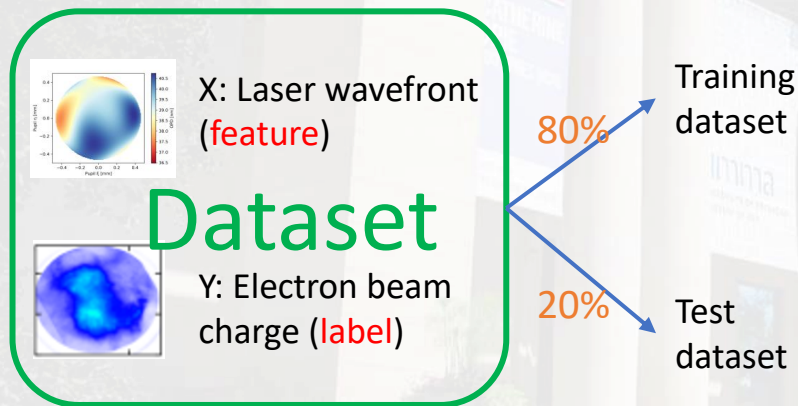
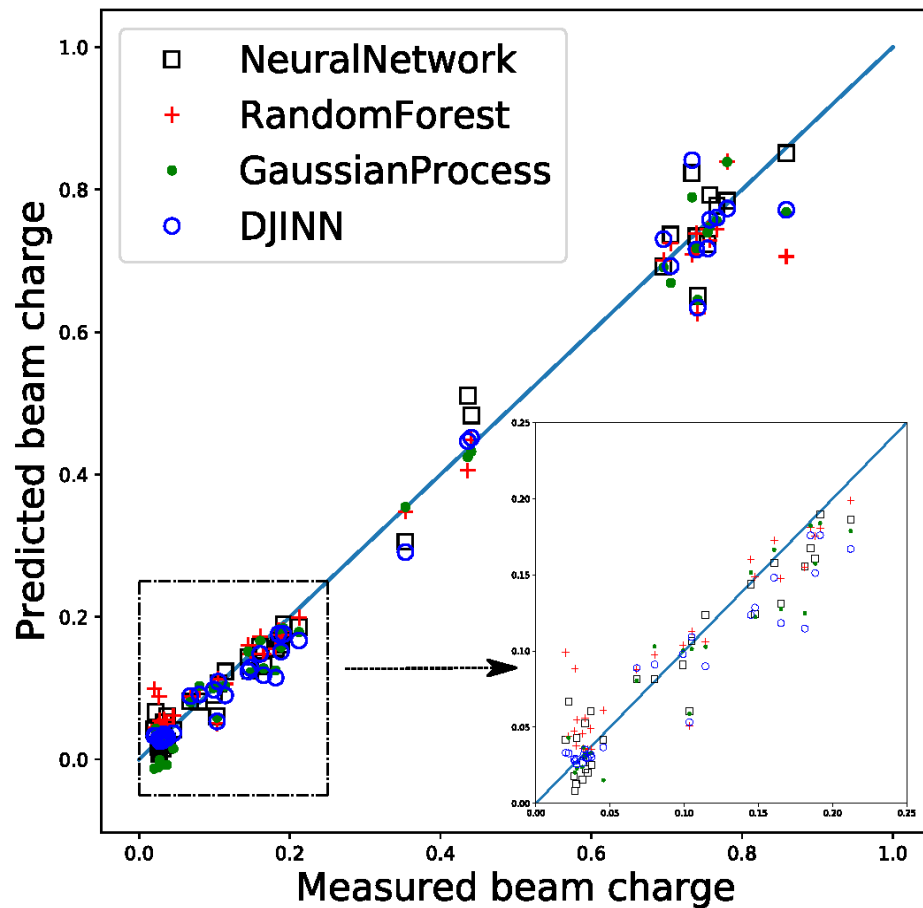


Predicted vs. measured electron beam charge



Model	MSE	MAE	R^2	Ex Var
Random Forest	0.00132	0.0268	0.986	0.987
Neural network	0.00162	0.0292	0.983	0.984
DJINN	0.00154	0.02741	0.98403	0.98404
Gaussian Process	0.00185	0.0305	0.981	0.981





