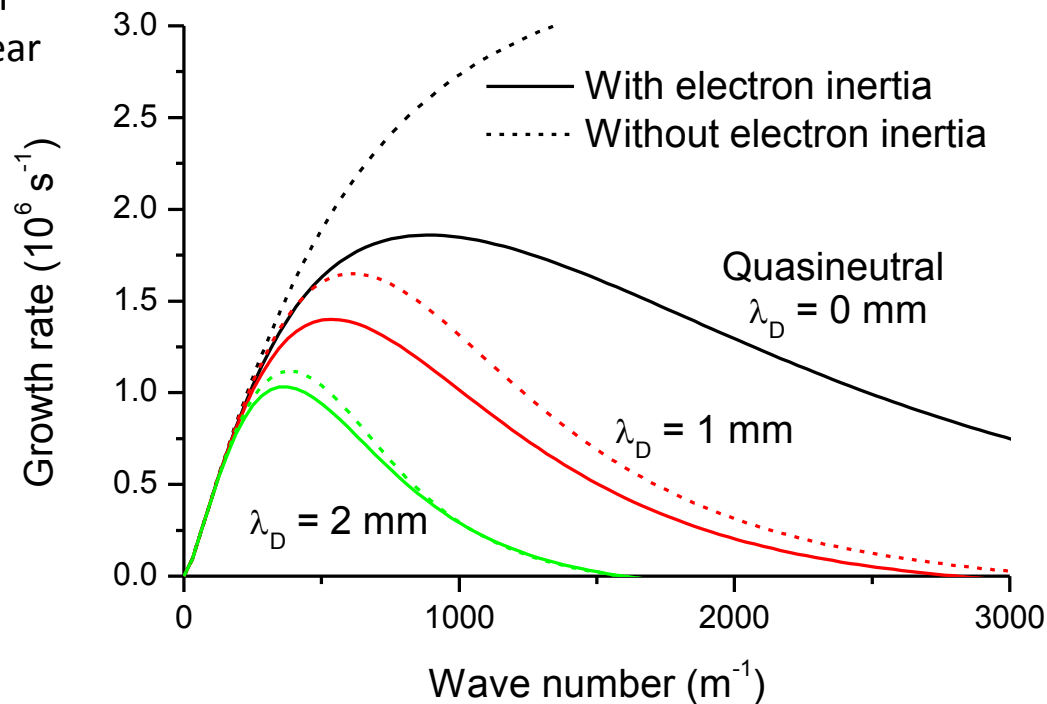
Measure growth rate, frequency and wave vector by Fourier analysis of MAGNIS results (during onset of instabilities) and check against general dispersion relation



Missing small scale physics?

- Kinetic effects e.g. due to electron cyclotron motion
- Space charge effects due to $n_e \neq n_i$ or $\nabla n_e \neq \nabla n_i$ or $\frac{\partial n_e}{\partial t} \neq \frac{\partial n_i}{\partial t}$

Effect of Poisson equation on linear growth rate



Conclusions & questions

- Plasma instabilities pose unavoidable challenges for (fluid) simulations of partially magnetized plasma devices
- Numerical constraints, convergence?
- Physical relevance, validity, limitations?
- How to patch or improve fluid closures, suppress unphysical artefacts?
- Linear stability analysis for simple geometry shows that:
 - MAGNIS code correctly solves the system of model equations during onset of instabilities
 - Physical model tends to produce instabilities at infinitely small scales (where model is not valid)
 - Model must be modified to capture small-scale physics: Poisson equation, electron gyro-viscosity, ion viscosity, ...

Conclusions

- Fluid models of magnetized plasma sources are capable of describing part of the drift-related dynamics: Hall effect, large scale fluctuations, spokes ...
- However, they also tend to produce unrealistic small-scale instability modes, which can be mathematically ill-behaved – how to get rid of these modes?
 - Add small scale physics: Poisson equation, gyro-viscosity, ...
 - Add artificial diffusion