Feature analysis in relativistic laser-plasma experiments utilizing machine learning methods

Jinpu Lin¹, Qian Qian¹, Jon Murphy¹, Abigail Hsu², Alfred Hero³, Alexander G.R. Thomas¹, and Karl Krushelnick¹

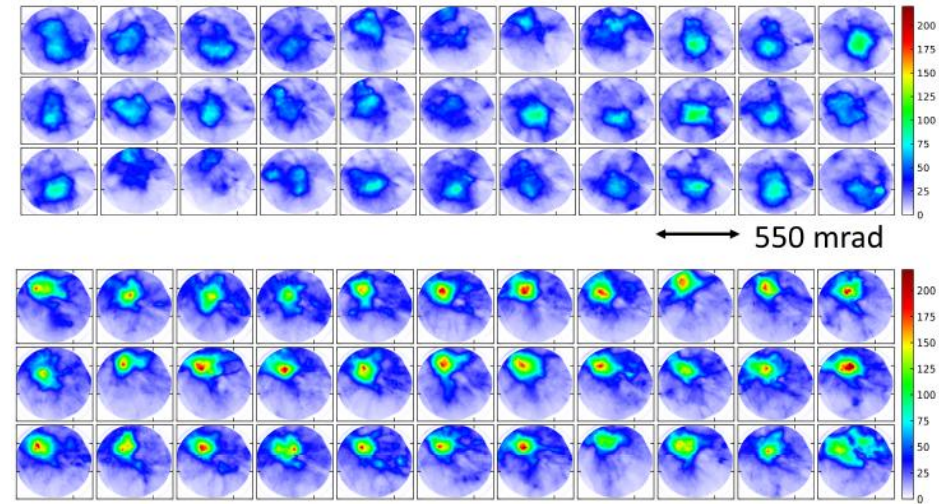
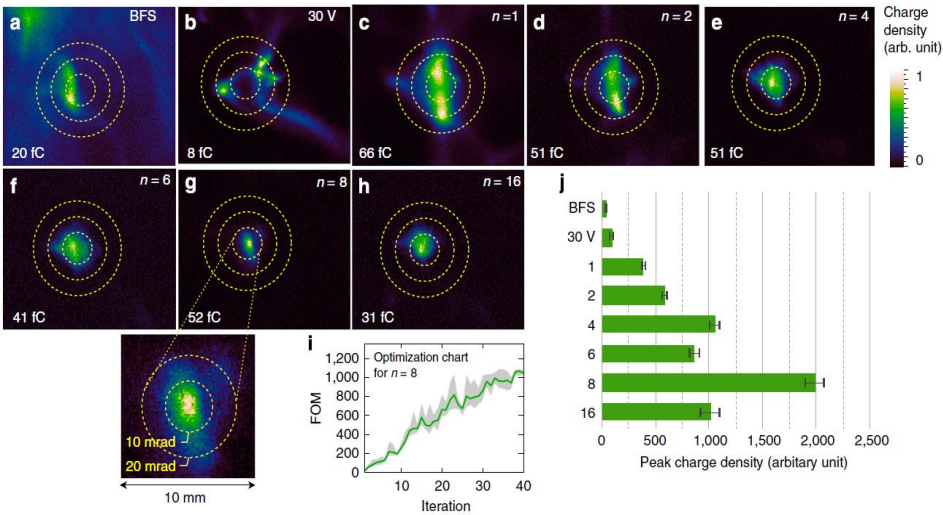
1) Center for Ultrafast Optical Science, University of Michigan

2) Department of Applied Mathematics and Statistics, Stoney Brook University

3) Department of Electrical Engineering and Computer Science, University of Michigan

November 12, 2020, 62nd Annual Meeting of the APS Division of Plasma Physics

Statistical methods in high-rep-rate LWFA – genetic algorithm

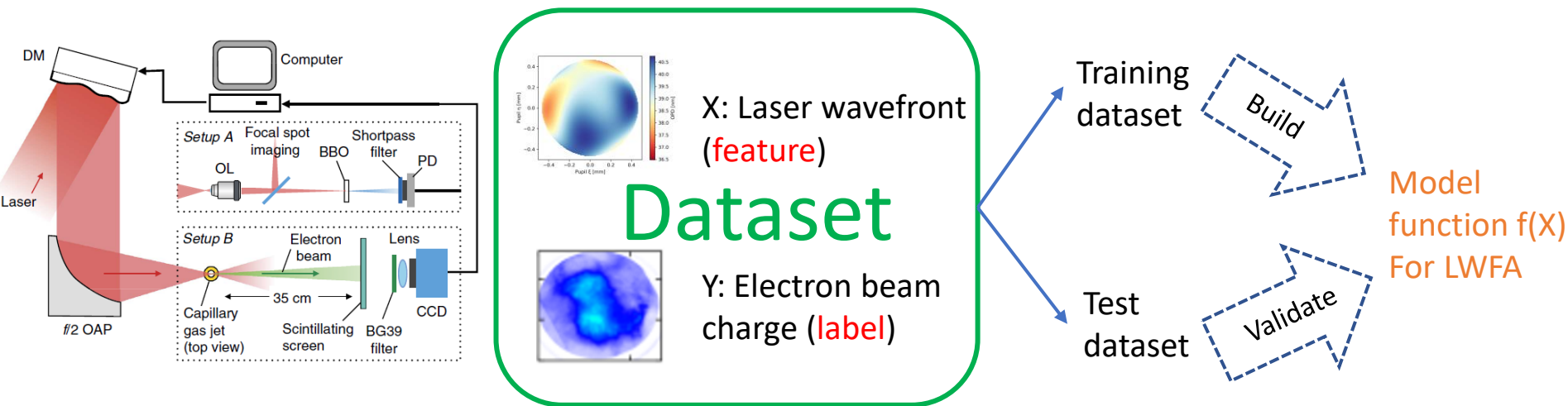


He, Z-H., et al. "Coherent control of plasma dynamics." *Nature communications* 6.1 (2015): 1-7.

Lin, J., et al. "Adaptive control of laser-wakefield accelerators driven by mid-IR laser pulses." *Optics express* 27.8 (2019): 10912-10923.

Apply machine learning to LWFA experimental data

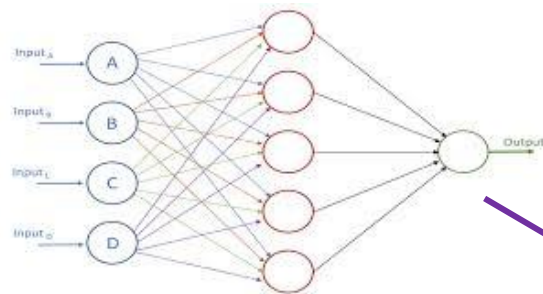
- Machine learning vs. genetic algorithms?
 - GA provides only local optimum while ML allows feature analysis
 - GA throws away 90% of the data while ML utilizes all



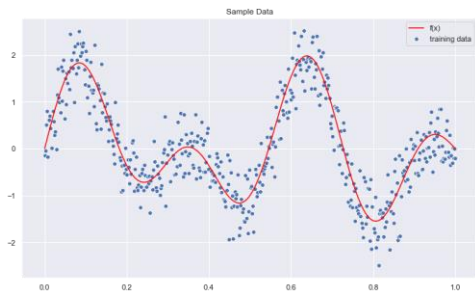
He, Z-H., et al. "Coherent control of plasma dynamics." *Nature communications* 6.1 (2015): 1-7.

Maris, A. D., et al. *Finding Correlations in Inertial Confinement Fusion Experimental Data Using Machine Learning*. No. LLNL-PROC-791967. Lawrence Livermore National Lab.(LLNL), Livermore, CA (United States), 2019.

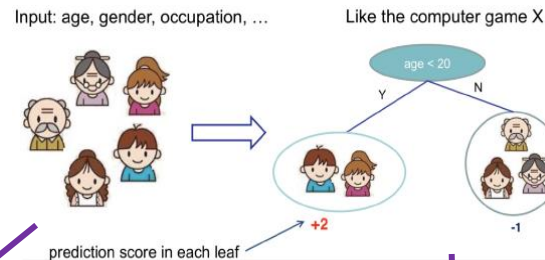
Supervised learning architectures



Neural network

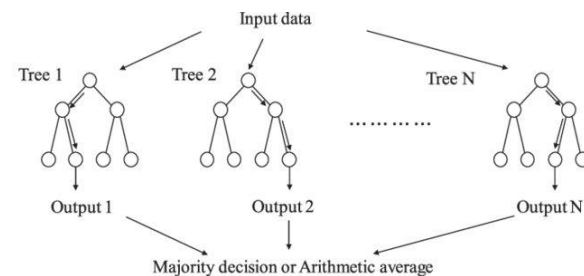


Gaussian process regression with kernels



Decision tree

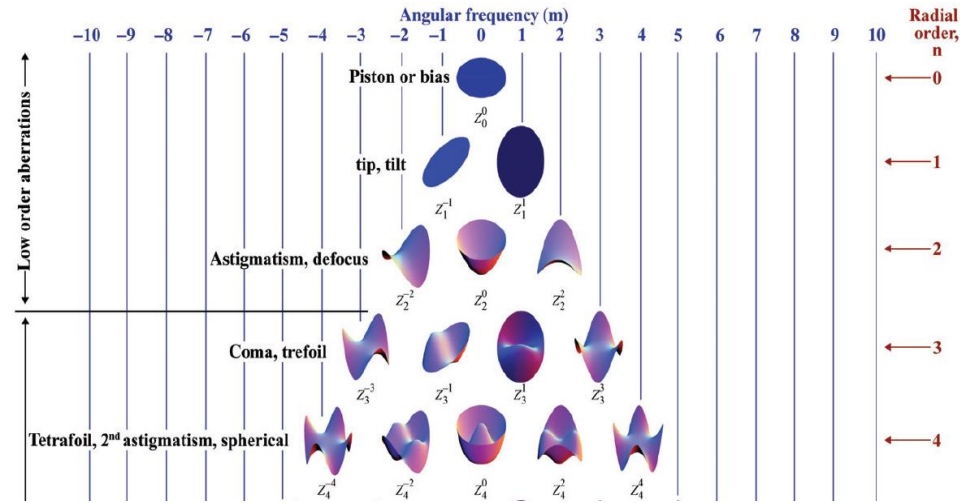
Deep Jointly-Informed Neural Networks¹



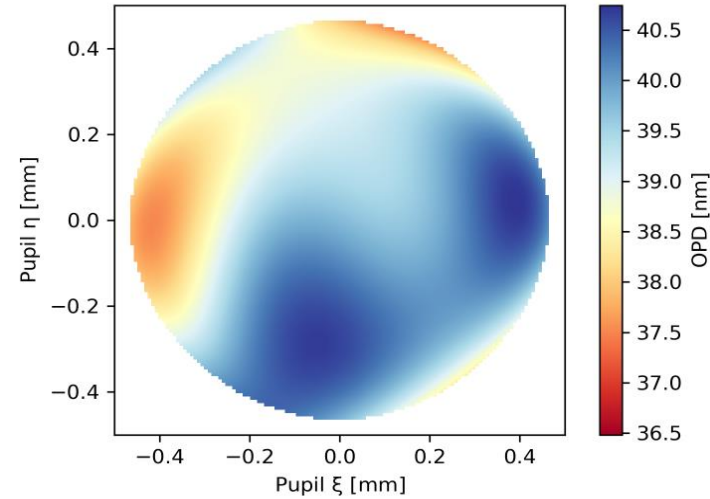
Random forest

¹Humbird, Kelli D., et al. "Deep neural network initialization with decision trees." *IEEE trans. on neural networks and learning systems* 30.5 (2018): 1286-1295.

Feature dimensionality reduction



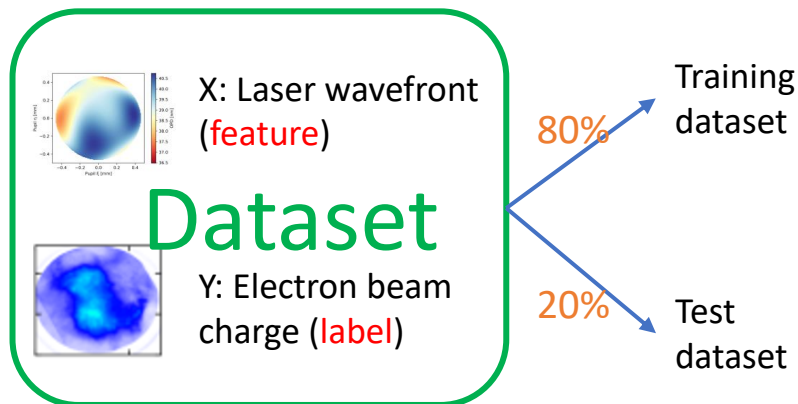
Zernike expansion



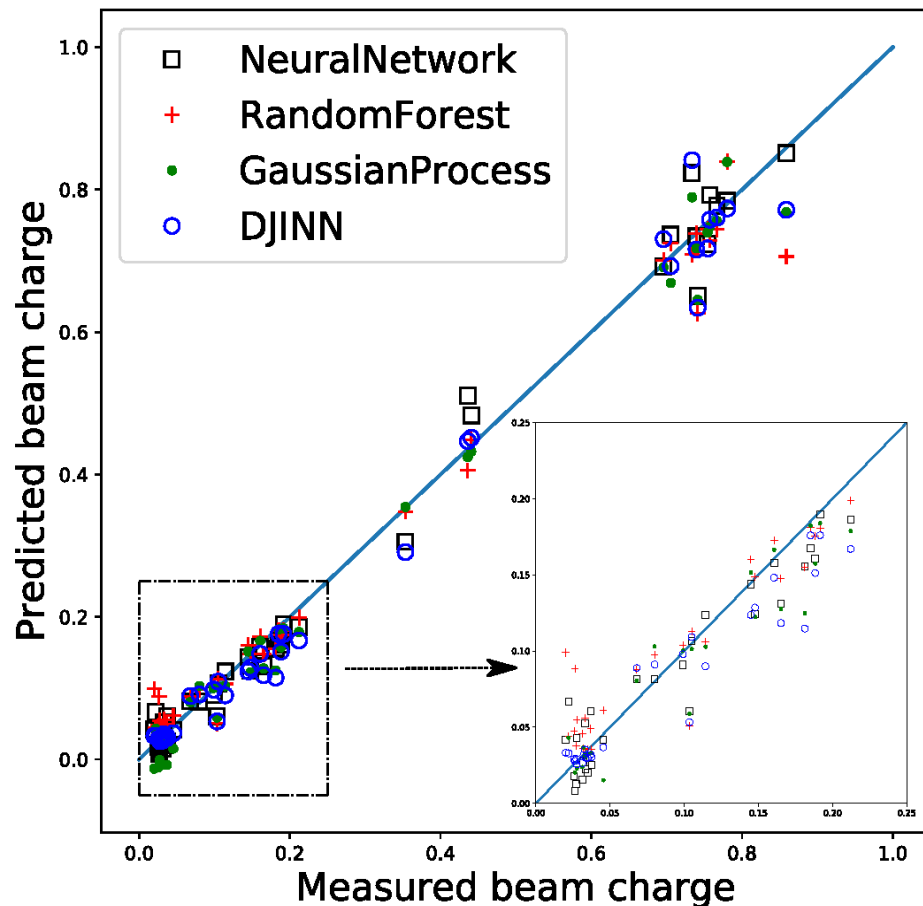
Reproduced laser wavefront

Reduced feature (X) dimension = 15,
Label dimension = 1

Predicted vs. measured electron beam charge



Model	MSE	MAE	R^2	Ex Var
Random Forest	0.00132	0.0268	0.986	0.987
Neural network	0.00162	0.0292	0.983	0.984
DJINN	0.00154	0.02741	0.98403	0.98404
Gaussian Process	0.00185	0.0305	0.981	0.981



Feature Importance of Zernike coefficients

- ❑ Calculate prediction error of model f:
$$e = L(y, f(X))$$
- ❑ For each feature i:
 - ❑ Permute data values in feature i
 - ❑ Generate permuted feature matrix X^{perm}
 - ❑ Calculate prediction error of model f:
$$e^{perm} = L(y, f(X^{perm}))$$
 - ❑ Obtain importance of the i^{th} feature =
$$e^{perm} - e$$
- ❑ Sort importance of features in descending order

Molnar, Christoph. Interpretable Machine Learning. Lulu. com, 2020.

Importance

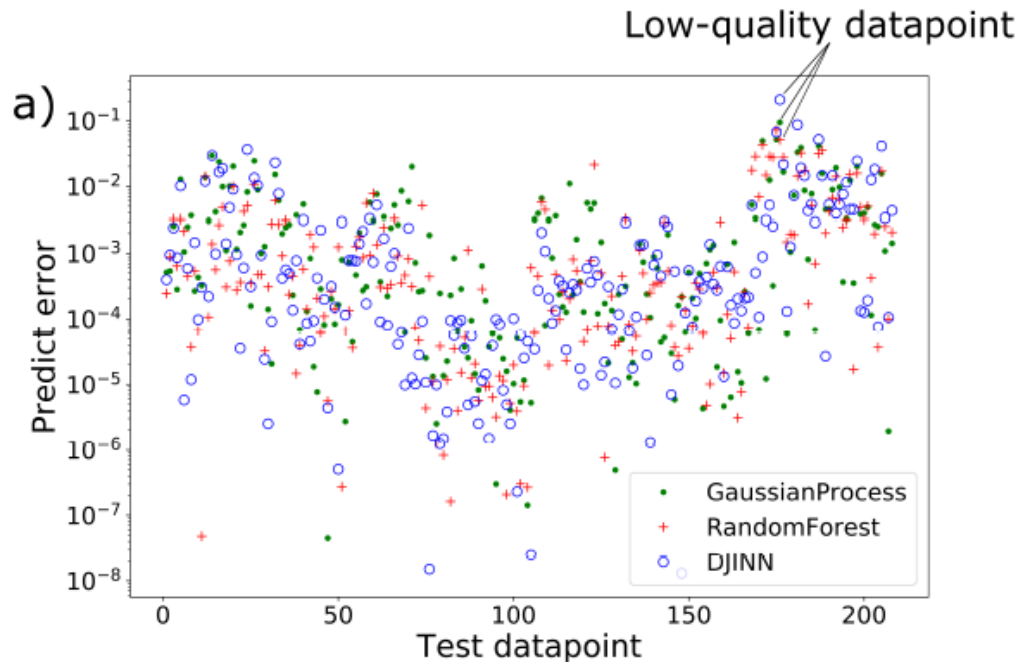
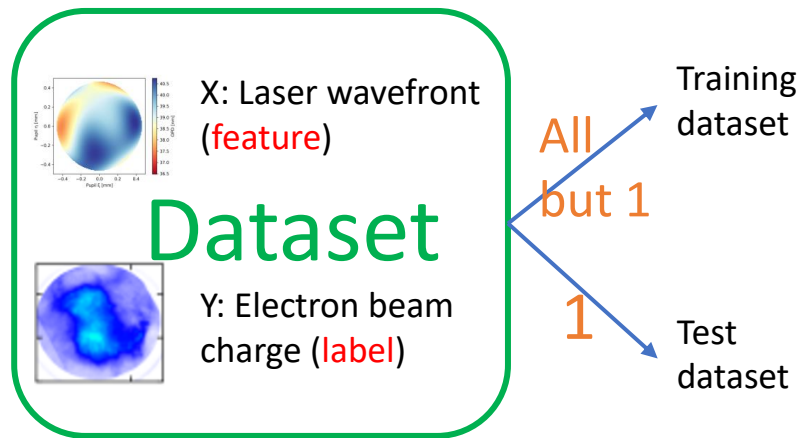


Model	1 st	2 nd	3 rd	4 th
Random Forest	0	6	1	10
Gaussian Process	1	10	0	6
Neural Network	0	10	1	3
DJINN	1	10	0	6
Correlation	0	10	1	13

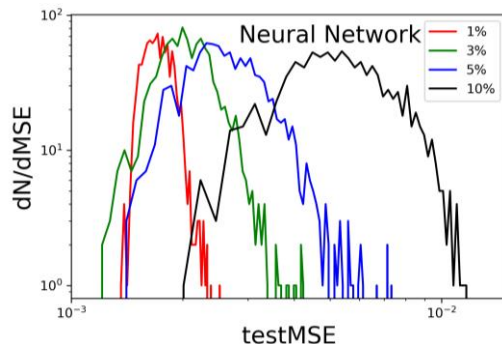
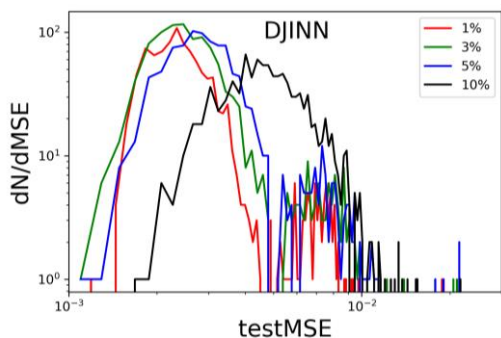
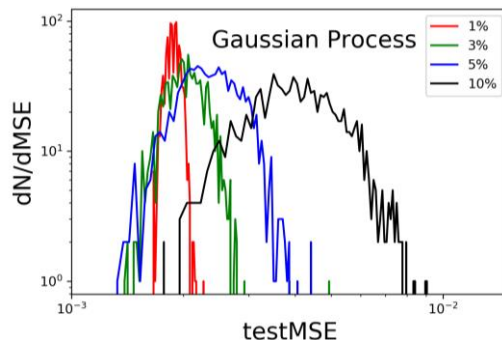
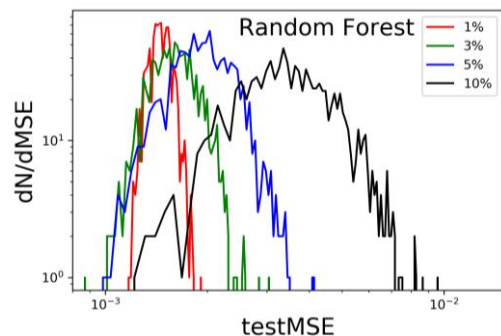


Order of Zernike polynomials

Evaluate model performance on every datapoint



Model performance against virtual measurement errors



RF

Measure. Error	Mean	σ	Within σ	Within 2σ	Within 3σ
1%	1.47E-03	1.11E-04	67.8%	95.2%	99.6%
3%	1.64E-03	2.65E-04	69.8%	95.9%	99.3%
5%	2.04E-03	4.40E-04	70.8%	95.1%	99.2%
10%	3.83E-03	1.21E-03	70.2%	95.7%	99.4%

GP

Measure. Error	Mean	σ	Within σ	Within 2σ	Within 3σ
1%	1.88E-03	8.34E-05	69.0%	96.1%	99.5%
3%	2.07E-03	2.72E-04	70.4%	97.0%	99.6%
5%	2.46E-03	4.67E-04	68.5%	95.5%	99.5%
10%	4.37E-03	1.22E-03	69.6%	95.6%	99.5%

DJINN

Measure. Error	Mean	σ	Within σ	Within 2σ	Within 3σ
1%	2.9E-03	1.6E-03	91.3%	93.2%	97.4%
3%	3.2E-03	2.0E-03	90.1%	94.1%	98.3%
5%	3.6E-03	2.0E-03	88.3%	94.8%	98.4%
10%	5.3E-03	2.0E-03	74.6%	96.6%	98.7%

NN

Measure. Error	Mean	σ	Within σ	Within 2σ	Within 3σ
1%	1.73E-03	1.60E-04	69.8%	96.3%	98.9%
3%	2.14E-03	4.15E-04	74.1%	95.8%	98.9%
5%	2.83E-03	8.05E-04	75.2%	96.0%	98.2%
10%	5.93E-03	2.00E-03	70.9%	96.8%	98.9%

Summary

- Built four supervised learning regression models to predict the electron beam charge from the laser wavefront change
 - Evaluated model prediction : $MSE < 2 * 10^{-3}$ for all models
 - Random Forest fit our dataset better considering performances and computational cost
- ML enables feature analysis beyond just optimizing a target
 - Ranked the importance of Zernike coefficients in this LWFA process
- Analyzed data quality by evaluating models on every datapoint
- Included virtual measurement error

<http://arxiv.org/abs/2011.05866>

linjinp@umich.edu